PHYSICAL MEASUREMENT.

A COURSE OF EXPERIMENTS IN PHYSICAL MEASUREMENT.

In Four Parts.

PARTS I., II., III., AND IV.

COMPLETE IN ONE VOLUME FOR THE USE OF

TEACHERS AND STUDENTS.

BY HAROLD WHITING, PH.D.,

FORMERLY INSTRUCTOR IN PHYSICS AT HARVARD UNIVERSITY.

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PREFACE.

THIS book is intended to aid in the preparation of students for courses in civil, electrical, or mechanical engineering, and for advanced work in all branches of science requiring the use of accurate methods and instruments of precision. To this end, the course of experiments described, unlike that contained in most manuals published in this country, is exclusively devoted to quantitative physical determinations. Comparatively little use is made of the ordinary experimental demonstrations of well-known physical laws and principles, which, it is believed, are better suited to the lecture-room than to the laboratory. Most of the experiments consist in the determination of magnitudes wholly unknown to the students, and are made with instruments which they themselves have tested, in order that they may learn to depend upon their own observations.

Attention has been paid throughout this book to the *general methods* which underlie all physical measurement, rather than to the special devices by which particular difficulties are overcome. It is considered of greater advantage to show how comparatively accurate measurements may be made with rough apparatus, than to explain the use of instruments of precision which, in the hands of students, are apt to give erroneous results. The apparatus required for this course is, accordingly, of the simplest possible description.

Most institutions are obliged, by considerations of expense, to limit either the quantity or the quality of the instruments provided for the laboratory. When the supply of apparatus is insufficient, the work of a given student at a given point of time is obviously determined, to a greater or less extent, by the instruments which happen to be free for him to employ, and the systematic instruction of large classes becomes impracticable. This book is intended especially for use in laboratories which are or can be provided with a liberal supply of moderately accurate apparatus. Effort has been made to devise inexpensive instruments, especially when several copies of a given kind are likely to be needed; and it has been found that, notwithstanding the expense of all necessary reduplications, a considerable saving may be effected by the "Collective System" of instruction in the cost and labor of conducting an elementary laboratory course. The experiments are accordingly such as can be once for all explained to, and within a reasonable length of time performed by a large class of students. They are moreover arranged in a connected and progressive order.

The care and accuracy required to obtain concordant results in Physical Measurement, the continual use of experimental, inductive, and controlled methods, give to that science a peculiar educational value, aside from the natural laws and principles with which the student must become familiar. The course of experiments has been adapted, in so far as possible, to the needs of students who, having little or no previous training either in mathematics or in physics, wish to obtain a general scientific education. Every branch of physics is accordingly represented by typical examples. In order, however, not to exceed the natural bounds of an elementary treatise, the author has limited his selection to such experiments as have been proved, practically, in his own experience, to yield the most satisfactory results from an educational point of view. It is hardly necessary to add that these experiments involve physical measurement in every case.

The amount of mathematics required in the use of this book is not so great as might be supposed from a casual examination of its pages, since many proofs are given in full which in other text-books are taken for granted. The course of one hundred experiments involves only the simplest propositions in arithmetic and geometry, and little or nothing of algebra or trigonometry beyond the mere notation. Problems presenting any special difficulty are treated separately in a portion of the Appendix (Part IV.) not intended for general use.

The first part of this book relates especially to hydrostatics, thermics, optics, and acoustics; containing measurements of mass, density, length, temperature, heat, light, and wave-lengths of sound.

The second part contains all such measurements as involve motion or acceleration. That part of acoustics which relates to the measurement of time is also included; then follow dynamics, magnetism, and a comparatively extended series of electrical measurements. A few experiments intended (with certain exceptions) for advanced students are added, together with a description of certain instruments of precision.

The third part contains notes on the general methods of physical measurement, and on physical laws and principles. An extended series of mathematical and physical tables is also included in this part.

The fourth part, or Appendix, contains suggestions to teachers in regard to laboratory equipment, apparatus, expenses, and methods of instruction. It includes a full set of examples, showing how the observations in the course of one hundred experiments should be recorded and reduced. These examples embody results a great part of which were actually reported by students. There are also three working lists of experiments, of different lengths and degrees of difficulty, and proofs of certain important mathematical formulæ.

The text of the first and second parts is divided into short chapters, distinguished by the names of the experiments (Exps. 1–100) to which they relate. The experiments are still farther divided into sections (¶¶ 1–270), devoted in some cases to the practical, in other cases to the theoretical treatment of the subject. It has not been thought necessary or desirable to indicate in all cases just what portions of an experiment the student is expected to perform, and what portions it is sufficient for him to read. This must, of course, depend largely upon circumstances. Full directions for each of the one hundred regular experiments, or for each part of which it consists, will usually be found in a separate section headed by the word "Determination." In the case, however, of outside experiments mentioned only for the sake of illustration or continuity, directions are either entirely omitted, or replaced by a mere outline of the methods involved, with which it is important that the student should become acquainted. Examples will be found under the "Peculiar Devices employed in Calorimetry" (¶ 97), and the "Velocity of Light" (¶ 247), which, though obviously impracticable, even for advanced students, furnish reading matter which is none the less instructive.

More than half of the sections in the first and second parts relate to principles involved in the experiments, the construction of the necessary apparatus, or the calculation of results. These should be read or omitted by the student at the discretion of the teacher. The references to the third part (§§ 1–156), which occur throughout the experiments, should be looked up by the student in the order in which they are met, and afterward read consecutively. The teacher should make sure that these references are understood, in the case especially of students who may have had no previous training in physics.

The examples in the fourth part are intended to aid the teacher in preparing a list of the data required for a given determination, and in explaining the reduction of these data. The calculations are made, for the most part, by purely arithmetical processes, and in so far as possible, by one step at a time, so that the student can hardly fail to understand them. The author has found in his own experience that such examples can be safely trusted in the hands of students; but, for obvious reasons, it was thought better that they should be contained in the fourth part or Appendix, copies of which, separately bound, can be used by teachers who prefer to keep the examples at certain, or at all times, in their own hands.

The three lists of experiments, proposed by the author with a view of preparing students for various requirements of Harvard College, may be useful also to teachers who wish merely to shorten the course of experiments described in this book, without interrupting the continuity of the course.

The mathematical portions of the Appendix contain proofs which may be of interest to ambitious students and a convenience to teachers who find it desirable to step *beyond the limits* of this book.

Few references are given to works of other authors. It has been thought better in an elementary book to incorporate in the text such abstracts from the best authorities as it may be necessary for the student to refer to. The course of experiments here described was elaborated from one previously given by Professor Trowbridge, and outlined in his "New Physics" (Appleton, 1884). In this course frequent reference was made to the well known works of Everett, Kohlrausch, and Pickering. It is impossible to say to what extent the author may be indebted to these sources for the ideas contained in this book.

The advanced sheets of a "Syllabus" of experiments arranged by the author were distributed to his class in the year 1884–1885, before the works of Glazebrook and Shaw, and Stewart and Gee, could be obtained. While considerable assistance was derived from these works in the preparation of this book, the "Syllabus" mentioned above was taken as the basis for most of the experiments. The notes contained in the third part were first distributed to students in 1888–1889, but largely rewritten in 1890. The tables were condensed, by permission, from those of Professors Landolt and Börnstein, and from other sources elsewhere acknowledged. The first part was printed in 1890; the remaining three parts in 1891. In the same year a corrected edition of the first three parts was prepared for the use of students, and all four parts were combined in a single volume for the use of teachers and students.

The author is indebted to Professor Trowbridge for an outline of many successful experiments; to Professor Hall for a revision of a part of the proof-sheets, for numerous useful and practical suggestions, and for parts of experiments taken from his elementary course; to the late Mr. Forbes, of the Roxbury Latin School, for important criticisms; and

to Mr. Edgar Buckingham, Assistant in the Jefferson Physical Laboratory of Harvard University, for valuable aid in preparing the course of experiments.

The author wishes also to acknowledge several errata kindly pointed out to him in earlier copies, and to state that he will gladly receive from any source further corrections or criticisms which may be of service in preparing a revised edition of this book.

CAMBRIDGE, November, 1891.

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Part First.

MEASUREMENTS RELATING TO DENSITY, HEAT, LIGHT, AND SOUND.

EXPERIMENT I.

MEASUREMENT OF DENSITY.

¶ 1. **The Density of a Rectangular Block.** — The volume of a rectangular block may be defined as the product of its length, its breadth, and its thickness. If, accordingly, each of its three dimensions has been measured (§ 1) in centimetres (§ 5), we may find the volume of the block in cubic centimetres by multiplying these three dimensions together. When two blocks are of exactly the same size, but of unequal weight, as for instance a block of wood and a block of metal, they are said to differ in respect to density. Obviously, to determine the density of a body, we must find its weight as well as its volume. For convenience in calculation, the weighing should be made in grams (§ 6), since density is customarily expressed in grams per cubic centimetre (§ 9). To calculate the density of a body, we divide its weight in grams by the number of cubic centimetres contained in its volume, and thus find the weight of one cubic centimetre. This is the density (or average density) in question, expressed in absolute units of the C.G.S. system (§ 8). It should be noted that in this system *the density of a body is equal to the weight in grams of a cubic centimetre of the substance of which it is composed.*

The density of a fluid cannot, for obvious reasons, be determined like that of a solid, by *direct* measurements of its weight and linear dimensions; but when the volume of a block has been found, there are various methods by which the weight of an *equal bulk* of a fluid may be determined. We may, for instance, find the weight of the fluid necessary to fill a mould or vessel into which the block exactly fits; or we may fill a vessel with the fluid, and weigh the quantity which runs over when the block is immersed; or we may load the block [1] until it neither floats nor sinks in the fluid, — the weight of the block being in this case equal to that of an equal bulk of the fluid (§ 64). Other methods will be described in experiments which follow. The density of a fluid is always calculated, like that of a solid, by *dividing its weight by its volume.* We have seen how one may find the *weight* of a certain quantity of a fluid equivalent in volume to a rectangular block; the *volume* of the fluid in question (being equal to that of the block) is calculated by multiplying together the length, breadth, and

[1] In a wooden block, auger-holes bored parallel to the grain may be nearly filled with lead, and closed with a wooden plug even with the surface. A cube measuring 10 *cm.* each way and weighing 998 *g.* will be found useful to illustrate the density of water. The block should be coated with oil or other material impervious to water.

thickness of the block. All measurements of density will be found to depend more or less directly upon linear dimensions as well as upon weight.

The density of water may be found, approximately, by any of the methods suggested above; but the exact measurement of the density of water is one of the most difficult problems in physical measurement. We shall need continually to refer to the values in Table 25, which have been obtained by combining the results of the most careful observers. The student will of course accept these values in preference to any which he himself may obtain; but to use them intelligently, he must thoroughly understand both what they represent and how they are found. He should convince himself that the density of water is not far from unity; or that, in other words, 1 *cu. cm. of water weighs nearly* 1 *g.* (see § 6); and he should familiarize himself with the fundamental method of measuring density by weight and linear dimensions, applicable, as we have seen, either to a solid or to a liquid.[1] In case that a rectangular block is used, the necessary data are its weight in grams, and its length, breadth, and thickness in centimetres. The observations are made as stated below.

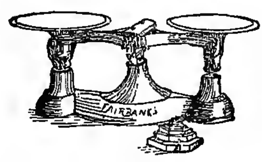


FIG. 1.

¶ 2. **Determination of Weight by the Method of Trial.** — The block is to be weighed with rough scales, such as are represented in Fig. 1, and which should be affected by a decigram. To

[1] See the Harvard University List of Chemical Experiments, Exp. 1.

select the weights necessary to balance a given body requires in general many trials. The number of trials may be greatly reduced, in the long run, by a strict adherence to the method here described. (See § 35, 2d ed.) We first place the block on one scale-pan, and a single weight, which we judge to be nearly equal to it, on the other. If this weight is too small, that is, if it is insufficient to lift the block, we add to it another weight of about equal magnitude, if any such exist in the set of weights; or should there be no weight equal to the first, we add one of the next greater magnitude. If the two weights together fail to lift the block, we add a third as nearly equal to the sum of the other two as may be convenient, and thus by doubling the weight in one scale-pan as many times as may be necessary, we find a quantity capable of lifting the load in the other scale-pan. If on the other hand, the first weight tried lifts the block, that is, if it is too heavy, we substitute for it one half as great, if any such be contained in the set; otherwise, the largest weight less than half of the first; and if the second weight is too great we substitute in the same way a third weight not greater than half of the second, and so continue to halve the weight until finally it is lifted by the block.

The weight of the block thus becomes known between two limits. We next try a weight as nearly half-way between these limits as may be obtained by the addition or subtraction of one weight at one time, or by the substitution of one weight for another; and thus gradually approximate to the weight of the block by successively halving the interval between the limits known to contain it.

By aimless departures from this method of approximation, the number of trials may be indefinitely increased; but certain modifications may be advisable when, from the slow motion of the scales or from any other cause, one has good ground to think that the true weight has been nearly found. In all such cases one should add or take away only so much weight as may be reasonably expected to turn the scales.

When the block has been exactly counterpoised by weights, it should be transferred to the other scale-pan and balanced against the same weights as before. (See § 44.) If the scales are as accurate as they are "precise," (§ 48, 2d ed.) the equilibrium will not be disturbed, otherwise a readjustment of the weights will be necessary. In the latter case the average of the two weighings is adopted as the true weight of the block. (See Experiment 8.)

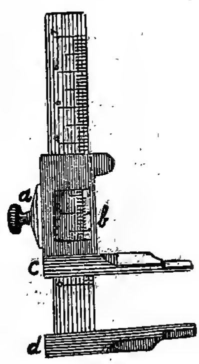


FIG. 2.

¶ 3. **Determination of Length, Breadth, and Thickness by a Vernier Gauge.** — We have seen in ¶ 2 how the weight of a block can be found; it remains to measure its length, breadth, and thickness, in order that its density may be determined. A Vernier gauge (Fig. 2) is suitable for this purpose. To obtain great accuracy with such a gauge, special precautions are necessary (see Experiment

19). For the purposes of this experiment, however, it will be sufficient to observe that the distance between the jaws (*c* and *d*) is directly indicated on the main scale of the instrument by the "pointer" or "zero" of the Vernier scale (*b*) on the sliding piece (*a b c*), to which the jaw (*c*) is attached. To identify the zero of the vernier, we bring the jaws (*c* and *d*) into contact; the zero of the vernier should then come opposite to the zero of the main scale. For convenience in reading the vernier, the zero is generally placed at a point (*b*) considerably beyond the movable jaw (*c*); but if, as in the figure, the main scale begins at an equal distance from the fixed jaw (*d*), the readings will not be affected. Evidently, in such a gauge, the edge of the sliding jaw (*c*) cannot be used as an index.

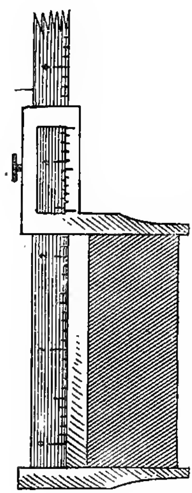


Fig. 3.

The whole number of millimetres between the jaws is equal to the number of the first millimetre division below the zero of the vernier, that is, between it and the zero of the main scale. The tenths of millimetres above this whole number may be read from the vernier as explained in § 40.

The block is first clamped lengthwise between the jaws of the gauge as in figure 3, and ten measurements are thus taken at different points. It is then clamped so as to obtain in a similar manner ten measurements of its breadth, and finally ten of its thickness. In each case the readings are made to millimetres and tenths. The object of taking a large number of measurements is to find the *average* length, breadth, and thickness with a degree of exactness (§ 48) corresponding to that attained in the weighing already performed (¶ 2). We finally calculate the volume and density of the block as explained in ¶ 1.

¶ 4. **Corrections Disregarded in Experiment 1.** — The vernier gauges which we usually employ are supposed to read correctly at 0° Centigrade; and hence will not be quite accurate at ordinary temperatures. For instance, if the gauge, having been cooled by melting ice to 0°, is fitted to the block as in Fig. 3, then allowed to become warm through contact with the air of the room, it will no longer fit the block as closely as it did, owing to expansion of the metal by heat. The block, though really unchanged in size, will appear to be somewhat smaller than before. This effect of expansion is barely perceptible; but we tend, nevertheless, to underestimate all the dimensions of the block, and hence also its volume. With brass gauges at 20°, the error in the volume would amount to about 1 part in 900 (see Table 8 *b*, also § 83).

Another source of error lies in the fact that the weighings are made in air, and not *in vacuo* (§ 65). In the case of a body weighing about one gram to the cubic centimetre, it is found (see Table 21), that the atmosphere exerts a buoyant action which apparently deprives it of about one 900th of its weight. We should therefore underestimate both the weight and the volume, in such a case, in the same proportion; and the density obtained by dividing the one by the other would not be affected. Even when the corrections in this experiment do not, as above, completely offset one another, they generally amount to less than one part in a thousand, and may be neglected in comparison with errors of observation. (See § 24.)

EXPERIMENT II.

TESTING A HYDROMETER.

¶ 5. **Determination of the Sensitiveness of a Hydrometer.**—A Nicholson's hydrometer is to be loaded as in Fig. 4, by placing weights in the upper pan, *a*, until a small ring round the lower part of the wire stem sinks just beneath the surface of the water; then small weights are added, say 5 centigrams, until by the sinking of the instrument, another ring round the upper part of the stem is brought just below the water level. The distance between the two rings, through which the hydrometer sinks under the action of the weight added,

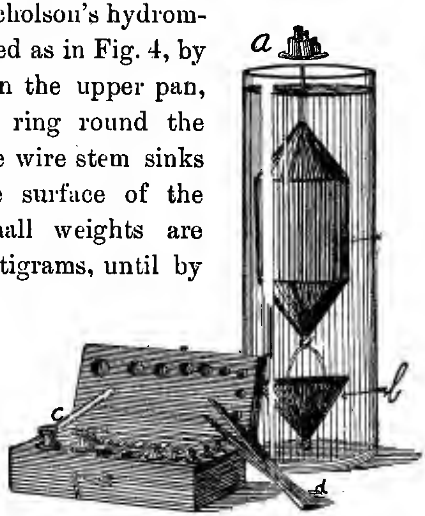


Fig. 4.

is estimated roughly by a small millimetre scale. We now calculate the effect of one centigram in sinking the instrument. This is called the sensitiveness (§ 41) of the hydrometer, and is useful in determining the degree of precision with which the adjustments of the instrument should be made (see § 48). Thus if the effect of one centigram is distinctly perceptible, we should try to avoid errors even less than a centigram in magnitude.

In using a Nicholson's hydrometer, several precautions should be observed. It frequently happens that through friction against the sides of the vessel, or through capillary phenomena where the surface of the water meets the stem, the hydrometer is unaffected by any slight change in the load. To avoid the first difficulty, the instrument should be kept floating in the middle of the jar, by the use of a guide of some sort. Such a guide may be conveniently constructed of wire, as in Fig. 5. To avoid the uncertainty of capillary action, the stem of the hydrometer should be kept wet, by a camel's-hair brush, for at least a centimetre above the water level.

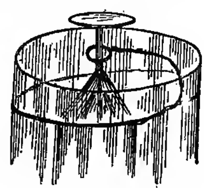


FIG. 5.

In water freshly drawn bubbles of air are apt to form, clinging to the sides of the hydrometer. These should be removed by the same brush. The formation of air bubbles may generally be prevented by using either distilled water, or water which has been standing for some time in the room.

It is important to keep the upper part of the stem, the pan, and the weights absolutely dry. The guide (Fig. 4) should prevent the hydrometer from sinking completely below the surface.[1]

¶ 6. **Accurate Adjustment of a Nicholson's Hydrometer.** — A mark is made near the middle of the stem of the hydrometer and the load is altered, a centigram at a time, until this mark is floated as nearly as possible in line with the surface of the water. If a glass jar is used, it is better to sight this mark by the under surface of the water, as shown in Fig. 6.

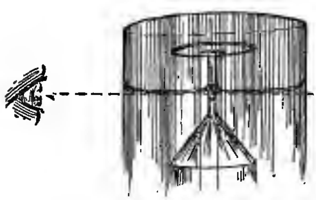


Fig. 6.

In the absence of weights smaller than one centigram, we estimate and record fractions of a centigram as follows: when the mark is floated exactly on a level with the surface of the water, the fact is expressed by placing a cipher in the third decimal place (belonging to the milligrams). If, however, a given weight fails to sink the mark to this level, while the addition of one centigram sends it as much below the surface as it was before above it, half a

[1] A sheet of cardboard or metal with a hole in the middle, is recommended by some authorities (see Pickering's Physical Manipulation, Article 45) to serve as a guide, and at the same time to prevent the weights from falling into the water. A student relying upon this safeguard is apt, however, not to acquire a sufficient degree of skill to prepare him for the manipulations of a delicate balance. (Exps. 6–14.)

centigram or 5 milligrams is obviously the weight to be added; hence the original weight should be followed by a 5 instead of a 0 in the last place. Thus if with 25.99 *g.* the mark is 2 *mm.* above the surface of the water, and with 26.00 *g.* it is 2 *mm.* below it, the weight sought must be 25.995 *g.* Again, if the lesser of two weights differing by one centigram is evidently nearer than the other to the weight desired, we substitute a figure 2 or a 3 for the 5 in the last place, or if the greater weight is more accurate, we write a 7 or an 8 instead. Any distinct information of this kind should always be recorded when possible, by means of a figure in the last place, even if that figure be extremely doubtful (§ 55). Closer estimates will hardly be justified in the case of a Nicholson's hydrometer.

¶ 7. **Effect of Temperature on a Nicholson's Hydrometer.** — The temperature of the water in the jar is now taken. The water is then cooled with ice to about 10°, and the weight required to balance the hydrometer is determined as before, with a new observation of temperature. Then the jar is filled with tepid water (at about 30°) and the experiment is repeated. A comparison of the different results shows how much the buoyancy of water is affected by temperature. For this purpose the observations which we have now obtained at three different temperatures are to be represented graphically on coordinate paper by three points, *A B* and *C*, as explained in § 59, and through these points the curve *A B C* is to be drawn with a bent ruler. (See Fig. 7.)

The ambitious student may supplement this experiment by using water hotter than 30° and colder than 10°, also water at intermediate temperatures. He will thus obtain data for plotting a more com-

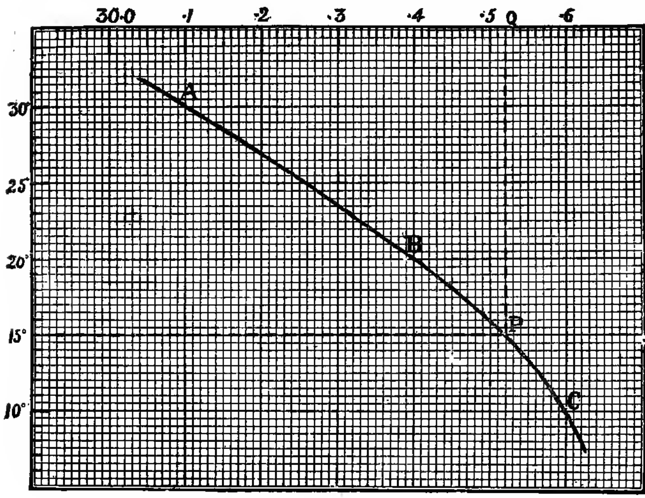


FIG. 7.

plete curve than that shown in the figure. This curve, if we neglect the expansion of the metal of which the hydrometer is composed, represents the relative buoyancy, or (see § 64) the relative density, of water at different temperatures.

EXPERIMENT III.

WEIGHING WITH A HYDROMETER.

¶ 8. **Determination of Weight in Air by a Nicholson's Hydrometer.** — From the results of Experiment 2 it is possible to find (see § 59) the weight necessary to sink a hydrometer to a given mark in water of any ordinary temperature. It is obvious that in all determinations with a Nicholson's hydrometer, the temperature of the water must be observed at the time of weighing. To find the weight of a body, place it in the upper pan (*a*, Fig. 8), and with it enough weights from the box to sink it to the same mark as before. Evidently less weight will be required than at the same temperature without the body, and the difference will be equal to the weight of the body in question.

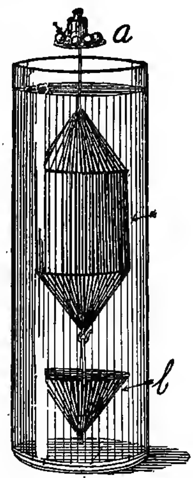


FIG. 8.

¶ 9. **Reasons for Neglecting Corrections for the Buoyancy of Air.** — Since the air buoys up both the brass weights and the body used, the result of this experiment is what we call the apparent weight of the body in air (§ 65). The amount of this buoyancy depends (see § 68) in one case upon the density of the brass weights, in the other case, upon that of the body in question; hence if these two densities are approximately equal, the air will exert nearly the same force in both cases. The result, obtained as we have seen by difference, will not therefore be affected to an appreciable extent.

For the purposes of this and other experiments which follow, we choose ten steel balls, perfectly round and uniform in size, such as are used in the bearings of the front wheel of a bicycle. The density of these balls (7·8) is not far from that of the brass weights (8·4), and it will be seen by reference to Table 21 that the correction for the buoyancy of air may be wholly disregarded.

EXPERIMENT IV.

WEIGHING IN WATER WITH A HYDROMETER.

¶ 10. **Determination of Specific Gravity by a Nicholson's Hydrometer.** — The steel balls used in the last experiment are now to be placed in the lower pan of the hydrometer (*l.* Fig. 9), which is lifted by the stem out of the water for this purpose. The instrument is then balanced with weights, and the temperature of the water observed as in the last two experiments.

In lowering the hydrometer into the jar, care must be taken to remove with a camel's-hair brush all bubbles of air from the steel balls, as well as from the sides of the hydrometer, and also, of course, not to spill any of the balls. In the adjustment of weights the same precautions must be used as in the last two experiments. We have already obtained the weight of the steel balls in air (¶ 8); we find similarly their weight in water from the results of Experiments 2 and 4, and finally their apparent specific gravity (see § 66).

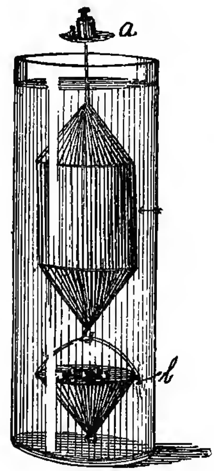


Fig. 9.

¶ 11. **Use of the Methods of Substitution and Multiplication.** — It will be noted that in Experiment 3 the unknown weight of a body takes the place of a known weight of brass used in Experiment 2; the one is in fact substituted for the other. The method of finding the weight of a body by a Nicholson's hydrometer is therefore essentially a method of substitution (§ 43). This statement also applies to the determination of weight in water by the same instrument; for the weight of a body in water is here substituted for a known weight of brass in air. The errors committed with a Nicholson's hydrometer depend upon the peculiarities of the instrument itself, rather than upon the quantities weighed. We are in fact liable to the same error in weighing one bicycle ball as in weighing ten. The proportion which the error bears to the total quantity weighed is, however, diminished when this quantity is increased. The use of a large number of bicycle balls for the determination of specific gravity in Experiments 3 and 4 is a good example of the accuracy gained by the method of multiplication (§ 39).

¶ 12. **Corrections Disregarded in Experiment 4.** — In the last experiment we disregarded the effects of the buoyancy of air on the steel balls and on the brass weights, because these effects were so nearly equal, both being in air. Here, however, the balls are in water and the weights in air.

There is, therefore, nothing to compensate for the buoyancy of air on the brass weights. It is seen by reference to § 65 that 7 grams of brass are buoyed up by the air with a force of about 1 milligram; and as a Nicholson's hydrometer can float only about 4 times 7, or 28 grams, the effect of buoyancy on the weights cannot be greater than 4 milligrams. This error may generally be disregarded in comparison with the errors of observation. The manner of applying a correction for the buoyancy of air is explained in Experiments 8 and 9, also in §§ 65–68.

In calculating apparent specific gravity, no corrections need be taken into account; but the result should be expressed as the apparent specific gravity of a given body at a given temperature referred to water at a given temperature. The result will be affected somewhat by the density of the air, but hardly to a perceptible extent. The student is advised, as a matter of habit simply, to note the conditions of the atmosphere in which his weighings are performed (see Experiment 5).

EXPERIMENT V.

ATMOSPHERIC DENSITY.

¶ 13. **Determination of Barometric Pressure.** — The three conditions of the atmosphere which affect the results of physical measurement are barometric pressure, temperature, and humidity. Let us first consider how barometric pressure is observed. A very rough but serviceable form of mercurial barometer consists simply of a glass-tube (*a b*, Fig. 10), which, having been filled with mercury,[1] is inverted in a cistern of mercury (*b*). The mercury sinks in the closed end of the tube to a level *a*, above which there will be a nearly perfect vacuum.[2] As there is no pressure at *a*, to counteract the atmospheric pressure below, the mercury stands in the tube at a level (*a*) above the level (*b*) in the cistern. It is found by experiment[3] that the atmospheric pressure is transmitted through the cistern of mercury and the open end of the tube to a point, *b*, on a level with the surface of the mercury in the cistern. The atmospheric pressure is accordingly determined by

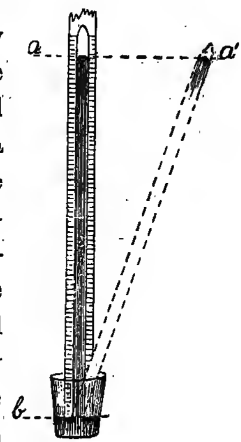


FIG. 10.

[1] The tube and the mercury must be perfectly clean and dry. For cleaning mercury, see Pickering's Physical Manipulation, I. 9.

[2] The "Torricellian vacuum."

[3] See § 62.

the length, *a b*, of the column of mercury which it sustains.[1] The distance *a b* can be measured by means of a graduated wooden rod, by which the tube is supported in a vertical position. The level *b* is first sighted in the ordinary manner with care to avoid parallax (§ 25); and the reading thus found is subtracted from that of the level *a*, obtained in a similar manner (see § 32).

In the case of a standard mercurial barometer the lower end of the column of mercury should always be looked at first, else a considerable error is likely to arise (§ 32); for even when the barometer ends in a large cistern of mercury, the level in this cistern must vary somewhat as more or less mercury rises into the tube. In some barometers this rise and fall is compensated by turning a screw (*d*, Fig. 11). This raises or lowers the mercury in the cistern, and when a certain steel or ivory point (*a*, Fig. 11), just touches its own reflection, the level of the mercury is known to be at the right height. When the lower end of the mercurial column in the tube has been thus adjusted, the height of the upper end is usually read by a movable sight, provided with a vernier (§ 40). The lower edge of the sight is to be set on a level with the highest part of the mercurial column, so as to appear to be tangent to the meniscus or curved surface of the mercury (Fig. 12, *a*). To avoid par-

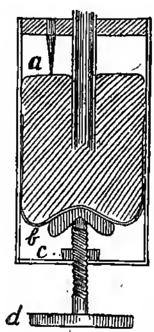


FIG. 11.

[1] See § 63.

allax (§ 25), a double sight is frequently used, consisting of two edges in the same horizontal plane, one in front of, the other behind the mercurial column. The student should find by direct measurement whether the distance from the zero-point (*a*, Fig. 11), to the lower edge of the sight (*a*, Fig. 12) is indicated correctly upon the scale of the barometer. If the reading of the barometer is in inches, it may be reduced to centimetres conveniently by Table 16.

FIG. 12.

Aneroid barometers are generally constructed so as to agree very closely with mercurial barometers. They will be found accurate enough for correcting the results of most physical measurements. If an Aneroid barometer is to be used, the student should compare its indication with that of a mercurial barometer, determined as explained above.

¶ 14. **Corrections of a Barometer.** — A small quantity of air almost always finds its way sooner or later into the space above the mercury in a barometer (*a*, Fig. 10), where it causes a slight depression of the column. To test a barometer for air, we tilt the tube *a b* (Fig. 10) into a new position *a′ b*, being careful to keep the mercury in the cistern at a constant level, *b*, either by raising the cistern or by adding more mercury to compensate for that which flows into the tube. In the absence of air, the mercury should follow the horizontal line *a a′*, and should completely fill the tube when the inclination is sufficiently increased.

A simple way of correcting for air in a barometer is to adjust the angle $a' b a$ (Fig. 10) by trial, so that the space above a' is half that above a. By thus reducing the air to half its original volume, the pressure will be doubled;[1] hence a' will be as much below a as a is below its proper level. By measuring the difference between the levels, a and a', we find accordingly the correction for air. A correction of 2 or 3 *mm.* may be disregarded, as it will probably be offset by other corrections which the accuracy of the instrument will not justify us in considering. In case the correction is much larger than this, the barometer should be refilled with mercury. The filling of a standard barometer should be attempted only by a skilled workman. Unless perfectly free from air, such a barometer is little better than the rough instrument shown in Fig. 10.

In all exact readings of a barometer, the three following corrections are usually applied: (a) for expansion, (b) for capillary depression, and (c) for the pressure of mercurial vapor.[2] The temperature of the mercury in a barometer is found by a thermometer beside it. Let t be this temperature, reduced if necessary to the Centigrade scale (see Table 39), and let h be the height in centimetres of the mercurial column; then the correction for expansion is $.00018\ ht$, which is to be subtracted from the observed height. The object of this correction is to find

[1] This follows from the law of Boyle and Mariotte (§ 79).

[2] The reduction of a barometric reading "to the sea level" is not required for the purposes of physical measurement.

how high the mercury would stand if its temperature were 0° Centigrade. Since 1 *cm.* of mercury when heated 1° Centigrade expands by the amount .00018 *cm.* (see Table 11), h *cm.* would expand h times as much; and h *cm.* heated $t°$ would expand ht times as much, whence we obtain the correction in question. At the ordinary temperature of a room (20°), and at the barometric pressure, 75 *cm.*, this correction for expansion would be $.00018 \times 20 \times 75$ *cm.* = 2.7 *mm.* It is therefore useless to read a barometer (as is often done) to tenths or hundredths of a millimetre, when no correction for temperature is made. The correction given above may be applied to barometers with wooden or glass scales, the expansion of which may be neglected. When, however, the body of the instrument consists of steel, the coefficient .00017 should be used instead of .00018; and if the barometer is mounted in brass or white metal, the factor .00016 will be still more accurate. These numbers represent the difference of expansion between the mercury and the scale by which it is measured. For more accurate values see Table 18 *a*.

When the tube of a barometer is less than a centimetre in diameter, there is found to be a perceptible depression of the mercurial column due to "capillarity," or "surface tension," the general nature of which will be investigated farther in Experiment 67. The internal diameter of the tube should be found if possible by measuring a plug which fits it in the part where the column of mercury ends (see *a*, Fig. 10). A different method of calibration will be considered in Experiment 26. When the internal diameter is known, the correction for capillarity may be found roughly from Table 18 *b*. Thus for a tube 5 *mm.* in diameter, in which the height of the mercury meniscus is unknown, the capillary depression may be taken as 1.5 *mm.* In various barometers which are constructed so that the internal diameter cannot be measured, we generally assume that the instrument-maker has allowed for capillarity in adjusting his scale, and we therefore neglect this correction. It is customary, also, to neglect the effect of capillary phenomena in the cistern of mercury.

Owing to the evaporation of mercury into the space above it in the tube of the barometer, that space is never quite empty. The quantity of mercurial vapor which it contains is found to increase when the temperature increases, and also the pressure which it exerts. To allow for the slight depression of the mercurial column due to this cause, Table 18 *c* has been constructed from the results of actual observation. Thus for a temperature of 20°, we find that the mercurial column is depressed to the extent of 0.02 *mm.* by the pressure of its own vapor.

We have found in a particular case that 2.7 *mm.* should be subtracted from the observed height of a barometer on account of expansion; that 1.5 *mm.* should be added for capillarity and also 0.02 *mm.* to offset the pressure of mercurial vapor. The resulting correction is 1.18 *mm.*, to be subtracted; or let us say, —1.2 *mm.* nearly. The student who employs a mercurial barometer should find in the same way an average correction for it. If an Aneroid is used, such a correction is found by comparing one reading at least with the corrected reading of a mercurial barometer. In the course of experiments which follow, readings of the barometer are needed only for slight corrections in the results of physical measurement. By applying to the barometer an average correction, much labor will be saved, and the error introduced will be insignificant.

¶ 15. **Determination of Atmospheric Temperature and Humidity.** — The temperature of the air of a room may be determined, with a sufficient degree of accuracy for most purposes, by an ordinary mercurial thermometer, the reading of which may be reduced from the Fahrenheit to the Centigrade scale by Table 39. The thermometer should be brought as near as may be practicable to the place where the temperature is required. It should, for instance, be inside of the balance case in very delicate weighings. It must not, however, be exposed to the rays of the sun, nor for any length of time to the heat radiated by a lamp or by the human body. When the greatest accuracy is desired, the bulb of the thermometer should be protected from radiation to or from surrounding objects, by a shield of polished metal.

The humidity of the atmosphere is most conveniently determined by a class of instruments of which the hygrodeik is an example. The indications of these instruments depend upon the cooling produced by evaporation (see § 88). It is found that when the bulb of a thermometer is covered with wet wicking

(a, Fig. 13), its reading differs from that of an ordinary thermometer (b) by an amount depending upon the dryness of the air. When the air is completely saturated with moisture, as in a dense fog, there is no evaporation from the wet bulb, hence the two thermometers agree; if the air, however is heated, the fog disappears, evaporation begins, and the wet-bulb does not rise so high as the dry-bulb thermometer. On the other hand, when the air of the room is cooled sufficiently, either fog is formed or dew is precipitated on various objects; and the two thermometers again agree. The temperature at which this occurs is called the dew-point, and is calculated from the readings of the wet and dry-bulb thermometers by reference to Table 15, or by a special mechanical device, for the operation of which directions are usually furnished by the instrument-maker.

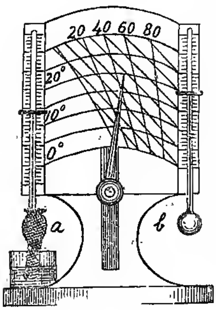


Fig. 13.

¶ 16. **Observation of the Dew-point.** — Unless a hygrodeik is known to give accurate indications, the latter should be confirmed by a direct determination of the dew-point, as follows: a polished metallic vessel is partly filled with water, and as much ice and salt are added as may be necessary to make a film of moisture condense on the surface. The temperature at which this first occurs is just below the dew-point. Soon, however, the contents of the vessel become warmer through contact with the air, and the film begins to disappear. The temperature is now a little above the dew-point. By observing carefully a thermometer with which the cold contents of the vessel are continually stirred, the dew-point may be determined within two limits, differing by less than one degree.

Care must be taken not to breathe on the metallic vessel, since the breath is much damper than the air of the room; and as there is more or less evaporation from all parts of the human body, even the hand should be kept as far away as possible.

¶ 17. **Relation of Relative Humidity to Dew-point.** The actual amount of moisture in a given quantity of air has been determined by extracting it through the action of certain hygroscopic substances, such as chloride of calcium, and measuring the gain in their weight. It is found that hot air can hold more moisture without forming fog than cold air. We have a common instance in the air of a room which, though apparently dry while warm, deposits moisture upon the window-panes by which it is cooled.[1] The ratio of the amount of moisture actually held in the air (at a given temperature) to the maximum amount (which can be held at that temperature) is called the relative humidity of the air. The relations between temperature, dew-point, and relative humidity do not follow any simple law; but if any two of these quantities are given, the third may be found by

[1] For a further illustration see list of Experiments in Elementary Physics, published by Harvard University, Exercise 22.

referring to Table 15, containing the results of various experiments.

It may be noted that the dew-point depends solely upon the amount of moisture in the air; that dry air has a lower dew-point and less relative humidity than moist air at the same temperature, while for a given dew-point the relative humidity increases with a fall of temperature, until fog is finally formed, or decreases as it becomes warmer until the air is practically dry. It should also be noted that dry air is denser than moist air. We must regard the latter as a mixture of air, not with water, but with steam, which is only about two-thirds as heavy as air. Hence in Table 20 the correction for moisture is negative.

¶ 18. **Determination of Atmospheric Density by means of a Barodeik.** — From the temperature, pressure, and humidity of the atmosphere, the determination of which has been explained above, the density of air may be calculated by the data of Tables 19 and 20. Whenever great accuracy is desired this calculation must be performed.

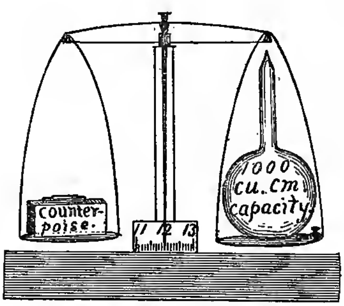


Fig. 14.

For most purposes, however, the density of the atmosphere may be found from a single observation of a barodeik (Fig. 14), the principle of which is spoken of in § 71. It is important to compare the indication of the instrument in at least one case with the calculated density of the atmosphere. A reading of the barodeik should accompany every weighing in which more than three figures are to be preserved, except when the pressure, temperature, and dew-point have been determined.

EXPERIMENT VI.

TESTING A BALANCE.

¶ 19. **Manipulation of a Balance.** — The delicacy of a balance depends upon the sharpness of the knife-edges (*a* and *c*, Fig. 15) from which the pans are suspended, also upon the sharpness of the central knife-edge (*b*) upon which the beam (*a c*) turns. In order that these edges may not become dull, the pans should be supported by some mechanical device at all times except when an observation is actually being taken. It is particularly important that they should be so supported when they are being loaded or unloaded, or when the balance is liable to be jarred in any other manner. In an ordinary prescription balance (Fig. 15), the pans rest upon the bottom of the case when the instrument is not in use. Such a balance is thrown into operation by turning a milled head outside of the case. The beam is thus raised as slowly as possible, so as not to injure the knife-edges by suddenly throwing weight upon them. It is not necessary in every case to raise the beam as far as it will go. As soon as the pointer moves decidedly to one side or the other, the beam should be slowly lowered again. In other cases a prolonged observation of the pointer must be made in order to decide in which direction the beam tends to incline. During such observations the beam should be raised to its fullest extent. Whenever accuracy is desired, the

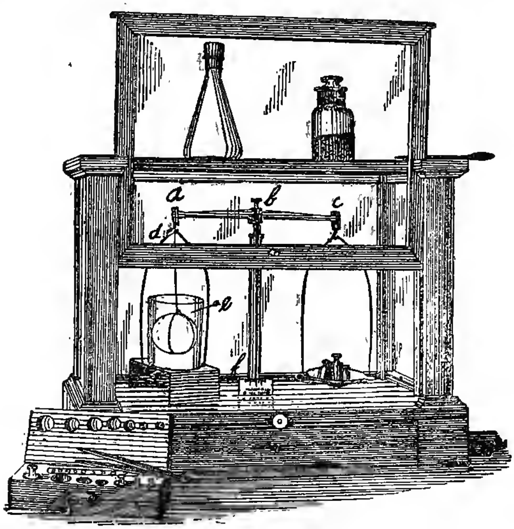


Fig. 15.

door of the balance case should be closed, in order to cut off currents of air; in fact, the door should never be opened except when the purposes of manipulation actually require it. This precaution is necessary to protect the instrument from moisture and dust, and is especially important when the air within the balance case is kept artificially dry by chloride of calcium or other hygroscopic material. The glass case should be cleaned when necessary with a damp cloth, to avoid charging it with electricity.[1]

Before weighing with a balance the case should be levelled and firmly supported, the scale-pans should be scrupulously cleaned and returned to their places, and any dust which may have collected on the knife-edges or their bearings should be cautiously removed with a camel's-hair brush. The beam is now thrown into operation by the mechanism already alluded to. If the instrument is correctly adjusted, the pointer attached to the under side of the beam will oscillate slowly and for some time through nearly equal arcs on either side of the central division of a scale (*f*, Fig. 15) directly behind it. If it tends to one side, that side is the lighter; and bits of paper or tinfoil should be fastened to the scale-pan until an exact balance is established.[2]

In loading the pans, pincers should be used as much as possible. In the case of the smaller weights, especially, contact with the fingers should be avoided. It makes no difference, theoretically, where the loads in the pans are placed; but many practical difficulties will be avoided by keeping them as nearly as possible in the centre. Both loads should be at the same

[1] By rubbing the glass at one side of a balance case with a piece of silk, a considerable error may be introduced into a weighing. The student should be cautioned, in general, against the effect of charges of electricity on delicate instruments. An eye-glass rubbed on the sleeve has been known to cause serious errors in physical measurement.

[2] See, however, first footnote, ¶ 26.

temperature as the air within the balance case; for though heat weighs nothing, a hot body may be lifted slightly by upward currents of hot air around it. With non-metallic loads we should avoid friction, which, as we have seen, may generate charges of electricity. When magnetic matter (as iron or steel) is to be weighed, all magnets (§ 126) should be removed from the immediate neighborhood. In an actual weighing, the scale-pans should be prevented from swinging, both on account of currents of air and because of the irregular motion given to the pointer.

¶ 20. **Method of Weighing by Oscillations.** — The reading of a pointer is usually taken while it is in motion, since much time would be lost in waiting for it to come to rest, and even then friction might stop it somewhat on one side of its true position of equilibrium. While in motion the pointer swings first to one side of its position of equilibrium, then to the other. The furthest point reached in a given swing to the right or to the left is called as the case may be a right-hand or a left-hand turning-point. Owing to friction, each swing is smaller than the one before it; hence the position of equilibrium is not exactly midway between any two successive turning-points. To avoid errors from this source we adopt the following rule: *observe any* ODD[1] *number of consecutive turning-*

[1] The object of making an odd number of observations is that the first and last may be on the same side; for in this case the turning-points on one side are on the whole neither earlier nor later than on the other side, and the gradual diminution of the swing affects each average alike.

points; find the average of those on the right and the average of those on the left; add these averages algebraically and divide by 2. The result is the point about which the oscillation is taking place, and at which the index tends eventually to come to rest.

It is convenient for many reasons to call the middle scale-division number 10, not 0, since otherwise plus and minus signs must be employed. In practice it is sufficient to observe three consecutive turning-points of the index.

It is frequently impossible to balance a given load exactly by any combination of weights which we are able to obtain. Let us suppose that with a weight, w, the index tends to rest at a distance from the middle-point equal to x scale-divisions; while with the smallest possible addition of weight, a, it tends to rest on the other side of the middle-point and at a distance from it equal to y scale divisions. Then the exact weight indicated for the load, l, is (see § 41),

$$l = w + \frac{ax}{x + y}.$$

The quantity $x + y$ is called *the sensitiveness of the balance to the weight* (a) *under the load* (l); and as it occurs in all exact estimations of weight by interpolation, it may be made properly the subject of further investigation.

¶ 21. **Determination of the Sensitiveness of a Balance.** — To test the sensitiveness of a balance with the pans empty, after carefully adjusting it as suggested in ¶ 19, we add a small weight, let us say 2 *cg.*

to the left hand pan. Instead of swinging about the middle scale-division, which we have agreed to call number 10, it will swing about a new point corresponding, let us say, to number 12·6 on the scale. This would show that the balance is sensitive to the extent of 12·6 — 10, or 2·6 divisions for 2 *cg.*, or 1·3 divisions per *cg.*, when the pans contain little or no load besides their own weight. This fact is recorded by making a cross (as in Fig. 16) on a piece of co-ordinate paper at the right of the number 0, representing the load, and below the number (1·3) representing the sensitiveness in question.

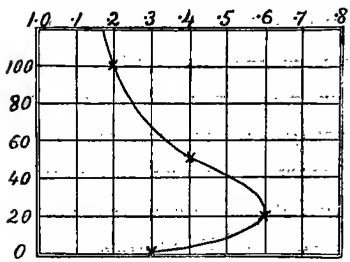


FIG. 16.

We now place, let us say, 20 grams in each pan, and find as before the sensitiveness per centigram. It will not necessarily be the same as when the pans are empty; in fact, a difference is almost always observed.[1] The sensitiveness is then found with 50 grams in each pan, and finally with 100 grams in each pan. Thus, in an actual case, a balance which was sensitive with the pans empty to the extent of 1·3 divisions per *cg.*, was affected to the extent of 1·6 divisions per *cg.* with 20 *g.* in each pan, 1·4 divisions

[1] It will be shown in ¶ 22 that the effect of a load on the sensitiveness of a balance cannot be anticipated; hence the student who records faithfully what he sees, not what he expects to see, will here as elsewhere in Physical Measurement, be likely to obtain the most accurate results. (See § 30.)

per *cg.* with 50 *g.* in each pan, and 1·2 divisions per *cg.* with 100 *g.* in each pan. These results are recorded, as before, by crosses in the proper places (see Fig. 16), and a curve is drawn by a bent ruler through these crosses. This curve enables us to find approximately the sensitiveness of the balance under any ordinary load by the method explained in § 59.

When we know the sensitiveness (s) of a balance to 1 *cg.*, a single observation of the pointer is sufficient to determine exactly the weight indicated. If w is the lighter weight (in the pan toward which the pointer inclines) and x the number of scale-divisions between the resting point of the index and the middle of the scale, the load (l) indicated is found by substituting s for $x + y$ and .01 for a in the formula of ¶ 20; or

$$l = w + \frac{.01\,x}{s}.$$

¶ 22. **Conditions on which the Sensitiveness of a Balance Depends.** — In order that a balance may move perceptibly under the influence of a very small weight added to either pan, the central knife-edge (b, Fig. 15) on which the beam turns must not only be sharp (¶ 19), but must pass nearly through the centre of gravity. If the centre of gravity is above this knife-edge, the balance will be "top heavy." This difficulty must be remedied by attaching a bit of sealing-wax to the pointer below the knife-edge b, or by lowering the centre of gravity in any other manner.[1] If on the other hand the centre of gravity is too low, the balance will be too steady, and it will not respond sufficiently to a small change in the load. In this case it is necessary to fasten a small weight to the balance beam, somewhere above the knife-edge b, or otherwise to raise its centre of gravity.

When the balance-pans are loaded, new considerations come in. Since in all positions of the beam the loads hang vertically beneath their respective knife-edges, the result is the same as if they were concentrated at those knife-edges. Let us suppose that the instrument has been adjusted so as to be sufficiently sensitive when the pans are empty. In order that it may remain equally sensitive when loaded, the three knife-edges must be in the same straight line, as in A, Fig. 17. If the two outer knife-edges which

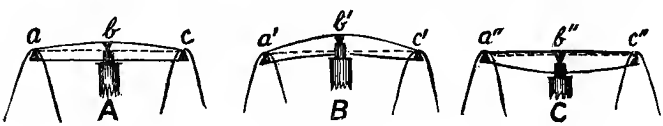


Fig. 17.

bear the loads (see a'', c'' in C) are distinctly above the central knife-edge (b''), the combined effect of the loads will be towards unstable equilibrium; or if the outer knife-edges (see a', c' in B), are below the central knife-edge (b'), the combined effect of the loads will be to steady the balance, and hence to diminish its sensitiveness. There are therefore three types to which a balance beam may belong, repre-

[1] A movable screw or counterpoise is provided in some balances for the purpose of raising or lowering the centre of gravity.

sented by the three diagrams, *A*, *B*, and *C*. In the first, the load does not affect the sensitiveness, except in so far as friction may be concerned; in the second, it lessens it; in the third, it may increase the sensitiveness until the balance actually becomes "top heavy."

A common balance may belong successively to all three of the types, *C*, *A*, and *B*. Let us suppose that with the pans empty the extremities of the beam are bent upward, as in *C*. With a medium load, the beam may be straightened, as in *A*, and with a still greater load the ends may be bent downward, as in *B*.

Such a balance would be more sensitive with a small load in each pan than when the pans were empty; because a small load, being insufficient to straighten the beam, would raise its centre of gravity[1] as in *C*; but when already heavily loaded, so that the beam is bent downward as in *B*, the further addition of weight would lessen its sensitiveness. The curious shape of the curve found in the last section (Fig. 16), is thus accounted for.

¶ 23. **Determination of the Ratio of the Arms of a Balance.**— The balance is now readjusted if necessary as in ¶ 19, so that the pointer swings accurately about the central division of the scale when the pans are empty, and the 100 gram weight is balanced against its equivalent as before, only that small weights are added to one side or to the other to

[1] A balance, though stable with a heavy or with a medium load, as well as when the pans are empty, may actually become "top heavy," with a small load in each pan. In such a case, the centre of gravity should be permanently lowered.

bring the pointer as nearly as possible to the central division, and the exact weight estimated as in ¶ 21, considering as the load, l, that weight which is apparently the larger. The loads in the two pans are now interchanged, readjusted by the use of the small weights, and compared exactly as before. The pans being once more emptied, the pointer should swing about the central division, otherwise the balance must be readjusted and the process described in this section must be repeated until the equilibrium of the balance remains undisturbed.

The object of testing the balance, as above, with equal weights in the opposite scale-pans, is to discover any inequality which may exist in the length of the balance arms ($a\,b$ and $b\,c$, Fig. 17). Such an inequality might seriously affect the accuracy of results, and we have no right to neglect it even in ordinary weighings without some test similar to the one described. It is true that by the method of double weighing (see § 44), errors due to the inequality of the balance arms may be eliminated; but double weighings are sometimes impracticable, as in the case of a body of variable weight, or in a very long series of determinations. In such cases the inequality of the balance arms should be found by a careful and extended series of observations. For the purposes of this course of experiments, a single determination will suffice. The ratio of the balance arms is calculated therefrom as explained in the next section.

¶ 24. **Calculation of the Ratio of the Balance Arms.** — If the arms of a balance are unequal, it is important to know from which arm the unknown weight is suspended. To avoid the necessity of mentioning in each case the pan containing the load in question, it is customary to place the unknown weight at the left hand whenever a single weighing is to be made. In this way the known weight, consisting generally of several small pieces, is conveniently adjusted by the right hand.

To find the proportion which the weight on the left arm always bears to the weight on the right arm, we need only a single comparison between two known weights. As these weights are inversely as their respective arms (see § 113), the proportion in question is equal to the ratio of the right arm to the left arm. Thus if (in an extreme case) 101 grams in the left-hand pan balance 100 grams in the right-hand pan, the right arm must be $\frac{101}{100}$ or 1.01 times as long as the left arm. All weights in the left-hand pan are therefore 1% greater than those which balance them in the right-hand pan; hence to find the value of an unknown weight in the left-hand pan we multiply that of the known weight in the right-hand pan by 1.01. The ratio of the balance arms is in general that number by which the known weight must be multiplied in order to find the unknown weight which balances it. We usually require, as we have seen, the ratio of the right arm to the left arm. This is found by dividing a known weight in the left-hand pan by a known weight in the right-hand pan which balances it.

The object of interchanging the two weights in

¶ 23, each nominally equal to 100 grams, is to avoid mistakes arising from a difference between the two weights in question. If no such difference exists, the interchange will not affect the result. Otherwise to find the ratio of the balance arms, we take the average of the two weights in the left-hand pan, and divide it by the average of the two weights in the right-hand pan. In taking these averages we accept the nominal values of the weights in question, any errors in which are practically eliminated by the method of interchange (§ 44) here adopted.

EXPERIMENT VII.

CORRECTION OF WEIGHTS.

¶ 25. **Process of Testing a Set of Weights.** — The brass 1 gram weight is first balanced against all the smaller weights, which should together be equal to 1 gram; then each 2 gram weight against the 1 gram plus the smaller weights; then the 5 gram weight against the two 2 gram weights plus the 1 gram; then in the same way the 10, 20, 50, and 100 gram weights, each against its equivalent. Whenever there are two ways of making an equivalent, that selection is made by which the fewest weights may be employed. (See § 36, 2d ed.) The 100 gram weight is finally balanced against a standard.[1] In

[1] The standard should be of the same material as the set of weights employed, that is, of brass; but if any other material is used, a correction must be made for the unequal buoyancy of the atmosphere upon the loads in the two pans. See § 67 and Table 21.

each case, where two weights are balanced, the difference between them is estimated by the method of vibration (¶ 20), and recorded as will be explained below. To avoid corrections named in the last experiment, the method of double weighing is used in every case.

¶ 26. **Estimation of Tenths in Weighing.** — In a long series of weighings, as in testing a set of weights, it is hardly thought to be advisable (see, however, § 33) to record each turning-point of the index as in ¶ 20. The student who wishes to make any extended use of the balance should learn to estimate correctly the point of the scale about which the index is swinging, and hence the number of divisions from the middle of the scale[1] to the point where the index tends to rest; to carry this number in the head while finding by inspection of figure 16 (see ¶ 21 and § 59) the sensitiveness of the balance under the load in question,[2] and to divide mentally the number thus carried in the head by that representing the sensitiveness of the balance, or the effect of 1 *cg.* (See general rules for interpolation, § 41.) He will thus find the fraction of a centigram necessary to make the index swing about the middle-point of the scale, and will

[1] Instead of adjusting the balance as in ¶ 19, so that the index may swing about the middle-point of the scale, the advanced student may often prefer to observe accurately the point about which the index actually oscillates when the pans are empty, and *to measure all distances from this point.*

[2] It is sometimes quicker to add one centigram to the lighter pan, and thus to re-determine the sensitiveness. In many cases the sensitiveness may be recalled from memory with a sufficient degree of exactness.

record the number of milligrams nearest to that fraction with the proper algebraic sign.

Thus if with a weight marked 10 g_1 in the left-hand pan and with 10 g_2 in the right-hand pan, the index swings about a point corresponding to 10·3 of the scale, — that is, 0·3 divisions to the right of the middle-point, — and if the sensitiveness of the balance with a load of 10 grams is about 1·5 divisions per centigram (see Fig. 16, ¶ 21), the weight 10 g_1 is clearly heavier than 10 g_2 by $0{\cdot}3 \div 1{\cdot}5 = \frac{1}{5}$ *cg.* or 2 *mgr.* We record such an observation as follows:

$$10\ g_1 = 10\ g_2 + 2\ mgr.$$

In the same way we enter the result of placing 10 g_1 in the right-hand pan and 10 g_2 in the left-hand pan; and if there is any difference, we find the average excess of 10 g_1 over 10 g_2, or the reverse.

¶ 27. **Calculation of the Corrections for a Set of Weights.** — Any one familiar with algebra can find the relations existing between the different weights of a set from a series of equations obtained as in the last section. The following suggestions may however be useful. Call the value of the 1 gram weight G; find the total value of the smaller weights (100 *cg.*) in terms of this. For instance, let

$$100\ cg. = G + 1\ mgr.$$

Then find the value of the 2 gram weights, 2 g_1 and 2 g_2 in terms of G. If for example,

$$2\ g_1 = 100\ cg. + G - 1\ mgr.,$$

we find, substituting for 100 *cg.* its value, G + 1 *mgr.*,

$$2\,g_1 = G + 1\ mgr. + G - 1\ mgr. = 2\,G;$$

and if still further, it has been observed that

$$2\,g_2 = 2\,g_1 + 2\ mgr.,$$

we find similarly

$$2\,g_2 = 2\ \mathrm{G} + 2\ mgr.$$

Again, if by observation

$$5\,g = 2\,g_1 + 2\,g_2 + \mathrm{G} + 1\ mgr.,$$

we have

$$5\,g = 2\,\mathrm{G} + 2\,\mathrm{G} + 2\ mgr. + \mathrm{G} + 1\ mgr.$$
$$= 5\ G + 3\ mgr.$$

In the same way we find the values of all the weights in terms of G, until we come finally to the standard. Knowing the standard in terms of G, we find G in terms of the standard. The corrected value of G should be expressed in grams and carried out to five places of decimals. Substituting this value in all the equations, we obtain finally the correction in *mgr.* for each weight belonging to the set from 1 gram upwards.

This method of framing and reducing equations is not peculiar to a set of weights. The student may substitute for it, if he prefers, the correction of a set of standard electrical resistances, which he will learn how to compare in Experiment 87. The same method may be applied to any other standards capable of being arranged like a set of weights, so that each one may be compared with an equivalent made up of the others below it. The general principle by which such a standard set is corrected is one of the best illustrations of the method of multiplication (§ 39) upon which nearly all measurements are founded.

EXPERIMENT VIII.

WEIGHING WITH A BALANCE.

¶ 28. **Determination of Weight in Air by a Balance.** — The apparent weight of a body in air may be found approximately, as has been explained in Experiment 1, by placing it in one pan of a balance — the left being understood unless otherwise stated (see ¶ 24) — and finding by trial (¶ 2) the requisite number of weights to counterpoise it. The accurate determination of weight in air differs from this rough method chiefly in the delicacy of the instrument employed, and in the consequent care of manipulation (see ¶ 19). In this, as in all other accurate determinations with the balance, unless otherwise stated, it is assumed that the method of weighing by oscillations is employed (¶ 20).

The object recommended for this experiment is a glass ball, the weight of which will be needed later on in the course. To prevent it from rolling out of the pan, it may be set in the middle of a small ring of known weight, which we will suppose to be counterpoised with one of equal weight in the opposite pan.

It is necessary in this experiment either to know the ratio of the balance arms (see ¶ 23), or to employ the method of double weighing (§ 44) as in Experiment 7. The density of air must also be determined by an observation of the barodeik (¶ 18), or by an observation of the atmospheric pressure, temperature, and humidity (¶¶ 13–15). We must also know the material, and hence approximately the densities of both the object weighed and the weights with which it is counterpoised. These densities may be found with a sufficient degree of accuracy by referring to Tables 8–11. The correction of apparent weights to *vacuo* is then made as explained in § 68.

EXPERIMENT IX.

THE HYDROSTATIC BALANCE, I.

¶ 29. **Determination of the Density of Solids by the Hydrostatic Balance.** — An arch is placed over a balance pan as in Fig. 18, so as not to interfere with its free vibration; and on the middle of the arch is set a beaker. The glass ball weighed in the last experiment is now bound in a network of fine wire and suspended by a single strand from the hook of the balance, so as to clear the bottom of the beaker. The latter, being moved if necessary so that its sides may not touch the ball, is filled with a quantity of distilled water sufficient to cover,[1] in all positions of the balance, both the ball and its network of wire. All bubbles of air

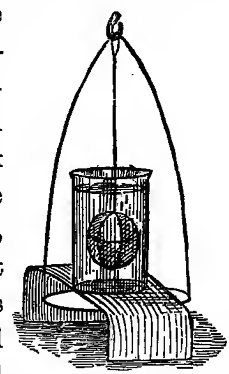


Fig. 18.

[1] A small loop of wire, projecting above the surface, may completely ruin a determination.

clinging to the ball, or wire, must now be removed with a camel's-hair brush. The suspending wire, being likely to attract grease or other foreign matter which repels water, is cleaned if necessary, so that it may be kept wet for a distance of about one centimetre above the level of the water, by the continual oscillation of the balance. The capillary phenomena already noticed in ¶ 5 are thus reduced to a small and nearly constant amount.[1]

By these adaptations the instrument which we employ has been completely transformed into a "hydrostatic balance," by which the weight of the ball and wire in water may now be found, as in the last experiment, by counterpoising it with weights in air (see Fig. 15, ¶ 19). The method of weighing by oscillations is not, however, recommended in the case of a hydrostatic balance; but rather a direct observation of the pointer in its position of equilibrium, which, owing to fluid friction, is quickly reached.

Apart from friction, the sensitiveness of a hydrostatic balance is always somewhat less than that of the same balance when used for measuring weights in air,[2] and must therefore be re-determined by adding a centigram to the smaller of the two loads when nearly balanced and observing the result (see ¶ 21). In this, as in all experiments with the hydrostatic

[1] The use of spirits of wine to diminish still further the capillary action (Trowbridge, "New Physics," page 17), is not recommended to beginners, on account of the danger of its mixing with the water and thus affecting its density.

[2] The variable amount of water displaced by the suspending wire tends to increase the stability of the balance.

balance, the temperature of the liquid should be observed both before and immediately after finding the weight of a solid in it.

The weight of the wire in water must be found separately in the same manner and under the same conditions as before.[1] The ball is removed from the network of wire so as to leave the latter undisturbed in so far as possible, and water is added to the beaker in order that the same amount of wire may be submerged in each case. It may even be necessary, if a coarse wire is used, to adjust the level of the water exactly to a given mark, and if the network is bulky, to raise or lower the temperature of the water to the same point as before.

The apparent weight of the ball in water is found by subtraction, and reduced to *vacuo* by the principle of § 67. The difference between the apparent weights in air and in water gives the apparent weight of water displaced (§ 66), and hence the volume displaced (see Table 22). The difference between the weight of the ball *in vacuo* (¶ 28) and its weight in water (reduced to *vacuo* as explained above) gives, by a strict interpretation of the Principle of Archimedes (§ 64), the weight *in vacuo* of water displaced, and hence also its volume (by Table 23). We have thus two methods of calculating volume, of which the first is more generally useful, as it does not require any previous reduction of weights to *vacuo;* but the

[1] Precautions similar to those which follow are necessary whenever a method of difference is employed. For further illustration see § 32.

second is more rigorous, because, depending upon weights *in vacuo*, the results will not be affected by variations of apparent weight due to changes in atmospheric density. The latter should therefore be employed when any considerable time elapses between the determinations of weight in air and in water. The density (or average density) of the ball is finally calculated (see ¶ 1) by dividing its weight *in vacuo* by its volume. (See ¶ 4, also § 68.)

EXPERIMENT X.

THE HYDROSTATIC BALANCE, II.

¶ 30. **Determination of the Density of Liquids by the Hydrostatic Balance.** — The experiment consists essentially of a repetition of Experiment 9, substituting, however, for distilled water some other liquid of greater or less buoyancy.

Various modifications of this experiment may be necessary according to the nature of the liquid used; for instance in the case of strong acids, platinum wire must be substituted for iron, which would be speedily dissolved, and even platinum cannot be used in *aqua regia*. To avoid fumes in the balance case, the suspending wire is sometimes carried down through a series of small holes to a beaker below. To avoid evaporation, in the case of volatile liquids, the beaker should always be covered with cork or cardboard perforated for the suspending wire. The same precaution should be taken when moisture is likely to be absorbed. In some liquids scarcely any bubbles are formed; in others, such as glycerine, it may take hours to remove them, though their formation may be prevented if the glycerine is poured in a continuous stream down the sides of the beaker. In most liquids the effects of temperature are greater than in the case of water (see Table 11), hence the thermometer must be read with the greatest care. It is well to warm or cool the liquid (and hence also the ball) to the temperature of the water in Experiment 9, to avoid all corrections for temperature.

¶ 31. **Calculation of the Density of Liquids by the Hydrostatic Method.** — We find in the same way as in Experiment 9, the apparent weight of the ball in the liquid, allowing for the wire as before; and from this we subtract the weight of air displaced by the brass weights (see § 67), to find the true weight of the ball in the liquid. The difference between its true weight in the liquid and that *in vacuo*, already found (¶ 28), is equal to the weight *in vacuo* of the liquid displaced. This follows from the Principle of Archimedes (§ 64).

The volume of liquid displaced is of course equal to the volume of the ball, which will not differ perceptibly from the value previously determined (see end of ¶ 29) if the temperatures of the two experiments are nearly the same. If this is not the case, it is necessary to allow for an expansion or contraction of the glass, at the rate of about one part in 40,000 for every degree Centigrade. (See Table 8 *b* and § 83.)

The weight *in vacuo* of the liquid displaced is finally divided by its volume to find its density.

The weight *in vacuo* may be checked by calculating the apparent weight of the liquid displaced, as in Experiment 9, then reducing at once to weight *in vacuo* by applying the necessary factor from Table 21, as explained in § 68, using the density already calculated. This latter method is slightly inaccurate, as has been stated before (¶ 29), on account of its disregarding variations of atmospheric density during the course of experiments.

In determining the density of water by the hydrostatic balance, the weight displaced may be found as in Experiment 9 or 10; but the volume displaced cannot be calculated in the manner explained above, because the tables which we employ themselves depend upon the density of water. It is necessary to calculate the volume of the solid immersed from actual measurements of its dimensions[1] (see ¶ 1). By this method, essentially, with the aid of instruments of precision, accurate determinations of the density of water have been made (see Table 25). The student will have an opportunity in Experiment 19, to confirm these determinations within the limit of accuracy of the instruments which he employs.

[1] The volume, v, of the glass ball may be calculated from its diameter, d, by the formula, $v = \cdot 5236\ d^3$. In place of the glass ball we may use, for purposes of illustration, the rectangular block whose volume has already been determined in Experiment 1. If it floats in water, a lead sinker may be attached to it. The sinker must remain in place after the block is removed, in order that its weight may be allowed for. A spring balance may be used to find roughly the weight of water displaced. See Exercises 7–10 in the Descriptive list of Experiments in Physics published by Harvard University.

EXPERIMENT XI.

CAPACITY OF VESSELS.

¶ 32. **Determination of the Capacity of a Specific Gravity Bottle.** — Any bottle with a solid stopper of ground-glass may be used for finding the specific gravity of liquids; but when solids are to be introduced, one with a wide mouth will be needed. The capacity of the bottle is determined in the following manner. The bottle is first washed in perfectly pure water, then dried with a cloth inside and out, and afterwards still more thoroughly dried with a hot air-blast.[1] The weight of the bottle is found within a centigram, then the bottle is alternately dried and weighed until by the agreement of two successive weighings, the drying is known to be complete. The last weight found, if confirmed by the method of double weighing as in ¶ 28, is the apparent weight of the bottle in air. It is understood that the stopper is always weighed with the bottle. In this case, it should be placed in the scale-pan beside the bottle, so that the density of the air may be the same inside and out. The bottle, which will be warmed by the hot air-blast, must be allowed time to cool to the temperature of the room before the weighing is completed, since otherwise currents of hot air might seriously affect the result (see ¶ 19).

[1] When a hot air-blast cannot be had, the bottle may be dried by rinsing it out several times with a small quantity of alcohol, and exposing it for a few minutes to a draught of air.

The bottle is then filled with distilled water at an observed temperature, not far from that of the room; then closed in such a manner (see Fig. 19) as to allow all bubbles of air to escape.[1] The outside of the bottle is then carefully dried with a cloth or blotting-paper. The weight is again found with the same degree of accuracy as before, and immediately afterward the temperature of the water and the density of the air (¶ 18).

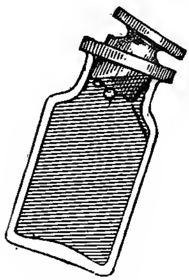


Fig. 19.

The difference between the two apparent weights of the bottle containing air and water, respectively, is equal to the apparent weight in air of the water which it contains (§ 66); this weight of water multiplied by the space occupied (at the higher of the two observed temperatures, see ¶ 33) by a quantity of water weighing apparently 1 gram (in air of the observed density, see Table 22), gives the total space occupied by the water, or in other words the capacity of the bottle at the observed temperature.

¶ 33. **Effects of Varying Temperature on a Specific Gravity Bottle.** — It is hardly necessary, in the experiments which follow, to allow for the expansion of the glass bottle due to changes of temperature which

[1] If the shape of the stopper makes this impossible, it must be altered by grinding or by filling up any hollows in it with paraffine or other material not acted upon by ordinary liquids. In this case the weight in air must be re-determined.

it is likely to undergo.[1] In a laboratory, maintained as it should be at a nearly constant temperature, these changes will be slight. Unless, however, special precautions are taken to keep the water in the bottle at a constant temperature, serious errors are likely to arise. These errors will be still greater in the case of certain other liquids which we shall employ. The expansion of alcohol, for instance, will be found to be several hundred times as great as that of glass (see Table 11).

Let us first suppose that the liquid which fills a closed bottle is gradually cooling, and hence in the process of contraction. A bubble will soon be formed. This need not, however, give rise to apprehension if the initial temperature (at which the bottle was filled) has been correctly observed; for the weight of the liquid will not be changed by its contraction, and the bubble weighs practically nothing. We may therefore determine the weight of a liquid which fills a bottle at an observed temperature, after it has fallen below that temperature.

Now, let us suppose that the liquid is growing warmer; and hence, expanding, that it forces its way out by the stopper, yet clings to the bottle. Unless the liquid is volatile or hygroscopic,[2] its weight

[1] The capacity of a vessel increases by the same amount as the volume of a solid of the same material which would exactly fill the vessel. In the case of glass, this increase is at the rate of about 1 part in 40,000 per degree Centigrade.

[2] Hygroscopic liquids, such as sulphuric acid or chloride of calcium, should be slightly warmed before the experiment, so that they may be weighed while cooling.

will be unchanged, and hence may be determined at leisure. If, however, the liquid evaporates immediately (as many liquids do) on contact with the air, there will be a continual loss of weight. In such cases, we must find the temperature as nearly as possible at the time of weighing, when it will be seen that the quantity of liquid weighed exactly fills the bottle.

In practice, both the initial and final temperatures are usually observed; the former just before the insertion of the stopper, the latter immediately after completing the weighing. We notice that with a non-volatile liquid, the initial temperature is always required; and the same statement applies to a volatile liquid which is cooling; but with a volatile liquid in general it is the *maximum* temperature which we wish to determine. In no case do we take the mean of the two temperatures before and after the experiment.

The liquids which we employ should be warmed or cooled if necessary, so that they may be nearly at the same temperature as the room; since otherwise the rapid changes of temperature which must ensue (§ 89) would make an accurate observation of the thermometer impossible. Errors in weighing might also be introduced, owing to currents of hot or cold air (¶ 19). In the case of certain liquids (as ether) which are apt to become cold through evaporation,[1]

[1] Care must be taken in general to prevent evaporation; and especially in the case of impure liquids, the strength of which would be affected by the escape of the more volatile ingredients.

there is danger that moisture may be condensed on the sides of the containing vessel (see ¶ 17). Particular care must be taken in the case of water, when below the temperature of the room; lest through the humidity of the air or from other causes it should fail to evaporate as fast as it is driven out of the bottle. Any moisture collected around the stopper should be removed with blotting-paper before making a final adjustment of the weights.

EXPERIMENT XII.

DISPLACEMENT I.

¶ 34. **Determination of Displacement by the Specific Gravity Bottle.** — The experiment consists essentially of a repetition of Experiment 11, with a bottle already partly full of sand, or any other substance insoluble in water. The capacity of the bottle for water is evidently less than before by an amount exactly equal to the space which the sand takes up; hence the latter can be found by subtracting the new capacity from the old. This method of determining volume is especially convenient in the case of powders, which cannot easily be suspended from a hydrostatic balance.

Certain modifications of the methods used in Experiment 11 are introduced when finely divided substances are employed. Even with sand considerable difficulty may be found in removing the bubbles of air which cling to it under water. By continual shaking with water in a well-stoppered bottle, this air may generally be freed from the sand.[1] To obtain dry sand, it should be heated before the experiment to a temperature above 100°.

The same process may be used to dry various powders not easily melted or decomposed by heat; but others require special precautions belonging to the province of Chemistry rather than Physics.

It may be observed that the apparent weight of the solid used in this experiment is incidentally determined; for we have only to subtract from the apparent weight of the bottle with it that of the bottle without it as found in the last experiment. The density of the solid may therefore be calculated as in Experiment 9.

¶ 35. **Illustration of the Principle of Archimedes.** — To understand what is meant by the water displaced by a solid, the bottle may be filled with water as in Experiment 11, then the solid may be introduced; water will be literally displaced, and if the whole quantity thus driven out of the bottle could be collected and weighed, we should have a direct measurement of the water displaced by the solid. In practice we prefer to find this by difference.

If we call s the apparent weight of the sand, b that of the bottle, w that of the water which fills it, and d that of the water displaced by the sand, the weights observed are (1) b and (2) $b + w$ in Experiment 11,

[1] An air-pump greatly facilitates the process, but unless special precautions are taken the water is apt to bubble over into the receiver and to find its way into the valves of th pump.

(3) $b + s$ and (4) $b + s + w - d$ in Experiment 12. The apparent weight of water which fills the bottle is the difference between the first and second observations, or (2) — (1), but when the sand is already in the bottle the quantity of water required is the difference between the last two observations, or (4) — (3); hence the quantity displaced is [(2) — (1)] — [(4) — (3)].

Now the weight of the sand in air is evidently the difference between the first and third observations, or (3) — (1); its apparent weight in water is the difference between the second and fourth,[1] or (4) — (2); its loss of weight in water is therefore [(3) — (1)] — [(4) — (2)]. This is seen by comparison to be identical with the expression above for the weight of water displaced.

The student who finds difficulty in realizing how the apparent weight or loss of weight of a solid in water can be found by the specific gravity bottle may repeat these measurements with a hydrostatic balance, using a cup to hold the sand in place of the network of wire employed in Experiment 9 to hold the glass ball; or he may find the weight and loss of weight in water of the steel balls used in Experiment 4 by means of the specific gravity bottle. The Principle of Archimedes (§ 64) states that loss of weight

[1] In both observations we have the same weight of the bottle, and the same hydrostatic pressure of the water upon the bottom or sides of the bottle (§ 63); the only difference is the downward pressure of the sand, which is present in (4) and absent in (2). This pressure exerted under water is what we call the weight of the sand in water.

in water (which we think of as determined by hydrostatic methods) is equal to the weight of water displaced (which we think of as determined by a specific gravity bottle). The agreement of the results obtained by hydrostatic methods with those from the specific gravity bottle may serve therefore either as an illustration of this principle or as a mutual confirmation of these results.

EXPERIMENT XIII.

DISPLACEMENT II.

¶ 36. **Determination of the Volume and Density of Solids Soluble in Water.** When owing to the solubility in water of the substance employed, the method explained in the last experiment cannot be applied, it remains only to find some other fluid of known density in which that substance is insoluble. The various products of the distillation of petroleum are especially suited to this purpose, since they dissolve few (if any) ordinary substances which are soluble in water. We may occasionally, with great care, use a saturated aqueous solution of the substance whose density is to be determined, or a liquid which has been allowed to act chemically upon an "excess" of that substance, since in either of these cases the liquid will have no further action on the solid. Gases may also be employed; but on account of the difficulty of measuring their weight correctly even by the most delicate balances, it is customary to estimate the quantity present by a direct or indirect measurement of its volume.[1] Owing, however, to the tendency of certain substances to absorb large quantities of gas, all such methods may lead to erroneous and even absurd results.

For sake of simplicity we will choose the liquid whose density has been determined in Experiment 10, and for the solid some substance insoluble in that liquid; and in order that the density of the liquid may be the same as before, it should be warmed or cooled if necessary to the temperature observed in Experiment 10. With such a solid and liquid, Experiment 12 is to be essentially repeated.

¶ 37. **Calculation of Volume and Density by the Use of Specific Volumes.** We have already seen how the weight of water displaced by a solid may be found either by the hydrostatic balance (Experiment 9) or by the specific gravity bottle (Experiment 12). By the same methods we may obtain the weight of any other fluid displaced by a solid. We have already applied this principle in Experiment 10 for determining the density of a liquid. Knowing the weight in grams and the number of cubic centimetres displaced, we found by division the weight of 1 *cu. cm.* It would have been equally simple to interchange the divisor and dividend, and thus to find the space in *cu. cm.* occupied by 1 gram. This is sometimes called the *specific volume* of a liquid.

The mutual relations existing between the weight

[1] For a description of the "Volumenometer," see Trowbridge's New Physics, Experiment 31.

w, the volume v, the density d, and the specific volume s, of any substance are given by the equations

$$d = \frac{w}{v},\ s = \frac{v}{w},\ \therefore\ s = \frac{1}{d},\ v = w\,s,\ etc.$$

The specific volume is therefore technically the "reciprocal" of the density. To find it we divide unity by the density already determined in Experiment 10, or by that which we may find from Experiment 14.

We have already used specific volumes in Table 23 (see ¶ 29), and we know that the weight *in vacuo* of the liquid displaced, multiplied by its specific volume,[1] gives the actual volume displaced, which is of course equal to that of the solid causing the displacement. The volume of the solid enables us to reduce its apparent weight to *vacuo* (§ 67), and hence to calculate its density (§ 68).

EXPERIMENT XIV.

DENSITY OF LIQUIDS.

¶ 38. **Determination of the Density of a Liquid by the Specific Gravity Bottle.** We have already found the weight of a bottle containing water and air, and we have calculated its capacity; it remains only to find its weight when filled with any other fluid, in order

[1] The student should bear in mind that the specific volume here employed is the space occupied by a quantity of liquid weighing 1 gram *in vacuo*, not that which weighs apparently 1 gram in air. True specific volumes must be multiplied by true weights *in vacuo* to find actual volumes. Apparent specific volumes (see Table 22) are intended to give the same result with apparent weights in air.

that the density of that fluid may be determined. For the purpose of comparison we will choose the liquid already used in Experiments 10 and 13, and warm or cool it, as nearly as may be convenient, to the temperature of those experiments. The actual temperature should be observed for reasons explained in ¶ 33, both before and immediately after weighing. The barodeik should also be read, in order to make sure that no great change has taken place in the course of our experiments with the specific gravity bottle, since otherwise its apparent weight in air must be re-determined.

The apparent weight of a quantity of alcohol sufficient to fill the bottle is found by subtracting that of the bottle with air from that of the bottle filled with alcohol, and is reduced to *vacuo* as explained in § 67. The density is then calculated by dividing the weight *in vacuo* by the capacity of the bottle, from ¶ 32. The strength of the alcohol is finally found by reference to Table 27, using a process of double interpolation (see § 58). The strength of the alcohol may also be calculated from the data of Experiment 14; and even if the temperatures in Experiments 10 and 14 differ considerably, the two results should agree in respect to strength.

EXPERIMENT XV.

THE DENSIMETER.

¶ 39. **Hydrometers and Densimeters.** — There are various kinds of hydrometers employed in the arts.

Nicholson's has been already described, and is the type of a "hydrometer of constant immersion;" that is, one which in use is always made to sink in a liquid to a given mark. A common glass hydrometer is, on the other hand, an example of "variable immersion." The distance it sinks in a fluid depends upon the density of the fluid, and is read by a scale attached to the stem of the instrument. The scales used in the arts are generally arbitrary. The principal ones are those invented by Baumé, Beck, Cartier, and Twaddell, which are compared in Table 40 with a scale of density. The instruments most convenient for scientific purposes carry a scale which indicates at once the density of the liquid, and hence bear the name of densimeters.

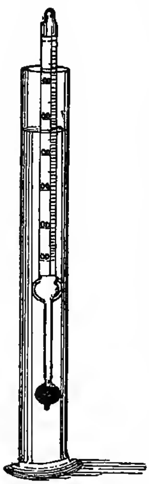


FIG. 20.

The sensitiveness of a densimeter evidently depends upon the smallness of the graduated stem, compared with the whole displacement of the instrument; but if we make the stem too small, a single hydrometer of the ordinary length can cover only a very limited range of densities. A set of three instruments is often used,—one for liquids lighter than water, one for liquids heavier than water, and one for liquids of intermediate density. There are also sets of twelve or more hydrometers, covering together the whole range of densities from sulphuric acid (1.8) to ether (0.7). With these great accuracy and rapidity may be attained, even without applying any of the ordinary corrections;[1] but if rapidity be the chief object, a single instrument with a "specific gravity scale" will be found most convenient. Such a one is often called by dealers a "Universal hydrometer" (see Fig. 20).

The errors of such instruments are not so great as one might expect, considering that the scales are printed in quantities from originals none too carefully made, fitted to tubes of by no means uniform bore, regardless within certain limits of their size, and fastened to these tubes at a point too high or too low, as the case may be. Still, even if the reading in water is found to be nearly correct, considerable errors may be discovered in other parts of the scale. As these errors depend largely upon the calibre of the tube, the process of correcting them may be properly called calibration (§ 36).

¶ 40. **Calibration and Use of a Densimeter.** — The reading of the instrument is taken while floating successively in at least three standard liquids of known density, such as water, alcohol, and glycerine (see Tables 25–27), then in a number of other liquids whose density is to be determined. As with a Nicholson's hydrometer, the under surface of the liquid is (when possible) used as a sight (see Fig. 6, ¶ 6); and the same precautions are taken to avoid friction against the sides of the jar, and the effects of capil-

[1] It should be remembered that changes of atmospheric density influence only that portion of a hydrometer which is above the liquid, and hence will not generally affect even the fourth place of decimals. The effect of a narrow range of temperature in changing the volume of a glass hydrometer is equally unimportant.

lary action due to the stem's becoming dry near the surface of the liquid. Both the densimeter and the thermometer (which is invariably read in every observation) must be washed after immersion in each liquid, either under the faucet or in three changes of water; they should also be carefully dried before immersion in a new liquid; otherwise more or less dilution or mixture is sure to take place. The corrections of the densimeter are then calculated and applied as explained in the next section.

¶ 41. **Treatment of Corrections by the Graphical Method.** — Correction and error are by definition (§ 24) equal and opposite. If the observed value of a quantity is greater than its real value, we say that the error is positive, the correction negative. Thus, by subtracting the observed from the tabulated densities of water, alcohol, and glycerine at a given temperature, we find the several corrections for the instrument by which these densities were observed. The correction of an instrument will generally vary according to the reading in question; hence, to find the correction for every reading, it is necessary to construct either a table

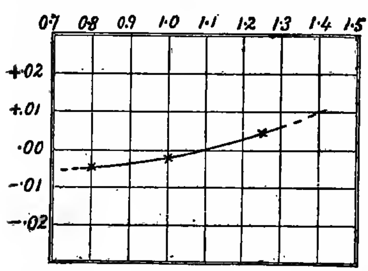


Fig. 21.

of corrections or a curve. Thus, in Fig. 21 the three points indicated by crosses represent (see § 59) corrections of a particular densimeter corresponding to three densities: namely, for alcohol, density 0.80, correction —.004; for water, density 1.00, correction — .002; for glycerine, density 1.25, correction +.004. The curve drawn by a bent ruler through the crosses enables us to find approximately the correction of this instrument for all intermediate densities by the general rules of the graphical method (§ 59). Thus for an ammoniacal solution of the density 0.9 or thereabouts, the correction would be not far from —.003. Corresponding corrections should be applied to each of the liquids whose density has been determined by means of the densimeter.

EXPERIMENT XVI.

BALANCING COLUMNS.

¶ 42. **Determination of Density by Methods of Balancing Columns.** The ordinary method of balancing columns is illustrated in Figure 22. Some mercury, for instance, is poured into a U-tube, then into the longer arm some water. Suppose the mercury is thus forced up to a level, *b*, in the shorter arm, and down to a level, *c*, in the longer arm, by a column of water reaching from *a* to *c*, and let the vertical distances, *bc* and *ac*, between the corresponding levels be measured; then since the density of water is known, the density

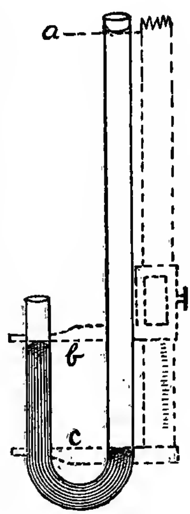


FIG. 22.

of mercury will be determined (see ¶ 43). When the two liquids are miscible this method cannot be applied.

Another method in which this difficulty is avoided is illustrated in Figure 23. A tube in the form of an inverted Y is plunged into two vessels, *c*, containing water, and *d*, let us say, glycerine. The two liquids are then sucked up cautiously to the respective levels *a* and *b*; and held there by closing a stop-cock in the stem of the Y. The relative density of the glycerine will then be determined by measuring the distances *ac* and *bd*. These distances are measured vertically, in the case of each liquid, between its level in the tube and its level in the cistern.

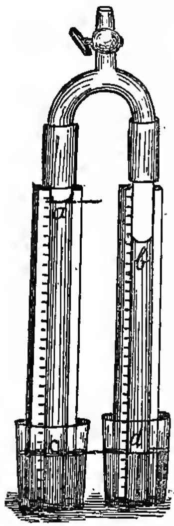


FIG. 23.

For measuring long distances, as *ac* or *bd*, a millimetre scale behind the tubes will suffice; for short distance (as *bc*, Fig. 22) a vernier gauge may be preferable; but special care must be taken to have the shaft vertical. To diminish the effects of capillary attraction, the tubes should have a diameter of a centimetre if possible,[1] and the level should be read

[1] If smaller tubes are used, two experiments must be made. In one, the columns of liquid should be as long as may be convenient; in the other as short as possible. The effects of capillary action are then eliminated in the usual manner by taking differences (see § 32). Thus instead of the column *ab* (Fig. 22) we find the difference between two such columns in the two experiments; and in the same way we find the difference between the two columns *bc*. These differences evidently balance one another.

by the middle point of the surface, whether convex or concave (see case of the Barometer, ¶ 13).

The common temperature of the two balancing columns may be found by a mercurial thermometer midway between them. An observation of the barodeik will be unnecessary.

¶ 43. **Theory of Balancing Columns.** Two liquid columns are said to balance one another when they exert equal and opposite pressures at a given point. Since pressure is affected by the density as well as by the depth of a fluid, the greater height of one column must counterbalance the greater density of the other. In other words, the densities of two balancing columns must be to each other inversely as their vertical heights.

It is evident that, in Figure 22 of the last article, the vertical height of the water is equal to *ac*, the total length of the column; but that of the mercury which balances it is only a portion of the whole column of mercury, namely, *bc;* for the part in the bend of the tube having the same level, *c*, at both ends, exerts no pressure to the right or to the left (§ 62), and serves simply to transmit pressure from one column to the other. For the same reason, we disregard in Figure 23 the portions of the liquids below *c* and *d,* and find that the balancing columns are *ac* and *bd.*

The balance between the two liquid columns in the first method (Fig. 22) will not be disturbed by the atmospheric pressure, provided that it affects both columns alike, as is very nearly the case; but strictly we must observe that the barometric pressure is greater at b than at a. There is, as it were, a column of air of the height ab acting on the mercury without any equivalent acting on the water. Since the density of air is about 800 times less than that of water, we should subtract from the apparent length of the column of water, ac, one 800th part of the distance ab, to find the column of water which would balance the mercury *in vacuo*.

In the second method (Fig. 23), supposing c and d to be on the same level, we find in the same way an unbalanced column of air, ab, acting on the shorter of the two columns of liquid. If the longer column is as before, water, we subtract from it one 800th of ab. If the shorter is water we add one 800th of ab. In applying this correction, we neglect the fact that the air within the tube is slightly rarefied, since the accuracy of the instrument employed will not justify more than a rough approximation to the density of the air in question.

If in either method l is the length of the column of liquid whose density, D, is to be determined, w the length of the column of water which balances it *in vacuo*, and d the density of this water at the observed temperature (see Table 25), we have, solving the inverse proportion mentioned above,

$$D = \frac{wd}{l}.$$

EXPERIMENT XVII.

DENSITY OF AIR.

¶ 44. **Determination of the Density of Air.** — A stout flask provided with a stop-cock (Fig. 24) is made thoroughly dry (see ¶ 32), and weighed with the stop-cock open. The flask is then connected with an air-pump, and as much air as possible is exhausted. The stop-cock is now closed; and the flask, having been disconnected from the air-pump, is re-weighed. It should be left on the balance long enough to prove that there is no perceptible gain of weight from leakage of air into it, then quickly opened under water as in Fig. 25. The stop-cock is closed by some mechanical contrivance while the flask is still completely submerged; then the flask is dried outside and weighed with the water which has entered. The temperature of the water is now observed. Finally the flask is filled completely with water and re-weighed. When all these observations have been recorded, an observation of the barodeik (see ¶ 18) is made for purposes of comparison. Having found the proportion of air exhausted, we calculate its density, as explained below.

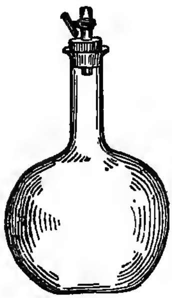


FIG. 24.

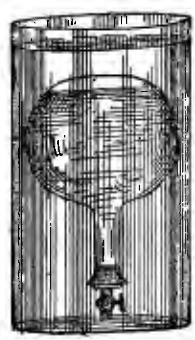


FIG. 25.

¶ 45. **Theory of the Partial Vacuum.** — When a flask from which the air has been partially exhausted is opened under water as in Figure 25, the water is forced inwards until the residual air is sufficiently compressed to resist the atmospheric pressure from outside. If the temperature is constant, as will be essentially the case when the flask is surrounded by water, the pressure depends chiefly on the density (see § 78); hence the residual air is compressed until its density is the same as that of the outside air. The space which it then occupies, compared with the whole capacity of the flask, will then represent the proportion of air remaining in it; and the amount of water which enters compared with the total amount necessary to fill the flask will represent the proportion of air exhausted.

The flask must not be plunged too deep below the surface of the water, for if it is the air within it may be perceptibly compressed; but it is well to submerge it to a depth of 10 or 20 *cm.*, to offset the expansion of the air caused by its taking up vapor from the water with which it comes in contact (see Table 13). The less air there is, the less will be its expansion. To obtain accurate results, we must therefore exhaust nearly all the air, or else substitute for water some less volatile fluid.

It may be observed that the water which enters the flask replaces, bulk for bulk, that portion of the air which has been exhausted. The weight of this air is the difference between the weights of the flask before and after exhaustion; the weight of the equivalent bulk of water is the difference between the last two weighings, — before and after the admission of water. We notice that in this experiment, unlike those which precede it, the water enters the flask without displacing any air whatever; hence no allowance is made for the weight of air displaced. Both the weight of air exhausted and that of the water which takes its place are affected by the buoyancy of the atmosphere upon the brass weights (§ 65), and in the same proportion; hence their quotient is unaffected, and represents the true specific gravity of the air referred to the water. This should agree closely[1] with the atmospheric density indicated by the barodeik.

EXPERIMENT XVIII.

DENSITY OF GASES.

¶ 46. **Determination of the Density of a Gas.** — A light flask, as large as the balance pans will admit, is made perfectly dry (see ¶ 32), and weighed with its stopper beside it. To determine the density of the air within the flask, an observation of the barodeik is made (see ¶ 18). Then the flask is filled with coal-gas conducted through a rubber tube reaching as far as possible into the flask. To prevent the escape of the coal-gas, which is lighter than air, the

[1] The true specific gravity of any substance referred to water at any temperature must strictly be multiplied by the density of water at that temperature (see § 69), to find the density of the substance in question. In the present case, the multiplication will hardly affect the last significant figure of the result.

flask is held in an inverted position throughout the process; after which the tube is drawn slowly out of the flask without checking the flow of gas (see

Fig. 26.

b, Fig. 26), and the stopper (*a*) is immediately inserted. The weight of the flask is again determined. More gas is then passed into the flask as before until it reaches a constant weight. The temperature of the gas in the flask is then found by a thermometer inserted through a bored stopper; and the pressure is determined by an observation of the barometer. Finally the flask is filled with water and weighed for the purpose of finding its capacity.

The last weighing and the observation of temperature which should accompany it may be comparatively rough; but the weighings with air and with gas should be made with the utmost precision, since the difference between them, upon which the result depends, is so slight that even a small error would affect this result in a very considerable proportion (see § 36). If ordinary prescription scales are used, the result should depend upon the mean of at least five double weighings in each case. When great accuracy is desired, a counterpoise should be used consisting of a second flask, hermetically sealed, equal to the first in volume and nearly equal in weight. Small weights added to the counterpoise should bring about an exact adjustment. By using such a counterpoise, changes in atmospheric density are eliminated, since the air will buoy up the contents of both pans alike.

The capacity of the flask is then calculated as in ¶ 32, and the density of coal-gas at the observed temperature and pressure is found by the formula of § 70, using the density of the air indicated by the barodeik. The result is then reduced to 0° and 76 *cm.* pressure by the formula of § 81.

EXPERIMENT XIX.

MEASUREMENT OF LENGTH.

¶ 47. **Selection of a Standard of Length.** — A careful comparison of the various scales which we have hitherto employed for the measurement of length will generally show cases of disagreement. These may sometimes be explained as the result of expansion by heat (see Table 8 *b*); for, though a scale should be correct at 0°, unless otherwise stated, there is no agreement to this effect among manufacturers.[1] In other cases errors are discovered which may be traced to the machine by which the scales are divided. It will not do to assume that the most carefully finished scales are the most accurate. Those printed in large quantities on wood compare very

[1] English measures are generally adjusted (if at all) to a temperature of about 62° Fahrenheit. Certain French manufacturers maintain that all standards are supposed to be correct at 4° Centigrade. In the case of brass metre scales, discrepancies of nearly half a millimetre may sometimes be traced to the temperatures at which they have been adjusted.

favorably with common varieties of "vernier gauge" (see Fig. 27). The latter, in particular, need to be tested as will be explained below. For this purpose, "end standards" are made by various manufacturers with a considerable degree of precision. In place of these, however, the student will find it more instructive to use one depending, as follows, upon his own measurements.

The volume, v, of a glass ball has already been determined (¶ 29); from this the diameter, d, may be calculated by geometry, using the formula [1]

$$d = 1.2407 \sqrt[3]{v}.$$

In calculating the diameter of a sphere from the cube root of its volume, great accuracy may be obtained (see § 36). Thus if the volume is really 40.00 *cu. cm.*, and owing to an error of 1 *cg.* in weighing, the observed value is 40.01 *cu. cm.*, the calculated diameter will be 4.2435 *cm*, instead of 4.2432 *cm.* The difference (.0003 *cm.*) between the calculated and the true value would be imperceptible.

If the ball which we employ is not perfectly spherical, an *average* diameter will be given by the formula. We shall see in ¶ 50, I. how slight irregularities can be allowed for. We may therefore obtain from our experiments in hydrostatics a standard, in the form of a sphere, by which it is possible to correct the reading of a vernier gauge, or any other kind of caliper.

[1] This is derived from the ordinary formula —

$$v = \frac{\pi}{6} d^3$$

¶ 48. **Testing Calipers.** A caliper is an instrument intended especially to determine by contact the diameter of bodies, generally the outside diameter. It is provided with two points called "teeth" or "jaws," one of which at least is movable. In one class of calipers the jaws are hinged together, their motion being magnified in some cases by a long index; in another class there is a sliding motion, as in the vernier gauge used in Experiment 1 (see Fig. 27); in a third class the motion is produced by a screw, as in the micrometer gauge (Fig. 28).

The instrumental errors (§ 31) likely to arise differ, of course, according to the special construction of the gauge in question; but there are certain classes of defects common to all calipers, and hence it is well, before beginning any series of measurements, to make a regular examination of each instrument, covering the following points: —

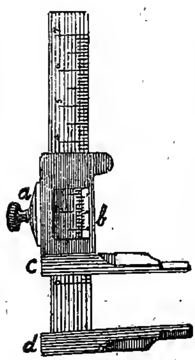


Fig. 27.

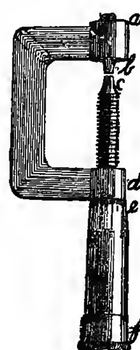


Fig. 28.

(*a*) Distortion. The shank of a vernier gauge (*ad*, Fig. 27) should appear perfectly straight to the eye, when "sighted" in the ordinary manner, and perfectly free from twist. A micrometer screw (*cd*, Fig. 28) should similarly appear straight, so that the tooth *c* may be accurately centred in all positions.

(*b*) CONTACT. The jaws of a gauge must be able to touch each other at some point (as *pp′* Fig. 29)

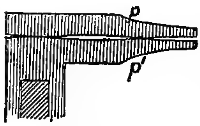


FIG. 29.

convenient for measurement. The shape of these jaws may be modified, if necessary, by the use of a file, or by the application of solder, in order that this condition may be fulfilled. The location of the point of contact is found by examining the streak of light between the jaws.

(*c*) PERPENDICULARITY. The surfaces of the teeth or jaws at the point of contact should be at right angles with the shank of the gauge. In the case of a micrometer, any obliquity immediately appears when the screw is rotated. To detect it in a sliding gauge it is necessary to reverse one of the jaws (as *b* in Fig. 30), and to see whether the two inner surfaces remain parallel.

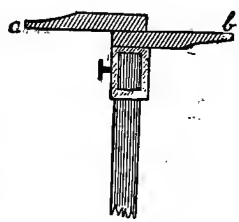


FIG. 30.

(*d*) GRADUATION. The uniformity of the thread of a micrometer screw is sufficiently established if it turns in the nut, when well oiled, with equal facility throughout its entire length. The graduation of a vernier gauge is most easily tested by the vernier itself; for if the latter always subtends exactly the same number of divisions on the main scale, these may be assumed to be sensibly uniform.

(*e*) LOOSENESS. A gauge should slide freely from one position to another; but any looseness in the moving parts must be prevented. For this purpose a set screw (*a*, Fig. 27) is usually attached to a vernier scale. In the absence of any equivalent arrangement, a nut may often be tightened successfully by pinching it slightly in a vice.

If the defects here mentioned cannot be overcome, the caliper should be discarded for the purposes of the exact measurements which follow.

¶ 49. **Precautions in the Use of Calipers.**

(*a*) WARMTH. In ordinary measurements with a vernier gauge, the warmth of the hand will hardly cause a perceptible expansion; but with micrometers, considerable care must be taken to avoid errors from this source. The usual method is to hold the instrument with a cloth, but it is still more effective to mount it in a vice, and thus to leave both hands free for making the necessary adjustments.

(*b*) CLAMPING. When a caliper has been "set" on a given object, it is customary to clamp it before making a reading, lest in the mean time dislocation should take place. There is danger, however, that in the very act of clamping any instrument, its "setting" may be disturbed. Vernier gauges, unless specially provided with springs to keep the moving parts in place, are troublesome in this respect. The difficulty is lessened by keeping a moderate pressure on the clamp while the setting is taking place. In all instruments, the accuracy of a setting should be tested after clamping.

(*c*) STRAIN. The teeth or jaws of a caliper must obviously not be bent forcibly apart by the pressure between them and the object on which they are set;

for the bending will introduce an error in the reading. One may judge whether the pressure is excessive or not by the muscular force required to produce it, or by the hold which the caliper seems to have upon the object in question. The best micrometers are provided with a friction head (*f*, Fig. 28) which slips when the required pressure is obtained. A most important result is thus secured, namely, a uniform pressure in all settings of the gauge, including the zero reading (see § 32) whereby the effects of strain may be eliminated.

(*d*) ROUGHNESS. If the surfaces of the teeth or jaws of a caliper are not perfectly smooth and flat, an object may fit between them with greater facility in some places than in others. To eliminate the effects of any such irregularity, the diameter which is to be measured should terminate in the points (*p* and *p′*, Fig. 31) which determine the zero reading of the gauge (see Fig. 29).

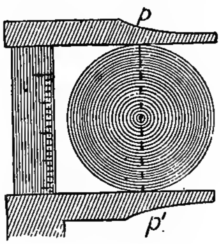


FIG. 31.

These are generally the most prominent points of the inner surfaces; hence the rule, *place the object to be measured where it fits with the greatest difficulty.*

(*e*) OBLIQUITY. The line *pp′* (Fig. 31) is necessarily parallel to the shank of the gauge; hence also the diameter of any object which coincides with it. If, however, through any mistake in the above adjustment, the diameter to be measured is perceptibly inclined with respect to the line *pp′*, a considerable error is likely to be introduced into the result. It may be shown by trigonometry that if the inclination is less than 1°, the error will be less than one six-thousandth part of the quantity measured; and hence practically insensible. Since the eye can detect under favorable circumstances an obliquity even less than 1°, the following rule will be found sufficiently accurate: *make the diameter to be measured sensibly parallel to the shank of the gauge.*

(*f*) Position. An object may be fitted between the teeth of a caliper in various ways, and care must be taken that the diameter thus measured is the one sought. In the case of a rectangular block, for instance, a minimum diameter is usually required, and care must be taken not to place it cornerwise; in the case of a sphere, however, a maximum measurement is wanted, and to secure this, especially when the teeth are rounded (as in Fig. 32), many trials must be made and with the greatest care.

Fig. 32.

(*g*) Parallax. Errors of parallax (§ 25) may be avoided when two scales are mutually inclined, by holding the eye or the gauge in such a position that the lines appear parallel, as in *A*, Fig. 33, not inclined as in *B*.

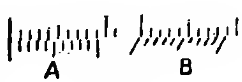


Fig. 33.

¶ 50. **Correction of Calipers**—It is important to determine the reading of a gauge or caliper when the jaws are in contact (see Fig. 29). This is called the "zero reading," because it corresponds to a distance zero between the points p and p' where contact takes place. A gauge need not be condemned simply because the "zero reading" is not exactly zero. The fulfilment of this condition is in fact exceedingly rare. It is only necessary that the zero reading shall be accurately determined, in order to avoid (by subtraction) all errors from this source (§ 32).

I. Vernier Gauge. The general method of reading a vernier gauge has been explained in ¶ 3. We have seen in § 37 how the tenths of the millimetre divisions on the main scale are read by means of a "vernier."

In case, however, the indication of the vernier lies between two numbers, it becomes necessary in all exact measurements to estimate fractions of tenths. We have already found a rough way of representing such fractions (see ¶ 6). A more exact method is described in § 37. To obtain success in applying this method to a vernier reading to tenths of a millimetre, the rulings of the scale should be fine, and a hand lens (such as is represented in Fig. 34) should be used to magnify the vernier and main scale divisions so that the difference between them may be plainly visible to the eye. The student will find it difficult, at first, to select the diagram in § 37 most resembling the case of coincidence in question;[1] but with

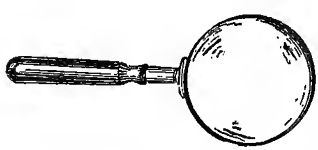


Fig. 34.

[1] One of the chief difficulties in conducting this experiment lies in the tendency of students to hold a gauge more or less obliquely, so that all cases of coincidence may appear to be exact, or (what is nearly as hopeless) precisely alike. To an accurate observer, no two settings present in general exactly the same appearance.

a little practice most of his errors should be confined to a range of one or two hundredths of a millimetre.

If the zero of the vernier comes opposite a point below the zero of the main scale, the reading is negative. For convenience, however, the negative sign is applied (as in logarithms) only to the whole number indicated on the main scale, — the fraction remaining positive. Thus if the zero on the vernier passes the zero on the main scale by .02 *mm.* when the jaws are brought into contact, the reading of the vernier should be .98; and in this case, the zero reading is $\bar{1}.98$, according to the general rule given in ¶ 3.

When the zero reading has thus been found within one or two hundredths of a millimetre, a body of

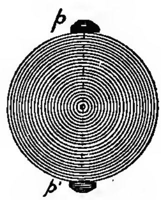


FIG. 35.

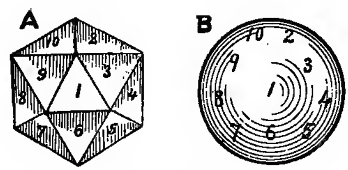


FIG. 36.

known diameter is set between the jaws of the gauge. The glass ball, for instance, used in Experiments 8 and 9 is to be placed (see Fig. 31), so as to reach between the points p and p' by which the zero reading was determined (see Fig. 29). Looking at the jaws endwise, we should see the ball symmetrically situated, as in Fig. 35.

If the ball is not perfectly round, we shall need at least 10 measurements of its diameter; and these measurements should obviously be distributed as uniformly as possible over the surface of the sphere. The student will do well to mark in ink ten points upon the ball as in B, Fig. 36, which are to be brought successively under the point *p* (Fig. 35), in one jaw of the gauge. After each measurement, the corresponding mark should be erased, to prevent confusion. As to the manner of spacing the ten points in question, the student is advised to begin with a 20-sided paper weight (*A*, Fig. 36), to place a number in the middle of each of the ten faces visible from a given point of view, then to copy these marks on the glass ball *B*, so that they may appear to be spaced in the same manner in both cases. The geometrician will observe that there is one way and only one way of distributing ten diameters uniformly over the surface of a sphere, and that this way has been here practically adopted.

In each of the ten measurements, a reading is made to hundredths of millimetres; then the zero reading is re-determined. From the mean of the ten measurements above, the mean zero reading is subtracted. We thus find the average diameter of the ball according to the gauge. Dividing this observed diameter by that obtained by the hydrostatic method (which we will suppose to be the true diameter — see ¶ 47), we obtain an important factor, namely, the average space in millimetres occupied by each millimetre division in a certain part of the gauge. If the gauge is uniformly graduated (see ¶ 48, *d*), it is obviously possible to correct all measurements made with the gauge at the same temperature by means of the factor thus found. In practice, however, it may be assumed that a gauge has been selected in which these corrections are too small to be considered.

II. MICROMETER GAUGE. — In place of the glass ball of Experiments 8 and 9, the student may use the steel balls of Experiments 3 and 4, provided that the displacement of these balls has been confirmed by the specific gravity bottle, as suggested in ¶ 35. The joint volume of these balls is then found by the use of Table 22 (see ¶ 29), then the average volume, from which (the balls being uniform in size) the average diameter is calculated by the formula of ¶ 47.

The diameters of these balls may now be measured by means of a micrometer gauge (see Fig. 28). The tests to be applied to a micrometer and the precautions to be followed are essentially the same as with any other kind of caliper (see ¶ 48 and ¶ 49). The zero reading is found as in the case of a vernier gauge by bringing the teeth into contact. Then the teeth are separated by turning the head of the screw (Fig. 28) until the ball whose diameter is to be measured fits symmetrically between these teeth as in Figure 37.

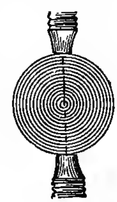


FIG. 37.

The whole number of revolutions of the screw should correspond with the number of main scale divisions on the nut *d*, uncovered by the barrel *e*. The hundredths of a turn may be read by the graduation on the edge of the barrel, using as an index a mark running along the nut. Care must be taken to avoid a mistake of a whole turn in reading the gauge; if, for instance, nine whole divisions (nearly) are uncovered by the barrel, and the index points to 98 hundredths, the reading is 8.98 (not 9.98). It is safer with many micrometers to confirm the *whole* number of revolutions by actually counting them.

In reading the micrometer the divisions corresponding to hundredths of a revolution should be divided into tenths by the eye (§ 26). A micrometer with a millimetre thread thus indicates the thousandth part of a millimetre. In the case of a negative zero reading, as with the vernier gauge, the minus sign should be applied only to the whole number of turns.

The diameter of each of the steel balls is determined in this way to thousandths of a turn of the screw; and from the average reading we subtract the average zero reading, observed before and after the above with an equal degree of precision. We find in this way the average number of turns and thousandths of a turn actually made by the screw. Dividing the average diameter of the balls (from the hydrostatic method) by the corresponding number of turns of the screw, we have finally the distance through which the micrometer screw advances in each revolution. This is called the "pitch of the screw." We shall assume that a micrometer has been found, reading to millimetres and thousandths so accurately that in the case of objects of small diameter, no correction need be applied.

EXPERIMENT XX.

TESTING A SPHEROMETER.

¶ 51. **Determination of the Zero Reading of a Spherometer.** A spherometer (Fig. 38) is essentially a micrometer (see ¶ 50, II.) supported by three legs (*d*, *f*, *g*). The vertical screw (*ce*) has a head (*b*) divided into a hundred parts, the tenths of which may be estimated by the eye (§ 26). The thousandths of a revolution may thus be read by means of an index (*a*). This index carries a vertical scale (*af*), on which the head of the micrometer (*b*) registers the whole number of revolutions made by the screw. Both on the scale (*af*) and on the micrometer, the indications should increase as the screw is raised. It is well to renumber the main scale if necessary, so that negative readings may be avoided.

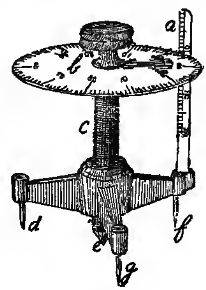


FIG 38.

The zero reading of a spherometer is its reading when the point of the central screw is in the plane of the three feet. To find it, the instrument is set on a piece of plate glass (Fig. 39) of sensibly uniform thickness, selected by the aid of a micrometer gauge, and the screw of the spherometer is raised or lowered until all four points seem to touch the glass at the same time (see Fig. 40). If the central screw is driven too far forward, the instrument will not stand firmly upon the glass, but will have a tendency to rock. This will be noticed especially if one of the feet be held down by the finger, while the other two feet are subjected to an alternating pressure. In fact, the conditions upon which rocking depends are so delicate that a change of a thousandth of a millimetre may cause it to appear or to disappear. When the instrument has been adjusted so that rocking is barely perceptible, the reading is estimated in millimetres to three places of decimals, in the same manner as in the case of a micrometer gauge.

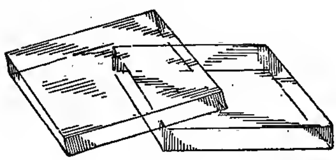


Fig 39.

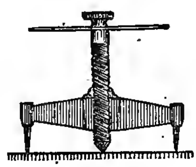


Fig. 40.

On account of possible irregularities in the glass, at least five readings should be taken in different parts of one surface; and as plate glass is apt to warp slightly in the process of manufacture, five more readings should be taken on the other surface. The mean of the values thus found on a piece of glass of uniform thickness gives the zero reading of the spherometer, and should be determined after as well as before any series of measurements such as will be described in the next section, in order to avoid errors due to change of temperature and to the wearing away of the points upon which the instrument rests.

¶ 52. **Determination of the Pitch of the Screw.** A spherometer with a screw of known pitch can be used in place of a micrometer to measure the diameter of small objects. These are placed upon the plate glass already used to determine the zero reading, and the screw is adjusted so as to touch them from above (see Fig. 41). If the point of the screw is very sharp, and the surface of the object in question convex, great care is needed in finding the maximum diameter.

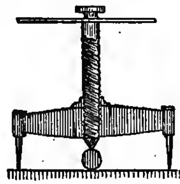


FIG. 41.

To determine the pitch of the screw, we select an object of known diameter by means of a vernier or micrometer gauge; we may determine, for instance, the diameter of a steel bicycle ball. This is then fitted as above (Fig. 41) beneath the point of the screw, and the reading of the spherometer accurately determined. Subtracting the zero reading, we have the number of turns made by the screw in traversing the diameter of the ball. Dividing this diameter by this number of turns, we have (as in ¶ 50 II.) the pitch of the screw.

Assuming that the screw has a uniform pitch, it is evident that the distance traversed by the point of the screw will always be given by the product of the number of turns and the pitch of the screw.

¶ 53. **Determination of the Span of a Spherometer.** The span of a spherometer, or the average distance of its three feet (*d*, *f*, and *g*, Fig. 38) from the central screw (*e*) in its zero position (Fig. 40) is an important element in all calculations relating to curvature (see next experiment). It may be determined roughly by a series of measurements with an ordinary vernier gauge. If difficulty is found in measuring directly the distances in question from centre to centre, an impression of the feet and central screw may be taken on paper, and the distances thus indirectly determined.[1] For this purpose the student will doubtless prefer to use a glass scale, if one can be obtained, graduated in millimetres and tenths. In such a scale the rulings should be placed next the paper, and examined with a magnifying glass.

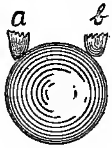


FIG. 42.

If the feet are blunt (as *a* and *b* in Fig. 42), the point of contact will be uncertain. In such a case the feet should be sharpened, and the zero reading re-determined.

¶ 54. **Testing a Spherometer.** We have seen that a spherometer may be fitted to a plane surface (¶ 51); in the same way it may be adjusted to a curved surface. To bring this about, the central screw must be driven forward, if the surface is concave, or turned backward if the surface is convex. The distance through which it must be moved obviously depends upon the curvature of the surface in question. The spherometer can therefore be used to determine the curvature of surfaces. There are, however, various sources of error in the use of a spherometer, and to

[1] Some authorities prefer not to measure directly the distances (*ed*, *ef*, *eg*, Fig. 38) of the three feet from the central screw, but to calculate the span by multiplying the average of the three distances (*df*, *fg*, *gd*) between the feet by the square root of one third, or 0.57735.

detect these, the instrument is first of all adjusted to a surface of known curvature, as for instance that of the sphere used in Experiments 8 and 9, (see Fig. 43), or if that is not large enough, to some other sphere of known diameter. The central screw is set as in ¶ 51, so that rocking is barely perceptible, and the reading of the instrument is determined with the same degree of precision as before. At least ten set-

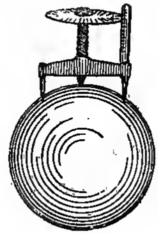


FIG. 43.

tings should be made on different portions of the spherical surface. In reducing the results we find first the average reading of the spherometer, then subtracting the zero reading we find the number of turns which the screw has made, and hence the distance in millimetres through which the point of the screw has retreated from its zero position, since the pitch of the screw has been already determined in ¶ 52. If this distance is d, and the diameter of the sphere D, the square (s^2) of the span of the spherometer may be calculated by the formula (see ¶ 56, II.), —

$$s^2 = Dd - d^2.$$

In this formula, all measurements should be expressed in millimetres. The result should confirm that obtained by squaring the span actually observed in ¶ 53. Slight discrepancies may sometimes be traced to obliquity or excentricity of the central screw, or to irregularities in the shape of the three feet.

EXPERIMENT XXI.

CURVATURE OF SURFACES.

¶ 55. **Determination of the Radius of Curvature of a Spherical Surface.** It is frequently required in optics to know the curvature of the surfaces of a lens; for this curvature, together with the nature of the glass of which a lens is made determines its power of bringing light to a "focus" (§§ 103–104); and conversely, if the curvature and focussing power are known, we may find what sort of glass the lens is composed of. This subject will be fully treated of in Experiments 41 and 42. It is necessary at present only to point out that as the surfaces of lenses are generally ground to resemble portions cut out of a sphere, their curvature may be determined in the same way as that of any other spherical surface.

The spherometer is set upon the lens as in Figure 44, and adjusted so that rocking is barely perceptible as in ¶ 51 and ¶ 54. Ten settings are thus made on each side of the lens, the curvatures of which, even if both are convex, are by no means necessarily the same. Between successive measurements the position of the spherometer should be varied somewhat, so as to determine as well as possible the average curvature of each surface.

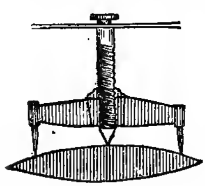


Fig. 44.

The results are then averaged for each surface; the mean zero reading subtracted from each, and the distance (d) between the point of the screw and the plane of its three feet thus determined. From this, the diameter, D, of the sphere of which the surface in question forms a part is calculated by the formula (see ¶ 56, I.),

$$D = d + s^2 \div d.$$

where s^2 is the square of the span already calculated in the last article.

The "radius of curvature" is found by halving the diameter.

¶ 56. **Theory of the spherometer.** The formulae of the last two articles depend upon the following considerations: Let a, Figure 45, be the point of the central screw of a spherometer, and b one of the three feet lying in the plane bc, and let ad be a diameter of the sphere abd intersecting the plane bc at c; then if the screw is properly adjusted, acb and bcd will be right triangles. Now abd is also a right triangle, being measured by half the semicircular arc ad; hence the angles cba and bdc are equal, both being complementary to cbd; the right triangles abc and bdc are therefore similar and we have —

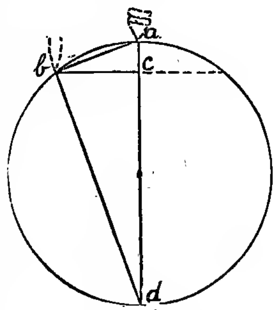


FIG. 45.

$\overline{ac} : \overline{bc} :: \overline{bc} : \overline{cd}$, whence

$\overline{cd} = \overline{bc}^2 \div \overline{ac}$, and

$$\overline{ad} = \overline{ac} + \overline{cd} = \overline{ac} + \overline{bc}^2 \div \overline{ac}. \qquad \text{I.}$$

We are thus able to calculate the diameter of a sphere (ad) if we know the span of the spherometer

(bc), and the distance, ac, between the point of the screw, a, and the plane of the three feet, bc. We can also calculate the square of the span bc, by the formula, easily derived from the above,

$$\overline{bc}^2 = \overline{ad} \times \overline{ac} - \overline{ac}^2. \qquad \text{II.}$$

EXPERIMENT XXII.

EXPANSION OF SOLIDS.

¶ 57. **Determination of the Coefficient of Linear Expansion.** — By measuring the length of a rod at two different temperatures, the amount of linear expansion due to heat may obviously be determined.

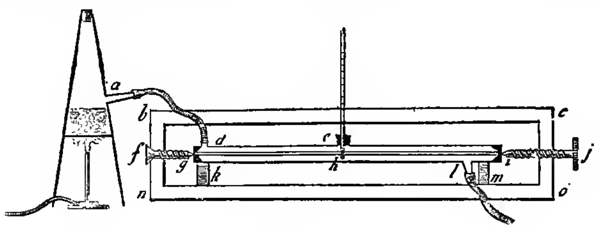


FIG. 46.

To make the expansion measurable, a long rod must be employed; and even then delicate instruments are needed to measure the expansion accurately. A micrometer gauge, especially constructed for this purpose, is represented in Fig. 46. It consists of a rectangular wooden frame, $bcon$, capable of admitting a metallic rod, gi, 1 metre long, between the fixed point fg and the point of the micrometer screw, ij.

The rod is surrounded with a tube, also 1 metre long, held in place by the supports, *k* and *m*. The tube is closed at both ends with corks, thinner near the middle than at the edges, and serving to keep the rod in position.

A setting of the micrometer is first made with the rod in position, and the reading determined (see ¶ 50, II.); the temperature of the rod is then found by means of a thermometer, *h*, passing through a cork, *e*, in the side of the tube. To determine the pitch of the micrometer,[1] it is turned backward (as in ¶ 52) until an object of known diameter fits between it and the end of the rod. A new reading is then made, and the pitch of the screw is calculated as in the case of an ordinary micrometer gauge (¶ 50, II.).

The screw of the micrometer is now withdrawn, to allow room for the expansion of the rod, and steam from a generator (*a*) is passed through the tube from the inlet (*d*) to the outlet (*l*). As soon as a steady current of steam appears at the outlet, a new setting of the micrometer is made.

Subtracting from the last reading of the micrometer the original reading, we find the number of turns made by the screw. From this, knowing the pitch of the screw (¶ 52), we find the expansion of the rod in *mm*. Subtracting the original temperature (let us say 20°) from the final temperature (100°, nearly, —see, however, Table 14) we find the rise of temper-

[1] By using the same micrometer as in ¶ 52, a determination of pitch will be rendered unnecessary.

ature which has caused this expansion. To find the expansion of 1 *mm.*, we divide the total expansion by the length of the rod in *mm.* (1,000 *mm.*) ; and we divide the quotient by the rise of temperature in degrees (80° in this instance) to find the expansion in *mm.* of 1 *mm.*[1] for 1°. The result is called the coefficient of linear expansion of the material of which the rod is composed (§ 83).

¶ 58. **Errors in the Determination of Linear Expansion.** — In determining the temperature of a metallic rod by a thermometer beside it, a considerable error is likely to arise unless the temperature of the surrounding air is constant, and the observation prolonged. Air is, as we shall see (Experiment 31), a comparatively poor conductor of heat. To attain greater accuracy in this experiment, the tube may be filled with water, as it is found that an equilibrium of temperature is reached much more quickly with water than with air (see ¶ 65, (6)). A still more accurate method is to replace the tube by a trough packed with melting ice or snow. The mixture should be stirred vigorously for a few minutes, so that the rod may acquire a nearly uniform temperature, not far from 0°. If this method is followed an observation of the thermometer will be unnecessary.

For rough purposes, the temperature of the steam which fills the tube in the second part of the experiment may be assumed to be 100° ; but this tempera-

[1] The student should note that the expansion of 1 *mm.* in *mm.* is numerically the same as that of 1 *cm.* in *cm.* The result does not therefore need to be reduced to the C. G. S. System.

ture really depends more or less upon the barometric pressure. The thermometer cannot be depended upon to give this temperature correctly, particularly if the bulb only is surrounded by steam. When accuracy is desired, an observation of the barometer must be made (see ¶ 13). The true temperature of the steam may then be found by Table 14, as will be explained in Experiment 25.

It is obviously impossible for the *whole* rod, *gi* (Fig. 46), to be in contact with the steam or ice surrounding it; for even when the corks are hollowed out, as shown in the figure, so as to leave nearly the whole surface of the rod uncovered, there must still be a small portion at each end which the steam or ice can never reach. The expansion of the rod will not therefore be as great as it should be.

On the other hand, the points *fg* and *ij*, being heated by contact with the rod, will expand somewhat, and thus make the expansion of the rod appear to be greater than it really is. To diminish the conduction of heat, the teeth may be protected by the use of insulating material, or by simply pointing them. In all cases contact should be maintained only as long as may be necessary to make a reading of the micrometer. There is always more or less uncertainty as to temperature when a hot and a cold body are in contact. To eliminate errors arising from this source, it would suffice to construct a new apparatus, which should be as short as possible, but otherwise similar to the first, and to calculate the results from the *difference* of expansion in the two cases, according to the general method suggested in § 32.

There is, however, no way to allow for the expansion of the sides of the gauge, caused by the warmth of the steam jacket. We meet here, in fact, one of the fundamental difficulties in the accurate measurement of expansion, — namely, changes in the length of the instruments by which expansion is measured. To avoid errors from this source, a glass tube is sometimes substituted for the metallic tube represented in Fig. 46, so that the expansion of the rod may be observed from a distance. In the most accurate determinations, the gauge or standard used for comparison is insulated from all sources of heat, and even, in some cases, maintained artificially at a uniform temperature.

The expansion of a gauge constructed, like that shown in Fig. 46, principally of wood (see Table 8, *b*), and with sufficient space for the circulation of air, will be found in practice to be very slight; but, in the absence of special precautions, the student should not expect his results to contain more than three significant figures (§ 55).

EXPERIMENT XXIII.

EXPANSION OF LIQUIDS, I.

¶ 59. **Determination of the Coefficient of Expansion of a Liquid by the Method of Balancing Columns.** — A convenient form of apparatus for this experiment (see Fig. 47) consists of two vertical metallic tubes, *ch* and *fj*, about one metre long, with horizontal elbows (*cd*, *ef*, *hi*, and *ij*) at each end. The lower elbows are connected together with a rubber tube (*i*), while each of the upper elbows is joined to one end of a differential gauge (*ab*) by one of the rubber couplings (*d* and *e*). Each of the tubes *ch* and *fj* is surrounded with a larger tube, or "*jacket*," which can be filled either with melting ice, with water, or with steam. The spouts *g* and *k* are to be used either as inlets or as outlets, as the experiment may require.

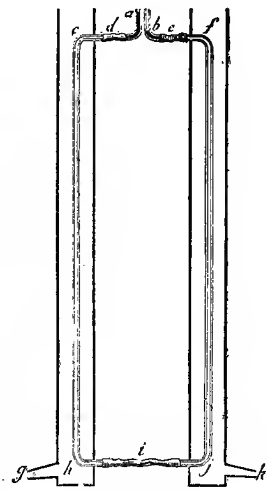


FIG. 47.

The liquid whose expansion is to be investigated is first freed from any air which may be held in solution, by boiling it, then poured steadily through a funnel into the tube *a* until, after completing the circuit (*a d c h i j f e b*), it issues in a continuous stream from *b*. The whole apparatus is now inclined first to the right and then to the left, so that any bubbles of air which may be lodged in the horizontal tubes may have an opportunity to escape. A little liquid is next poured out, until the column stands at the level *b*. This level should be the same, at first, on both sides of the gauge.

Steam is then admitted to the jacket *cg* through the spout *g*; and the jacket *fk* is filled with water from a faucet by a tube connected to the spout *k*. The temperature of the water is observed after it reaches the top of the tube, *f*. The height of the liquid in each side of the gauge (*a* and *b*) is measured as soon as it becomes stationary, by means of a millimetre scale, as in Experiment 16. (See ¶ 42.) The vertical length of the tube (*ch*) is finally measured between the elbows (*cd* and *hi*), from centre to centre, as close as possible to the jacket. This measurement should (strictly) be made while the tube is still heated by steam.

When the apparatus has become sufficiently cool, the water is emptied out of the jacket *fk*, which is, in its turn, filled with steam, while the jacket *cg* is cooled by water from the faucet. The temperature of the water and the reading of the gauge are observed as before; in this case, however, the vertical distance *fj* is measured. The object of interchanging the jackets is (see § 44) to eliminate errors due to capillarity, or in fact any cause which might tend constantly to raise or lower the level of the liquid on one particular side of the gauge.

Instead of admitting steam to one of the jackets, melting ice may be employed, or water at various temperatures, which must, of course, be observed. The other jacket is always maintained at a temperature not far from that of the room, by the water with which it is filled.

¶ 60. **Precautions in determining Expansion by the Method of Balancing Columns.** — It is evident that the temperatures employed in this experiment must not be higher than the boiling-point nor lower than the freezing-point of the liquid in question, and that this liquid must not be such as to act chemically on the tubes which contain it. Even a very slight action may generate a quantity of gas sufficient to impair the accuracy of the results. The air dissolved in the liquid must be completely boiled out before the experiment, since otherwise bubbles are apt to form when heat is applied. The tubes should be large enough to allow the escape of any air which may be carried into them while they are being filled; but small bubbles can sometimes be dislodged only by jarring the whole apparatus.

The tubes should be completely surrounded with the steam, water, or melting ice by which their temperature is to be regulated. There should be a free vent through one of the spouts (g or k) for the water formed by the melting of the ice, otherwise the temperature of the mixture may rise above 0°. If steam is admitted through one of these spouts, the jacket should be partly covered, leaving only a small opening through which the steam should escape in a slow but continuous stream. If the jackets contain water, the latter should be stirred vigorously to secure a uniformity of temperature. It is well also, in this case, to find the reading of a thermometer at different levels. This will require either a self-registering thermometer, or one with a very long stem. If the temperature is not uniform, the average temperature must be calculated.[1]

The jackets (*cg* and *fk*) should be made vertical by a plumb line, as nearly as the eye can judge, and also both branches of the gauge (*ab*). The tubes *cd*, *ef*, and *hj* should be perfectly horizontal, in those portions at least which are affected by the flow of heat to or from the jackets. The gauge (*ab*) should be maintained at a uniform temperature (the same always as that of one of the jackets) by surrounding it, if necessary, with water. The tubes of which this gauge is constructed should be of the same uniform calibre, and both perfectly clean, otherwise the effects of capillary action may not be perfectly eliminated. It is well to make sure, both before and after the experiment, that the liquid stands at the same level on both sides of the gauge when the temperature in the two jackets is the same.

To obtain the most accurate readings of such a gauge, a double sight should be employed, as in the case of a standard barometer. The setting is always made so that the plane of the sights may be tangent to the meniscus, or curved surface of the liquid (see ¶ 13 and ¶ 42). The sights may be provided with a vernier reading to tenths of a millimetre.

¶ 61. **Theory of Balancing Columns at Unequal Temperatures.** — The difference in hydrostatic pressure between the two liquid columns, *ch* and *fj*, is balanced

[1] The average temperature will be indicated at once by an air thermometer of sufficient length, which the student himself may be interested to construct. See Experiment 26.

by the pressure of a column of liquid reaching from *a* to *b*, or more strictly, by the difference between the hydrostatic pressure of such a column and that of an equally long column of air. The latter, being exceedingly light, may be left out of the account. To simplify calculations, we will suppose all the tubes to have a cross-section of 1 *sq. cm.* Then if d is the difference in *cm.* between the two levels (*a* and *b*) in the gauge, when it is maintained at the same temperature (t) as the jacket *fj*; and if l is the length of the column *ch* at a higher temperature, t_2; then l *cu. cm.* of the liquid at the temperature t_2 plus d *cu. cm.* at the temperature t_1, balance l *cu. cm.* at the temperature t_1. It follows that l *cu. cm.* at $t_2°$ must balance $(l - d)$ *cu. cm.* at $t_1°$. Now two columns of liquid of the same cross-section cannot balance one another unless they have the same total weight; hence the same quantity of liquid which occupies $(l - d)$ *cu. cm.* at $t_1°$ must expand by the amount d *cu. cm.* when heated to $t_2°$, since it then occupies l *cu. cm.* If an expansion of d *cu. cm.* is caused by a rise of $(t_2 - t_1)$ degrees, 1° would cause an expansion in the average $(t_2 - t_1)$ times less than d *cu. cm.*; and since the expansion of 1 *cu. cm.* would be $(l - d)$ times less than that of $(l - d)$ *cu. cm.*, the expansion (e') of 1 *cu. cm.* for 1° would be

$$e' = \frac{d}{(l - d)\ (t_2 - t_1)}. \qquad \text{I.}$$

This expression becomes somewhat modified when the gauge is at the higher temperature, t_2. We have, then, $(l + d)$ *cu. cm.*, all at the temperature t_2, balancing l *cu. cm.* at the temperature t_1. The expansion is as before, d *cu. cm.*; but the quantity expanding is no longer $(l - d)$, but l *cu. cm.* The expansion e'' per *cu. cm.* per degree is therefore

$$e'' = \frac{d}{l\,(t_2 - t_1)}. \qquad \text{II.}$$

We have assumed so far that the tubes have a cross-section of 1 *sq. cm.*; but the principles of hydrostatic pressure are independent of cross-section (see § 63); hence the solutions found in one case may be applied to all. The method of balancing columns is the only one which enables us to measure the expansion of a liquid without taking into account changes in the capacity of the vessel in which the liquid is contained.

The object of this method is to determine an *average* coefficient of expansion between two temperatures rather than the true coefficient of expansion (§ 83) at any particular temperature. The results may differ considerably from those contained in Table 11, which refers in nearly all cases to the expansion of liquids from 0° to 1° Centigrade. We consider, moreover, the expansion of a quantity of liquid measuring 1 *cu. cm.* at the lower of the two temperatures observed instead of at 0°. The result given by the formulæ of this section should, therefore, be designated as *the relative coefficient of expansion from* t_1° *to* t_2°, that is, from the lower to the higher temperature.

EXPERIMENT XXIV.

EXPANSION OF LIQUIDS, II.

¶ 62. **Determination of the Coefficient of Expansion of a Liquid by means of a Specific Gravity Bottle.** — The experiment consists essentially of a repetition of Experiment 14, with a given liquid at two or more different temperatures. These temperatures should be separated from one another as widely as possible, in order that the densities observed may differ by an amount large enough to be accurately measured. The temperatures themselves must be determined with the greatest care, particularly if they are far above or far below the temperature of the room; for in this case rapid changes will take place and must be guarded against.

A convenient way of heating a liquid in a specific gravity bottle to a uniform temperature, is to surround the bottle up to the neck with hot water. To prevent evaporation, the bottle should be closed temporarily by a cork, with a hole made in it sufficiently large to admit freely the stem of a thermometer, to which a brass fan is attached (see Fig. 50, ¶ 65). By this means the liquid is continually stirred until a maximum temperature is reached. As soon as the reading of the thermometer has been observed, the stopper is inserted, with due care not to enclose bubbles of air (see ¶ 32, Fig. 19). The bottle is then carefully dried, and weighed at leisure (see ¶ 33), after cooling to the temperature of the room.

The student is advised not to attempt determinations of density below the temperature of the room, on account of the obvious difficulty of preventing the loss, especially in the case of a volatile liquid, of the portion which is forced out of a specific gravity bottle by its gradual rise of temperature. He should, however, make at least two determinations of density above the temperature of the room, with the liquid already employed in Experiment 14; and he should repeat rapidly the determination made in that experiment at the temperature of the room, to make sure that the result has not been seriously affected by atmospheric changes, or by variations of the density of the liquid due to evaporation or other causes. Coefficients of expansion are then calculated and reduced as explained in the next section.

¶ 63. **Calculation of Coefficients of Expansion.** — Let t_1, t_2, t_3, etc., be the temperatures at which the densities d_1, d_2, d_3, etc., respectively, have been determined and calculated, essentially as in ¶ 38. The results are first represented by points plotted on coordinate paper (see Fig. 48), and connected by a curve drawn with a bent ruler, essentially as in § 59. The necessary forces should be applied to the ruler as near the ends as possible, in order that the curve may be continued downward as far as 0°. The density of the liquid (d_0) at 0° is now *inferred* by means of this curve (see § 59).

The specific volumes, v_0, v_1, v_2, v_3, etc., corresponding to the densities d_0, d_1, d_2, d_3, etc., are now found by the formulæ derived from ¶ 37, —

$v_0 = 1 \div d_0$; $v_1 = 1 \div d_1$; $v_2 = 1 \div d_2$; $v_3 = 1 \div d_3$, etc. Evidently a certain quantity of liquid expands by the amount $(v_2 - v_1)$ *cu. cm.* when heated from the temperature t_1 to the temperature t_2; that is, $(t_2 - t_1)$ degrees. The expansion per degree is therefore $(v_2 - v_1) \div (t_2 - t_1)$. Since the quantity of liquid

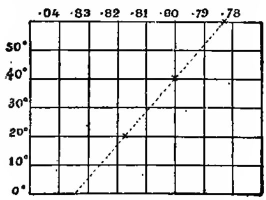


FIG. 48.

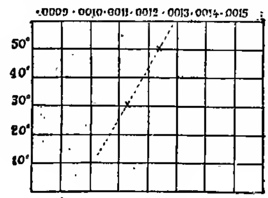


FIG. 49.

thus expanding occupies v_0 *cu. cm.* at 0°, the expansion (e) of a quantity occupying 1 *cu. cm.* at that temperature would be one v_0th as large, or

$$e = \frac{v_2 - v_1}{v_0\,(t_2 - t_1)}.$$

The coefficient e which determines the expansion of a quantity of liquid occupying the unit of volume at the standard temperature (0°) is a true as distinguished from a relative coefficient of expansion (see ¶ 61); it expresses, however, the average expansion between the two temperatures t_1 and t_2. We find in the same way the average coefficient of expansion from t_2 to t_3 by substituting, in the formula above, t_2, t_3, v_2, and v_3, for t_1, t_2, v_1, and v_2, respectively. Each result may be represented on co-ordinate paper by a cross, at the right of a point half-way between the two temperatures in question, and under the corresponding coefficient of expansion (see Fig. 49). A line drawn through these points represents approximately the coefficient of expansion at any given temperature. It is clear, however, that with only two determinations of the coefficient of expansion, we cannot tell even whether this line should be straight or curved.

EXPERIMENT XXV.

THE MERCURIAL THERMOMETER.

¶ 64. **Preservation of a Mercurial Thermometer.** — It would seem hardly necessary to point out that a mercurial thermometer is an exceedingly fragile instrument; but in the processes of manipulation about to be described, it is frequently required that a thermometer should be subjected to forces very near the limit of its strength, and which, even in skilled hands, may break it. The student is therefore advised to experiment with thin tubes or strips of window-glass, before attempting the calibration of a thermometer; and to examine the almost microscopic thickness of the glass constituting the bulb, before subjecting it to any considerable pressure. In respect to its resistance to a blow endwise, the bulb of a thermometer may perhaps be compared to the point of a lead-pencil when moderately sharp. In attempting to move the mercury in the thermometer by centrifugal force, the student should limit himself to such velocities as he might give to a palm-leaf fan. More thermometers are broken by suddenly arresting than by suddenly creating the necessary velocity. If a glass thermometer be temporarily mounted on a wooden support, like an ordinary house thermometer, it may be much more roughly treated with the same safety.

The full heat of a flame should never be applied immediately to any glass instrument, since fracture will almost inevitably result. By giving to a flame a waving motion, heat may be applied as slowly as may be desired. As soon as the glass acquires a dull-red heat the danger of fracture is past. There will, however, be no occasion for so high a temperature in the case of a thermometer. The student is particularly cautioned against plunging a cold thermometer into hot mercury,[1] or a hot thermometer into any cold liquid whatsoever.

In applying heat to the bulb of a thermometer, care must be taken not to drive out more mercury than there is room for in the expansion chamber at the top of the instrument. The temperature of the mercury should not be raised above its boiling-point[2] (350° C.) *in any part* of the thermometer; for the pressure of the vapor, being transmitted to the bulb, will be likely to cause an explosion.

¶ 65. **Precautions in the Use of a Mercurial Thermometer.** — (1) Temper. — In addition to the dan-

[1] The thermometer should be placed in the mercury while cold, and gradually heated with the mercury. On account of its rapid conduction of heat, mercury is more likely to cause fracture than other liquids.

[2] Special thermometers are now constructed so as to read safely as high as the boiling point of sulphur (440° C.).

ger of fracture, the accuracy of a thermometer may be greatly impaired by any wide change of temperature, especially if the change be sudden. After a thermometer is freshly made, there is found to be a gradual contraction of the bulb, which continues perceptibly for months and even for years. This accounts for the fact that nearly all old thermometers stand somewhat too high, although they are not supposed to be graduated until the contraction of the bulb has ceased. The value of a thermometer evidently depends partly on its age or "temper." This value may be completely destroyed by a sudden change of temperature.

(2) CHANGE OF FIXED POINTS. — In fact, when a thermometer is simply heated to the temperature of steam, then cooled as gradually as possible, the readings are almost always affected to the extent of one or two tenths of a degree. In the course of a month the thermometer may return to its former reading, but the change is gradual. It is therefore customary to test a thermometer — in ice, for instance — (see ¶ 69, II.) *after* testing it in steam (see ¶ 69, I.), or in fact after subjecting it to any considerable change of temperature.

(3) CONTINUITY OF THE MERCURIAL COLUMN. — Errors in reading a thermometer frequently arise from a break in the mercurial column, which can be guarded against only by inspection. A slight jarring is usually sufficient to make the column reunite; but when a small bubble of air interrupts the column, or when in the expansion chamber a globule

becomes separated from the rest of the mercury, special precautions are necessary (see ¶¶ 65, 67).

(4) TEMPERATURE OF THE STEM. — To make an accurate determination of temperature with a mercurial thermometer, it is necessary that the mercury, in the stem as well as in the bulb, should be raised to the temperature in question. In a thermometer reading to —10° C., for instance, if the bulb only is heated, the errors, even if the thermometer is correctly graduated, will be as follows: at 50°, — 0°.5; at 100°, — 2°.0; at 200°, — 7°.6; at 300°, — 17°; etc. As the temperature rises, more mercury flows into the stem, and it becomes still more important to heat this mercury to the given temperature (see ¶ 84).

(5) UNIFORMITY OF TEMPERATURE. — In nearly all determinations of the temperature of liquids, it is necessary to make use of some stirring apparatus, to secure a uniformity of temperature. A small fan of thin sheet brass is customarily attached to the stem of the thermometer, just above the bulb. The stirring is accomplished by twisting the stem of the thermometer. Special devices are necessary when finely divided substances are employed, though the stem of the thermometer itself may (with due care) occasionally be used, especially in mixtures, as of powdered ice and water, where the resistance will be exceedingly small.

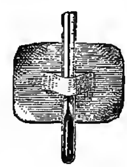


FIG. 50.

(6) TIME REQUIRED. — The length of time required to attain an equilibrium of temperature depends largely upon the conductivity of the surrounding medium, and

upon the degree of accuracy which is aimed at. Let us suppose that a thermometer is taken out of a mixture of ice and water, and placed in air at 32°; if at the end of one minute it rises 16°, that is, half-way towards its final temperature, we may expect it to accomplish in another minute half of what is left, or 8°, according to the general law explained in § 89. The temperatures attained would thus be as follows: in 1 m., 16°; in 2 m., 24°; in 3 m., 28°; in 4 m., 30°; in 5 m., 31°; in 6 m., 31½, etc. At the end of 10 minutes the reading would differ from 32° by only $\frac{1}{32}$ of a degree, a quantity hardly perceptible to the eye on an ordinary thermometer. Now, if the thermometer had been placed in water at 32° instead of in air, the temperature would have reached 16° in a few seconds; and at the end of a minute it would have indicated 32° within a very small fraction of a degree. Again, a mixture of hot lead and cold water may take several minutes before the temperature is practically equalized.

One almost always knows, at least roughly, what the final temperature will be. A useful rule is to observe how long it takes the temperature to reach a point half-way between its original and its final value; then to allow from ten to twenty times as long a time before making a determination of the temperature, according to the degree of accuracy required.

(7) OTHER PRECAUTIONS. — The necessity of shielding a thermometer from radiation has been already alluded to (¶ 15). Delicate thermometers

may be perceptibly affected by mechanical, hydrostatic, or even barometric pressure on the bulb, and by mercurial pressure from within. Such thermometers should be tested both in a vertical and in a horizontal position. Other special precautions will be mentioned as the necessity for them arises.

¶ 66. **Selection of a Mercurial Thermometer.** — For the purpose of calibration, it is best to select a glass thermometer, graduated on its own stem (*bc*, Fig. 51), in degrees at least 1 *mm.* long, from 0° to 100° centigrade, with a few divisions above 100° and below 0°. The bulb (*ab*) should have a volume[1] of nearly 1 *cu. cm.*; and the expansion chamber (*c*) at the top of the thermometer should have about $\frac{1}{10}$ of this

Fig. 51.

capacity. The bulb (*ab*) should for convenience be elongated as in the figure, so as to pass freely through a hole in a cork fitted to the stem of the thermometer. The expansion chamber should be pear-shaped (see *c*, Fig. 51), since otherwise particles of mercury are likely to lodge there. The shape and size of the tube must be such that mercury may be made to flow, with a little jarring, from one end to the other; and the quality of the mercury such that there is no tendency for the column to break up into small fragments.

[1] The volume of a thermometer bulb may be estimated by the quantity of water displaced in a small measuring glass (¶ 85). A small bulb usually implies a stem of small calibre, which may give rise to difficulty in calibration.

¶ 67. **Manipulation of a Thread of Mercury.** — It is frequently required in the calibration of a thermometer to separate from the rest of the mercury in the stem of the thermometer a thread or column of a given length, and to place it in a given part of the stem. When a thread has been broken off, it may be easily moved (by sufficiently inclining or swinging the thermometer) under the influence of its own weight or inertia. For slight motions, jarring is often efficient. The place where the thread breaks off is generally determined by a microscopic bubble of air. To find the location of this bubble, the thermometer is inverted. If a thread of mercury separates at once from the rest, the position of the bubble is evident; if the mercury runs in an unbroken column into the expansion chamber, a small quantity of air will probably be found in the bulb; and if the mercury flows easily back again, there is probably a little air in the expansion chamber.

The (nearly) empty space in the bulb caused by the flow of mercury into the expansion chamber has in any case the appearance of a bubble, which may be made to rise into the neck (*b*, Fig. 51) by suddenly turning the thermometer into an upright position. If it really contains air, it may be worked up into the stem by jarring the thermometer, especially before all the mercury has had time to flow back from the expansion chamber. If the experiment has been successful, a thread of mercury may now be broken off by inverting the thermometer, and tapping it gently on the table.

In the absence of air in the bulb or in the stem, it remains only to make use of air in the expansion chamber. As much mercury as possible is first made to flow into the expansion chamber, and detached from the rest by jarring the thermometer while in a horizontal position. Then the rest of the mercury is returned to the bulb. If there is any air in the expansion chamber, a part of it will now flow into the bulb; and when the globule of mercury is once more returned to the bulb by centrifugal force (see ¶ 64), a thread of mercury can probably be separated.

The presence of a bubble of air[1] in the neck of the bulb (*b*) greatly facilitates the adjustment of the length of the thread of mercury which will break off when the thermometer is inverted. If the bulb is *slowly* heated or cooled by a certain number of degrees, the mercury will usually flow *by* the bubble without dislodging it, thus lengthening or shortening the thread by that same number of degrees. The surest way, however, of shortening a thread of mercury by a few degrees is to hold the thermometer upright and jar it slightly (see ¶ 64), so that the bubble may rise farther and farther into the stem. If at the same time the bulb is gradually cooled, one may be perfectly sure of shortening the thread to any extent. There is no certain method of increasing the length of a thread of mercury, except by transferring it to the expansion chamber, and adding to

[1] Few, if any, thermometers will be found to be entirely free from air.

the globule thus formed more or less mercury from the stem. The globule is then detached and forced backward into the stem, as has been previously described. To prevent it from all returning to the bulb, the latter should be warmed somewhat. The thread will now, probably, be much too long; but may, as we have seen, be shortened at pleasure.

Certain difficulties which are occasionally met in these manipulations may be avoided by the cautious application of heat (¶ 64). It is sometimes impossible to force mercury from the expansion chamber into the stem either through its weight or through its inertia, especially when through accident the expansion chamber has been allowed to become *completely* full. Heat should then be applied to the *top* of the expansion chamber until the mercury is driven out by the pressure of its own vapor. When a thread of mercury can be broken off in no other way, heat may be applied to the stem of the thermometer at the point where a separation is desired. When the mercury refuses to leave the bulb, the flow may be started by slightly warming it; in fact, any desired quantity of mercury may be forced into the expansion chamber in this way (see, however, ¶ 65, (1)).

When the calibration of a thermometer has been finished, as will be explained in the next section, it is well to remove the bubble of air from the mercury. This is done either by cooling the bulb in a freezing mixture (as, for instance, ice and salt) until no mercury remains in the stem; or if this is impossible, by heating the bulb until the air is driven into the expansion chamber. In either case a slight jarring should free the bubble from the mercury. If the bubble is too small to respond to this treatment, it will hardly affect the accuracy of results, unless it actually causes a break in the mercurial column (see ¶ 65, (3)).

¶ 68. **Calibration of a Mercurial Thermometer.** — A thread of mercury, about 50° in length,[1] is placed so as to reach first from 0° upwards, then from 100° downwards. The reading of the end near 50° is taken to a tenth of a degree in both cases, as will be explained below. This enables us to detect any difference in calibre between the upper and lower parts of the thermometer. Next, a thread about 25° long is made to reach first from 0°, then from 50° upwards, then also from 50° and from 100° downwards, with exact readings of the end near 25° or 75°, as the case may be. These will enable us to compare the different quarters of the tube from 0° to 100°. It is not necessary, for most purposes, to carry the process of calibration any further.

To avoid parallax (§ 25) the eye may be held so that the divisions of the scale seem to coincide with their own reflections in the thread of mercury. One end of the thread is always placed so as to coincide exactly with a given division line of the scale (0°, 50°, or 100°), so that any error in the estimation of tenths of degrees will be confined to the reading of the other end. To reduce this error to a minimum,

[1] A thread from 49° to 51° will answer. In cases presenting special difficulty, a greater latitude may be allowed.

the student is advised to study or to construct for himself diagrams like the following (Fig. 52), showing the appearance of a mercurial column when dividing the space between two lines into a given number of tenths, and to identify the reading in each case with the diagram which it most resembles.

Before calculating a table of corrections (see ¶ 70) from the results of calibration, it is necessary to determine two "fixed points" on the scale of the thermometer, as will be explained in the next section.

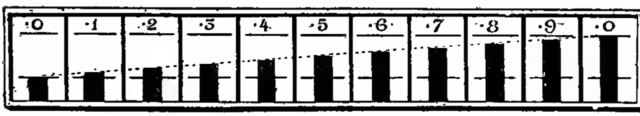


Fig. 52.

¶ 69. **Determination of the Fixed Points of a Thermometer.**[1] — I. The mercurial thermometer is placed in a steam generator (Fig. 53) so that the bulb and nearly the whole of the stem may be surrounded with steam. Only the divisions above 99° project above the cork (*a*) by which the thermometer is held in place. When the greatest accuracy is desired, the sides of the generator are made double, as in Fig. 54. By this means the inner coating, being surrounded on both sides with steam, will have a temperature of 100° nearly, and there will be no radiation of heat between it and the thermometer, since radiation depends upon a difference of temperature (§ 89). It is

[1] The student who is interested in the changes produced in a thermometer by the application of heat will do well to observe the freezing-point before as well as after the boiling-point.

important also to construct a shield of some sort so that the boiling water in the bottom of the apparatus may not be spattered upon the bulb of the thermometer. Such a shield is moreover useful in preventing the thermometer from dipping into the water. It must be borne in mind that the temperature of boiling water is very uncertain, being sometimes

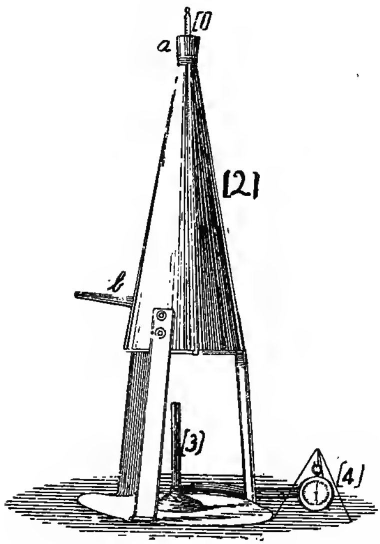


Fig. 53.

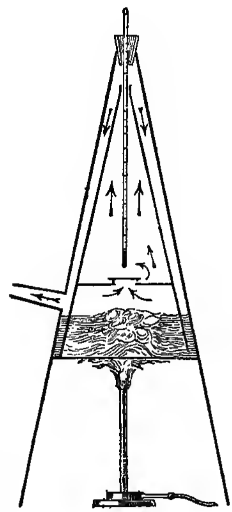


Fig. 54.

several degrees above the true boiling temperature, even when the water is perfectly pure, owing to the adhesion of the liquid to the sides of the vessel containing it. On the other hand, the temperature at which steam condenses depends only upon the pressure to which it is subjected.

It is possible, with an apparatus like that shown in Fig. 53, particularly if the spout (*b*) be small, to generate steam so rapidly that the pressure may be perceptibly greater within the generator than it is outside. Care must be taken to check the supply of heat until the feeblest possible current of steam issues continuously from the spout. The atmospheric pressure is then to be observed by means of a barometer ([4] Fig. 53), and the reading of the thermometer determined within a tenth of a degree (see ¶ 68, Fig. 52). If the barometer happens to stand at 76 *cm.*, this reading is called the "boiling-point" of the thermometer, otherwise a correction must be applied, as will be explained in the next section.

II. The thermometer is now allowed to cool as slowly as possible to the temperature of the room, so as not to destroy its "temper" (¶ 65, (1)), then surrounded in a beaker with a mixture of water and finely-powdered ice (Fig. 55), well stirred and covering the scale within one or two divisions of the zero mark. The melting-point of ice is not perceptibly affected by barometric or ordinary mechanical pressure. The ice must be pure and clean. The bulb of the thermometer must not be jammed by the ice (¶ 65, (7)). The reading is to be accurately observed (¶ 68). This reading is called the "freezing-point" of the thermometer.

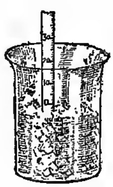


FIG. 55.

The boiling and freezing points are called the two "fixed points" of a thermometer, and from them, with the results of calibration, a complete table of corrections should be calculated, as will be explained in the next section.

¶ 70. **Calculation of a Table of Corrections for a Thermometer.** — The correction of a thermometer at 0° is found at once by reversing the sign of the reading in melting ice (see ¶ 69, II., also ¶ 41). If, for instance, the reading in melting ice is +0°.9, the correction at 0° is — 0°.9. The correction at 100° is found by subtracting (algebraically) the actual reading in steam from the true temperature of steam corresponding to the barometric pressure observed. (See Table 14.) Thus if the thermometer reads 99°.0 when the barometer stands at 72 *cm.*, since the true temperature of steam at this pressure is 98°.5, the thermometer stands too high by 0°.5, and the correction is — 0°.5. It is obvious that under the normal pressure (76 *cm.*) the thermometer would indicate 100°.5 instead of 100°.0; hence the standard boiling-point is 100°.5 on this thermometer. We find the standard boiling-point in general by adding (numerically) to 100°.0 the correction (at 100°) if the thermometer is found to stand too high, or subtracting the same if the thermometer stands too low.

Let us now suppose that in the calibration of the thermometer a given thread of mercury reached from 0° to 49°.5; if the bottom of this thread had been placed at the observed freezing-point (+0°.9) instead of at the mark 0°, it would evidently have reached farther up the tube. Since the length of the thread can hardly vary by a perceptible amount when it is moved less than one degree, even in a tube with considerable variations of calibre, we may assume that the thread would reach a point just nine tenths of a degree higher than before; in other words, it would reach from 0°.9 to 50°.4. In the same way, if the thread is found to reach from 100° to 50°.7, we infer that it would have reached from the standard boiling-point (found by observation to be at 100°.5) to a point five tenths of a degree above 50°.7, or 51°.2. Between 50°.4, and 51°.2 we find a half-way point[1] on the thermometer, namely 50°.8. If the thread of mercury had been four tenths of a degree longer it would have reached to this half-way point, either from the freezing-point or from the boiling-point. We infer that the volume of the tube included between the boiling and freezing points is exactly halved at 50°.8. Now, by definition, the temperature at which the mercury reaches this point is 50°.0, *according to a perfect mercurial thermometer;* hence the correction for the thermometer at 50° is —0°.8.

In the same way we find the correction of the thermometer at 25°, then at 75°, by considering how far the shorter thread (25° long) would have reached if one end had been placed at +0°.9 instead of 0°, at 50°.8 instead of 50°, or at 100°.5 instead of 100°. We thus find two points near 25°, and half-way between them a third point, showing where the thermometer would stand at a temperature of 25°,

[1] This point is sometimes called the "middle point" of a thermometer; but some authorities mean by the "middle point" one *half-way between the divisions numbered* 0° *and* 100° *respectively.*

according to a perfect mercurial thermometer; we find also the indication of the thermometer for a temperature of 75°; and hence also the corrections at 25° and 75°.

The corrections at 5°, 10°, 15°, etc., up to 100° are finally calculated by interpolation. Thus if the correction at 25° is found to be —0°.8, and at 75°, —0°.7, we should find the following table:—

TABLE OF CORRECTIONS.

0°	—0°.9	25°	—0°.8	50°	—0°.8	75°	—0°.7
5°	—0°.9	30°	—0°.8	55°	—0°.8	80°	—0°.7
10°	—0°.9	35°	—0°.8	60°	—0°.8	85°	—0°.6
15°	—0°.8	40°	—0°.8	65°	—0°.7	90°	—0°.6
20°	—0°.8	45°	—0°.8	70°	—0°.7	95°	—0°.5
25°	—0°.8	50°	—0°.8	75°	—0°.7	100°	—0°.5

EXPERIMENT XXVI.

THE AIR THERMOMETER, I.

¶ 71. **Calibration of an Air Thermometer.** — A simple form of air thermometer consists of a glass tube (*ac*, Fig. 56) about 40 *cm.* long, and 2 *mm.* in diameter, closed at one end (*a*). The tube has an

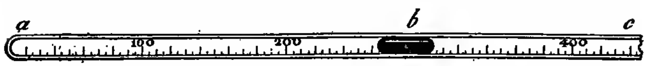


FIG. 56.

engraved millimetre scale, on which an index of mercury (*b*) shows any change in the volume of the enclosed column of air (*ab*). Before closing the end of the tube (*a*), the tube should be thoroughly cleaned and dried.

To test the calibre of the tube, we first weigh it when empty; then we pour in some pure mercury (see ¶ 13) to a depth, let us say, of 5 *cm.*, working it well into the bottom of the tube by means of a fine steel wire. The depth of the mercury is then found as accurately as possible by the millimetre scale, and the tube is re-weighed. Then more mercury is added, a little at a time. After each addition, the depth is recorded, and the corresponding weight is found. This process is continued until the tube is nearly filled with mercury, when the calibration is complete.

Subtracting from each weighing that of the empty tube, we find the amount of mercury contained at each step in the process. Multiplying each weight of mercury in grams by the space in *cu. cm.* occupied by each gram (0.0738 at 20°) we have the capacity of the tube corresponding to the different depths observed. The results are to be entered on co-ordinate paper in the usual method (§ 59). Thus in Fig. 57 the crosses represent volumes from ·1 to ·7 *cu. cm.* corresponding to depths from 0 to 50 *cm.* The curve enables us to find the volume of air enclosed by the index of mercury (*b*, Fig. 56) at any point of the tube. It is easy to show by geometry that unless the crosses all lie in the same straight line, the tube cannot be of uniform calibre.

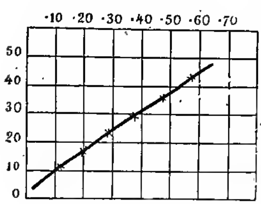


Fig. 57.

¶ 72. **Precautions in the Use of an Air Thermometer.**— To obtain accurate results with an air thermometer, it is necessary that the tube should be perfectly clean; for any foreign matter may interfere with the free motion of the mercury index. If in the process of calibration the tube has become coated with the impurities which mercury sometimes contains, it should be scoured with a small wad of cotton on the end of a fine steel wire. Moisture in the tube must be avoided with the utmost care, on account of the vapor which it generates when heated; and in case the slightest trace of condensation appears, the tube must be heated, and dried by a current of air conducted through a still finer tube to the very

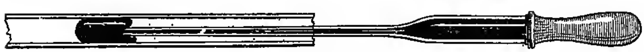


Fig. 58.

bottom of the thermometer. The tube must be large enough to allow a free motion to the mercury index, but not so large that bubbles of air may force their way through the mercury.

The mercury used should be of the purest,—at least twice distilled, and perfectly clean and dry. It may be introduced into the tube by means of a medicine dropper drawn out in a flame so as to have a long fine point (Fig. 58). By piercing the mercury, as in Fig. 59, and inclining the tube, the position of the globule may be varied at pleasure. It will be found convenient to place the index so that the lower end may point to a number on the millimetre scale corresponding to the "absolute temperature" (§ 76). Thus if the temperature of the room is 20°, the lower end may be placed at a distance of 273 + 20, or 293 *mm.* from the bottom of the tube. "Absolute temperatures" are indicated approximately[1] by an air thermometer thus constructed; but as the thermometer is affected by barometric changes as well as by changes in temperature, the indications should always be corrected by the method explained in the next section.

To eliminate the effect of the weight of the index, the experiment should be arranged so that the air thermometer may be observed always in the same position. It is necessary, also, that the whole col-

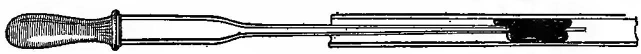


Fig. 59.

umn of air, as far as the index, should be heated or cooled to the temperature which is to be measured. The index must therefore be partly covered in many observations by the heating or cooling apparatus, so that an observation of the upper or outer end will alone be possible. In such cases the length of the index must be allowed for, as what we wish to find is the space occupied, not by the air and the mercury together, but by the air alone. The length of the index must be found by a separate observation in each case, as it is not necessarily the same in different parts of the tube.

[1] Within a few degrees. The air thermometer here described is affected to the extent of about 4° for a rise or fall of 1 *cm.* in the barometer.

¶ 73. **Determination of Temperature with an Air Thermometer.** — The reading (r) of an air thermometer is observed, let us say, in a horizontal position, and compared with that of a mercurial thermometer beside it. The air thermometer is then surrounded in a horizontal trough by melting snow or ice, and the reading (r) of the lower end of the index either directly or indirectly determined (see ¶ 72). Then it is surrounded by steam, in an apparatus similar to that shown in Fig. 46, ¶ 57, and the reading (r_1) is again observed. The air thermometer is finally allowed time to cool to the temperature of the room, and again compared with the mercurial thermometer. We will assume, in the absence of any marked change in the barometer or in the temperature of the room, that the air thermometer returns to its original reading, r; if it does not, the experiment should be repeated.

Referring to the curve found in the calibration of the tube (Fig 57, ¶ 71), we now find the volumes v, v_0, v_1, of the confined air corresponding respectively to the observed readings, r, r_0, r_1, of the lower end of the index. The temperature (t) indicated by the air thermometer is then calculated by the formula

$$t = 100 \, \frac{v - v_0}{v_1 - v_0},$$

which is, however, strictly accurate only when the barometer stands at 76 *cm.* (see ¶ 74, VIII.). It is interesting to compare the reading of a mercurial thermometer with the true temperature as indicated by an air thermometer, even if (as will probably be the case) the accuracy of the observations will not justify a correction of the mercurial thermometer.[1] Instead of air, coal-gas or hydrogen may be employed in a thermometer, or in fact any gas not easily liquefied. The results are essentially the same as with the air thermometer. At the same time that air thermometers have for various reasons (see ¶ 74) been adopted as standards of temperature, it is found, by carefully comparing them with mercurial thermometers, that the difference in their indications at ordinary temperatures is generally small in comparison with errors of observation. On account of their greater convenience and precision, mercurial thermometers are therefore employed in most scientific determinations.

¶ 74. **Theory of the Air Thermometer.** — The air thermometer depends upon the Law of Charles (§ 80), that the volume of a gas under a constant pressure is proportional to its "absolute temperature" (§ 76); that is, to its temperature when reckoned from a certain point, about 273° centigrade below freezing, at which it is supposed that all substances would be completely devoid of heat. If T, T_0, and T_1 represent respectively the *absolute* temperature at which the volumes v, v_0, and v_1 were observed, we have, according to the law stated above,

$$T_1 : T_0 : : v_1 : v_0 \qquad \text{I.}$$

$$T : T_0 : : v : v_0 \qquad \text{II.}$$

[1] To lend interest to this experiment, the student may be provided with a very inaccurate mercurial thermometer.

From I. and II. we find by one of the ordinary rules of proportion,

$$\frac{T_1 - T_0}{T_0} = \frac{v_1 - v_0}{v_0}, \qquad \text{III.}$$

and

$$\frac{T - T_0}{T_0} = \frac{v - v_0}{v_0}. \qquad \text{IV.}$$

Dividing IV. by III. we have

$$\frac{T - T_0}{T_1 - T_0} = \frac{v - v_0}{v_1 - v_0}. \qquad \text{V.}$$

Now the difference between the freezing and boiling temperatures, T_1 and T_0, under the normal barometric pressure (76 *cm.*) is divided on the centigrade scale into 100 parts, called degrees, or

$$T_1 - T_0 = 100°, \qquad \text{VI.}$$

and any ordinary temperature, t, is measured by the excess of the corresponding absolute temperature (T) above the freezing point (T_0); that is,

$$T - T_0 = t. \qquad \text{VII.}$$

Substituting the values of $T_1 - T_0$, and $T - T_0$ in VI. and VII. for their equivalents in V., and multiplying by 100°, we have (at 76 *cm.* pressure),

$$t = 100° \, \frac{v - v_0}{v_1 - v_0}. \qquad \text{VIII.}$$

If the barometer does not stand at 76 *cm.* we substitute for 100° in the equation the actual number of degrees between freezing and boiling (see Table 14).

The student may test the accuracy of his work by calculating the "absolute zero" (z), in this case, the temperature at which the index would reach the bottom of the tube, provided that there were no change in the rate at which the air contracts. Substituting in equation VIII. $v = 0$, we have at 76 *cm.* pressure,

$$z = -100° \frac{v_0}{v_1 - v_0}, \qquad \text{IX.}$$

in which the factor 100° should strictly be corrected as in VIII. for barometric pressure. The meaning of this equation is particularly evident in a special case. If, for example, in a perfectly uniform tube, the index falls from a reading of 373 *mm.* in steam to a reading of 273 *mm.* in ice, — that is, 100 *mm.* for 100°, or 1 *mm.* per degree, — it is clear that to reach the bottom of the tube it must traverse still farther a distance of 273 *mm.*, corresponding to 273° of the same length. The result of this experiment, when accurately performed with any of the so-called "permanent gases" is invariably to indicate a temperature not far from —273° C. for the absolute zero. It is evident that, if the volume of a gas contracts by an amount equal to one 273d part of its volume at the freezing-point for every degree which it is cooled, the volume will be reduced to nothing at the temperature of 273° below zero; and conversely, if z is the absolute zero, that the gas must gain or lose one zth part of its volume at zero degrees when it is heated or cooled 1° centigrade. The *coefficient of expansion* (e) (§ 83) is therefore numerically equal to $1 \div z$; and may be calculated by the formula

$$e = \frac{v_1 - v_0}{100° \times v_0}. \qquad \text{X.}$$

The coefficient of expansion of all permanent gases is in the neighborhood of .00367.

EXPERIMENT XXVII.

THE AIR THERMOMETER, II.

¶ 75. **Construction of an Absolute Air-Pressure Thermometer.** — A form of air thermometer dependent almost entirely upon pressure is represented in Fig. 60. It consists of a U-tube (*abc*), with a large bulb (*c*) blown at the end of the shorter arm, and a somewhat smaller bulb (*a*) at the end of the longer arm. The apparatus is sealed at the atmospheric pressure with enough mercury to fill the smaller bulb more than half-full.

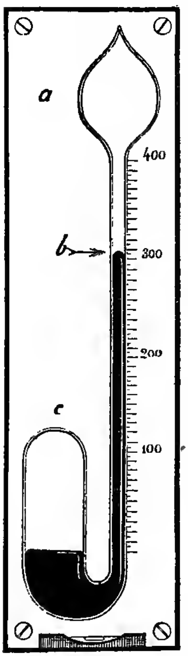


Fig. 60.

It is evident that at the absolute zero of temperature (see § 75), in the absence of any pressure in either bulb, the mercury must stand at the same level in both arms of the U. To locate the absolute zero accordingly, mercury is poured back and forth from one bulb to the other until no difference in the level is observed when the thermometer is returned

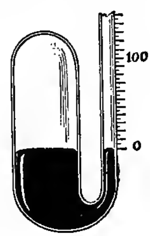


Fig. 61.

to a vertical position. The zero of a millimetre scale is now adjusted to this level (see Fig. 61). By pouring mercury into the bulb *a* (Fig. 60), and suddenly restoring the thermometer to an upright position, the mercury in the tube will be found to stand above its level in the cistern, owing to the compression of air in *c* and its rarefaction in *a*. This process is repeated with more or less mercury in *a* until the column reaches a point *b* on the scale corresponding to the absolute temperature (see ¶ 72). The thermometer should now indicate any temperature correctly on the absolute scale, and has the advantage over that employed in Experiment 26 of being unaffected by atmospheric pressure.

In practice, the bulb *c* is made so much larger than the tube (*b*) that no account need be taken of the variation of the mercury level in *c*. The height of the mercurial column is measured accordingly by a fixed scale. The expansion of the air in the bulb *c* is also disregarded, together with the compression of the air in *a*. All these causes tend to diminish the sensitiveness of the thermometer.

The air thermometer represented in Fig. 60 depends upon the principle (§ 76) that the pressure of a gas which is prevented from expanding increases in proportion to the absolute temperature. When both bulbs (*a* and *c*) contain gas, the pressure in each increases, and hence also the difference in pressure between them increases with the absolute temperature. It follows that the height of the mercurial column which can be maintained by the difference of pressure in question itself varies as the absolute temperature.

¶ 76. **Determination of Temperature by the Pressure of Confined Air.**[1] — A tube (*c*, Fig. 62), already employed in ¶ 71, is to be connected with a mercury manometer (*ab*) constructed as follows: two bottles, *a* and *b*, are each provided with two siphons passing through an air-tight stopper, one to the top, the other to the bottom of the bottle. The long siphons

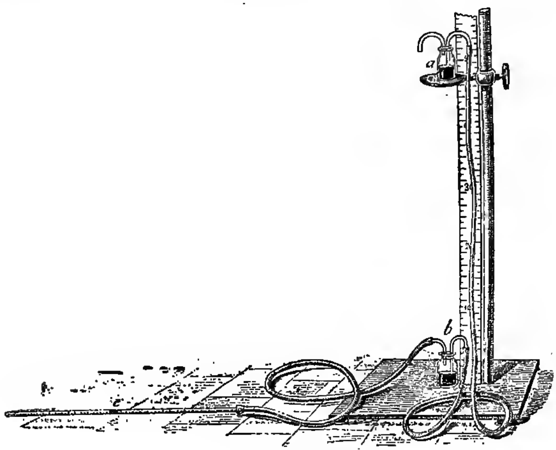


Fig. 62.

and a thick-sided rubber tube connecting them are filled with mercury, and enough more is added to fill both bottles half-full. The mercury stands naturally at the same level in the two bottles; and without disturbing this level, the tube *c* is connected to the short siphon of one of the bottles, *b*, by a thick

[1] An experiment illustrating the increase of pressure produced by temperature will be found in Exercise 25 of the "Elementary Physical Experiments," published by Harvard University.

rubber tube, and the reading of the index determined. All the joints must be carefully wound with string to prevent leakage.

The tube c is now surrounded with melting ice, which may be contained in a horizontal trough (see ¶ 57), leaving only the outer end of the mercury index uncovered. The position of the index is then accurately observed. A reading of the barometer is made. The tube (c) is next surrounded with steam, in a steam jacket (Fig. 46, ¶ 57). The air within c is prevented from expanding by raising the bottle, a, on an adjustable platform to a certain height above b (see Fig. 62). The height of b is to be adjusted so that the mercury index in the tube c may stand at exactly the same point as before. The vertical distance between the mercury levels in a and b is then measured with a metre rod. The tube c is now cooled by filling the jacket with water, the temperature of which is to be found approximately by a mercurial thermometer. The height of the bottle, a, is again adjusted so that the index may return to its original position; and the difference between the two mercury levels is measured as before.

Let h_0 be the height of the barometer, h_1 the height of mercury required to prevent the air from expanding when heated to 100° (nearly), and h the height required to confine it at the (true) temperature, t; if we call the pressures of the air v_0, v_1, and v at the absolute temperatures T_0, T_1, and T, respectively; then by definition (§ 74) we have, as in ¶ 74, I. and II.,

$$T_1 : T_0 :: v_1 : v_0 \text{ and } T : T_0 :: v : v_0;$$

from which we may find, as before, the temperature, t (¶ 74, VIII.), the absolute zero, z (¶ 74, IX.), and a coefficient, e (¶ 74, X.), which determines in this case the proportion in which the *pressure* of confined air increases when heated 1° centigrade. Substituting the values of v_0, v_1, and v, we find

$$t = 100^\circ \frac{h}{h_1} \quad z = -100^\circ \frac{h_0}{h_1} \quad e = \frac{h_1}{100^\circ\, h_0}.$$

It is believed that in the case of a perfect gas the coefficient which determines the increase of pressure per degree should be the same as the coefficient of expansion (Experiment 26). In practice, differences are observed even with the most permanent gases; but these differences are small in comparison with the errors of observation which the student is likely to make.

It is interesting to compare the temperature, t, indicated by an air-pressure thermometer with that indicated by a mercurial thermometer, and to test the accuracy of the work by calculating the temperature (z), at which air would be wholly devoid of pressure, as well as the coefficient e, relating to change of pressure. If the results agree with the values given in ¶ 74, within one or two per cent, the student will be justified in applying a correction to the mercurial thermometer.

EXPERIMENT XXVIII.

PRESSURE OF VAPORS, I.

¶ 77. **Application of the Law of Boyle and Mariotte in the Air Manometer.** — One of the most important applications of the Law of Boyle and Mariotte (§ 79) is in the construction of a pressure-gauge, or manometer. A simple form is represented in Fig. 62. It consists of a U-tube, closed at one end and filled with mercury up to a certain level, corresponding to No. 1 on the gauge. The open end of the U-tube is connected with the interior of a vessel, the pressure in which is to be determined. If the mercury stands as before at No. 1, we know that the vessel must be at the ordinary atmospheric pressure. If, however, the air in the closed arm is compressed to half its original volume, we know that the pressure must amount to 2 atmospheres; if the air is reduced to one-third its original volume, the pressure is 3 atmospheres, etc. If, on the other hand, the air expands, the pressure must be less than 1 atmosphere. The pressure in atmospheres may therefore be indicated directly on a scale properly spaced. No. 2 is, for instance, half-way between the closed end of the tube and No. 1; No. 3 is one-third way; No. 4 one-quarter way, etc. Such a gauge is useful in experiments where it is necessary to know roughly the pressure in a closed vessel, as, for instance, a

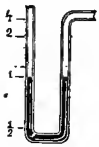


Fig. 63.

steam boiler. When accuracy is desired, it is necessary to increase the length of the tube, to calibrate it (see ¶ 71), and to allow for the hydrostatic pressure of the liquid in the bend.

The tube already calibrated (¶ 71), for the purpose of measuring the expansion of air, may serve as a manometer. The manometer may be surrounded (if necessary) with water, to prevent the temperature from varying perceptibly in the course of the experiment.

¶ 78. **Testing an Air Manometer.** — The tube (c) is to be connected, as in ¶ 76, with the bottle b (Fig. 62), and the reading of the index determined. When the bottle a is raised, by means of an adjustable platform, above the bottle b, the air in b, and hence that in c will be subjected to a pressure which can be determined by measuring the distance between the two mercury levels in a and in b by means of a vertical metre rod (see Fig 62). The reading of the manometer c is again determined. The bottle b is now raised above a, so that the air in b and hence also in c will be rarefied by an amount determined in the same way as before. To find the original pressure in c, an observation of the barometer is made (¶ 13).

Let h be the height of the barometer, h_1 that of the column (ab) producing compression, h_2 that producing rarefaction; and let the corresponding volumes of air enclosed by the index in c be respectively (see ¶ 71, Fig. 57) v, v_1, v_2, at the pressures p, p_1, p_2; then evidently $p = h$; $p_1 = h + h_1$; $p_2 = h - h_2$.

Now, according to the law of Boyle and Mariotte (§ 79),

$$vp = v_1 p_1 = v_2 p_2;$$

hence we should find

$$v \times h = v_1 \times (h + h_1) = v_2 \times (h - h_2).$$

If these products differ by an amount greater than can be attributed to errors of observation, the determinations upon which they depend should be repeated before making use of the manometer.[1]

¶ 79. **Determination of the Pressure of a Vapor by an Air Manometer.** — The air manometer which has just been tested, is first read at the atmospheric pressure, then connected with a thick rubber tube to

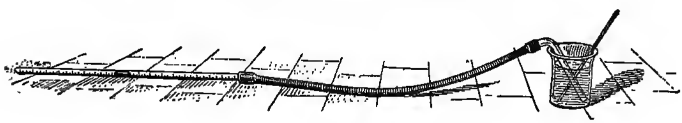


Fig. 64.

a stout tube of glass, closed at one end, and containing ether, already boiling (Fig. 64). The boiling may be effected with safety[2] by hot water, between 50° and 60°. The manometer should be horizontal, but raised somewhat, so that the ether condensing in the rubber tube may run back into the boiler. As soon as the ebullition is checked by the pressure of the vapor generated, an observation of the manometer is made; and at the same time, as nearly as pos-

[1] In testing an air manometer from ½ to 2 atmospheres, the errors due to departure from the Law of Boyle and Mariotte will not amount to one fourth of one per cent.

[2] On account of the danger of fire, all flame should be removed from the immediate neighborhood.

sible, the temperature of the water is accurately recorded. When the water has cooled 5°, 10°, etc., new observations of the manometer are made. If the ether ceases to boil, the rubber tube should be cooled, or air let out of it. It is well to put fresh ether in the boiler from time to time. The results are accurate only so long as boiling continues.

The pressure, p_1, corresponding to any reading of the manometer at which the volume, v_1, of air is enclosed, may be calculated from the volume, v, at the atmospheric pressure, p, by the formula expressing the Law of Boyle and Mariotte (§ 79),

$$p_1 = \frac{v}{v_1} p.$$

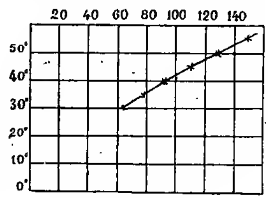


FIG. 65.

The results are to be plotted on co-ordinate paper, as explained in § 59, and a curve drawn, as in Fig. 65, to illustrate the pressure of the vapor at various temperatures.

EXPERIMENT XXIX.

PRESSURE OF VAPORS, II.

¶ 80. **Dalton's Law.** — We have seen in the last Experiment that the vapor of a liquid may exert a pressure either greater or less than that of the atmosphere, according to the temperature at which the liquid is maintained. The pressure of a volatile liquid is measurable even at the ordinary temperature of the room. To prove this, one has only to inject a few drops of ether with a medicine-dropper, properly bent (see Fig. 66), into the tube of a barometer constructed as in ¶ 13. The ether will form bubbles of vapor even before it rises to the top of the mercurial column; and the pressure of this vapor will cause the barometer to fall some thirty or forty centimetres. By measuring the fall thus produced, the pressure of the vapor of various liquids at different temperatures may be determined.

Fig. 66.

Another way to illustrate the pressure exerted by the vapor of a liquid is to pour a little of the liquid into a flask, so that it may evaporate into the air which the flask contains. If the flask is corked tightly as soon as the liquid is poured in, a considerable pressure may be generated. In fact, explosions sometimes occur from this cause. To measure the pressure, a tube may be passed through the cork into some mercury in the bottom of the flask (see Fig. 67), and the liquid should be injected by means of a medicine-dropper passing through the cork beside this tube, so as to avoid losing the pressure generated by evaporation before the cork can be put into its place.

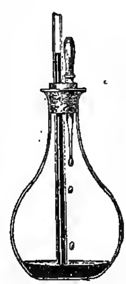


Fig. 67.

It has been found by experiment that the quantity of liquid which evaporates in a flask already containing air, and the pressure which it generates, are exactly the same as in a space from which the air has been completely exhausted. This discovery (known as Dalton's Law) is believed to show that the molecules of a gas occupy very little space in comparison with the space between them, into which a liquid may evaporate. In any case, the height to which the mercury column is raised in Fig. 67 is the same as its depression in Fig. 66, other things being equal. We shall make use of this fact to determine roughly the pressure of a vapor at various temperatures.

We have seen that when a liquid evaporates into a confined space filled with air, the pressure of the air is increased. It is evident that in an open flask the air must expand until the combined pressure of the air and the vapor inside becomes equal to the atmospheric pressure outside. If therefore we know the pressure of the air within the flask, and that of the air outside of it, the difference must be equal to the pressure of the vapor in question. To find the pressure of the air within the flask, it is necessary first to absorb or to condense the vapor which it contains.

¶ 81. **Determination of the Pressure of a Vapor in the Presence of Air.** — To find the pressure of aqueous vapor in an open flask, a small quantity of water is heated in it by submerging the flask up to the neck in a jar of hot water. The temperature of the water within the flask is now determined by means of a thermometer, and a rubber cork is tightly inserted. When the flask has become sufficiently cool it is weighed, then inverted, opened under ice-water, corked, dried, and reweighed with the water which enters it. Finally, it is filled with water and weighed again. A reading of the barometer is made.

Let w_1, w_2, and w_3 be the first, second, and third weights in grams, t the temperature, and h the barometric pressure in *cm.* within the flask; then the capacity (c) of the flask in *cu. cm.* for air or vapor is

$$c = w_3 - w_1 \text{ nearly};$$

and since the volume of air at 0° is nearly $w_3 - w_2$ *cu. cm.*, its volume (v) at $t°$ is (see § 80)

$$v = \frac{(w_3 - w_2) \times 273 + t}{273}.$$

The pressure of this air at $t°$ is $v \div c$ atmospheres (§ 79), or $hv \div c$ *cm.* Hence the pressure (p) of the vapor [1] must be

$$p = h - \frac{hv}{c}.$$

¶ 82. **Evaporation and Boiling.** — The student will notice the regular increase of the quantity of aqueous vapor in the air as the temperature is increased, until finally, as the water approaches its boiling-point, scarcely any air remains in the flask. It is interesting to push the experiment still further, and to expel all the air by actually boiling the water. Boiling may be distinguished from evaporation by the presence of bubbles of pure steam. Unlike the bubbles of air set free from the water by the application of heat, the bubbles of steam may at first completely condense with a crackling sound before reaching the surface of the liquid. When, however, the whole liquid is raised to the boiling-point, the bubbles expand as they escape from the liquid, and if the supply of heat

[1] We neglect in this formula the pressure 4.6 *mm.* of aqueous vapor at 0°.

is sufficient, furnish a steady current of steam which issues from the neck of the flask. The stopper is inserted before boiling has ceased, but, to avoid explosion, not until the source of heat has been removed. When the vapor is condensed by pouring cold water on the bottom of the flask (Fig. 68), ebullition will take place even after the water within the flask is no longer warm to the touch. If the experiment has been successful, a peculiar metallic sound will be heard on shaking the water in the flask. This sound is called the water-hammer, and indicates an almost total absence of air. If the flask is opened under water, it should be completely filled. If opened in air, the space not already occupied by water will be filled with air. The student may be interested to make a rough determination of atmospheric density by weighing the flask before and after the admission of air (see ¶ 44). The capacity of the flask for air is found from the quantity of water which must be added to that already present in order to fill the flask (see ¶ 45). The principal objection to a determination of density by this method lies in the fact that an unknown quantity of aqueous vapor may be taken up by the air which enters the flask. Its advantage consists in the nearly perfect vacuum which is produced by the condensation of aqueous vapor. For further illustrations of evaporation and boiling, see Exercise 22 of the "Elementary Physical Experiments," published by Harvard University.

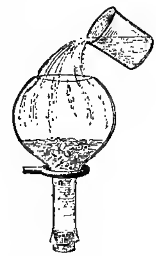


FIG. 68.

EXPERIMENT XXX.

BOILING AND MELTING POINTS.

¶ 83. **Determination of Boiling and Melting Points.** — The heater already used to determine the boiling-point of water on a mercurial thermometer may also be employed to find the boiling-points of other liquids. The chief objection to this apparatus is the change of composition which results from boiling away an impure liquid, owing to the fact that the more volatile ingredients are the first to escape. It becomes necessary to condense the vapor before it escapes from the spout, and to make the liquid thus formed return to the boiler. There are, moreover, two practical objections to the use of such an apparatus, — the difficulty of obtaining a sufficient quantity of liquid to fill the boiler, and the danger of fire.

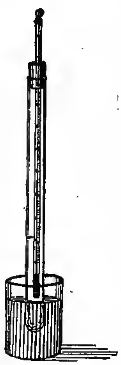


Fig. 69.

These objections are met by boiling the liquid in a long test-tube, as in Fig. 69. The vapor condenses on the sides but does not escape, and the danger of fire is avoided by the use of hot water instead of a flame as a source of heat.

Alcohol, for instance, will boil freely if the test-tube is plunged in water at or near the temperature of 100°, since the boiling-point of alcohol is between 78° and 80°. As the water cools it may be used successively to find the boiling-points of chloroform

(58°–61°), bisulphide of carbon (47°–48°), and ether (35°–37°). It is well to have the water about 20° warmer than the boiling-point of the liquid which is to be determined.

The same apparatus, or one with a shorter tube, may be used to determine melting-points. A piece of a paraffine candle may be melted in the test-tube by hot water; then, as it begins to harden, the temperature is observed. Again, by the use of hot water, the paraffine is gradually heated, and the temperature noted at which it begins to melt. Owing to impurity of the paraffine, certain constituents usually congeal more easily than others. It has, therefore, no definite melting point. A certain variety of commercial paraffine melts, for instance, between 53° and 57°. The results are to be corrected as explained below.

¶ 84. **Precautions and Corrections in Determining Boiling and Melting Points.** — To prevent radiation to or from the bulb of the thermometer, and to avoid all danger of spattering (see ¶ 69, I.), a shield may be constructed out of thin sheet brass, small enough to fit into the test-tube. The bulb must not dip into the liquid, but must be surrounded with vapor. The level of the vapor will be distinctly visible through the sides of the tube. It should reach a point a little beyond the end of the mercurial column in the stem of the thermometer, but must in no case reach the open end of the test-tube. A slight escape of the vapor, due to evaporation, cannot be avoided; but a continuous current must be instantly arrested by removing the source of heat.

In finding melting-points, the bulb and stem of the thermometer should be surrounded with liquid up to a point just below the end of the mercurial column. If the stem be dipped any farther into the liquid, it may become impossible to read the thermometer.

The student is advised not to attempt the determination of boiling-points above 100° C.,[1] on account of the danger of accidents. It may, however, be instructive to explain how a temperature above 100° can be determined with a thermometer reading only to 100°. A thread of mercury not over 100° in length is first broken off and stored in the expansion chamber (*c*, Fig. 51, ¶ 66). The thermometer is then tested in steam (¶ 69, I.). Its reading will be somewhat above 0°; let us say 15°. Then all the readings of this thermometer will be about 85° too low. It is possible, therefore, to determine temperatures up to 185°.

We should, however, remember that a column measuring 85° at a temperature of 100° will measure more or less than that amount, according to the temperature in question. Let the length of the thread of mercury, in degrees, be l, and let the temperature at which this thread is actually observed be t (100° in the instance above); then if t_1 is the temperature to be determined, the correction in degrees is $.00018\,l\ (t - t_1)$. This follows from the value of the coefficient of expansion of mercury; for if a thread

[1] Chloroform should be substituted for turpentine (which boils at about 160°) in the second Experiment in Physical Measurement in the list published by Harvard University.

1° long when heated 1° centigrade expands by the amount 0°.00018, then a thread l° long when heated $(t - t_1)$° would expand $l \times (t - t_1)$ times as much.

Thus the correction in determining the boiling-point of turpentine (160°) with a thread 85° long, broken off and measured at the temperature 100° instead of 160°, would be $.00018 \times 85 \times (160 - 100)$, or a little over 0°.9. Instead, therefore, of adding 85° to the reading of the thermometer (let us say 74°) we should add, strictly, 85°.9, — that is, the actual length of the thread of mercury at the temperature observed. Instances have already been given (¶ 65, (4)) of errors resulting from heating only the bulb of a thermometer to a given temperature. The corrections in such cases are calculated by the rule given above. That is, we multiply the length of the thread exposed to the air by the difference in temperature between the air and the bulb of the thermometer, to find the correction which should be applied.

In all determinations of temperature, the readings of the thermometer are made to tenths of a degree (¶ 68), and corrected by the table already calculated (¶ 70). The boiling-points of all liquids are affected more or less by atmospheric pressure. A reading of the barometer should always accompany such determinations.

EXPERIMENT XXXI.

METHOD OF COOLING.

¶ 85. **Determination of Rates of Cooling.** — A calorimeter (Fig. 70) is usually constructed of two (or more) metallic cups, one inside of the other. A vertical section of the calorimeter is shown in Fig. 71, and a horizontal section in Fig. 72. The inner cup, generally made of thin brass, has its outer surface brightly polished to lessen radiation; and for the same reason the outer cup should be polished inside. To prevent the conduction of heat from one

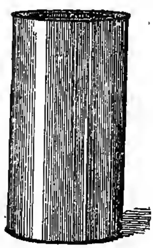


Fig. 70.

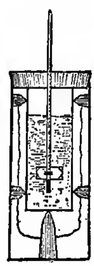


Fig. 71.

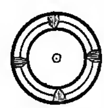


Fig. 72.

cup to the other, the cups are separated by pieces of cork, which should be sharpened to a point, and held in place by wires. A large flat cork serves to cover both cups, and thus in a great measure to prevent loss of heat; for if the top of the calorimeter were open, a considerable quantity of heat would be carried away by currents of air. In some cases a small stopper is also used, to close the inner cup water-tight.

We prefer for most purposes a calorimeter depending (like that shown above) upon air spaces for its insulation, to one in which these spaces are filled with wool, or other non-conducting material;[1] for though air transmits more heat than wool, it absorbs much less. The heat absorbed by insulating materials is a continual source of error in calorimetry, because there is no simple way of allowing for it. On the other hand, the heat transmitted through the sides of a calorimeter can, as we shall see, be easily determined.

(1) The inner cup is to be filled with hot water, between 90° and 100°, and the temperature of the water is to be found by a thermometer passing through a hole in the cork cover (Fig. 71). The stirrer attached to the stem of the thermometer is used to keep the water in continual agitation; and a stopper is employed to prevent any of it from being spilled over the edges of the cup. Observations of temperature are made at intervals of one minute,[2] and should be continued until the thermometer indicates 30 or 40 degrees. The temperature of the room is then observed; and the quantity of water which has

[1] When no allowance is to be made for loss of heat by the calorimeter, the use of felt is to be recommended. See Experiment 10 in the Descriptive List of Chemical Experiments published by Harvard University.

[2] A clock especially constructed to strike a bell once a minute will be found serviceable in the determination of rates of cooling. Simultaneous observations of time and temperature may thus be made (see § 28).

been used is determined by weighing the calorimeter with and without it.

(2) The experiment is now to be repeated with a much smaller quantity of water, just enough, let us say, to cover the bulb of the thermometer and the stirrer. The calorimeter is to be inclined in every possible direction between the observations of temperature, so as to bring the hot water in contact with every part of the inner cup.

(3) The experiment is again repeated with the same quantity of water as in (2), but without inclining the calorimeter. The stirrer is to be used as in (1), but simply to secure a uniform temperature in the water.

(4) Finally, the calorimeter is to be filled with glycerine or turpentine, warmed by hot water (see ¶ 83). The depth of the liquid, and the method of agitation should be the same as in (1). The temperatures and weights are to be observed as before.

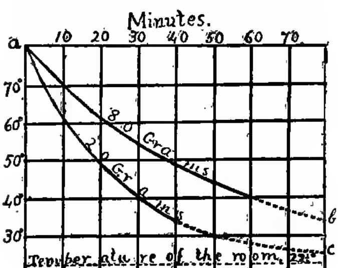


Fig. 73.

The results of (1), (2), (3), and (4) are to be reduced as will be explained in ¶¶ 86–89.

¶ 86. **Effect of the Temperature and Thermal Capacity of a body on its Rate of Cooling.** — (1) The results of ¶ 85 (1) are to be represented by a curve (*ab*, Fig. 73), drawn on co-ordinate paper as in § 59. The scale at the top of the paper corresponds to the number of minutes which have elapsed since the first observation was taken; the scale at the left of the paper represents the observed temperature of the water in degrees. The temperature of the room ($22\frac{1}{2}°$) is shown by the dotted line, which the curve (ab) should approach as a limit, — that is, without ever reaching it.

It is advantageous for many purposes that the scale of degrees at the left of the paper should represent, not the temperatures actually observed, but the differences between those temperatures and that of the room;[1] since the rate of cooling depends upon the differences in question (see § 89). If this method is adopted, the first observation should be one about 50° above the temperature of the room.

In any case the student should satisfy himself that Newton's Law of Cooling (§ 89) is approximately fulfilled.[2] Thus the calorimeter may cool (see ab, Fig. 73) between the 5th and the 10th minute from 75° to 70°, that is, 5° in 5 minutes; while between the 50th and the 60th minute it may cool only from 44° to 40°, or 4° in 10 minutes. In the first case, when the average temperature ($72\frac{1}{2}°$) is 50° above that of the room ($22\frac{1}{2}°$) we have a rate of cooling equal to 1° per minute; in the second case, with an average temperature (42°) nearly 20° above that

[1] This method of plotting the curves must be adopted if the temperature of the room varies considerably in the course of the experiment (¶ 85).

[2] Departures of 20% have been observed in a range of 60°. See Everett's Units and Physical Constants, Art. 143.

of the room, the rate of cooling is $\frac{2}{5}°$ per minute. Obviously,

$$50 : 20 :: 1 : \tfrac{2}{5}.$$

In the same way, with 20 grams of water in the calorimeter, the rate of cooling should be found to vary in proportion to the excess of temperature above that of the room. The rate of cooling is, however, very different in different cases, as it depends upon the quantity of water which the calorimeter contains. Let us next consider the relation between this quantity of water and the rate of cooling.

(2) The fundamental principle underlying all determinations by the method of cooling is that the *number of units of heat* (§ 16) lost by a calorimeter per unit of time is proportional to the *difference in temperature* between the inner and outer cups. It does not, therefore, depend upon the contents of the calorimeter except in so far as the nature or quantity of these contents may modify the temperature of the inner cup.

Let us first suppose that in both experiments, ¶ 85 (1) and (2), the water is agitated sufficiently to bring it in contact with every portion of the inner cup, so that a perfectly uniform temperature is the result; then if the outer cup is unchanged in temperature the *flow of heat* from one cup to the other corresponding to a given reading of the thermometer must be in both cases the same. How, then, do we account for the marked differences which we observe in the *rates of cooling?*

The supply of heat in a calorimeter may be compared to the quantity of water in a leaky pail. Given the rate of the stream flowing out of the pail, the time it takes for the water-level to fall one inch is evidently proportional to the horizontal section of the pail. In the same way, with a given flow of heat from a calorimeter, the time required for the temperature to fall 1° must be proportional to what we call the *thermal capacity* (§ 85) of the calorimeter and its contents.

It is obvious from Figure 73 that with 80 grams of water the cup must cool more slowly than with 20 grams. In the first case it takes, for instance (see *ab*, Fig. 73), 60 minutes to cool from 80° to 40°; if in the second case only 20 minutes are required to cover the same range of temperature, the natural inference is that the thermal capacity in the first case is to that in the second case as 60 is to 20, or as 3 is to 1.

The thermal capacity in question is in no case simply proportional to the quantity of water which the calorimeter contains; for the inner cup, the thermometer, and the stirrer all possess a certain capacity for heat. We may estimate this capacity roughly by the method of cooling. Let us call it c. Then in the first case the total thermal capacity is $80 + c$; and in the second case it is $20 + c$; hence we have

$$80 + c : 20 + c :: 3 : 1,$$

a proportion which can be satisfied only if $c = 10$. We infer, therefore, that the calorimeter, thermometer, and stirrer are together equivalent, in thermal capacity, to about 10 grams of water.

We may assume provisionally that this inference is correct; but for accurate calculations, we prefer a determination of thermal capacity made as will be described in Experiment 32.

¶ 87. **Calculations concerning Loss of Heat by Cooling.** — We have found in the last section (¶ 86, 1), that when a certain calorimeter contains 80 grams of water at an average temperature 50° above that of the room, the rate of cooling is 1° per minute. We have also found (¶ 86, 2) that the calorimeter itself is equivalent in thermal capacity to about 10 grams of water; hence the total thermal capacity is $80 + 10$, or 90 units. The heat lost under these conditions is therefore 90×1, or 90 units per minute. Let us now suppose that the average temperature is only 1° above that of the room, instead of 50°; then by Newton's Law (§ 89) the rate of cooling will be $\frac{1}{50}$ of 1° per minute; hence the loss of heat will be $90 \times \frac{1}{50}$, or 1·8 units per minute.

It follows from the fundamental principle of the method of cooling (¶ 86, 2) that the loss of heat at a given temperature is the same, no matter what substance or substances the calorimeter may contain, provided that every part of the inner cup is brought in contact with the mixture. The rate of flow corresponding to difference in temperature of one degree between the inner and outer cups is accordingly an important factor in calculations (see ¶ 93, 3) relating to loss of heat by cooling.

Unless the calorimeter is filled, as in ¶ 85 (1), or its contents sufficiently agitated, as in (2), the inner cup will not be uniformly heated throughout. When a glass vessel is used (as in Exp. 38), only those portions nearest the liquid may be perceptibly warmed or cooled by it; and even with metallic vessels, especially when thin, differences of temperature can frequently be recognized by the touch. The result is a considerable diminution in the rate of cooling. To estimate the effect in question, we may utilize the results of ¶ 85 (3).

From these results the curve *ac* (Fig. 73) is to be plotted in the same manner as *ab* (¶ 86, 1). If in both curves (as in Fig. 73) the first observation utilized is about 80°, we shall find a point of intersection, *a*, nearly opposite 80° and 0 minutes. We may notice that *ab* takes 60 minutes to fall from 80° to 40°, while with *ac* only 30 minutes are required; hence the rate of cooling represented by *ac* is twice as great as in the case of *ab*, so that when reduced to 1° difference in temperature, it will be $\frac{2}{50}$ of 1° per minute. Now let the weight of water be 20 grams; then since the calorimeter is equivalent to 10 grams,[1] we have a total thermal capacity of 30 units. The loss of heat is therefore, not 1·8, as before, but $30 \times \frac{2}{50}$ or 1·2 units per minute.

These figures are sufficient to show the importance, in the method of cooling, of comparing two quantities

[1] We should remember, strictly, that if only a portion of the inner cup is heated, the thermal capacity will be somewhat less than 10 units.

under exactly the same conditions. Let us suppose that we were to calculate the thermal capacity of the calorimeter from the results of ¶ 85 (1) and (3), in which the conditions are not the same. Since the rate of cooling is twice as great in (3) as in (1), we might infer that the thermal capacity of the calorimeter with 80 grams was twice that with 20 grams. This would make the thermal capacity of the calorimeter alone 40 units instead of 10 (see ¶ 86, 2).

¶ 88. **Construction of a Series of Temperature Curves.** — From an extended series of results[1] it would be possible to construct a series of curves similar to those shown in Fig. 74. It is not, however, necessary that each of these curves should be the result of observation. From two of them, the rest may be obtained with more or less accuracy by different processes of interpolation.

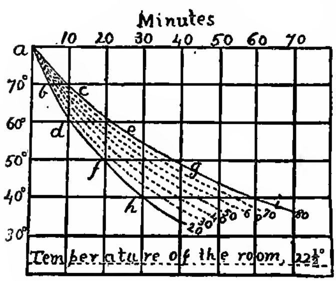


Fig. 74.

Let *acegi* and *abdfh* be the two curves already obtained (see Fig. 74), corresponding respectively to 80 grams and to 20 grams of water, and let it be required to draw a curve corresponding to 50 grams of water. Then since 50 is midway between 80 and

[1] The teacher may, for the sake of illustration, have a series of curves constructed from the results of a large class of students using different quantities of water.

20, the curve in question may be placed (roughly) midway between the other two; and in the same way other curves may be drawn so as to divide the distance equally into still smaller parts. This method of interpolation is, however, obviously inaccurate, and especially so between such wide limits.

A more accurate method depends upon the principle (see ¶ 86, 2) that the time of cooling is (other things being equal) proportional to the thermal capacity of the calorimeter and its contents. Since 80 grams require, for instance, 10 minutes to cool from 80° to 70°, and 20 grams take only five minutes (see Fig. 74), we may infer that 50 grams would require $7\frac{1}{2}$ minutes; or in other words, that the distance *bc* would be bisected by the 50-gram curve. In the same way the other horizontal distances, *de*, *fg*, *hi*, etc., would be bisected. To obtain the intermediate curves, accordingly, the *horizontal* distances, *bc*, *de*, *fg*, etc., are each to be divided into a given number of equal parts. The curves may then be drawn through the points of division.

It is easy to show that this method of interpolation, though more accurate than the first, may still lead to considerable errors, when we consider differences in the flow of heat from the calorimeter. With 80 grams of water, 1° above the temperature of the room, we have calculated that the loss of heat amounts to 1·8 units per minute (see ¶ 87); with 20 grams we have found similarly 1·2 units per minute. Let us assume that with 50 grams the loss is midway between these two numbers, or 1·5 units per minute. Then since the total thermal capacity is 60 units, the temperature must fall at the rate of $1 \cdot 5 \div 60$ or $\frac{1}{40}$ of 1° per minute. The time required to fall 1° at this rate would be 40 minutes; in the case of 80 grams it would be 50 minutes (see ¶ 87); in the case of 20 grams it would be 25 minutes. The times required for 80, 50, and 20 grams to fall through a given range of temperature would be, accordingly, proportional to the numbers 50, 40, and 25, respectively. Since 40 is by no means midway between 50 and 25, the 50-gram curve must be considered as only approximately bisecting the horizontal distance between the other two.

It is evident that if the system of curves shown in Fig. 74 were to be relied upon for exact calculations, it would be necessary to confirm the position of the 50-gram curve, at least, by direct observations. As a matter of fact we shall refer to Fig. 74 only for the purpose of making small corrections for cooling; so that we may disregard any errors in these curves which are likely to arise from an interpolation depending upon a division of horizontal distances into equal parts.

¶ 89. **Calculation of Specific Heat by the Method of Cooling.** — I. A set of curves is to be constructed essentially as in ¶ 88, using, however, in connection with the curve *acegi* (Fig. 74) representing the results of ¶ 85 (1), a curve *abdfh*, derived from the results of ¶ 85 (2), and not (as in Fig. 74) from the results of ¶ 85 (3). The intermediate curves will then represent rates of cooling corresponding to different quantities of water when brought in contact with every part of the inner cup. The results of ¶ 85 (4) are next to be plotted *on tracing-paper*, with a horizontal line (as in Fig. 73 to) represent the temperature of the room. This line is then superposed (by moving the tracing-paper) over a similar line in the new series of curves; and at the same time the curve on the paper is made to pass through the common point of intersection of the series in question (see *a*, Fig. 74).

A curve thus obtained with, let us say, 75 grams of turpentine, may be made to coincide, not with the 70-gram curve, nor with the 80-gram curve (see Fig. 74), but with one rather which would correspond to 30 or 40 grams of water. Under the conditions of the experiment, the heat lost by the calorimeter must be the same whether it contain turpentine or water (see ¶ 86, 2); hence equal rates of cooling imply equal thermal capacities (*ibid.*). Since the calorimeter has the same total thermal capacity with the turpentine as with the water, the 75 grams of turpentine must be equivalent to 30 or 40 grams of water; and 1 gram of turpentine must be equivalent to a quantity of water between $\frac{30}{75}$ and $\frac{40}{75}$ of a gram; or let us say 0.4 + grams. In other words, the specific heat (§ 16) of turpentine must be 0.4+. In the same way the specific heat of any other liquid might be calculated.

It is evident that the curves of ¶ 88, if thus treated, would not have given an accurate result. 20 grams of water might be found, for instance, under the conditions of ¶ 85 (3), to cool as slowly as the 75 grams of turpentine in ¶ 85 (4); but this would be due, not simply to the fact that water has a greater thermal capacity than turpentine, weight for weight, but also to the fact that a much smaller amount of surface is heated by the water. Obviously the 20 grams of water cannot be equivalent in thermal capacity to the 75 grams of turpentine, because their rates of cooling, though equal, have been compared under dissimilar conditions.

II. Another method of calculating specific heat depends upon a comparison of the rates of cooling of two liquids when *equal volumes* are employed. Let us suppose that the time occupied by 75 grams of turpentine in cooling from 80° to 60° in ¶ 85 (4) is really the same as that of 20 grams of water in ¶ 85 (3),— that is, 10 minutes (see *ac*, Fig. 73),— while that required in ¶ 85 (1) for 80 grams of water (see *ab*, Fig. 73) is 20 minutes; then since the conditions are nearly the same in (1) and (4), the total thermal capacities in question must be to each other as 10 is to 20 (¶ 86, 2). If the calorimeter is equivalent (see ¶ 86, 2) to 10 grams of water, we have with 80 grams of water a total thermal capacity of 90 units; hence with the turpentine the total thermal capacity must be $\frac{10}{20}$ of 90 units, or 45 units. Subtracting from the 45 units the 10 units due to the calorimeter, we find a remainder of 35 units, which must be the thermal capacity of 75 grams of turpentine. Hence the specific heat of turpentine is $35 \div 75$, or $0.4+$.

The method of cooling has been applied to the determination of the specific heats of solids in the form of powder, as well as to liquids; but it is generally thought to be less reliable than the methods of mixture about to be described (Exps. 33 and 34).

EXPERIMENT XXXII.

THERMAL CAPACITY.

¶ 90. **Determination of the Thermal Capacity of a Calorimeter.** — (1) We have already seen that the thermal capacity of a calorimeter may be calculated roughly from data obtained by the method of cooling (see ¶ 86, 2); but that a very slight change in the conditions of the experiment may make the result worthless. For this reason the method of cooling is hardly to be counted as a practical method for finding the thermal capacity of a calorimeter. The experimental determination of thermal capacity may be made by either of the following methods: —

I. The whole calorimeter is to be weighed, including (see ¶ 85, Fig. 71) the inner and outer cups, the cork supports and cover, and the thermometer and stirrer. The temperature of the inner cup is now found by observing the thermometer, after it has remained within this cup for some time (see ¶ 65, 6). Then water at an observed temperature, between 30° and 40°, is poured rapidly (¶ 92, 4) into the cup until it is nearly full (¶ 92, 8). The cork is immediately inserted (¶ 92, 6) and the time noted (¶ 92, 9). The water is then stirred (Fig. 50, ¶ 65) by twisting the stem of the thermometer, until two successive observations of the thermometer a minute apart (see ¶ 92, 10) agree as closely as in ¶ 85 (1), at the same temperature (see ¶ 92, 8). The resulting temperature is then observed, and the time again noted (¶ 92, 9). The whole apparatus is then re-weighed to find how much water is in the calorimeter (see also ¶ 92, 5).

There are two practical objections to the method just described: first, that the change in temperature of the water is almost too small to be measured accurately with an ordinary thermometer; and second, that the quantity of heat absorbed by the calorimeter may be small in comparison with that lost by cooling (¶ 93), which can only be roughly allowed for.

The change of temperature of the water may be increased by using a smaller quantity of it; but this is objectionable, as will be seen by comparing the results of ¶ 85, (2) and (3), unless the water can be *well shaken* in the calorimeter, or unless the object of the experiment be a determination of thermal capacity of the calorimeter when *partly full*. A thermometer graduated to tenths of degrees will be found useful in this and other experiments where it is necessary to measure small changes of temperature.

II. Another method of finding the thermal capacity of a calorimeter consists in heating the inner cup instead of the water. This may be done by filling the cup with hot lead (or better, copper) shot, the temperature of which is to be determined by two or three observations of a thermometer at intervals of a minute (see ¶ 92, 10). The shot must be well shaken between these observations, to secure a uniformity of temperature (see ¶ 92, 8); it is then poured out, and immediately replaced by water at an observed temperature near that of the room. The resulting temperature is then determined, and the weight of water used is found as before.

The change in temperature of the water may be made practically five or ten times as great in II. as in I., and the correction for its cooling will be comparatively slight. The principal source of error in this experiment is the rapid cooling of the inner cup while empty (see ¶ 92, 4).

(2) The results of an experimental determination of thermal capacity should in all cases be confirmed by a calculation based upon observations of the weights and specific heats of the substances employed in the construction of the calorimeter. The inner cup is to be weighed, also the stirrer (Fig. 50, ¶ 65); and the amount of water displaced by the thermometer is to be found by the aid of a small measuring-glass (Fig. 75). The glass should be filled with water so that the thermometer may be immersed to the same depth as when it is used to determine the temperature of liquids in the calorimeter. The level of the water is then carefully observed with and without the thermometer. It will be assumed that the thermometer is constructed of glass and mercury;

FIG. 75.

the calorimeter and stirrer of brass; otherwise the materials in question must be noted. From these data the thermal capacity of the calorimeter may be calculated (see ¶ 91, III.).

¶ 91. **Calculation of Thermal Capacity.** — We have already considered a method by which thermal capacity may be roughly computed through a comparison of rates of cooling (¶ 86, 2). This section relates to the calculation of thermal capacity from the observations made in ¶ 90.

If, as in the first method (¶ 90, I.), t_1 is the original temperature within the calorimeter, w the weight of water used, t_2 its temperature just before it is poured into the calorimeter, and t the resulting temperature, then, since w grams of water cool $(t_2 - t)$ degrees by coming in contact with the calorimeter, they must give up to it $w \times (t_2 - t)$ gram-degrees, or units of heat (§ 16). This raises the temperature of the calorimeter $(t - t_1)$ degrees; hence to raise it 1° would require a quantity of heat, c, given by the formula

$$c = \frac{w \times (t_2 - t)}{t - t_1}. \qquad \text{I.}$$

This is, by definition (§ 85), the thermal capacity of the calorimeter. To find the temperatures t and t_2, at the time when the water is introduced into the calorimeter, allowances for cooling should be made (see ¶ 93).

The second method (¶ 90, II.) differs from the first in that w grams of water are *warmed* $(t - t_2)$ degrees, and hence must *receive* $w \times (t - t_2)$ units of heat from the calorimeter, the temperature of which is thereby *reduced* $(t_1 - t)$ degrees; hence to reduce it 1° would require a quantity of heat, c, given by the formula

$$c = \frac{w \times (t - t_2)}{(t_1 - t)}. \qquad \text{II.}$$

This formula is evidently reducible to the same form as I.

In the last method (¶ 90, 2) if w_1 is the weight of the inner cup, w_2 that of the stirrer, and w_3 the weight (or volume) of the water displaced by the thermometer; if furthermore s_1 and s_2 are the specific heats, respectively, of the materials of which the inner cup and the stirrer are made,[1] and s_3 the thermal capacity of a quantity of mercury and glass equal in volume to a gram of water;[2] then the thermal capacity of the inner cup is $w_1 s_1$; that of the stirrer, $w_2 s_2$; that of the thermometer, $w_3 s_3$; hence the total thermal capacity of the calorimeter (c) is given by the formula,

$$c = w_1 s_1 + w_2 s_2 + w_3 s_3. \qquad \text{III.}$$

If, for example, the inner cup contains 100 *g.* of brass, of the specific heat .094, its thermal capacity is

[1] The inner cup and stirrer are usually made of brass (an alloy of copper and zinc), the specific heat of which may be taken as .094.

[2] It will be noted that though the specific heat of mercury (.033) differs greatly from that of glass (0.19), the thermal capacity of *equal volumes* is very nearly the same. Since 1 *cu. cm.* of mercury weighs 13.6 grams, it will require 13.6 × .033, or 0.45 units of heat, to raise it 1°. In the same way, since 1 *cu. cm.* of ordinary glass weighs not far from 2.5 grams, it would require about 2.5 × 0.19, or 0.47 units of heat to raise it 1°. In calculating the thermal capacity of a thermometer, there will be, accordingly, *no appreciable error* in assuming for s_3 a mean value, 0.46.

$100 \times .094$, or 9.4 units; if the stirrer is made of thin brass weighing 2 grams, its thermal capacity is similarly 0.2 units; and if the thermometer displaces 0.9 grams of water, its thermal capacity is (see 2d footnote, page 161) 0.9×0.46, or about 0.4 units. The total thermal capacity of a calorimeter thus constructed would be $9.4 + 0.2 + 0.4 = 10.0$ units.

The first method is apt to give too high results, since the cooling of the water, due to evaporation and other causes, is attributed to contact with the calorimeter.

The second method usually gives too low results, on account of the rapidity with which heat escapes from the calorimeter while empty. If, however, the outer cup becomes heated indirectly by the shot, a portion of this heat may be radiated back to the inner cup when filled with water. It is possible, therefore, that the results may be too great.

The last method generally gives too small a result, because we neglect the heat absorbed by the materials surrounding the inner cup. If, however, only a portion of the inner cup is to be heated, we may easily over-estimate its thermal capacity.

In the latter case, we prefer an experimental determination of thermal capacity; but when the inner cup is made of very thin metal (as is desirable for accurate work), the thermal capacity may be so slight that it cannot be exactly determined by experiment. In such cases, we usually depend upon a calculation based, as in the last method, upon the weights and specific heats of the materials composing the calorimeter.

¶ 92. **Precautions Peculiar to Calorimetry.**— In nearly all experiments in calorimetry two bodies, of known weights and temperatures, are brought together so that by the flow of heat from one to the other (see Experiments 33 and 34) or by the action of one on the other (see Experiments 35–38) a third temperature results. There are, accordingly, many precautions common to these various experiments.

(1) CHEMICAL ACTION. — It is evident that the substances employed should exert no chemical action on the sides of the calorimeter. With strong acids, a glass vessel should generally be employed. Instead of a brass stirrer, one of platinum may be used. In the case of mercury, iron will do even better. A coating of asphaltum is often sufficient to prevent metals from being attacked by acids.

When two substances are placed together in a calorimeter, neither should act chemically upon the other unless the object of the experiment be to measure the heat developed by the reaction. The chemical relations between two substances thus employed must frequently be investigated by a separate experiment.

(2) COMPARISON OF THERMOMETERS. — The general precautions necessary to the accurate observation of temperature have been already considered (¶ 65), and must be observed. In addition to these precautions, certain others are required when *simultaneous* observations of temperature are to be made. In such cases it may be necessary to employ as many thermometers as there are temperatures to be determined; and these thermometers have to be compared with

one already tested by a process of calibration (¶ 68). To do this, the several thermometers are to be placed in boiling water, in ice-water, and in water of at least three intermediate temperatures. A large quantity of water should be used (see (3)), and it must be well stirred in each case. The indications of each thermometer are to be read in turn; then again read *in the inverse order.* There should be regular intervals (let us say 30 seconds each) between the observations. The two readings of each thermometer are to be averaged, and the averages compared. Knowing (from Experiment 25) the corrections for one of the thermometers, we may easily calculate the corrections for the others. For example, if three thermometers, A, B, and C, gave the following readings:

$$A,\ 76^\circ.0;\ B,\ 75^\circ.7;\ C,\ 75^\circ.1;\ C,\ 74^\circ.7;\ B,\ 74^\circ.5;\ A,\ 74^\circ.0;$$

the average for A would be 75°.0; for B, 75°.1; for C, 74°.9. These averages evidently correspond to the same point of time. We should therefore subtract 0°.1 from the correction of A at 75° to find that of B; and we should add 0°.1 to find that of C.

The object of making observations in the order given above is to eliminate errors due to cooling.

(3) Constant Temperature.—The difficulty of making accurate observations of temperature at a given point of time increases with the rate of cooling. The use of large masses of water (see (2)) is one of the most general methods of avoiding rapid changes of temperature. In certain experiments in

calorimetry, special devices are frequently employed. When, for instance, one of the temperatures to be observed is in the neighborhood of 100°, a steam-heater may be employed (see Fig. 77, also Fig. 79, ¶ 94). Again, a body may be maintained at 0° by surrounding it with melting ice; or it may be kept indefinitely, without special precautions, at the temperature of the room, provided that the latter be constant.

By the use of devices for maintaining a constant temperature, thermometric observations become greatly simplified. One or more temperatures may be known by definition, — as in the case of ice, or steam at a certain pressure (§ 4). In the absence of cooling, a series of observations for each temperature will not be required, and the temperatures of several bodies at a given point of time may be found from successive observations with the same thermometer. The least constant temperature should be observed nearest the time in question.

(4) Exposure to the Air. — When a body is transferred from a heater or from a refrigerator to a calorimeter, there is always more or less heat gained or lost from exposure to the air. The time of exposure should evidently be made as short as possible. In pouring liquids, a glass funnel may be employed; but the funnel must be warmed to the same temperature as the liquid, otherwise it would take from it more heat than the air. Water may be guided conveniently from a beaker to a calorimeter by a wet glass rod, *abc*, bent as in Figure 76. To

prevent the water from following the side of the beaker, the lip should be greased at the point *b*.

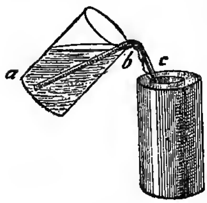


Fig. 76.

The wet stem of a thermometer may also be used as a conductor, and with this advantage, that, since the thermal capacity is easily found (¶ 90, 2) the heat required to raise it to a given temperature may be calculated. We may notice, however, that if the thermometer is immediately afterward placed in the calorimeter, it will give up most if not all of the heat which it has absorbed, and that the remainder may be neglected. Hot shot may be poured directly from a heater suitably shaped (see Fig. 79, ¶ 94) into a calorimeter; but it is safer to use a paper funnel, to prevent the possibility of losing a portion of the shot. Most of the shot should enter the calorimeter without touching the funnel; and the remainder should be in contact with it only for an instant. In this case the heat absorbed by the paper may be neglected. A hot body may also be suspended by a thread, and thus transferred from one place to another.

It is obvious that the calorimeter should be brought as near the heater or refrigerator as is possible without danger that its temperature may be affected by radiation, conduction, or convection from the heater (§ 89). A common pine board makes an excellent shield. In Regnault's apparatus[1] (Fig. 77) the

[1] For a fuller description of Regnault's apparatus, see Cooke's Chemical Physics, page 470.

calorimeter (at the left of the figure) can be brought directly under the large steam heater (at the right of the figure). The steam heater rests upon a support, serving to shield the calorimeter from radiation. The support is made hollow, so that it may be kept cool by a current of water. The inner chamber of the heater contains hot air. The temperature within it is observed by means of a thermometer passing through a cork by which the top of the chamber is

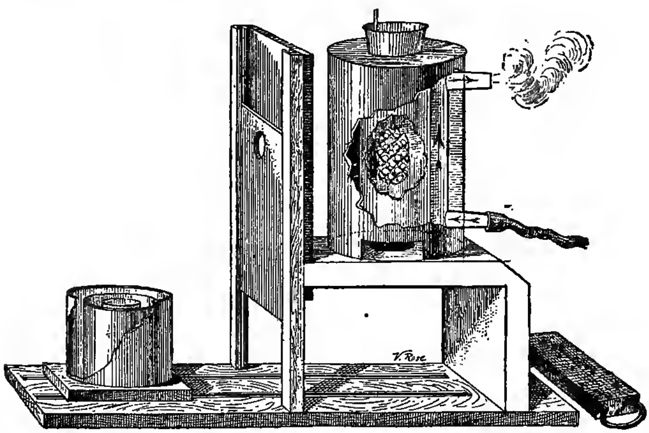


FIG. 77.

closed. The bottom of the chamber is closed by a non-conducting slide. By drawing the slide a body suspended by a thread in the hot-air chamber may be lowered directly into the calorimeter. The calorimeter is then immediately removed to a sufficient distance from the heater, so that the resulting temperature may be accurately determined.

By devices similar to those alluded to, the gain or loss of heat by exposure to the air may be almost indefinitely reduced, but never completely avoided. The student is advised not to attempt any correction for this heat; because a greater error might easily result from applying such a correction than from neglecting it altogether. At the same time, it is well to estimate roughly the quantity of heat gained or lost, with a view to determining what figures of the final result are likely to be affected.

For this purpose two experiments may be made. In one, a body is transferred in the ordinary manner from the heater or from the refrigerator to the calorimeter. In the second experiment, it is passed back and forth let us say 5 times each way, and finally placed in the calorimeter. The body is thus to be exposed to the air in one case about 11 times as long as in the other case, and under similar conditions; so that from the difference in the results we may infer the effect of an ordinary exposure (see ¶ 93, 4).

(5) Loss of Material. — In rapidly pouring a liquid into a calorimeter, or in rapidly lowering a hot solid into a liquid already contained in a calorimeter, there is danger that a portion of the liquid or solid may be lost. It is accordingly desirable to weigh, both before and after each addition to the contents of the calorimeter, not only the calorimeter itself, but also the vessel in which the substance in question was originally contained. The student will do well also to make sure that the space between the inner and outer cups is empty, both before and after

the experiment; for if any of the substance finds its way into this space, its loss will not be apparent from the weighings.

(6) Evaporation. — A considerable portion of the heat lost by a liquid when poured into a calorimeter may be caused by evaporation. When once the liquid has been transferred to the calorimeter, all further loss of heat by evaporation should be prevented by immediately corking the inner vessel. It will be assumed that the inner vessel is never uncorked, except when necessary for the purposes of manipulation. Of two liquids, the denser is usually the less volatile, and hence should be heated in preference to the other. For the same reason, a solid should be heated in preference to a liquid. A combustible liquid should, as we have seen (Exp. 30), never be heated directly by a flame, but indirectly by hot water.

(7) Temperature of the Room. — The loss of heat which takes place from the gradual cooling of a calorimeter and its contents depends, as we have seen in Experiment 31, upon the difference of temperature between the inner cup and its surroundings. To diminish the loss of heat in question, it has been proposed that the outer cup should be placed in water at the same temperature as the inner cup. More accurate results might be expected from calorimetry if some means were perfected by which we could adjust the temperature of surroundings to the needs of an experiment. In practice, however, the experiment must be adapted to the temperature of the air in

which it is to be performed. When considerable time is required to obtain an equilibrium of temperature (see (8)), it is important that the average temperature within the calorimeter should agree as closely as possible with that of the room. The weights and temperatures of the substances employed in calorimetry, are, therefore, frequently chosen so as to give a final temperature between 20° and 25°.

It is much easier to prevent than to allow for losses of heat by cooling; and it may be stated as a general rule in calorimetry that we must avoid in so far as possible all differences of temperature between bodies under observation and the objects by which they are surrounded.

(8) Equilibrium of Temperature. — It has already been pointed out that to obtain a uniform temperature throughout the inner cup of a calorimeter, the cup should be completely filled. If this is not done, special precautions must be taken to bring its contents into contact with every portion of its surface (see ¶ 85, 2). The necessity of stirring these contents has also been alluded to (¶ 65, 5). When a mixture (like lead shot and water) is of such a nature that an ordinary stirrer cannot be used, the inner cup must be closed water-tight, so that the contents may be shaken. The thermometer should in this case fit tightly into the stopper which closes the inner cup, and should reach into the body of the mixture. Solids, if any be used, should be finely divided, so that there may be no risk of breaking the thermometer.

We prefer, moreover, finely divided solids, on account of the comparative rapidity with which an equilibrium of temperature may be reached, or a process of fusion, solution, or chemical combination completed. When a solid sinks in a fluid (as is generally the case), it is well if it can be warmer than the fluid, on account of the manner in which convection currents are formed; and for the same reason we prefer that the denser of two liquids should have the higher temperature. It is always desirable that the denser of two substances should be poured into the other, so that, as it passes through, as much heat as possible may be communicated from one to the other. The various processes in calorimetry should in general be completed in the shortest possible time, especially when they cannot be conducted at the temperature of the room, since otherwise large losses of heat are apt to occur.

Throughout the processes in question, stirring must be interrupted from time to time, in order that rough observations of temperature may be made. When two successive observations agree, or when they differ by an amount which may be attributed to the regular cooling of the calorimeter (see Exp. 31), the equilibrium of temperature should be complete. The student will do well, however, to make sure that the temperatures at the top and bottom of the calorimeter are the same, before proceeding to make exact observations of the thermometer.

(9) TIMING OBSERVATIONS.—When observations of temperature are taken regularly at intervals of one

or two minutes throughout an experiment, we may infer the time when a given process begins and when it ends; but to avoid errors due to the possible omission of one or more observations, it is well to note the beginning and end of each process in *hours, minutes, and seconds*. In any case, the time should be thus noted, (1st) when all the bodies have been transferred to the calorimeter, and (2d) when, after an equilibrium of temperature has been reached, the resulting temperature is first observed.

(10) SERIES OF TEMPERATURES. — It is well in all cases to make several observations of the final temperature within a calorimeter, in order that the result may not depend upon one alone (see § 51). The series should be made at intervals of one minute, so that, as in ¶ 93 (2), the rate of cooling may be found and allowed for. If the calorimeter contains water only, we may utilize the temperature curves already plotted (see ¶ 93, 1); or if we have determined, as in ¶ 87, the flow of heat from the calorimeter, we may make an allowance for the heat lost as in ¶ 93 (3). In the absence of any previous determination under the same conditions as in the actual experiment, a series of observations of the temperature of the calorimeter will be required.

In the same way, if the temperature of a body is changing perceptibly before it is placed in a calorimeter, it must be determined by a series of observations. The intervals in all such series would naturally be one minute each; but when the temperatures of two or more bodies are to be found, the observations must be taken in turn. When special precautions concerning equilibrium of temperature (see (8)) have to be observed, the student is advised not to attempt observations at intervals of less than one minute. The temperatures of the several bodies concerned are to be reduced in all cases, as in ¶ 93 (1), to the time when they are *first enclosed in the calorimeter*. After this time, losses of heat are to be calculated as above, from the known rate of cooling of the calorimeter.

¶ 93. **Corrections for Cooling.** — (1) GRAPHICAL METHOD. — When a calorimeter contains water only, as in the determination of thermal capacity above (¶ 90, I.) or in parts of various experiments which follow, the temperature at one point of time may be inferred from an observation taken at another point of time by using one of the curves in Fig. 74, ¶ 88. Let *ab* (Fig. 78) be the curve corresponding to the quantity of water which the calorimeter contains, and let *c* be the observed temperature. We first find a point *d* on the curve at the right of *c*, then a point *e* above *d*. Then we measure off a distance *ef* on the scale of minutes corresponding to the length of time during which the calorimeter has been cooling. Then we find a point *g* on the curve below *f*, and finally the temperature *h*, at the left of *g*.

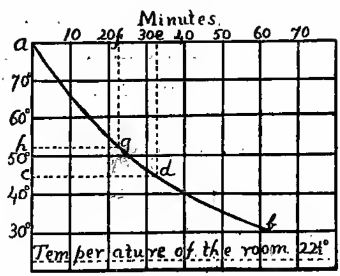


FIG. 78.

This temperature corresponds in the figure to a time *f* earlier than *e*; but by laying off the distance *ef* to the right of *e*, we could find, if we chose, the temperature at a later point of time.

A more exact method would be to start with a point *c* (in Fig. 78), corresponding to a temperature as far above that of the room (22½°, Fig. 78) as the actual temperature observed was above the observed temperature of the room. The number of degrees included between *c* and *b* gives approximately, in any case, the fall of temperature which takes place in an interval of time corresponding to the number of minutes between *e* and *f*.

(2) Analytical Method. — When several temperatures have been recorded at regular intervals, we may infer the temperature at a point of time before the beginning or after the end of the series as follows: The observations are first written down in a column, as in the example below; then the temperature of the room is subtracted from each, and the results entered in a second column; then a third column is formed from the differences between each pair of consecutive numbers in the second column; then each number in the third column is divided by the one just below it in the second column, to find what per cent must be added to that number in order to obtain the one above it; these per cents are arranged in a fourth column and averaged; then each number in the third column is divided by the number in the second column just above it, to find what per cent must be subtracted from that number to obtain

the number just below it; the per cents to be subtracted are then arranged in a fifth column and averaged. We may now extend the second column upwards by adding to the first number in it the average per cent from the fourth column, and we may extend it downward by subtracting from the last number the average per cent found in the fifth column. When the second column has been thus extended, the corresponding numbers in the first column may be found by adding in the temperature of the room. The temperature at a time which would come between the observations in the series thus extended may evidently be found by simple interpolation.

For example, when the temperature of the room is 26°, the observations below would be reduced as follows: —

Temperatures Observed.	Temperatures less 26°.	Fall of Temperature.	Per Cent to be Added.	Per Cent to be Subtracted.
66°.0	40°.0			
		2°.0	5.3	5.0
64°.0	38°.0			
		1°.9	5.3	5.0
62°.1	36°.1			
		1°.6	4.6	4 4
60°.5	34°.5			
		1°.5	4.5	4.3
59° 0	33°.0			
		1°.6	5.1	4.8
57°.4	31°.4			
		1°.4	4.7	4.5
56°.0	30°.0			
		Average	4.9	4.7

To extend the second column upwards we add to the first number in it 4.9 per cent of itself. Since 4.9 of 40°.0 is 2°.0, the number above 40°.0 should be 40°.0 + 2°.0, or 42°.0; and since 4.9 per cent of 42°.0 is 2°.1, the next number should be 44°.1, etc.

To extend the second column downwards, we subtract from the last number (30°.0) in it not 4.9 per cent but 4.7 per cent of 30°.0; that is 1°.4; this gives 28°.6; and subtracting from this 4.7 per cent of itself, or 1°.3, we find 27°.3 for the number following, etc.

Adding 26° to the new numbers in the second column, we infer, finally, that the temperatures preceding 66°.0 in the first column should be 68°.0 and 70°.1, while those following 56°.0 should be 54°.6 and 53°.3, etc.

Let us suppose that the temperatures were observed at intervals of one minute; then to represent the temperature for instance 1.5 minutes before the first recorded observation, we should take a number half-way between 68°.0 and 70°.1, or 69°.0 nearly. If, however, the intervals between observations were two minutes each, then 1.5 minutes would be three fourths of one interval, and we should add to 66° three fourths of the difference (2°) between it and the next temperature above it in the series to find the temperature (67°.5) in question.

The discovery of various methods by which the calculations described above may be shortened, especially by the use of logarithms, may be left to the ingenuity of the student. The method here described is important, as an illustration of the fact that when a body is steadily cooling its temperature falls, not a given amount in each minute, but a certain *per cent* (approximately) of the number of degrees which lie between it and the temperature of the room (see ¶ 86, 1).

The accuracy with which a series of observations may be extended by analytical methods evidently grows less as the number of new terms increases. It may be said in general that the new terms should not be more numerous than those obtained by actual observation.

(3) Heat Lost by Cooling. — We must distinguish between the rate of cooling of a calorimeter and the number of units of heat lost by it. The latter may be found without knowing the nature of the mixture which the calorimeter contains, provided that the inner cup is completely filled by the mixture, or filled to a known depth; for we have only to refer to the results already found with water at the same depth in Experiment 31.

If, for example, a calorimeter, nearly filled with a mixture of lead shot and water, has been cooling for ten minutes at an average temperature about 20° above that of the room, we reason that since at a temperature 1° above that of the room it was found (¶ 87) to lose 1.8 units of heat per minute, at a temperature 20° above that of the room it would lose 20 times 1.8, or 36 units per minute; that is, 360 units in ten minutes. If, therefore, the first accurate observation of temperature was taken ten minutes after the introduction of the mixture, we should add 360 units to the amount of heat apparently given out by the hot body, or if more convenient we may subtract 360 units from the quantity of heat apparently absorbed by the cool body (see ¶ 98).

(4) Method of Multiplication. — When two experiments are made, in one of which a body is exposed,

let us say, 11 times as long or 11 times as often to the air as in the other experiment, in which we give it the ordinary exposure, the difference between the results obtained in the two cases should correspond to the effect of 11 less 1, or 10 ordinary exposures. Hence, if this difference be divided by 10, we may estimate roughly the correction to be applied to the result obtained with the ordinary exposure.

If, for example, the thermal capacity of a calorimeter is found to be 10.1 units when warm water is poured into it directly, and 11.1 units if the water is first poured back and forth five times each way, then the effect of cooling due to 10 transfers is 11.1–10.1, or 1 unit in the result; and the effect of a single transfer is about 0.1 unit. The true thermal capacity is, therefore, about 10.1–0.1, or 10.0 units. If the cooling due to transferring a substance from one place to another is thought to affect the figure in the tenths' place, as in the example, it is evident that the hundredths will not be significant (see § 55).

EXPERIMENT XXXIII.

SPECIFIC HEAT OF SOLIDS.

¶ 94. **Determination of the Specific Heat of a Solid by the Method of Mixture.** — I. A quantity of lead shot sufficient to half fill the calorimeter (Fig. 70, ¶ 85) is first weighed, then put into a steam heater (Fig. 79), and covered by a cork. A thermometer, passing through the cork into the midst of the shot, is allowed to remain there until it ceases to rise. Meanwhile the temperature within the calorimeter is determined by a second thermometer (¶ 92, 2). The calorimeter is then weighed, and a vessel containing a mixture of ice and water is also weighed. This vessel should be provided with a strainer, so that water may be poured from it without danger of particles of ice following the stream. The ice and water should be thoroughly stirred just before the experiment, to secure a uniform temperature of 0°. The time should now be noted (¶ 92, 9).

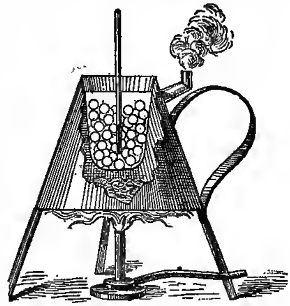


Fig. 79.

The thermometer and corks are then removed from the heater, and the shot is poured as rapidly as possible (¶ 92, 4) into the calorimeter. Immediately ice-cold water is added, — the quantity being nearly sufficient to fill the calorimeter. A thermometer is then pushed cautiously into the middle of the shot through a small stopper, closing the inner cup water-tight (¶ 92, 8). The large cork cover (Fig. 71, ¶ 85) may then be added, and the time again recorded. The mixture must now be carefully shaken. The temperature indicated by the thermometer is to be noted at intervals of 1 minute, until it begins to fall steadily (¶ 92, 8 and 10). Then the calorimeter is re-weighed with its contents; and the vessel originally containing the water is also weighed (¶ 92, 5).

II. Instead of finding the temperature of the shot in the heater, as in I., we may determine it by a series of observations in the calorimeter, before the ice-water is added (¶ 92, 10). It is necessary in this case to cork the inner cup, and to shake the shot between the observations of temperature (¶ 92, 8), in order that there may be a uniform temperature not only in the shot, but also in the inner cup, the thermal capacity of which must be considered. The ice-water is finally added, and the temperature of the mixture determined as before.

III. Instead of pouring the hot shot first into the calorimeter, we may begin by introducing ice-water. In this case the proper quantity of water must be determined beforehand. It will probably be found that the water should fill the calorimeter about half-full. In other respects this method is the same as I.

IV. Instead of assuming that the temperature of the water is the same as that within the vessel originally containing it (that is, 0°), we may find its temperature after it has been transferred to the calorimeter. In this method, however, as in the second method, the thermal capacity of the cup must be considered. To avoid the necessity of making a separate series of observations (¶ 92, 10) between which the water in the calorimeter must be shaken up (¶ 92, 8), it is customary to use water at the temperature of the room. In this case, the mixture will be above the temperature of the room; hence its rate of cooling must be allowed for (¶ 93).

V. Other methods of determining specific heat may easily be devised, depending upon the use of hot water and cold shot. We have in fact already made use of such a method in finding the thermal capacity of a calorimeter (¶ 90, I.). On account, however, of the practical difficulties arising from evaporation (¶ 92, 6), the high temperature of the mixture (¶ 92, 7), and the small change of temperature produced, these methods are generally avoided. The principal use which can be made of them is as a check (§ 45) upon results obtained in the ordinary manner.

The student may observe that in the second method the shot falls suddenly in temperature, on account of the heat which it gives up to the calorimeter. This heat is subsequently restored to the mixture when the calorimeter is cooled to its original temperature; hence in the first method no account need be taken of the thermal capacity of the calorimeter. Again in the fourth method, the cold water may at first rise rapidly in temperature on account of the heat imparted to it from the calorimeter, but this heat is restored to the calorimeter when it is again raised by the mixture to its original temperature; hence in the third method no account need be taken of the thermal capacity of the calorimeter.

Instead of lead shot, copper or iron rivets may be employed with very slight modifications of the experiment. In the case however of solids which are soluble in water, we must substitute for water some other liquid of known specific heat in which the solids are insoluble (¶ 92, 1). The student may be guided in his choice of methods by obvious considerations of practical convenience as well as by the principles explained below in ¶ 95; but he should make at least one determination of specific heat of a solid by the method of mixture and reduce it as will be explained in ¶ 98.

¶ 95. **Comparison of Methods for the Determination of Specific Heat.** — The principal difficulty in the first method (¶ 94, I.), for the determination of specific heat, is to avoid a great loss of heat while the shot is being transferred from the heater to the calorimeter.

In the second method (¶ 94, II.) there is no opportunity for a loss of heat on the part of the shot, since its temperature is determined by a series of observations within the calorimeter, from which its temperature at any point of time may be found (¶ 93, 2). The principal objection to the second method is the difficulty of determining accurately a series of temperatures in which rapid changes take place; and the necessity of allowing for the thermal capacity of the calorimeter, which is always a more or less uncertain quantity, and bears a considerable proportion to the thermal capacity of the shot.

The third method (¶ 94, III.) has the same practical advantages and disadvantages as the first.

The fourth method (¶ 94, IV.) is the one commonly employed for the determination of specific heat. Since the temperature of the water is found when within the calorimeter, there is no opportunity (as in the other methods) for heat to be imparted to it in the act of pouring. There is however difficulty, as in the second method, in determining accurately a temperature which is changing (¶ 92, 3), and still further difficulty in maintaining a uniform temperature throughout the calorimeter with a quantity of water which only half fills it (¶ 92, 8). When the latter difficulty is avoided by using water at the temperature of the room, the mixture must have a temperature considerably above that of the room, and one therefore which is hard to determine (¶ 92, 3). The thermal capacity of the calorimeter must also, as in the second method, be taken into account.

By comparing the results of the first and second methods, we are able to estimate the effect of the heat lost in pouring the shot into the calorimeter (see also ¶ 93, 4), and by comparing results of the third and fourth methods, we are able to estimate the effect of the heat absorbed by the ice-cold water when it is poured from one vessel to another. This will be found to be small in comparison with the heat lost by shot at 100° under similar circumstances. The second method, in which the latter is eliminated, is therefore preferable to the fourth. In the first and third methods, the heat lost by the shot is partly offset by that imparted to the water. Since the former is greater than the latter, the third method is preferable to the first; because the longer exposure of the water may compensate for the more rapid cooling of the shot. The choice between the second and third methods will depend largely upon the comparative accuracy with which we can determine the heat given out by the calorimeter (¶ 87) and the heat lost by the shot (¶ 93, 4). The advantages of using in any case hot shot and cold water have been already stated (¶ 94, V.).

EXPERIMENT XXXIV.

SPECIFIC HEAT OF LIQUIDS.

¶ 96. **Determination of the Specific Heat of a Liquid by the Method of Mixture.** — The specific heat of a liquid may be determined either by mixing it mechanically with water, or by bringing it in contact with a solid of known specific heat. The first method is the more direct, but cannot be employed with liquids which unite chemically with water, unless we know the amount of heat given out or absorbed by the reaction (see ¶ 92, 1). Before deciding which method we shall employ, we therefore mix together the contents of two test-tubes, each at the temperature of the room, one containing water, the other the liquid in question. If no change of temperature is observed, the first method is adopted. If the temperature rises or falls, we must either make a separate experiment to determine accurately the amount of this rise or fall (see Exp. 35), or else adopt the indirect method, using a solid instead of water.

I. The determination of the specific heat of an insoluble liquid by the method of mixture does not differ essentially from the case of a solid. A heavy oil may for instance be heated by the same apparatus (Fig. 79, ¶ 94) employed for the shot, and mixed with ice-cold water, according to either of the methods described (¶ 94). Instead of shaking the mixture, a brass fan or stirrer (Fig. 50, ¶ 65) may be employed.

The objections to mixing hot water with a cold liquid are not nearly as strong as in the case of solids (¶ 94, V.); for though most liquids have a specific heat less than that of water, the differences are very much less. By pouring a comparatively small quantity of water at a temperature not exceeding 40° or 50° into a liquid at 0° a mixture may be had not far from the temperature of the room. With liquids less dense than water this method is generally to be preferred (see ¶ 92, 6 and 8). The results may be reduced by the appropriate formula from ¶ 98.

Attention has already been drawn (¶ 92, 1) to precautions against chemical action in the case of corrosive liquids, and in the case of volatile liquids against evaporation (¶ 92, 6) and combustion (¶ 83).

II. In the case of liquids which mix with water, the ordinary methods of mixture cannot generally be employed, on account of the heat absorbed or developed by solution or combination. It is necessary to find some substance, of known specific heat, upon which such a liquid exerts no thermal action. This substance is then mixed with the liquid by either of the methods of ¶ 94. The data necessary for finding the specific heat of the liquid are as usual the weight of the two substances in question, the temperature of each before the experiment, and the resulting temperature of the mixture.

The lead shot already employed (¶ 94) may be used to determine in this way the specific heat of alcohol, glycerine, saline solutions, etc. For corrosive liquids, like nitric acid, glass beads (of specific heat about 0.19) may be similarly employed (see general formula, ¶ 98). Evidently this indirect method is more general than the ordinary method of mixture, since it can be applied to all liquids, whether soluble or insoluble in water. It has the advantage of eliminating almost completely the heat lost by the hot body between the heater and the calorimeter, since this loss is practically the same in the case of water as in the case of other liquids with which a comparison is made.

¶ 97. **Peculiar Devices employed in Calorimetry.** — In the method of mixture (Exps. 33 and 34) a thermal equilibrium between two or more substances is established by bringing them in contact. It is not, however, necessary that the two bodies should touch each other. The difficulties which arise from the mutual action of two substances may often be avoided by surrounding one of them with an envelope, through which, by the conduction of heat, an equalization of temperature takes place. If, for instance, a hot liquid contained in a glass bulb be surrounded by cold water, a certain quantity of heat will be given out. Having found by a separate experiment how much heat is derived from the bulb alone, we may calculate the specific heat of the liquid in the ordinary manner, that is, from the weights and changes of temperature involved (see general formula, ¶ 98).

The liquid in question may be contained in an ordinary thermometer bulb. In this case its change of temperature may be inferred very accurately from its contraction, as shown by the fall of a column of liquid in the stem of the thermometer. It is necessary, of course, to make a careful comparison of a thermometer containing an unknown liquid with an ordinary mercurial thermometer (see ¶ 92, 2). This method has obvious advantages in the case of costly liquids.

On the other hand, when the supply of a fluid is unlimited, it is frequently advantageous to use an envelope in the form of a spiral tube, or coil, through which the fluid in question may be passed in a continuous stream. We are thus enabled to bring a great volume of the fluid in thermal equilibrium with a small volume of water. This device is exceedingly important in the case of gases, since it would be otherwise impossible to bring enough gas in thermal equilibrium with a given quantity of water to affect the temperature of the water by a measurable amount.

The weight of the gas employed is not measured directly, but is determined from its density (see ¶¶ 44, 46) and from the volume employed. The volume is indicated by a gas-meter (*ab*, Fig. 80) through which the gas is first passed. The gas is then raised to the temperature of 100° by passing it through a steam jacket, *bd*. Then it circulates through a coiled tube surrounded with water, and escapes from an orifice where its final temperature can be observed. From the thermal capacity and rise of temperature of the calorimeter, we may calculate the quantity of heat given out by a known quantity of gas in falling through a known number of degrees, and hence the specific heat of the gas. It is found that the specific heat of air at the constant pressure of one atmosphere is about 0.238, or a

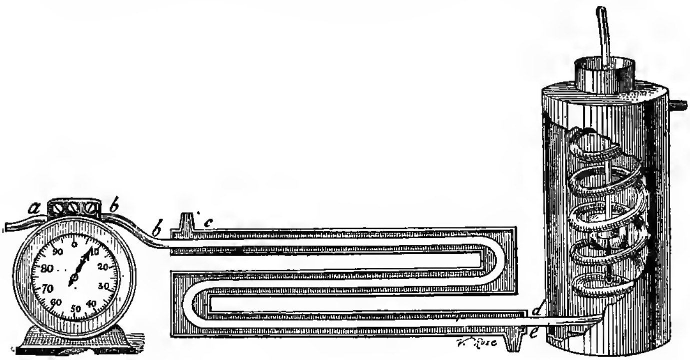


Fig. 80.

little less than one fourth that of an equal weight of water.

A much more difficult task consists in the determination of the specific heat of a gas when confined to a constant volume. The following method is suggested. It depends upon the fact that a given electric current passing for a given time through a given conductor generates in that conductor a given quantity of heat. This quantity may be found by experiment (see Exp. 86), or calculated by the principles of § 136. Let us suppose that a known quantity of heat is thus suddenly generated within a closed flask (Fig. 81); and that the increased pressure of the air is measured, as in ¶ 80, by the rise of mercury in an open tube. Then the average temperature of the air within the flask can be calculated (see § 76).

We may therefore find the thermal capacity of a known volume or of a known weight, and hence the specific heat in question (about .169).

FIG. 81.

It is found that the thermal capacity of a cubic metre of air is about 219 units at 0° and 76 *cm.* when prevented from expanding, as against 308 units when free to expand under a constant pressure. The thermal capacity of an *equal volume* of oxygen, of nitrogen, or of hydrogen is very nearly the same as that of air under similar conditions.

Instead of using an electrical current to generate heat (as illustrated in Fig. 81), we may employ various other agents, as for instance the combustion, the solidification, the fusion, the condensation, or the vaporization of a known weight of a given substance, or the conversion through friction of a given amount of work into heat (see Exp. 70). If, for example, the combustion of a gram of coal heats a kilogram of water 8°, and a kilogram of petroleum 16°; or if 100 grams of ice cool these liquids 8° and 16° respectively; the specific heats must be to each other as 2 to 1. The same inference would be drawn if the same quantity (100 grams) of steam which heats 1 kilogram of water 54° were found to heat 2 kilograms of petroleum by the same amount. The specific heats of different substances are to each other, in general, inversely as the changes of temperature produced by a given cause, and also inversely as the weights affected. The determination of specific heat is evidently capable of as many modifications as there are different methods by which a definite quantity of heat may be generated or absorbed.

Instead of using the pressure of air to measure its temperature, we may also employ its expansion (§ 80) as in the air thermometer (¶ 74). The specific heat of air under a constant pressure might obviously be determined by an apparatus similar to that represented in Fig. 81; hence, conversely, if this specific heat is known, we may measure quantities of heat by the expansion which they produce in air at a given pressure. It does not (as one might think) make any difference theoretically *how much* air is heated; because an increase in the quantity of air will be offset by a decrease in the temperature to which it will be raised by a given amount of heat; and for the same reason it is indifferent whether a small portion of the air is heated a great deal, or whether a considerable portion is heated by a proportionately small amount. In this method of estimating heat it is not necessary to wait for an equilibrium of temperature. We hasten in fact to make our observations before an equilibrium is reached, so as to avoid loss of heat by contact of the air with the sides of the vessel in which it is contained. It has been calculated that one unit of heat should in all cases cause in a body of air at 76 *cm.* pressure an expansion of about 12 cubic centimetres. Since an expansion of less than 1 cubic millimetre is easily detected, we have, in the air thermometer, a very delicate means of measuring small quantities of heat.[1]

Instead of air, we may use any other fluid which has a regular rate of expansion to determine quantities of heat. The principle above explained has been applied by Favre and Silbermann in the construction of their mercury calorimeter.[2] This is essentially a thermometer with a huge bulb. If even a small quantity of hot liquid be introduced into a cavity in this bulb, there will be a perceptible expansion of the mercury, by which we may measure the heat given out by the liquid in question; for it has been found that 1 unit of heat always causes in a body of mercury an expansion of about 4 cubic millimetres.

There are various other definite effects produced by a given quantity of heat, any one of which might theoretically be applied to the purposes of calorimetry. The only application of practical importance depends, however, upon the heat required for the fusion of ice (see Experiment 36). A rough form of ice calorimeter consists of a block of ice (Fig. 82) with a small cavity in which a hot body may be

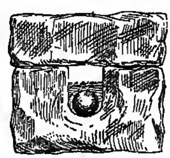


FIG 82.

[1] The air thermometer has been used in the Jefferson Physical Laboratory to measure minute quantities of heat generated in a carbon fibre by telephone currents.

[2] See Ganot's Physics, § 463.

placed. A second block may be used as a cover. The water formed by the liquefaction of ice is gathered by a sponge, and weighed by the usual method of difference. Since one unit of heat melts one-eightieth of a gram of ice, the quantity of heat given out by the body in falling to a temperature of 0° can easily be calculated. In Bunsen's ice calorimeter, the quantity of ice melted is estimated by the change *in volume* of a mixture of ice and water.

¶ 98. **Calculation of Specific Heat in the Method of Mixture.** — If w_1 is the weight of the body, the specific heat of which (s_1) is to be determined, and t_1 the temperature of this body, reduced to the time of mixing; if w_2 is the weight of the body the specific heat (s_2) of which is known, and if t_2 is its temperature, also reduced to the time of mixing; if c is the thermal capacity of the calorimeter, t_3 its original temperature and, t the temperature of the mixture; then if q is the quantity of heat lost by cooling, that is, absorbed by the air, etc., we have, by the principle of § 90, the general formula,

$$w_1 s_1 (t - t_1) + w_2 s_2 (t - t_2) + c (t - t_3) + q = 0.$$

From this formula we may obtain the solution of all problems in the determination of specific heat by the method of mixture.

In addition to s_2, c, and q (which are known, or may be calculated), we require at least five data for a determination of specific heat; namely, the two weights employed, w_1 and w_2, the two corresponding temperatures, t_1 and t_2, also the temperature, t, of the mixture. The original temperature, t_3, of the calorimeter must also be determined, unless by the nature of the experiment it is known to agree with one of the other temperatures.

When water is used $s_2 = 1$; hence we have, if the water used is colder than the mixture,

$$s_1 = \frac{w_2 (t - t_2) + c (t - t_3) + q}{w_1 (t_1 - t)}; \qquad \text{I.}$$

or if the water is warmer than the mixture,

$$s_1 = w_2 \frac{(t_2 - t) - c (t - t_3) - q}{w_1 (t - t_1)}. \qquad \text{II.}$$

If the temperature of the water is taken in the calorimeter, so that $t_2 = t_3$, we may combine the terms in the numerator, so that for cold water,

$$s_1 = \frac{(w_2 + c) (t - t_2) + q}{w_1 (t_1 - t)}; \qquad \text{III.}$$

or for hot water,

$$s_1 = \frac{(w_2 + c) (t_2 - t) - q}{w_1 (t - t_1)}. \qquad \text{IV.}$$

If the original temperature of the calorimeter is the same as that of the mixture, the terms $c (t - t_3)$ and $c (t_3 - t)$ disappear from I. and II. respectively; hence, for cold water,

$$s_1 = \frac{w_2 (t - t_2) + q}{w_1 (t_1 - t)}; \qquad \text{V.}$$

and for hot water,

$$s_1 = \frac{w_2 (t_2 - t) - q}{w_1 (t - t_1)}. \qquad \text{VI.}$$

If, finally, the temperature of the mixture is the same as that of the room, there is no loss of heat by cooling (§ 89), that is, $q = 0$; hence the term q disappears from all the formulæ. We have therefore in the simplest possible case, when the calorimeter is at the temperature of the room both before and after the experiment, if cold water is used,

$$s = \frac{w_2 (t - t_2)}{w_1 (t_1 - t)}; \qquad \text{VII.}$$

and if hot water is used,

$$s = \frac{w_2 (t_2 - t)}{w_1 (t - t_1)}. \qquad \text{VIII.}$$

The calculation of the thermal capacity of the calorimeter (c) is explained in ¶¶ 86 and 91; that of the heat lost (q) in ¶ 93, 3. The correction of the temperatures t_1 and t_2 to the time of mixing may be done either by graphical or by analytical methods (¶ 93, 1 and 2).

EXPERIMENT XXXV.

HEAT OF SOLUTION.

¶ 99. **Determination of Latent Heat of Solution.** — When a solid dissolves in a liquid, or when two liquids mix together, there is almost always a rise or fall of temperature. This is due probably to a molecular re-arrangement which takes place. The object of this experiment is to find how much heat is given out or absorbed, as the case may be, by one gram of a given substance when mixed with or dissolved in water.

I. LIQUIDS. — When equal volumes of alcohol and water are mixed together (see ¶ 96) a rise of temperature may be observed. To measure this rise accurately, a calorimeter is to be weighed empty, and re-weighed with a quantity of alcohol which fills it half-full, and which is at a temperature, accurately observed, not far from that of the room. An equal volume of water, heated or cooled if necessary so as to have exactly the same temperature, is then mixed with the alcohol in the calorimeter, and the resulting temperature accurately determined by a series of observations (¶ 92, 10). The weight of water is also to be found (see ¶ 92, 5). If the thermal capacity of the calorimeter and the specific heat of the liquid are both known, the latent heat of solution may be calculated by formula II., ¶ 100.

It is better, however, to repeat the experiment with water at a much lower temperature, which must be determined (see ¶ 92, 10) by a series of observations. The object aimed at is to offset in this way the heat due to mixture. When alcohol in a calorimeter at the temperature of the room is mixed with an equal volume of water, which is cooler than it by the right number of degrees, scarcely any rise or fall of temperature will be observed in the calorimeter. In this case a single observation will suffice.

Let us suppose, for example, that equal volumes of alcohol and water rise 8° when mixed at the same temperature, but that if the water is 9° cooler than the alcohol, the rise is 2°. Then since 9° in the water makes a difference of 8° — 2°, or 6°, in the mixture, 12° in the water would make a difference of 8° in the mixture. It follows that the alcohol could be mixed with an equal volume of water 12° below it in temperature without being warmed or cooled by the process.

It would be well to test the accuracy of such a conclusion by a third experiment. When the desired difference of temperature has been found, either by experiment or by calculation, the latent heat of mixing is easily computed. We multiply the weight of water by its rise of temperature to find the number of units of heat received, and divide by the weight of alcohol to find the amount given out by one gram; or we may use formula III., ¶ 100.

The experiment may be varied by using different liquids, or by mixing a given liquid with water in different proportions.

II. Solids. — When ammonic nitrate is dissolved in water a fall of temperature is observed. The amount of this fall may be determined as in the case of alcohol; but in order that the solid may be readily dissolved, it is better to use only one part of the salt in nine of water. To ensure rapid solution, the salt should be pulverized. In the first experiment the salt, the water, and the calorimeter should all start at the temperature of the room. The fall of temperature of the water may require a thermometer divided into tenths of degrees for its accurate determination. The use of a stirrer is very important (¶ 65, 5).

The experiment may now be repeated with water somewhat warmer than before, with a view to making the resulting temperature agree with that of the room. The water should, however, be placed first in the calorimeter, in order that the temperature of the latter may be accurately determined. A series of observations must be taken (¶ 92, 10). The salt is finally added, and the fall of temperature accurately measured. If the water has been heated too much or too little, the experiment may be repeated until the mixture agrees in temperature with the room; or the desired temperature of the water may be calculated by the same process of reasoning as was employed in I. In calculating the latent heat of solution by this method, the thermal capacity of the calorimeter must be taken into account, since part of the heat absorbed by the salt is supplied by the calorimeter. In other respects the reduction is the same as in I. (see also formula IV., ¶ 100).

If, for instance, 10 grams of salt cool 90 grams of water contained in a calorimeter with a thermal capacity equal to 10 units, from 22° to 20°, that is 2°, we have $(90 + 10) \times 2 = 200$ units of heat given out. Since 10 grams of the salt absorb 200 units, each gram must require 20 units of heat; hence the latent heat of solution is 20. The latent heat in question varies slightly according to the strength of the solution formed.

¶ 100. **Calculation of the Latent Heat of Solution.** — If w_1 is the weight of the substance whose latent heat of solution, l_1, is to be determined, s_1 its specific heat, and t_1 its original temperature; if w_2 is the weight of the solvent, s_2 its specific heat, and t_2 its original temperature; if c is the thermal capacity, t_3 the original and t the final temperature of the calorimeter (hence also of the mixture), then the quantities of heat absorbed are, (1) $w_1\ s_1\ (t - t_1)$ in raising the temperature of the substance dissolved; (2) $w_2\ s_2\ (t - t_2)$ in raising the temperature of the solvent; and (3) $c\ (t - t_3)$ in raising the temperature of the calorimeter and (4) $w_1\ l_1$ in the act of solution. Hence, by the principle of § 90,

$$w_1\, s_1\, (t - t_1) + w_2\, s_2\, (t - t_2) + c\, (t - t_3) + w_1\, l_1 = 0, \qquad \text{I.}$$

neglecting the heat lost by cooling.

This gives for the latent heat of mixing with water, which we consider positive if heat is absorbed, but negative if (as is usually the case when two liquids are mixed) heat is given out,[1] since $s_2 = 1$, and since t_1 and t_3 are the same (the temperature of the liquid being determined in the calorimeter),

$$l_1 = -\frac{(w_1\, s_1 + c)\, (t - t_1) + w_2\, (t - t_2)}{w_1}. \qquad \text{II.}$$

If the experiment is varied so that $t = t_1$ then we have simply

$$l_1 = -\frac{w_2\, (t - t_2)}{w_1}. \qquad \text{III.}$$

If, however, the temperature of the water is found within the calorimeter, so that $t_2 = t_3$, the substance

[1] The same formula may be used to determine the heat of combination, only that the sign must be reversed (see ¶ 106).

dissolved being as before unchanged in temperature, we have for the latent heat of solution, which we call positive when heat is absorbed, the formula

$$l_1 = \frac{(w_2 + c)\,(t_2 - t)}{w_1}. \qquad \text{IV.}$$

EXPERIMENT XXXVI.

LATENT HEAT OF LIQUEFACTION.

¶ 101. **Determination of the Latent Heat of Water.** — Latent heats of liquefaction are determined in essentially the same manner as latent heats of solution (Exp. 35, II.). Instead, however, of dissolving a solid in a fluid, the solid is simply melted by the fluid. Knowing the weights, specific heats, and changes of temperature of the substances in question, we may calculate by the general formula (¶ 100, I.) the heat required to melt one gram of the solid; or, in other words, its latent heat of liquefaction.

It is evident that the liquid must exert no solvent action on the solid, otherwise we should have to allow for heat of solution (see Exp. 35). It is also necessary that the mixture be at a higher temperature than the solid, else the solid will not melt. It is well that the solid should start at its melting-point, since otherwise we must allow for the heat necessary to raise it to the temperature in question. A considerable time must generally be allowed for the process of melting; to shorten this time as much as possible, the mixture should be vigorously stirred. Observations of temperature should be taken from time to time (¶ 92, 8) during the process.

When ice is the solid employed, difficulty will be found in obtaining sufficiently small pieces free from water. The ice should be cracked into fragments weighing a few grams each, which are then to be wrapped up in cotton-waste and weighed. Any moisture formed by the melting of the ice should be absorbed by the waste.

The calorimeter is weighed empty, and re-weighed when about half-full of warm water. The temperature of the water should be about 50°, and is determined by a series of observations (¶ 92, 10); then ice is added until the calorimeter is nearly full. The ice should be handled by means of a portion of the cotton waste which surrounds it, and each fragment should be wiped as dry as possible before placing it in the calorimeter. The time occupied by this process and by the fusion of the ice should be noted (¶ 92, 9). The resulting temperature of the water must be accurately determined. The quantity of ice used should be found both by re-weighing the cotton waste and by re-weighing the calorimeter (¶ 92, 5).

¶ 102. **Calculation of the Latent Heat of Water.** — If w_1 is the weight of ice employed, t_1 its original temperature (that is, 0°) and s_1 its specific heat in the liquid state (that is, 1); if w_2 is the weight of water employed, t_2 its temperature reduced to the time of mixing (¶ 93), and s_2 its specific heat (that is 1); if c is the thermal capacity of the calorimeter calculated as in ¶ 91, t_3 its original temperature (the same as t_2), and t the temperature of the mixture; we have, substituting these values in formula II., ¶ 100, —

$$l_1 = \frac{(w_2 + c)\ (t_2 - t) - w_1 t}{w_1}.$$

From the numerator of this fraction should be subtracted a correction expressing the number of units of heat lost by the warm water while the ice is being melted. Since the water begins at a temperature t_2, and ends at a temperature t, its average temperature is $\frac{1}{2}(t_2 + t)$, nearly. Subtracting the temperature of the room, we have, approximately, the average excess of temperature. Multiplying as in ¶ 93 (3), by the number of minutes required to melt the ice, and also by the heat lost per minute when the temperature is 1° above that of the room (see ¶ 87), we have the correction in question. Evidently, if the average temperature of the water is the same as that of the room, no correction for cooling need be made.

The truth of the formula for the latent heat of water may be seen by the following considerations: Since w_2 grams of water and the equivalent of c grams of water (in the brass and other materials composing the calorimeter) are cooled from $t_2°$ to $t°$, the heat lost by the hot bodies amounts to $(w_2 + c) \times (t_2 - t)$ units. Subtracting from this the correction for cooling, we have a remainder which must represent the heat absorbed by the cold bodies; that is, the ice and the water formed by its liquefaction. Now w_1 grams of ice form w_1 grams of water at 0°;

and to raise this to t° requires $w_1 \times t$ units of heat. Subtracting this from the previous remainder, we have, therefore, the heat required to melt w_1 grams of ice. Finally, dividing by w_1, we have the heat required to melt 1 gram, or the latent heat in question.

EXPERIMENT XXXVII.

LATENT HEAT OF VAPORIZATION.

¶ 103. **Determination of the Latent Heat of Steam.** — There are many points of resemblance between the determination of the latent heat of vaporization and that of the latent heat of liquefaction (Exp. 36). Instead of melting a solid in a liquid, a vapor is condensed in a liquid. From the weights, specific heats, and changes of temperature in question, latent heats of vaporization may be calculated by the *same general formula* (¶ 100, I.) as latent heats of liquefaction.

The vapor must evidently have no chemical affinity for the liquid. The liquid must be at lower temperature than the vapor, in order that the latter may be condensed. The vapor should start as nearly as possible at its temperature of condensation, otherwise an allowance must be made for the heat given out in reaching this temperature. Care must, however, be taken that the vapor is freed from particles of liquid formed by its condensation, before it passes into the calorimeter.

When steam is used, it is passed from a generator (*a*, Fig. 83) through a trap (*b*), where nearly all its moisture is deposited. It will be seen in the diagram that the exit tube is completely surrounded, either by steam or by cork, until it reaches the calorimeter. If, therefore, this tube is well heated by a current of steam before the experiment, there is no reason why any condensation should take place within it.

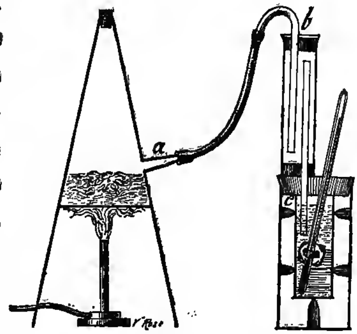


Fig. 83.

The calorimeter is weighed when empty, and reweighed with a quantity of water sufficient nearly to fill the inner cup, and as cold as possible. The temperature of this water is determined by a series of observations at intervals of one minute (¶ 92, 10); then the current of steam issuing from the trap is turned suddenly into the water. The water is stirred vigorously by twisting the stem of a thermometer to which a stirrer is attached. When the temperature of the water has risen as much above that of the room as it was below it before the admission of steam, the trap is taken away from the calorimeter, and the resulting temperature determined by another series of observations. The time used in heating the water to the required temperature should be as small as possible, to avoid errors due to gain or loss of heat; but if the *average* temperature agrees with that of the room, no correction for cooling need be applied (see ¶ 102). The weight of steam condensed is found by re-weighing the calorimeter, and the temperature of this steam determined by an observation of the barometer (see ¶ 69, II.).

¶ 104. **Calculation of the Latent Heat of Steam.** — If w_1 is the weight of steam condensed, s_1 the specific heat of the liquid formed by its condensation (that is, 1),[1] and t_1 its original temperature (let us say 100°, but see Table 14); if w_2 is the weight of water, s_2 its specific heat (that is, 1) and t_2 its original temperature; if c is the thermal capacity of the calorimeter, t_3 its original temperature (the same as t_2), and t the temperature of the mixture; we have, substituting these values in the general formula (¶ 100, I.), —

$$l_1 = \frac{(w_2 + c)\ (t - t_2) - w_1\ (100 - t)}{w_1}.$$

To the numerator of this fraction should be added the heat (if any) lost in cooling, since this is also at the expense of the steam.

The formula may also be established by a process of reasoning similar to that used in ¶ 102. To raise the equivalent of $w_2 + c$ grams of water $(t - t_2)$ degrees requires $(w_2 + c) \times (t - t_2)$ units of heat. Part of this was furnished by the w_1 grams of water at 100° (nearly) in cooling to t°. This part is clearly $w_1\ (100 - t)$. Subtracting this from the total heat

[1] The specific heat of water varies from 1.000 at 0° to 1.013 at 100°, having a mean value of about 1.005.

received by the water, we have that given up to it by w_1 grams of steam in the act of condensation; hence, dividing by w_1, we have the heat given out by one gram of steam at 100° when condensed into water at 100°; that is, the latent heat in question.

EXPERIMENT XXXVIII.

HEAT OF COMBINATION.

¶ 105. **Determination of Heats of Combination.** — The same method, essentially, is employed for the determination of heats of combination as for heats of solution (Experiment 35); the only difference being that the solvent has a chemical affinity for the substance dissolved. From the weights, specific heats, and changes of temperature of the materials involved, the heat of combination may be calculated by the general formula (¶ 100, I.). Heats of combination are, however, called positive when the result of mixture is to raise the temperature of the constituents.

(1) Zinc and Nitric Acid. — A gram of pure zinc filings is to be dissolved in at least fifty times its weight of dilute nitric acid. The student should determine by a preliminary experiment what strength of acid may be required to ensure rapid solution without danger of accident from excessive effervescence. This will depend largely upon the fineness of the zinc. When "zinc dust" is used, very dilute acids must be employed. The zinc dust should be

poured into the acid, not the acid on the zinc dust. The inner cup of the calorimeter (Fig. 71, ¶ 85) should be replaced by one of glass (¶ 92, 1), the thermal capacity of which must be calculated as in ¶ 91. The glass cup is then nearly filled (¶ 92, 8) with the dilute acid at a temperature below that of the room. This temperature must not, however, be so low as to arrest the chemical action. The process of solution may be greatly accelerated by the use of a platinum-stirrer;[1] but a brass stirrer coated with asphaltum may be employed (see ¶ 92, 1). The quantity of dilute acid used must be found by weighing the calorimeter with and without it; and the rise of temperature of this acid must be determined by a series of observations of temperature (¶ 92, 10) both before and after the experiment. It is well also to re-weigh the calorimeter after the experiment, to guard against any loss of material (¶ 92, 5). The loss of weight due to the escape of nitric oxide gas will hardly be detected.

(2) Zinc Oxide and Nitric Acid. — The experiment is now to be repeated with a quantity of zinc oxide which would be formed by the combustion of 1 gram of zinc. This quantity is 1.25 *g.*, very nearly. The same weight and strength of acid are to be used as before (1); but the temperature should be very little below that of the room.

[1] Currents of electricity generated by the contact of platinum and zinc assist the chemical action. It is, indeed, stated by some authorities that in the absence of such currents *perfectly pure* zinc is not attacked by dilute acids.

The density of the acid used should be determined roughly as in ¶ 40.

From the results of this experiment the student is to calculate (as in ¶ 106, below) the number of units of heat given out by 1 gram of zinc in uniting with an excess of dilute nitric acid, also what part of this heat is due to its uniting with the oxygen of the acid. The heat of combination of zinc with nitric acid will be found to have an important bearing upon problems relating to electric batteries in which zinc is the dissolving element and nitric acid the oxidizing agent (§ 145).

¶ 106. **Calculations relating to Heat of Combination.** — It is necessary, in general, to find the specific heat of the liquid used for a determination of heats of combination (see Experiment 34). The specific heats of certain solutions, amongst them nitric acid, may be found, when their densities are known, by Table 30. In calculating the thermal capacity of a calorimeter, the specific heat of the glass composing the inner cup may be taken as 0.19.

If w_1 is the weight of zinc employed, s_1 its specific heat (.095), t_1 its original temperature; if w_2 is the weight of acid employed, s_2 its specific heat (from Table 30), and t_2 its original temperature reduced (see ¶ 93, 2) to the time of solution; if c is the thermal capacity of the calorimeter, t_3 its original temperature (the same as t_2) and t the temperature of the mixture, we have for the heat of combination h (substituting h for l in the general formula of ¶ 100, and changing signs, since h would be negative if heat were absorbed),

$$h = \frac{(w_2 s_2 + c)(t - t_2) + w_1 s_1 (t - t_1)}{w_1}. \qquad \text{I.}$$

If, as in the experiment, a comparatively large quantity of acid is employed, the second term of the numerator may be neglected. When, moreover, 1 gram of zinc is used, $w_1 = 1$, and we have,

$$h = (w_2 s_2 + c)(t - t_2), \textit{ nearly}. \qquad \text{II.}$$

The truth of the last formula is sufficiently evident, since s_2 is the thermal capacity of 1 gram of the acid, $w_2 s_2$ must be that of w_2 grams; and this added to the thermal capacity (c) of the calorimeter must represent (neglecting the 1 gram of zinc) the total thermal capacity. In the formula (II.) the total thermal capacity is simply multiplied by the number of degrees rise in temperature. This must give the number of units of heat developed by the combination of the zinc with the acid.

The heat of combination of zinc oxide may be calculated by formula I. To find the heat given out by a quantity of zinc oxide (1.25 grams, nearly) which contains 1 gram of metallic zinc, this heat of combination must be multiplied by 1.25. The same result may be obtained directly by formula II. if, as in the experiment described, we have employed 1.25 grams of zinc oxide.

The chemical reaction which takes place when zinc is dissolved in nitric acid may be divided theoretically into two stages: first, the combination of 1 gram of zinc with oxygen, which is obtained by the decomposition of a part of the nitric acid,[1] thus:

$$\underset{\text{Zinc.}}{Zn} + \underset{\text{Oxygen.}}{O} = \underset{\text{Zinc oxide.}}{Zn\,O}; \qquad (1)$$

and, second, the combination of the 1 25 grams of zinc oxide thus formed with more of the nitric acid to form zinc nitrate, thus:

$$\underset{\text{Zinc oxide.}}{Zn\,O} + \underset{\text{Nitric acid.}}{N_2O_5{\cdot}H_2O} = \underset{\text{Zinc nitrate.}}{ZnO{\cdot}N_2O_5} + \underset{\text{Water}}{H_2O}. \quad (2)$$

We have already found the heat developed by the process as a whole. We have also found the heat developed in the second stage of the process, namely, the union of 1.25 grams of zinc oxide with nitric acid. The difference between these two quantities of heat must (by the principle of the conservation of energy) be equal to the heat developed by 1 gram of zinc in combining with oxygen extracted from nitric acid.

If, for example, 1 gram of zinc dissolving in 100 grams of nitric acid of a certain strength gives out

[1] Nitric acid, thus deprived of its oxygen, may be reduced to nitrous acid, nitric oxide (gas), or even to ammonic nitrate. The reactions are as follows: —

$2\,Zn + 3\,N_2O_5 \cdot H_2O = 2\,ZnO \cdot N_2O_5 + 2\,H_2O + N_2O_3 \cdot H_2O$ (nitrous acid).

$3\,Zn + 4\,N_2O_5 \cdot H_2O = 3\,ZnO \cdot N_2O_5 + 4\,H_2O + 2\,NO$ (nitric oxide).

$4\,Zn + 5\,N_2O_5 \cdot H_2O = 4\,ZnO \cdot N_2O_5 + 3\,H_2O + (H_4N)(NO_3)$ (ammonic nitrate).

Nitrous acid may be formed by the reduction of strong nitric acid. The presence of nitric oxide gas may usually be recognized by the red fumes which are generated when nitric acid is reduced. Ammonic nitrate is formed only in very weak solutions (Wurtz, Chimie Moderne, p. 169).

1,500 units of heat, while an equivalent (1.25 grams) of zinc oxide gives out only 400 units of heat, it is evident that 1500 − 400, or 1100, units of heat are due to the combination of 1 gram of zinc with the oxygen of the acid.

¶ 107. **Heat of Combustion.** — We have seen in the last section how we may find indirectly the amount of heat given out by a gram of a given material when it combines with the oxygen of an acid. This heat varies greatly according to the difficulty of extracting the oxygen in question. If, for instance, as in sulphuric acid, the oxygen must be taken away from hydrogen, for which it has a great affinity, nearly three fourths of the energy will be spent in decomposing the acid. In the case of nitric acid, less difficulty is encountered; since nitric acid is more readily decomposed (see footnote, ¶ 106). Even, however, in the case of chromic acid, in which the oxygen approaches very nearly its condition in the free state, the heat of combination with oxygen will differ somewhat from the result which we should obtain by burning a metal in oxygen gas.

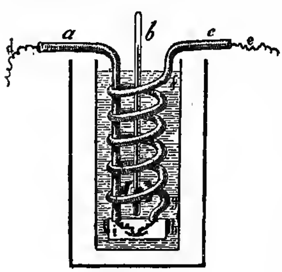


FIG. 84.

The heat given out by one gram of a substance when burned in oxygen is called its heat of combustion in oxygen. It may be determined directly by an apparatus shown in Fig. 84. The substance in question is placed in a deflagrating spoon, *i*, contained in a water-tight chamber, *h*; oxygen (or air) is admitted to this chamber by the tube *a*, and the gaseous products of combustion, if any, escape through the spiral tube *gfc*. The whole system of tubes is surrounded by water, contained in a calorimeter of the ordinary sort. When the temperature of the water has been observed, the substance is ignited by a current of electricity. From the rise of temperature and the thermal capacities of the calorimeter and its contents, the heat of combustion is calculated.

To determine the heat of combustion of a gas with this apparatus, a third tube must be added to supply the gas. A much simpler device consists, however, of a small metallic cone soldered into the bottom of a calorimeter. The cone ends above in a spiral tube, surrounded by water. A gas jet burned beneath this cone will give up nearly all of its heat to the water. The quantity of gas used is measured by a gas-meter. The determination of heats of combustion in general is an exceedingly difficult problem, but the ambitious student may be encouraged to attempt a rough determination of the heat of combustion of coal-gas or alcohol with a simple apparatus like the one described.

EXPERIMENT XXXIX.

RADIATION OF HEAT.

¶ 108. **The Pyroheliometer.** — A simple form of pyroheliometer (*πῦρ*, fire, heat; *ἥλιος*, sun; *μέτρον*, measure), or instrument for measuring the heat radiated by the sun, consists of a hollow tin box (Fig. 85) filled with water. One of the outer surfaces of the box is blackened, so as to absorb most of the heat which falls upon it. This surface is turned perpendicularly to the rays, the intensity of which is to be measured. The temperature of the water is observed by a thermometer passing through a hole in the side of the box. The number of heat units absorbed is calculated from the rise of temperature and thermal capacity of the vessel and its contents, as in other experiments in calorimetry. An allowance for cooling is made by watching the thermometer when the instrument is in shadow. It is found in this way that the solar radiation may amount to nearly 2 units of heat per minute on each square centimetre of surface.

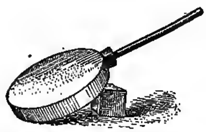


Fig. 85.

The pyroheliometer may also be used to measure the heat radiated by a candle, or any other source of heat; or it may be employed simply to compare two sources with each other. In all such experiments it is obvious that the distance of a given source of heat must be taken into account. It will be found, for instance, that the heat radiated by an ordinary candle-flame at a distance of about 2 *cm.* may be as intense as the sun's heat. At the distance of a decimetre, the heat from the candle could hardly be detected by a pyroheliometer.

¶ 109. **Application of the Law of Inverse Squares.** When a person stands midway between two sources of heat which are equal in every respect, he feels of course equal intensities of radiation. If, however, one of these sources is much more powerful than the other, he must approach the smaller of the two in order that the warmth from both may seem to be the same. Let the power of the first source be x, and the distance from it a; let the power of the second source be y, and the distance from it b; then according to the law of inverse squares (§ 94) the effects of the two sources will be proportional to $x \div a^2$ and to $y \div b^2$, respectively. If the two effects are equal, it follows that

$$x \div a^2 = y \div b^2; \text{ or } x : y :: a^2 : b^2.$$

It thus appears that the powers of any two sources of radiant heat are to each other *directly as the squares* of the distances at which they produce equal effects.

The same reasoning may be applied to two sources of light, to two sources of sound, or to any two sources of radiant energy, the effect of which diminishes as the square of the distance increases.

We have, accordingly, a principle by which we may compare any two sources of energy of the same kind; namely to find two distances, *a* and *b*, at which equal effects are produced.

To test the equality of two effects with any degree of precision, it is necessary to employ a "differential" instrument of some sort; that is, an instrument which is constructed especially to indicate the difference between two effects. The instrument must be so delicate that in the absence of any indication, we may assume that the two effects are equal. The methods for the comparison of two sources of heat about to be described, will be found to belong to the general class known as "null methods" (§ 42).

¶ 110. **The Differential Thermometer and the Thermopile.**—I. A differential thermometer, useful for the comparison of two sources of radiant heat, may be constructed as follows: two cylindrical metallic boxes, *d* and *e*, about 10 *cm.* in diameter, and 1 *cm.* deep, are made out of the thinnest brass, and fastened by a layer of wax to the support *bh*. The glass U-tube or gauge, *fg*,

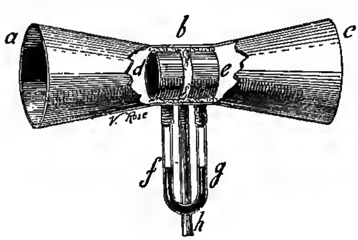


Fig. 86.

contains a little colored liquid, and is attached by rubber couplings to the boxes *d* and *e*, so that the system may be air-tight. The outer faces of the boxes, *d* and *e*, are coated with lampblack, to absorb heat; the sides may be covered with wool to prevent loss of heat. The two conical shields, *a* and *c*, blackened inside, are finally added to cut off lateral radiation.

A very slight amount of heat falling on the blackened surface of either of the cylinders, *d* or *e*, will cause an expansion of air within the cylinder in question. Unless this is offset by an equal expansion of air due to an equal amount of heat falling on the other cylinder, the level of the liquid in the gauge *fg* will be affected.

II. An instrument which may be made much more sensitive than a differential thermometer is represented in Fig. 87, and in *de*, Fig. 88. It consists of an alternate series of strips of bismuth and antimony, joined together in a sort of zigzag. Only four strips are shown in the figure, but a much greater number is generally used. The combination is known as a "thermopile," or "heat-battery." It is usually mounted on a support (Fig. 88),

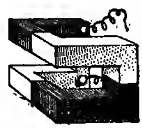


FIG. 87.

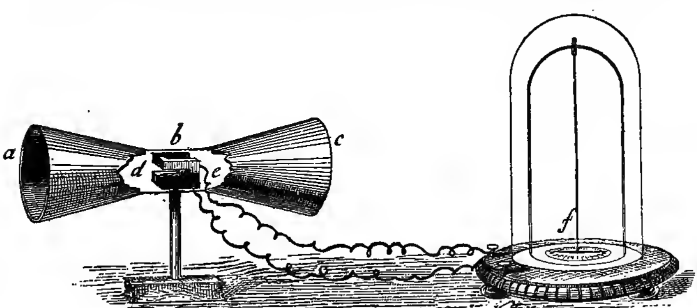


FIG. 88.

and provided with two conical shields, *a* and *c*. When heat falls on either set of junctions, as *d*, a current of electricity is generated (see Exp. 95). This current is measured by a galvanometer, *f*, the terminals of which are connected by wires with the terminals of the thermopile. The deflection of the galvanometer needle is reversed if heat falls on the opposite face of the thermopile, *e*. When equal amounts of heat fall on both the faces, *d* and *e*, the needle should not be deflected.

It would be out of place here to discuss the principles which underlie the phenomena in question. The student should for the present regard a thermopile and galvanometer simply as a convenient substitute for a differential thermometer and U-tube.

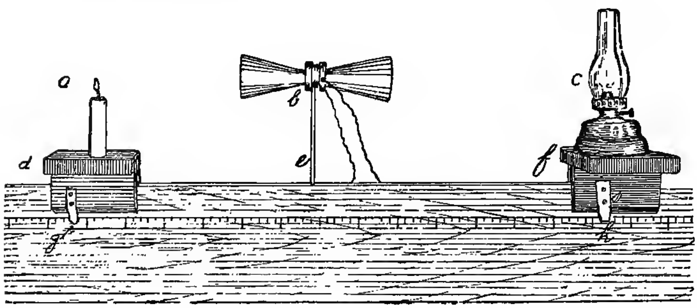


FIG. 89.

¶ 111. **Determination of Candle-Heat-Power.** — A thermopile connected with a galvanometer, as in Fig. 88, is mounted on a fixed support (*be*, Fig. 89), in the middle of a horizontal graduated rail (*gh*). The needle of the galvanometer is made to point to zero (¶ 112, 7). Two movable supports, *d* and *f*, constructed so as to slide along the rail, are placed one on each side of the thermopile. A candle (*a*) and a small kerosene lamp (*c*) are then mounted on the supports, *d* and *f* respectively, so that the flames may be on a level with the thermopile (¶ 112, 5). The supports are then to be set permanently at such distances from the thermopile (¶ 112, 2) that either flame alone will cause a deflection of the galvanometer of at least 45° (¶ 112, 1), but that both together will cause little or no deflection. The height of the lamp-flame is then adjusted, if necessary, until the deflection is reduced to zero.

The lamp and candle while still burning are next to be weighed as accurately as possible on a pair of open scales (Fig. 1, ¶ 2), and the time of weighing is to be noted in each case. The lamp and candle are then returned to their former positions on the supports *d* and *f*, where they are allowed to burn for, let us say, half an hour.

Meanwhile the distance of each from the nearer face of the thermopile is accurately determined by means of the markers (*g* and *h*), which should be just under the centres of the flames (¶ 112, 3). The distance (*de*, Fig. 88) between the faces of the thermopile must also be measured and allowed for (¶ 112, 4). If the needle of the galvanometer shows any deflection in the course of the experiment, it must be brought back to zero by increasing or diminishing the flame of the lamp. At the end of the half-hour, the candle and lamp are to be re-weighed in the same order as before, *while still burning.*

The candle and lamp are now to be replaced on their supports (*d* and *f* respectively), each of which is to be set permanently at the same distance from the thermopile as before, but on the other side of it (¶ 112, 8). The height of the lamp-flame is to be adjusted so as to neutralize the heat from the candle: and at the end of another half-hour, the lamp and candle are to be re-weighed, as before, while still burning.

Instead of a thermopile, a differential thermometer (¶ 110) may be employed, with essentially the same precautions (see ¶ 112). Instead of the kerosene lamp, an electric incandescent lamp may be used (Fig. 90). In this case it is necessary that the zinc-plates of the battery furnishing the electricity for the lamp should be weighed before and after the experiment. These plates should be well amalgamated with mercury to prevent unnecessary loss of material.

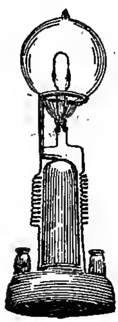


Fig. 90.

In any case the candle-heat-power of the lamp is to be calculated and reduced to the standard rate of consumption, as will be explained in ¶ 113.

¶ 112. **Precautions in the Determination of Candle-Power.** — (1) Before attempting an accurate comparison of two sources either of heat or of light, it is well to make sure that the instrument to be employed is sufficiently sensitive (§ 42). For this purpose it is first exposed to the radiation from the feebler source alone. To make a comparison, for instance, accurate within 1 per cent, the response must be 100 times as great as the minimum perceptible. The sensitiveness of the combination should, if necessary, be increased by bringing the source in question closer to the instrument until a sufficient response is obtained.

(2) It is important that one of the two sources compared should be at a fixed distance from the instrument throughout an experiment. When an oil-lamp or gas-flame is one of the sources, so that the height of the flame can be adjusted, it is well that both sources should be fixed; and for convenience in calculation, each distance may be made equal to some round number.

(3) The distance of the sources from the instrument may be most conveniently determined by means of markers (*g*, *h*, in Fig. 89). These markers should be in line with the centre of the source of light or heat (as, for instance, *h*), not at one side of it (like *g*). The student should confirm the indications of the markers by direct measurements. It should be remembered that the distances sought lie between the centre of a flame and the surface illuminated by it.

(4) Care must be taken in measuring the distances *ad* and *ec* to allow for the distance *de* (Figs. 86 and 88) between the two surfaces illuminated. This distance should be determined by direct measurement; for this purpose the conical shields must of course be removed.

(5) It is important that the rays of light or of heat should be equally inclined with respect to the two surfaces *d* and *e*. To help in securing this result, the surfaces should be made vertical, and the

sources of light or heat should be raised or lowered until they are on a level with these surfaces. Neither angle of incidence should exceed 20°. In this case slight differences in the angles of incidence, as in Figs. 96 and 98, will have no perceptible effect on the result.

(6) The conical shields *a* and *b* (Figs. 86 and 88) will serve to cut off lateral radiation. It is, however, necessary to place large black screens *behind* two sources of light which are being compared, so as to shut out light from all other sources. A dark room is of great service in photometry; a room of uniform temperature is equally important in measurements of radiant heat.

(7) Before comparing two sources of heat or light, it is well to make sure that the instrument to be employed is not affected by radiation from the windows or from the walls of the room (§ 32). The liquid in a differential thermometer should stand at the same level, for instance, in both arms of the gauge. If it does not, the gauge should be temporarily disconnected so that the air-pressure may be equalized. The needle of a galvanometer connected with a thermopile should point to zero, otherwise it should be made to do so by twisting the thread by which it is suspended, or by placing a magnet in its neighborhood. If the two surfaces of a photometer do not appear equally dark, it is necessary to make a rearrangement of the screens, by which at least equality of illumination may be secured.

(8) To eliminate all errors arising from unequal

radiation from surrounding objects, and from any inequality in the surfaces illuminated, two determinations should always be made (see § 44). In one of these a given surface is illuminated by the weaker source of light or heat; in the other, it is illuminated by the stronger source. An error in the adjustment of the markers may also be eliminated in this way.

¶ 113. **Calculations relating to Candle-Power.** — The standard candle is defined as one, seven-eighths of an inch in diameter (six to the pound), burning 120 grains of spermaceti per hour. A paraffine candle does not give out quite so much light as a sperm candle under similar circumstances. It is thought that no perceptible error will be committed by substituting for a standard candle one of paraffine burning 8 grams per hour ($123\frac{1}{2}$ grains, nearly). An ordinary candle may of course burn a little more or less than the standard. Since the heat or the light is very nearly proportional to the rate of consumption, we find that the actual candle-power of a paraffine candle[1] is equal to one eighth the weight in grams of the paraffine burned in one hour. This gives us the quantity, x, in the formula of ¶ 109. Hence, if a lamp at a distance b has the same effect as x standard candles at the distance a, as regards either heat or light, we may find the number of standard candles, y, to which this lamp is equivalent by the formula —

$$y = \frac{b^2}{a^2} x.$$

[1] The heat radiated in all directions by an ordinary candle amounts to about 2 units per second. This is only a small part of the total

By the "candle-power" of a lamp is ordinarily meant the number of standard candles to which it is equivalent in respect to light (see Exp. 40). The number of candles to which it is equivalent in respect to the radiation of heat may be called its "*candle-heat-power.*" It is evident that the thermopile and the differential thermometer, which absorb all rays alike (whether visible or invisible), are instruments for determining the candle-heat-power as distinguished from the candle-light-power of any source.

It is interesting to reduce the candle-power of a lamp to the normal rate of consumption of a candle (8 grams per hour). We first divide the actual candle-power of the lamp by the number of grams burned in one hour to find the candle-power corresponding to 1 gram per hour; then we multiply the result by 8. A surprising similarity exists between the candle-powers of different materials when thus reduced to a common standard.

EXPERIMENT XL.

PHOTOMETRY.

¶ 114. **Determination of Candle-Power by means of a Photometer.** — I. BUNSEN'S PHOTOMETER. — A very fair comparison of two sources of light may be made by means of a scrap of white paper rendered trans-

quantity of heat generated by combustion, which amounts to about 20 units per second. Less than 4% of the radiant heat is visible as light.

lucent at the centre by a drop of oil or varnish. When such a scrap is held up in front of a light, the oil-spot appears bright, as in Fig. 91; when held behind a light, it looks dark, as in Fig. 92. If both

FIG. 91.

FIG. 92.

sides of the paper are equally illuminated, the spot may nearly or quite disappear. Usually, however, the oil-spot seems a little darker than the rest of the paper. It is necessary, therefore, to look at it from both sides. When it appears equally dark from both points of view, we may infer that the two sides of the paper are equally illuminated.

To make use of an oil-spot for a comparison of two lights, the paper (*b*, Fig. 93) is provided with

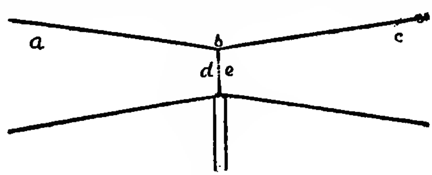


FIG. 93.

two shields, *a* and *c*, to cut off lateral radiation, and is mounted in the place of the thermopile (*b*, Fig. 89, ¶ 111) between a candle, *a*, and a lamp, *c*. The lamp-flame is adjusted as in ¶ 111 until the paper seems equally illuminated on both sides, *d* and *e*.

The distances of the lamp and candle, and the weights burned in one hour by each are found in the same manner as with the thermopile.

In practice the form given to the shields is not generally conical, as in the case of a thermopile, but barrel-shaped (see Fig. 94). The object of this is to facilitate the examination of the oil-spot through two openings, *a* and *c*. Such an instrument is called a Bunsen's photometer.

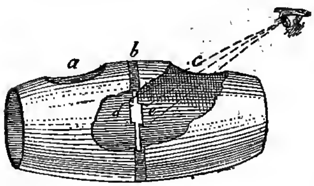


Fig. 94.

The general precautions in the use of a photometer have already been enumerated (¶ 112). Certain special precautions will be considered in ¶ 115. The results are to be reduced as in ¶ 113.

II. Rumford's Photometer. — If the diaphragm and shields used in Bunsen's photometer (Fig. 93) are removed, leaving only the rod by which they were supported, and if a piece of paper (*ac*, Fig. 95) is fastened to this rod so as to be equally inclined to the rays falling upon it from the lamp and from the candle (Fig. 89); then when the flames are placed at such distances as to give equal amounts of light at the point *b*, the shadows *ab* and *bc* (Fig. 95) cast by the rod should be equally dark. The instrument, thus arranged, is a form of Rumford's photometer, depending upon the principle that equal illuminations cause equal shad-

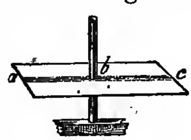


Fig. 95.

ows; it might be substituted for a Bunsen's photometer for a rough comparison of two lights.

It is obvious, however, that a slight inclination of the paper might expose it very unequally to the rays from the two sources, and thus vitiate the results. To lessen errors from this source, both lights are in practice placed on the same side of the rod, *b* (Fig. 96), the two shadows of which, *d* and *e*, are thrown horizontally on the vertical surface, *de*. When these shadows have been made equally dark by adjusting the distances of the lamp and candle, or the height of the lamp-flame the two lights are to each

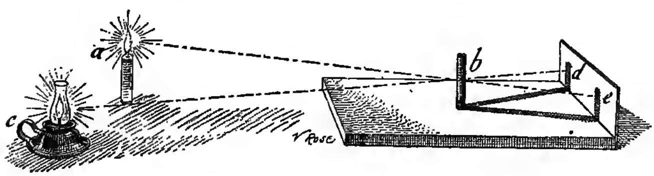


FIG. 96.

other as the squares of the distances *ae* and *cd*. These distances are therefore to be measured.

The student should observe that the distance of the rod from the screen may affect the sharpness of the shadows, but not their darkness, which depends simply on the distance of the lights *from the screen*. It is well to have the rod close to the screen, in order that the two shadows may be near together, but not so close that the shadows overlap. A small amount of light from the windows need not vitiate the result, provided that it casts no shadow on the screen. If it does, the light must be cut off.

The weights burned by the lamp and candle in one hour are found as with a Bunsen's photometer (I.), or with a thermopile (¶ 111); and the results are reduced in the same manner (¶ 113).

III. Box Photometer. — Instead of using a rod, as in Rumford's photometer (Fig. 96), it is sometimes advantageous to employ a partition (*b*, Fig. 97). One half (*d*) of the screen, *de*, may thus be illuminated by the candle (*a*), and the other half (*e*) by the lamp (*b*). The screen is made translucent, so that the intensities of illumination may be compared with the eye behind it.

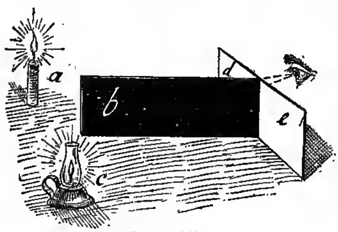


Fig. 97.

This form of photometer is particularly useful when it is possible to enclose the whole apparatus in a box. A horizontal section of such a box is shown

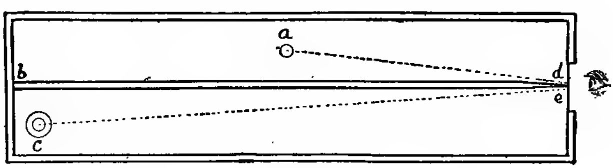


Fig. 98.

in Fig. 98. The distances *ad* and *ce* are measured directly by a metre rod.

If the angles of incidence, *adb* and *ceb*, differ by more than 10°, it may be well to alter the screen *de* slightly so that its inclination to both rays may be the same (¶ 112, 5).

An arrangement in which the distances of the lamp and candle are adjustable is represented in Fig. 99, which gives a view of the apparatus from above. The lights are contained each in one of the sliding boxes, *e* and *f*. The top of the main box is closed as far as the ends of the sliding boxes by a set of covers (*b*, *c*, and *d*). All direct light is thus excluded from the photometer. A cloth cover, *a*, may be thrown over the head when it is desired to compare very feeble illuminations.

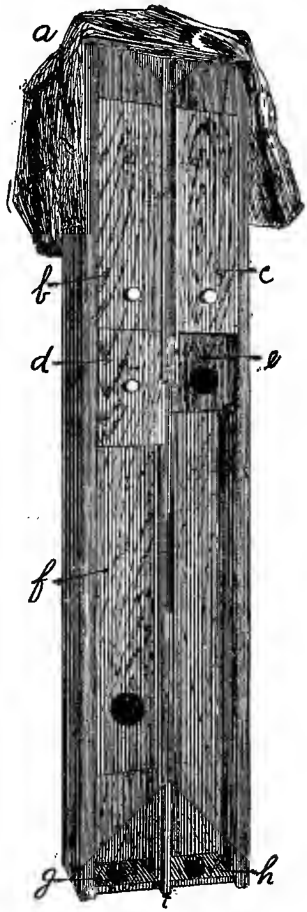


Fig. 99.

Box photometers may also be constructed on Bunsen's or on Rumford's principle. They have the advantage of a dark room without its expense or inconvenience.

The determinations of candle-power, and the reduction of the results, are made in precisely the same manner as in II. with a Rumford's photometer. See also ¶ 113.

¶ 115. **Errors in Photometry due to Color Blindness.** — Light is essentially a physiological as distinguished from a physical quantity. There is no standard by which we may prove that one kind of light is more brilliant than another. A person who is "color-blind" may consider a blue light brighter than a red light, which to a person of "normal vision" may seem much the brighter of the two. All eyes are in a certain sense color-blind, since the greater part of the rays which fall upon them are wholly invisible.

The modern theory of color may be stated briefly as follows: There are three principal effects produced on the eye by rays of light. The first is to excite in the retina a sensation which we call red. This is due mostly to waves of light between 60 and 70 millionths of a centimetre in length. The second is to excite a sensation which we call green. Nearly all rays of light produce this effect (green) to a certain extent; but it is caused most strongly by waves between 50 and 60 millionths long. The third effect is a sensation which we call violet, due to waves from 40 to 50 millionths in length. When waves[1] of different lengths are mixed, complex sensations are produced. Red and green rays together may produce, for instance, a sensation which we call yellow; violet and green may produce blue; red and violet may produce purple; while red, green, and

[1] The student must distinguish carefully the effects of mixing waves of light from the effects of mixing paints. These effects are in a certain sense opposite.

violet rays together may cause the sensation which we are familiar with in ordinary white light. Again, a single wave may produce two sensations: one 60 millionths of a centimetre long will, for instance, produce the double sensation which we call yellow; while one 50 millionths long will appear blue. The various hues which we find in different objects are due to the proportions, simply, in which the sensations of red, green, and violet are excited. The eye is capable of no fourth sensation by which the effect can be modified. According to this theory, two lights should be compared, (1) by means of the red rays, (2) by means of the green rays, and (3) by means of the violet rays which they emit.

The simplest way to compare the candle-power of two lights with respect to red rays is to hold a piece of ordinary "ruby glass" before the eye in observing the brilliancy of the two surfaces illuminated. Green and violet glasses may similarly be employed for the green and violet rays; but pure violet glass can hardly be obtained. It is better to use a piece of ordinary glass stained with violet-aniline containing a trace of Prussian blue.

With these precautions, personal errors in photometry might undoubtedly be diminished, particularly in the comparison of lights of different hues or tints; but as long as the eye alone is used to compare the brilliancy of two surfaces, it is doubtful whether the errors of a photometric comparison can ever be greatly reduced. The "probable error" of such a comparison may be estimated at about 5 per cent.

EXPERIMENT XLI.

PRINCIPAL FOCI.

¶ 116. **Determination of the Principal Focal Length of a Converging Lens.** — The principal focal length of a lens may be defined (see § 103) as the distance at which it brings parallel rays to a focus. An "optical bench," convenient for the measurement of focal lengths, is represented in Fig. 100. It consists of a wooden plank, set up edgewise, with three sliding supports, *d*, *e*, and *f*, the positions of which are deter-

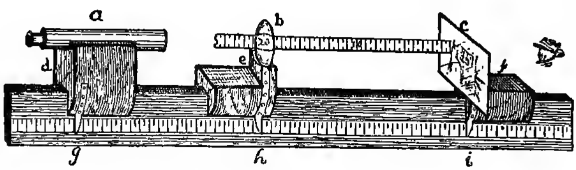


Fig. 100.

mined respectively by the markers *g*, *h*, and *i*. The apparatus is in fact the same as, or similar to, one already employed in Experiments 39 and 40 (see Fig. 89).

(1) Ordinary Method. — To find the principal focal length of a lens, it is mounted (see *b*, Fig. 100) on one of the slides (*e*), directly over the marker (*h*) (see ¶ 112, 3); and a translucent screen (*c*) is attached to another slide (*f*) directly over the marker (*i*). The third slide (*d*) is temporarily removed, so that the rays from distant points (at the

left of the figure) may be focussed by the lens (*b*) on the screen (*c*). That this may be possible, the bench should be set up in front of an open window commanding a distant view.[1] Either houses or trees may afford suitable images. It is assumed, however, that the objects in question are so far off that rays from any point in these objects may be considered parallel. They should be at least a hundred times as far from the lens as the lens is from the screen.

The distance between the lens and the screen is to be adjusted so that the image thrown on the screen may be as distinct as possible. The image may be viewed either from in front or (since the screen is translucent) from behind. The number of details visible in the image is the test of its distinctness most easily applied. When difficulty is found in the precise adjustment of distance, the screen is first brought so near the lens that the most minute details disappear; then it is placed so far from the lens that the same result is obtained. Midway between these two positions is the principal focus of the lens.

The distance of the principal focus from the centre of the lens is taken as the measure of its principal focal length. It is determined by observing the positions of the two markers, *h* and *i*, with respect to the scale close behind them. If either of the markers is out of line with the lens or screen, as the case may be, an error will evidently be introduced into the result

[1] In the absence of any suitable object, we may use a projecting lantern, focussed so as to give parallel rays. To obtain this result, the slide must be placed in the principal focus of the projecting lens.

(¶ 112, 3). To eliminate this error, we may interchange the places of the lens and screen. The whole bench must then be turned round so that an image may be formed by the lens on the back of the screen. The thickness of the screen should be so small that it need not be taken into account. If either of the markers is out of line, the distance between the lens and screen will apparently be increased in one case but diminished in the other case, and by an equal amount. The average of the two distances indicated by the markers is, therefore, the true distance from the centre of the lens to the screen.

If there is a second scale on the farther side of the bench, there will be no need of turning it round. We have only to turn round the slides *e* and *f*.

It is well to confirm the accuracy of the scale or scales in question by a direct measurement between the thin edge of the lens and the screen. The measuring rod must be held perpendicular to the screen, as in Fig. 100. One measurement should be taken from the farther edge of the lens, another from the nearer edge, and a third from the top of the lens. If any marked differences are observed, the lens should be readjusted until these differences disappear.

(2) Method of Parallax. — Instead of using a screen (*c*, Fig. 100), we may employ a wire netting or simply a vertical wire. If the wire coincides in position with the image formed by the lens, no "parallax" (§ 25) will be apparent when the eye is moved from side to side. If the wire is behind the image, it will seem to follow the eye; or if it is in

front of it, it will always appear to move in the opposite direction (see diagrams, Fig. 103, ¶ 118). The phenomena of parallax afford in fact a very delicate test by which a wire may be placed exactly in the image, and the position of the image thus accurately determined. This is called focussing by the method of parallax. The distance of the image from the lens is found from the indications of the markers, and confirmed by direct measurements as before (see 1).

(3) INDIRECT METHOD. — Another way of finding the principal focus of a lens involves the use of a telescope, which has been adjusted so that parallel rays striking the object-

FIG. 101.

glass (g, Fig. 101) are brought to a focus at a point c where cross-hairs are placed. The first step in focussing a telescope is always to make the distance of the eye-piece (b) from the cross-hairs (c) such that the latter may be seen as clearly as possible through the opening a. This is done by sliding the tube d within the tube e. Then the tube e is pushed into or drawn out from the tube f so that the cross-hairs may coincide with the image at c. In the last adjustment, care must be taken not to disturb the distance of the eye-piece from the cross-hairs, unless, as sometimes happens, the focus of the eye has changed so that the cross-hairs are no longer visible; in this case the first adjustment must be repeated before the second can be made. In some telescopes the method of focussing by parallax (see 2) can be used, but generally we have to depend simply on the distinctness of the image (see 1). If the telescope is accurately focussed, the image and the cross-hairs should both appear distinct to the eye.

A telescope thus focussed is mounted as in Fig. 100 at any point, *a*, in front of a lens, *b*. It will probably be found that a page of fine print replacing the screen, *c*, may be easily read through the combination. The distance of the page from the lens should be varied if necessary, so that the print may seem as distinct as possible.

The student should note that, owing to the parallelism of the rays from a given point in passing between the lens and the telescope, the distance between the lens and telescope does not affect the focus.

The principal focal length of a lens has been defined as the distance from the lens at which parallel rays are brought to a focus; it might also have been defined as the distance from an object at which rays diverging from it are rendered parallel by the lens. It is evident that the rays diverging from any point of the printed page (*c*) must be rendered parallel by the lens (*b*) in order to be visible in the telescope (*a*); for this telescope has been focussed for parallel rays, and cannot, therefore, be in focus for any others. It follows that the distance from the lens to the screen is equal to the principal focal length of the lens; the latter is, therefore, to be measured as in the methods previously described (see 1 and 2).

(4) Color Method. — Instead of depending entirely upon the distinctness with which the print can

be read, we may observe the colors with which each black letter seems to be surrounded. Unless the lens is of peculiar construction, so as to focus all rays alike, it will be found impossible to avoid this phenomenon. Let us suppose that the red rays are accurately focussed; then the green and violet rays will be just out of focus, and hence somewhat scattered. The spaces which would otherwise be perfectly black will, therefore, have a bluish tinge (¶ 115), particularly near the edges of the letters. In the same way, if the violet rays are just in focus, reddish or yellowish borders will encroach upon the spaces in question. It is thus evident that the principal focus of a lens depends upon the kind of light employed. Green light may be taken as the standard. To focus for the green rays, the distance of the lens from the print must be such that the black spaces have very narrow borders of a neutral tint; that is, one which inclines neither to red nor to blue.

To obtain the best results with the color-method, a perforated metallic lamp chimney should be substituted for the page of print (see Exp. 42). The measurements of distance are made and reduced as in methods previously described (see 1, 2, and 3).

The student should make at least two determinations of the principal focal length of a lens, — one by the ordinary method, the other by the indirect method, (3). The other methods will be met in experiments later on. The results of different methods should agree within limits which may be attributed to errors of observation.[1]

EXPERIMENT XLII.

CONJUGATE FOCI.

¶ 117. **Determination of Conjugate Focal Lengths of Lenses.** — A screen, *c* (Fig. 102), and a lens, *b*, are to be mounted on movable supports, as in Exp. 41; but in place of the telescope (*a*, Fig. 100) the support, *d*, is to carry a lamp, *a*, having a metallic chimney with several small holes in it. The marker, *g*,

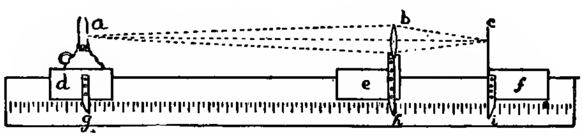


Fig. 102.

must be in line with the perforations in the chimney, not, as in ¶ 112, (3) with the flame, since the former and not the latter will be focussed upon the screen.

[1] If in (1) or (2) the object is too near, so that the rays from it striking the lens are perceptibly diverging, the distance of the screen from the lens must evidently be increased in order that these rays may be focussed upon it. On the other hand, if in 3 or 4 the telescope is focussed upon *the same object*, the distance of the print from the lens must be diminished in order that the rays which pass through the lens may be slightly divergent; for the telescope, being focussed for slightly divergent rays, can be in focus for no others. By averaging a result obtained by (1) or (2), with a result from (3) or (4), the true value of the principal focal length may be calculated, even when a distant view cannot be obtained.

Throughout this experiment the color method of focussing (see ¶ 116, 4) is to be used.

(1) The lens is first placed in the middle of the bench *gi*, with the lamp at a distance from it equal to twice its principal focal length, determined in Experiment 41. The screen is then moved until an image of the perforations of the chimney appears upon it; the distance between the lamp and screen is then measured. The lens will probably be found to be about half-way between the lamp and screen; if it is not exactly in the middle, it should be placed there, and the focus, if necessary, readjusted by increasing or diminishing the distance of both the lamp and the screen by an equal amount in each case. The distance of the screen from the lamp will be about four times the principal focal length of the lens.

(2) The lamp and screen are next separated by a distance equal to about five times the principal focal length of the lens; and the lens is placed so that the chimney may be focussed upon the screen as before. Two positions will be found, — one nearer the lamp, the other nearer the screen (see Fig. 102). In the first position, the image of the chimney will be magnified; in the second it will be diminished in size (see § 104). The second image will be the more distinct; the first, unless carefully searched for, may even escape detection. The distances *ab* and *bc* are to be determined in each case.

(3) The lamp and screen are finally separated as far as possible; and, as before, the lens is placed so as to throw first a magnified and second a reduced

image of the chimney upon the screen. In both cases, the distances ab and bc are to be determined.

The distances ab and bc in each of the cases (1), (2), and (3), are called conjugate focal lengths (§ 103). They may be determined by the readings of the markers g, h, and i. In (1) the *sum* of the distances ab and bc is alone needed, and should be confirmed by a direct measurement with a metre rod. If the markers are found to be tolerably accurate, the readings of the scale in (2) and (3) need not be confirmed by direct measurement.

From the results of each adjustment, the principal focal length of the lens is to be calculated by the formula derived from that in § 103: —

$$f = \frac{ab \times bc}{ab + bc}.$$

The results should agree with those obtained in Experiment 41 within a limit which may be attributed to the *thickness of the lens*, which has been disregarded in the formulæ.

The student should notice that it is impossible to focus the lamp upon the screen (1) when the distance ac is less than four times the principal focal length of the lens, no matter where the lens is placed; (2) when the distance (ab) between the lamp and the lens is less than its principal focal length, no matter where the screen is placed; and (3) when the distance (bc) between the screen and the lens is less than its principal focal length, no matter where the lamp is placed.

It should also be noticed that in (2) and in (3) the distances *ab* and *bc*, at which a magnified image is produced, are equal respectively to the distances *bc* and *ab*, at which we obtain an image reduced in size; and that in every case the distance between two perforations in the chimney is to the distance between their respective images as the distance of the lamp from the lens is to that of the screen from the lens (§ 104).[1] It is hardly necessary to call attention to the fact that all the images are inverted.

EXPERIMENT XLIII.

VIRTUAL FOCI.

¶ 118. **Real and Virtual Foci of Mirrors.** — Rays of light may be brought to a focus by a concave mirror as by a converging lens. If in Fig. 102 (¶ 117) we substitute for the lens, *b*, a mirror with its concave surface turned towards the lamp, *a*, and at a sufficient distance from it, an inverted image of the lamp will be formed at a point *c*, between *a* and *b*. This image, which will be reduced in size, may be received upon a screen, provided that the latter is not so large as to cut off all light from the mirror. Again, if the screen (*c*) is at a sufficient distance from the mirror (*b*), a magnified image of the lamp may be thrown upon it by placing the lamp at some point,

[1] It is instructive to prove this by actual measurement. See Experiment 38 in the Elementary Physical Experiments, published by Harvard University.

a, between *b* and *c* (as in Fig. 104), provided that the lamp does not intercept all the rays reflected by the mirror towards the screen. In any case the *real image* (*c*) formed by the mirror is on the *same side* of the mirror (*b*) as the object (*a*), not as in the case of a lens, on the opposite side of it.

The distances *ab* and *bc* are called, as in the case of a lens, conjugate focal lengths. The principal focal length of a concave mirror may be found by determining the distance at which parallel rays (or rays from a sufficiently distant object) are brought to a focus, or by the formula of ¶ 117, applicable to conjugate focal lengths. These methods are particularly valuable in the case of mirrors whose curvature cannot be determined by means of a spherometer (Experiment 21). Evidently the focal lengths of a mirror depend solely on its curvature. The material of which it is composed does not, as in the case of a lens, have to be considered.

The images thrown by a concave mirror upon a screen are instances of real images. The image of an object seen in a plane mirror is a typical case of a virtual image (§ 104). If the eye is placed *behind* the mirror (where the image seems to be) no light whatever is perceived. A thermopile would feel no heat there, nor would photographic paper be affected. And yet, as far as points *in front* of the mirror are concerned, the optical, thermal, and photographic effects are the same as if a real object existed behind the glass.

The simplest way to locate a virtual image is by the method of parallax (¶ 116, 2). A short wire is mounted in place of the lamp (*a*, Fig. 102) on a support, *d*; a longer wire, *c*, is attached to the support *f*, and a piece of looking-glass is placed between the wires on a support, *e*, instead of the lens *b*. The height of the wires should be such that the point of the long wire, *c*, may be visible above the image of the wire *a*, reflected (as in Fig. 103) by the mirror. As the eye is moved from the farthest left-hand point (see 1 and 3, Fig. 103) at which both wires are visible, to the farthest right-hand point (see 2 and 4, Fig. 103), both *a* and *c* (one being really, the other virtually, behind the mirror) will move from

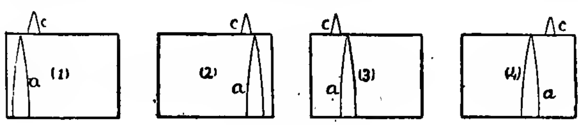


FIG. 103.

the left of the mirror to the right; but the one which is farthest off will apparently move farther than the other (see ¶ 116, 2). Thus if, as in (1) and (2), the point *a* moves completely across the mirror, while the point *c* only moves part way across it, we conclude that *a* is too far from (or *c* too near) the mirror, but if, as in (3) and (4), *c* moves wholly across while *a* moves only part way across, we conclude that *c* is too far from (or *a* too near) the mirror. By adjusting the distances *ab* and *bc* until no parallax (§ 25) is visible between *a* and *c*, the distance of the virtual image from the mirror may be determined.

It is found that the virtual image formed by a plane mirror is just as far behind it as the real object is in front of it.[1] If a mirror is slightly convex or concave, this will no longer be true. A comparison of the two distances *ab* and *bc* will serve therefore to detect any curvature in the surface of the mirror.

We notice that virtual images are never, like real images, inverted. When formed by a mirror they are always behind it. On the other hand, we shall see that the virtual focus of a lens is always on the same side as the object.

¶ 119. **Determination of Virtual Focal Lengths of Lenses.** — I. CONVERGING LENSES. — When the principal focal length of a lens exceeds the limit of the

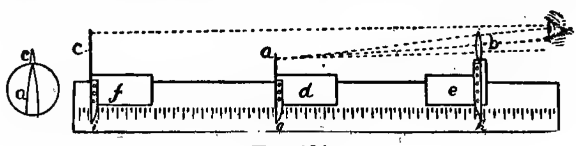


FIG. 104.

apparatus employed, it can be determined only by means of virtual foci. Two wires, *a* and *c*, are mounted on sliding supports, as in Fig. 104, on the same side of the converging lens (*b*) so that the top of the farther wire (*c*) may be visible just above the magnified image of the nearer wire (*a*) seen through the lens. The wires are then placed so that there may be no parallax (§ 25) between them when the eye is moved from side to side (see ¶ 118, Fig. 103). The virtual image of *a* then coincides with the real

[1] This may be shown by a simple geometrical construction. See Ganot's Physics, § 513, Deschanel, § 699.

point, c. The distances ab and ac are then measured, as in ¶ 117, and the principal focal length of the lens is calculated by the formula (see § 104),

$$f = \frac{ab \times bc}{bc - ab}.$$

II. Diverging Lenses. — With diverging lenses, focal lengths can be determined only by the method of virtual foci, since such lenses form no real images (§ 104). The method is essentially the same as that employed with converging lenses (see I.), except that the wire, a, viewed through the lens, b, must be further off than the wire, c, which is seen above or below it. It is well to substitute a broad netting or page of print for a, so that it may not be completely hidden by c.

The distances, ab and bc, are to be adjusted so that all parallax disappears between a and c; the virtual image of a will then coincide with c. The distances ab and bc are to be measured, and the value of f (which will be negative) is to be calculated by the same formula as before. It may be noted that a virtual image of distant objects is formed *between* a diverging lens and the objects in question, and at a distance (f) from the lens, which is sometimes called its (virtual) principal focal length.

The student should observe that a converging lens forms a virtual image *farther off* than the object looked at, while a diverging lens forms a virtual image *nearer* than the real object. Upon this fact depends in part the magnifying power of a converging lens, and the reducing power of a diverging lens.

The farther off an object is, the larger must it be in order that its image may occupy a given space on the retina; hence, the farther off we think it is, the greater will be our estimate of its dimensions.

In the arts, lenses are often numbered according to their principal focal length. A No. 12 spectacle lens is generally one which focusses distant objects at a distance of 12 inches. Near-sighted or diverging lenses are numbered on the same system. A No. 12 near-sighted lens combined with a No. 12 magnifying lens should form a perfectly neutral combination.

EXPERIMENT XLIV.

THE SEXTANT.

¶ 120. **Principle of the Sextant.** — A sextant may be constructed, as in Fig. 105, of two pieces of looking-glass, *ag* and *aj*, hinged together at *a* with their reflecting surfaces inward. The silvering is removed near *e* and near *i*, so that an object in the direction *x* may be seen through the two glasses; but enough silvering is left between *b* and *j* to make it possible also to see objects in the direction *y*, reflected by the mirror *ag* in the direction *hi*, then by *aj* in the

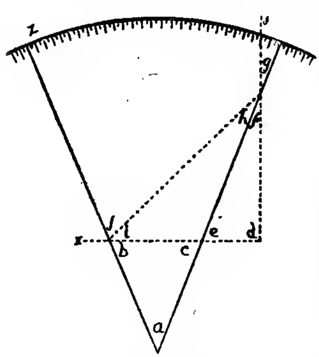


FIG. 105.

direction ie. The angle, a, between the mirrors may be measured by a graduated arc, yz.

Let us first find the relation between the angle d through which the ray y is bent and the angle a between the mirrors. The law of the reflection of light (§ 97) gives us the angles $b = j$ and $g = h$: The vertical angles c and e are equal by construction, also g and f; hence $f = h$. We have furthermore in the triangles abc and abh, —

$$a = 180° - b - c, \tag{1}$$

$$a = 180° - b - i - h. \tag{2}$$

Substituting equals for equals, we have, —

$$a = 180° - j - e, \tag{3}$$

$$a = 180° - b - i - f. \tag{4}$$

Adding (3) and (4),

$$2a = 360° - e - f - b - i - j;$$

or since b, i, and j together equal 180°,

$$2a = 180° - e - f. \tag{5}$$

But from the triangle def, we have, —

$$d = 180° - e - f; \tag{6}$$

hence, comparing (5) and (6), we find, —

$$d = 2a. \tag{7}$$

We see, therefore, that when a ray of light is reflected by two mirrors, the angle (d) between its original direction (yd) and its final direction (xd) is equal to twice the angle (a) between the mirrors.

Now let us suppose that the plane *ayz* is made vertical, and that the angle *a* is adjusted so that the rays of the sun[1] from the direction *y* may seem, after being twice reflected, to come from the direction *x*, let us say that of the horizon; then the altitude of the sun is evidently $2a$. The student should note that two objects in different directions may be visible *simultaneously* through a sextant. The sun may be made to appear, in fact, as if it were actually on the horizon.

¶ 121. **Description of an Ordinary Sextant.** — We have seen how a sextant may be constructed out

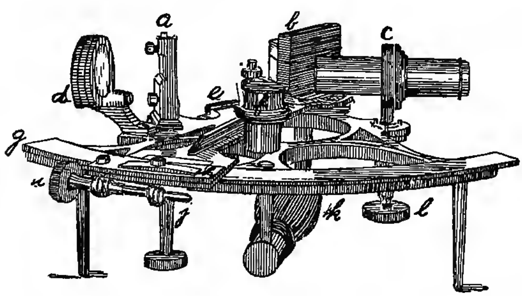


Fig. 106.

of two mirrors hinged together as in Fig. 105. In practice it would be necessary to remove most of the silvering between *j* and *z*, since it would otherwise interfere with the ray *yd* when the angle *d* is very small. In an ordinary sextant, this

[1] The mirror should be smoked near *f* and *g* before trying this experiment, in order that the brightness of the sun may be sufficiently diminished.

portion of the mirror is entirely removed. Of the two mirrors, *az* and *ag*, there remain in fact only the small portions, *bj* and *hg*, represented respectively by *a* and *b* in Fig. 106, or by *ac* and *df* in Fig. 107. The mirror *a* (Fig. 106) is fixed in position, and *b* is pivoted at its centre instead of an axis (as in Fig. 105) where the planes of the two mirrors intersect.[1] The angle between these planes is moreover measured, not by an arc (*zy*, Fig. 105) included by the angle (*a*), but by an arc *g* (Fig. 106) situated in quite a different part of the instrument. On this arc a vernier (*h*) connected by a movable arm with the mirror (*b*) serves to indicate the angles through which the mirror (*b*) is turned.

A tube or telescope, *c* (Fig. 106), permanently pointed toward the fixed mirror (*a*) serves principally as a guide for the eye. There is also, in most sextants, a set of dark glasses, *d*, which may be so placed as to diminish the light of the sun when looked at directly through the unsilvered part of the fixed mirror, *a*; there is also a set of dark glasses at *e* (not shown in the figure) to cut off excessive light reflected by the revolving mirror, *b*. A magnifying glass (*f*) is used for reading the vernier (*h*). The vernier is clamped by a thumb-screw (*j*), and slow motion is produced (only when clamped) by the tangent screw (*i*). There is also a screw (*l*) by which

[1] The fixed mirror, *a*, is called the "horizon-glass," because in nautical observations the horizon is usually seen through it; the revolving mirror, *b*, is called the "index-glass" because it carries the index. See Glazebrook and Shaw's Practical Physics, § 48.

the tube or telescope (c) may be either raised so as to come opposite the upper portion of the mirror, a, which is unsilvered, or lowered so as to be opposite the silvered portion. By this means, the relative brightness of the direct and doubly reflected images may be varied at pleasure. The handle k is of use especially in nautical observations.

¶ 122. **Adjustments and Reading of a Sextant.** — In order that a sextant may give accurate readings, certain conditions must be fulfilled.

(1) The tube or telescope, c, must be parallel to the plane of the graduated arc; for in demonstrating the relation between the angle (xdy, Fig. 105) through which a ray of light is bent and the angle (a) between the mirrors, we have assumed that the whole figure lies in one plane. This condition is fulfilled if a distant object, visible through the tube or telescope (c) in the middle of the field, appears, when sighted, to be in the same plane as the graduated arc. If this condition is not fulfilled, the position of the tube or telescope must be altered by an instrument-maker, so that the line of sight may be parallel to the plane of the graduated arc.

(2) The pivot on which the mirror (b, Fig. 106) rotates must be perpendicular to the plane of the graduated arc. This condition is fulfilled if the movable arm can be turned from one end of the arc to the other without either leaving it or binding against it. If it is not fulfilled, the sextant should be discarded.

(3) The revolving mirror should be perpendicular

to the plane of the graduated arc. This condition is fulfilled if the reflection of the arc in the mirror seems to be a continuation of this arc. If the reflected portion seems to slope upward or downward, the mirror leans forward or backward. The adjustment of the revolving mirror should not be attempted by the student, but should be left to the instrument-maker.

(4) The fixed mirror should be perpendicular to the plane of the graduated arc. This condition is fulfilled if, after the revolving mirror has been properly adjusted, the sextant can be set so as to give a single image of distant objects; for the fixed mirror is then parallel to the revolving mirror, and hence perpendicular to the arc. The reading of the sextant when so set is called its zero-reading (see ¶ 123). If no such setting can be made, the fixed mirror should be tipped a little forward or backward by turning one of the screws which hold it in place. This adjustment should be attempted only by persons who have acquired some skill in the use of a sextant.

(5) The fixed mirror should be nearly parallel to the revolving mirror when the index attached to the latter points to the zero of the graduated arc. This is the case if the sextant gives only a single image of distant objects when set as stated. If a double image is seen, one of the two mirrors should be rotated without disturbing the setting. A screw is usually provided for rotating the fixed mirror through a small angle. There is danger in so doing that the

last adjustment (1) may be disturbed. If it is, it must be repeated. The student is advised to omit the 5th adjustment altogether, since a slight error in it may cause a little inconvenience in allowing for zero error, but will not affect the accuracy of results.

(6) The arc and vernier must each be uniformly graduated. The uniformity of the arc may be tested (as in ¶ 48 *d*) by means of the vernier. If the latter subtends, for instance, 119 divisions in all parts of the arc, these divisions must have the same length. If the coincidences on the vernier follow in regular succession as the tangent screw (*i*) is slowly revolved, we may infer uniformity both in the main scale and in the vernier.

(7) The value of the main-scale and vernier divisions must be known. An accurate method of correcting the main scale will be considered (incidentally) in Experiment 45. To decide whether the divisions, of which every tenth one is usually numbered, are intended to be degrees, or only half-degrees, so as to represent the number of degrees through which a ray of light is bent (see ¶ 120, formula 7), a rough test will be sufficient. Thus if a string reaching from the pivot to the graduated arc also reaches from 0 to 120 on the arc, we may infer that the divisions are half-degrees. By calling them degrees we shall avoid the labor of doubling each reading of the sextant when measuring the angle through which a ray is bent by reflection in the two mirrors.

The divisions which represent degrees are divided in different instruments into two, three, four, six, and even twelve parts. The number of minutes corresponding to each part is easily calculated. The vernier usually contains lines of different lengths. There are as many of the longest lines as there are minutes in the smallest main-scale division. These lines are not usually so close together as the main-scale divisions, but by paying attention simply to the *number* of the long line which coincides most nearly with some main-scale division, we find the number of minutes to be added to the reading of the main scale (see § 40). Between the long lines, shorter lines are frequently placed, to represent fractions of a minute. Since a setting made by the eye, unaided by the telescope, is hardly accurate to a minute,[1] the student is advised to disregard these lines until he has mastered the reading of the sextant to degrees and minutes.

In angular as in linear measure, there is danger of making a mistake of a whole main-scale division (¶ 50, II.). If the reading of the main scale is thought to be about $x°$, and the vernier shows it to be a whole number plus 1′, we record this reading as $\underline{x°}$ $\underline{1'}$; but if the vernier indicates a whole number plus 59′, we record the reading, not as $\underline{x°}$ $\underline{59'}$, but $\underline{(x-1)°}$ $\underline{59'}$.

[1] A man four miles off would subtend an angle of about one minute. A minute corresponds to a distance of less than one three-hundredth of an inch on a piece of paper held at the ordinary distance (10 inches) from the eye.

The first degree-mark below zero is counted as *minus one;* the second, *minus two*, etc. The number of minutes is always positive, since the vernier is made to read this way. To avoid confusion, the negative sign is written over the number of degrees, which it alone affects (see ¶ 50, I.). Thus, a negative angle of —21′ would be recorded $\overline{1}° 39'$.

¶ 123. **Determination of the Zero-Reading of a Sextant.** — After a sextant has been adjusted as accurately as possible (see ¶ 122), its zero-reading must be determined. The index is first set at the zero of the main scale (as in ¶ 122, 5), the dark glasses are pushed out of the way, and the tube or telescope (*c*) is directed toward some distant object, — the smaller and brighter the better. A star is universally conceded to be the best object, but a distant electric arc-light will do. In the day-time, a church spire or the top of a flag-pole may answer. At sea the horizon line is frequently employed; in this case the plane of the sextant must be vertical. The angle between the mirrors should be so slight that the direct and doubly reflected images of the given object may at least be included in the same field of view. These images are then made to coincide by turning the tangent-screw (*i*, Fig. 106). Finally, the reading of the sextant is taken. This is called its zero-reading, because it corresponds to an angle, zero, between the direct and doubly-reflected rays.

It is easy to show that the fixed and revolving mirrors must be parallel when these rays (*yb* and *xg*, Fig. 107) are parallel; for the alternate interior angles b and e are equal by construction, hence their supplements, $a+c$ and $d+f$ must be equal. Now, the law of the reflection of light (§ 97) gives $a=c$, and $d=f$; hence, a being half of $a+c$ must be equal to d, which is half of $d+f$. Since c and d are alternate interior angles formed by the intersection of be with the mirrors ac and df, these mirrors must be parallel. Conversely, if the mirrors are parallel, the direct and doubly-reflected rays must be parallel.

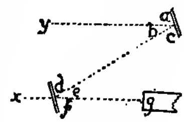


FIG. 107.

In order that the rays yb and xg may be sensibly parallel, let us say within one minute (1′) of angle, the object from which they come must be 3,438 times as far off from the sextant as these rays are from each other. Since the perpendicular distance, bg, is generally less than a twelfth of a metre, it may be safe to employ any object more than 300 metres off for the determination of the zero-reading of a sextant with the unaided eye. To obtain results accurate to half a minute, the minimum distance must be doubled; for accuracy within 10″ of angle the object should be at least 1,800 metres, or more than a mile away. For such results, a telescope (c, Fig. 106) must be employed.

¶ 124. **Determination of Small Angular Magnitudes by means of a Sextant.** — I. A sextant is to be set at or near its zero-reading; then turned so that the telescope (c, Fig. 106) may point directly toward the sun. The sextant is to be held so that its graduated arc may be in a vertical plane, below the revolving mirror (b). A sufficient number of dark glasses must be interposed in the paths of both the direct and the reflected rays. It is well to select these glasses so that the two images of the sun may differ in color, and thus be easily distinguished. The tangent screw (i, Fig. 106) is to be turned until the doubly-reflected image (R. I., Fig. 108) appears to be tangent to the direct image (D. I.) and below it, as in (1). The vernier is then read. Next, by turning the tangent screw the other way, the reflected image (R. I.) is made to move completely through the direct image, until it is tangent to, and above it, as in (2). The vernier is again read.

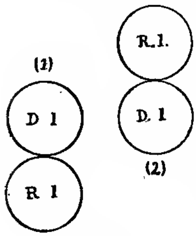


Fig. 108.

The first reading should be positive, the second negative. The average of the two should be found and compared with the zero-reading previously determined, with which it should agree. If the difference exceeds 1′, the measurements in ¶¶ 123 and 124 should be repeated.

The second reading is now to be subtracted (algebraically) from the first. The difference, divided by 2, is evidently the angular diameter of the sun. The semi-diameter, which is quoted in all nautical almanacs, varies from month to month, according to the earth's distance from the sun. Its mean value is not far from 16′.

II. The sextant may also be used for the determination of the angular diameter of small terrestrial objects. The plane of the graduated arc must be held in all cases so as to be parallel to the diameter which it is desired to measure. The object should be so small that a negative[1] as well as a positive reading may be obtained, as in the case of the sun. The average of the two readings should agree with a zero-reading obtained from the *same object*, or from one at an equal distance. The difference between the two readings is not affected by parallax, since the error in both readings is the same. This difference, divided by 2, is therefore the angular diameter of the object in question as seen from the pivot of the revolving mirror. The position of this pivot should be noted, or the results will have no meaning. It is well for the student to measure either the actual diameter or the distance of the object in question, still better, both of these quantities; for though either may be calculated from the other, the two together give him the means of testing his inferences as to the manner in which his sextant should be read and an opportunity of confirming his results.

EXPERIMENT XLV.

PRISM ANGLES.

¶ 125. **Determination of the Angles of a Prism.**—I. A small prism (*abc*, Fig. 109) is fastened to the revolv-

[1] A sextant should be capable of giving negative readings down to 3, 4, or even 5 degrees.

ing mirror (bc) of a sextant with its axis parallel, as nearly as possible, to that about which the mirror turns.[1] The mirror is then rotated so that the direct image of a distant object, seen in the direction ed, may coincide with the image of the same object reflected first by the face of the prism (ac) then by the fixed mirror (df). If the two images cannot be made to coincide, the face ac is probably not parallel to the axis of the mirror, and must be made so by tilting the prism either from b to c, or from c to b, without separating the two faces, bc, of the prism and of the mirror. When parallelism is established, an exact coincidence of the images may be brought about. A reading of the sextant is then made. This serves to determine the prism angle c. In the same way the other two angles are determined.[2]

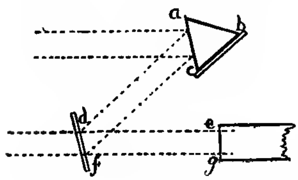


Fig. 109.

Subtracting from each reading of the sextant its zero-reading, determined as in ¶ 123, we have the indicated value of the angle corresponding to c (or acb) in the figure; for it is evident that the mirror cb in rotating from its zero position, ca, to the position

[1] The plane of the face, ac, should strictly pass through the axis of the mirror, to avoid errors of parallax. In practice, however, it is more convenient to mount the prism as in Fig. 109.

[2] To measure the three angles of a prism, one of which must be at least 60°, a sextant reading to 120° will be required. "Octants" are sometimes graduated to 120°; but do not read generally to more than 100°, on account of the space occupied by the vernier.

cb, turns through the angle *acb*. What we want, however, is the actual value of this angle, not the deviation of a ray of light striking the revolving mirror, which plays no part in the measurement. If, therefore, the sextant is found, as in ¶ 122, 7, to be graduated in half-degrees, half-minutes, etc., the indicated value of the angle must be halved in order to find the real value of *acb*.

The sum of the three prism angles should be 180°. A discrepancy of one or two minutes may be attributed (1) to errors of observation, (2) to pyramidal convergence of the sides of the prism, and (3) to errors in the adjustment or graduation of the sextant. If the measurements are several minutes in error, they should be repeated. If the same result is obtained, the parallelism of the prism faces should next be tested with a three-pointed caliper. With a perfect equilateral prism, we have evidently the means of detecting any error in the location of the 60° mark (or that numbered 120°).

II. Instead of a sextant, a spectrometer may be used, as will be explained in ¶ 126.

EXPERIMENT XLVI.

ANGLES OF REFRACTION.

¶ 126. **The Spectrometer.**—A spectrometer consists essentially of two telescopes (*ab* and *fg*, Fig. 110) capable of revolving about the centre of a graduated circle (*cde*). The eye-piece of the first telescope is generally removed, and a narrow slit (*a*, Fig. 110) is usually substituted for the cross-hairs (*c*, Fig. 101, ¶ 116). This slit is always at right-angles with the graduated circle, and at a distance from the lens, *b*, equal to its principal focal length; so that the rays from it may be rendered parallel by this lens (see ¶ 116, 3). The combination (*ab*) is called the "collimator" of the spectrometer. The telescope *fg* is focussed for parallel rays (¶ 116, 3), and carries an index with a vernier, by which its position on the graduated circle may be accurately determined.

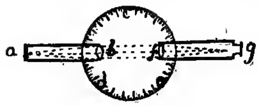


Fig. 110.

A zero-reading can be found by pointing the telescope toward the collimator as in Fig. 110, and adjusting it so that the image of the slit, *a*, may be visible in the centre of the field of view, which is determined by the intersection of cross-hairs.

Let us now suppose that it is desired to measure the angles of a prism. The latter is mounted as in Fig. 111 (*cde*), so that the face *ce* may reflect part of the light from *ab* in the direction *fg*, and so that at the same time the face *cd* may reflect light in the direction *f′g′*. The telescope is then set so as to receive first one, then the other of the images of the slit, thus formed, in the middle of its field of view, and in each case a reading of the vernier is made.

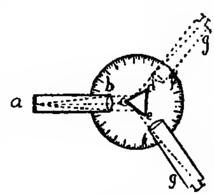


Fig. 111.

Let us suppose that the collimator is permanently set at 0° (or 360°) of the circle; that *fg* is at $x°$ and $f'g'$ at $y°$ of the circle. A radius of the circle perpendicular to *ce* would halve the angle x, on account of the law of reflection (§ 97); and hence would meet the circle at a point $\frac{1}{2}x°$. In the same way a radius perpendicular to *cd* would meet the circle half-way between $y°$ and 360°; or at $\frac{1}{2}y° + 180°$; hence if prolonged backward it would meet the circle at $\frac{1}{2}y°$. Now, the angle between two surfaces may be measured by the angle between two lines perpendicular to them; hence the difference between $\frac{1}{2}x°$ and $\frac{1}{2}y°$ measures the prism angle *dce*. In other words, the angle between two faces of a prism is equal to half the angle between the two directions in which they reflect parallel rays of light. (Compare ¶ 120, 7.)

The most important adjustments of a spectrometer are the accurate levelling and focussing of the telescope and collimator for parallel rays (see ¶ 116, 3). The faces of the prism must be made perpendicular to the plane of the graduated circle as in ¶ 125. An instrument especially adapted to measure the angle between two reflecting surfaces is sometimes called a goniometer.

¶ 127. **Determination of Angles of Refraction.** — I. The telescope (*fg*), and collimator (*ab*) of a spectrometer are slightly inclined as in Fig. 112, so that a spectrum (¶ 128) of the slit, *a*, may be formed in the telescope by a prism *dce*,

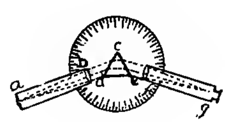


FIG. 112.

the angles of which have been determined (¶¶ 125, and 126). The angle *c*, causing the refraction, should be placed symmetrically with respect to the telescope and collimator. If *dce* is an equilateral prism, an image of the slit may also be formed in the telescope by reflection from the face *de*. It is found that when the faces *cd* and *ce* are as stated equally inclined to the rays *ab* and *fg*, the angle between these rays reaches a minimum.

To make sure that this position has been approximately found, the prism should be rotated a little. The violet of the spectrum should be replaced by blue, green, yellow, and red, until finally the spectrum disappears altogether. It should make no difference whether the prism is turned to the right or to the left. If the spectrum moves in opposite directions when the prism is turned in opposite directions, the desired position has not been found. In this case the rotation should be continued in one direction or the other until the spectrum seems to come to a standstill. The prism is then very nearly in its "position of minimum deviation."

The slit should now be illuminated with light from a sodium flame,[1] the reflected image if necessary cut off, and the telescope roughly set on the yellow refracted image of the slit. Then the prism is turned slightly so that this image may move as far as possible towards the red (or less refrangible) end of the spectrum. The telescope is again set on the yellow image

[1] A common Bunsen burner beneath a netting of fine iron wire sprinkled with nitrate of soda furnishes an excellent "sodium flame."

more carefully than before, and the prism turned first to the right, then to the left, so as to find if possible a position in which the yellow image is even less refracted than before. Thus by successive approximations, the telescope may finally be set upon an image of the slit formed by the prism in its position of minimum deviation.

Subtracting the zero-reading (¶ 126) of the telescope from its reading when set upon the refracted image, we have finally the angle of minimum deviation in question; that is, the least angle through which sodium light may be bent in passing through the prism angle, *dce*, in the figure.

The relation between angles of refraction and indices of refraction is considered in § 102.

In repeating the experiment, the prism should be rotated through 180°, so that the rays would be bent upward instead of downward as in the figure. If the position of the collimator is unchanged, any error in the zero-reading may be eliminated (see § 44) by averaging the result with that previously obtained.

II. Instead of the spectrometer a sextant may be employed for the determination of angles of refraction. The prism is to be mounted as in Fig. 113, so that a ray of light from a distant point may be refracted by the prism angle *c*, previously determined (¶ 125), then reflected by the revolving mirror *d*, and by the fixed mirror *e* into the telescope, *f*, where

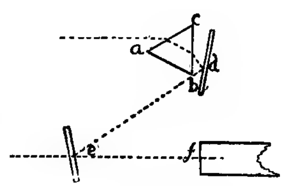


FIG. 113.

it is made to coincide with the direct ray, *ef*, from the same object. To obtain accurate results, monochromatic light should be employed; but a mean index of refraction may be found by making the direct image of a flame coincide with the yellow or green of its spectrum (§ 128). The prism must be placed by trial in the position of minimum deviation as with the spectrometer. The angle of deviation, being twice the angle between the mirrors, is indicated directly by the reading of the sextant, after the zero-reading has been subtracted.

The use of the sextant for the determination of angles of refraction is recommended only to those who have some skill in physical manipulation. For this reason a detailed description of the experiment has not been given.

¶ 128. **Spectra formed by the Dispersion of Light.** — The rays of light from a sodium flame, when bent (as in ¶ 127) by a prism, produce, with ordinary apparatus, a single yellow image of the flame. A flame colored with lithium gives similarly a red image, and one colored with thallium a green image. These images are not, however, in the same direction from the observer, owing to the fact that rays of different hues are unequally bent by a prism. Indeed, if a flame be colored by a mixture containing certain proportions of lithium, sodium, and thallium, three images of the flame — one red, one yellow, and one green — may be seen side by side, distinctly separated by dark spaces between them. Many substances, even when chemically pure, cause under the same circumstances several distinct images of a flame to be produced. Each of these images differs in hue from the rest. The images may be more or less bright and more or less widely separated. Together they constitute what is called the *spectrum* of the substance producing them. When, as in a common gas-flame, light of every hue is represented, an indefinite number of images are formed, and these necessarily overlap one another. The result is called a continuous spectrum.

An instrument intended simply to examine spectra with a view to observing the number of images present, is called a spectroscope. An instrument like that described in ¶ 126, especially adapted to the determination of angles of refraction, through settings made upon the differently colored images in a spectrum, is properly called a spectrometer.

Those substances which bend light the most usually produce the greatest separation or "dispersion" of rays of different colors. There is, however, no definite proportion between the effects of refraction and dispersion. Thus an equilateral prism of crown glass which bends rays of light about 40°, separates the extreme red and violet rays by about 4°; while a prism of flint glass, producing nearly double the dispersion, bends rays less than 50°.

To determine the dispersive power of a given substance, two indices of refraction are generally found (see § 102), one with red light, the other with violet light. The red light selected is of a peculiar wave-length (§ 98), namely, .00007604 *cm.*, being that which causes the line A in the solar spectrum. The violet light has similarly a wave-length .00003933 *cm.*, corresponding to the line H_2 of the solar spectrum. The difference between the indices of refraction of a given substance for these two rays is sometimes called the "index of dispersion" of the substance in question.

EXPERIMENT XLVII.

WAVE-LENGTHS.

¶ 129. **Theory of the Diffraction Grating.** — When a distant candle is looked at through a linen handkerchief, or through any fine network, several images of the candle are usually seen (Fig. 114). These are not, however, as one is at first apt to suppose, simply so many views of the candle through the meshes of the handkerchief; for each image represents the whole candle, and the distance between the images is not only disproportionate to the size of the meshes, but actually increases as the meshes become smaller. It is, moreover, unaffected by the distance of the handkerchief from the eye. The phenomenon is an example of *diffraction* (§§ 100, 101), and depends upon a re-

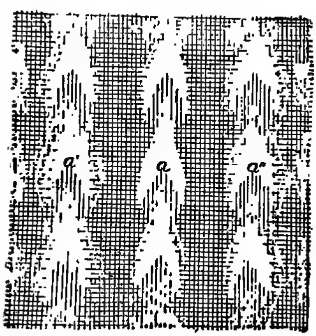


FIG. 114.

lation between the length of the waves of light and the distance between the threads.

The central image (a, Fig. 114) is the direct image of the candle. It may be distinguished from the side images, a' and a'', for instance, both by its greater distinctness and by the absence of color.

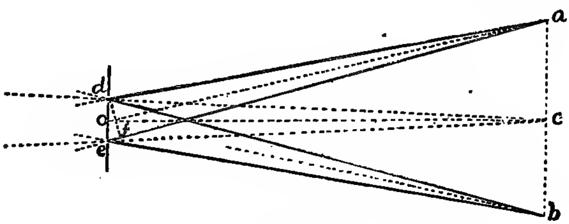


FIG. 115.

The side images will be found tinged with blue on the side toward a, and with red on the outer side. This is due to the fact that different colors are unequally bent by diffraction. Each of the side images is in fact a "spectrum" of the candle. It is interesting to place two candles at points corresponding

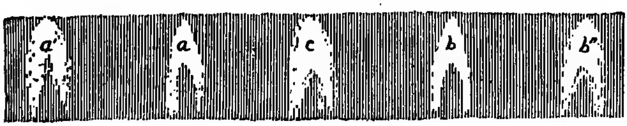


FIG. 116.

to a and b (Fig. 115) at such a distance that when the candles are viewed through a network de, the side image at the right of a may coalesce with the side image at the left of b, so as to form a single image at the point c similar to that represented in

Fig. 116. If o is one of the threads, d and e the spaces between it and the two parallel threads on either side of it, then drawing ad, ao, ae, cd, co, ce, etc., also df perpendicular to ao, we have (since ao practically bisects the angle a) $ad = af$. The path ae is accordingly longer than ad by the distance ef, which must therefore be the length of a wave of light (§ 101), since the rays do not interfere. Now, by similar triangles, we have,

$$ef : de :: ac : ao; \qquad \text{I.}$$

hence, if we know the distance, de, between the threads, the distance, ao, between the handkerchief and one of the candles, and the distance, ab, between the candles, so that by halving the latter the distance of the side image (ac) may be found, we may calculate the average length (l) of a wave of light by the formula,

$$l = \frac{de \times ac}{ao}. \qquad \text{II.}$$

The student may himself estimate wave-lengths in this way. For a human eye in its normal condition (see ¶ 115) the average wave-length in a candle-flame has been found to be about 60 millionths of a centimetre. We may make use of this fact to estimate the distance (d) between the threads of the handkerchief by the formula derived from I.,

$$d = .00006 \times \frac{ao}{ac}. \qquad \text{III.}$$

The angle aoc is called the angle of diffraction The ratio $ac : ao$ is by definition the sine of this angle;

hence if the angle be measured, the ratio can be found from Table 3.

¶ 130. **Determination of Angles of Diffraction.** — An ordinary diffraction grating (see § 101) consists of a set of parallel and equidistant lines ruled or photographed on glass (Fig. 117). A candle-flame viewed through such a grating gives several images, as in the case of a netting (Fig. 114); but these images are all in a single row (Fig. 116). The relation between the wave-length, distance between lines, and angle of diffraction is the same as in the case of a netting (¶ 129).

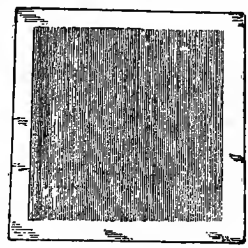


FIG. 117.

The angle of diffraction may be determined either by a sextant (¶ 124), or by a spectrometer (¶ 126). In any case the lines of the grating must be perpendicular to the graduated arc or circle by which this angle is to be determined.

I. A coarse diffraction grating, containing from 10 to 20 lines to the millimetre, is to be mounted directly in front of the tube or telescope of a sextant (*c*, Fig. 106, ¶ 121), which is then to be pointed at a distant sodium flame (¶ 127). When the fixed and revolving mirrors are nearly parallel, it should be possible to see the flame (either directly or by double reflection) with at least two images due to diffraction, one on each side of it (see *a′a″*, Fig. 114). The experiment consists in measuring the angular distance between the two side images next the flame, by the method already explained in ¶ 124.

Let *a* and *b* (Fig. 116) represent the direct and doubly reflected images of the flame. The revolving mirror is first set so that the side image at the left of *b* coalesces with the side image at the right of *a*, to form a compound image, *c*, as in the figure. Then the image (b'') at the right of *b* is made in the same way to coalesce with the side image (a') at the left of *a*. The two readings are then subtracted algebraically, one from the other, and the result is divided by 2 (as in ¶ 124), to find the angle subtended by the side images (a' and a'', Fig. 114). This angle must again be divided by 2 to find the angular distance of either of the side images from the flame. This angular distance evidently corresponds to the angle *aoc* (Fig. 115), and is, accordingly, the angle (a) of diffraction in question.

Since the wave-length of sodium light is .0000589, we have, substituting this value in formula III., ¶ 129, for the distance (d) between two lines of the grating.

$$d = \frac{.0000589}{sin\ a} \qquad \text{I.}$$

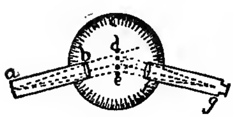


FIG. 118.

A grating, thus tested, serves as a convenient scale by which the diameters of small objects may be determined. Such a scale is interesting, because it represents the nearest approach to an absolute standard of length (see § 5).

II. Instead of a sextant, a spectrometer may be employed to measure angles of diffraction. If the grating is mounted in the centre of the graduated circle (Fig. 118), so as to be perpendicular to the collimator, *ab*, the reading of the telescope, *fg*, when set upon one of the side images, will determine the angle of diffraction in question. It is not very easy, however, to make the grating accurately perpendicular to the collimator, and the slightest deviation affects the angle of diffraction. A grating, like a prism (see ¶ 127) is found to have a position of minimum deviation, when it is equally inclined to the direct and diffracted rays (see *de*, Fig. 118). This position may be found by trial in the same way as with a prism.

When the method of minimum deviation is employed, the formulæ of ¶ 129 must be somewhat modified.[1]

The wave-lengths contained in Table 41 were determined by a method essentially the same as the one here given.

[1] In Fig. 115 each ray is supposed to lose one wave-length with respect to the next before reaching the grating. If, however, the grating is equally inclined to the incident and diffracted ray, the loss must be half a wave-length before, and half a wave-length after reaching the grating; that is, $ef = \frac{1}{2} l$. The angle *aoc* will represent also half the total angle of diffraction; or $aoc = \frac{1}{2} a$. If d is the distance *de* between the lines, we have, substituting $\sin \frac{1}{2} a = \sin aoc$ for $ac \div ao$ (see ¶ 129), and multiplying by 2,

$$l = 2 d \sin \tfrac{1}{2} a.$$

EXPERIMENT LXVIII.

INTERFERENCE OF SOUND.

¶ 131. **Determination of the Wave-Length of a Tuning-Fork by the Method of Interference.** — I. The two ends of a thick-sided rubber tube, about half a metre long, and with an internal diameter of at least 5 *mm.*, are joined together, as in Fig. 119, by a Y-joint, and a tube connected with the stem of the Y is held to the ear. A tuning-fork making from 400 to 600 vibrations per second (as for instance a "violin A-fork" or a "C-fork" just above it) is then touched lightly to the tube at different points, as in the figure. The note emitted will generally be plainly heard; but two or more points will be found at which the sound is nearly extinguished. These points are to be marked with ink on the rubber tube. Then the tube is to be disconnected from the Y-joint, straightened out, but not stretched, and the distance between adjacent marks carefully determined by a metre rod.

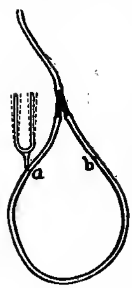


FIG. 119.

The extinction of the sound is due to the interference of vibrations reaching the Y-joint by the two different channels (§ 100), which differ either by half a wave-length, or by some odd multiple of half a wave-length. It follows that two adjacent points, *a* and *b* (Fig. 119), where the sound reaches a minimum, must be half a wave-length apart. To find the length of a wave of sound created in the tube by the vibration of the tuning-fork in question, we have therefore only to multiply the distance *ab* by 2.

Wave-lengths depend more or less upon the temperature of the air in the tube, which should therefore be noted. They are generally less in small tubes than in the open air, particularly if the sides of the tube be yielding. The interference is never complete, because the wave which travels the longer distance becomes weaker than the other, and hence cannot wholly destroy it. The points where the sound reaches a minimum may often be located more exactly when a fork is vibrating feebly than when it is sounding loudly.

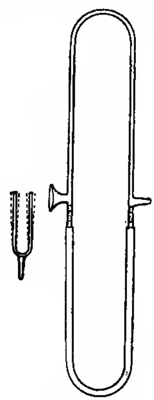


Fig. 120.

II. In place of a rubber tube, we may employ a pair of telescoping U-tubes (Fig. 120), forming a closed circuit. Near the junctions two openings are made. One of these is connected with the ear, the other receives vibrations propagated from a tuning-fork through the air. The two channels by which the sound reaches the ear may be made unequal in length by drawing out the tubes. The difference between them may be measured by graduations on the inner tube, or in any other obvious manner.

The smallest difference between the two channels which can produce interference is half a wave-length; hence, we multiply it by 2 to find the wave-length in question.

From the wave-length of a fork in air, we may calculate roughly its rate of vibration (¶ 134, formula II.).

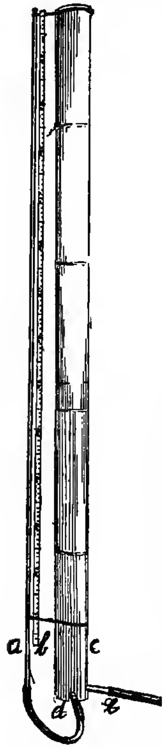


Fig. 121.

EXPERIMENT XLIX.

RESONANCE.

¶ 132. **Determination of Wave-Lengths by the Method of Resonance.** — A metallic tube or "resonator" $1\frac{1}{2}$ metres long and 10 *cm.* in diameter (*c*, Fig. 121) is filled with water; then a tuning-fork, making from 200 to 300 vibrations per second, is held near the mouth of the tube, while the water escapes by the spout, *e*. When the water falls to a certain level, the note emitted by the fork, instead of dying away, will suddenly swell out. The flow of water is then checked. Water from the faucet is now admitted to the resonator by the spout *e*, and again allowed to escape, with a view to finding at what level it gives the maximum resonance. The variation in the loudness should be observed both when the water is rising and when it is falling. By alternately increasing and diminishing the quantity of water in the tube, the desired level may be located within a millimetre. This level is then read by the gauge *ab*, consisting of a millimetre scale, *b*, and a glass tube, *a*, connected by a rubber tube (*d*) with the resonator.

The fork is now kept in vibration while the level of the water is allowed to fall to a much greater depth than before. A second point of resonance is thus located in the same way as the first. The temperature of the air within the tube should be carefully noted.

The distance between the two points of maximum resonance is found by subtracting one scale-reading from the other. This distance is (see § 99) exactly half a wave-length, and hence must be multiplied by 2 to find the wave-length of the fork.

The rate of vibration of the fork may now be calculated approximately, as will be explained in ¶ 134, by formula II. of that section.

EXPERIMENT L.

MUSICAL INTERVALS.

¶ 133. **Determination of Musical Intervals.** — I. METHOD OF INTERFERENCE. — The wave-lengths of two forks are to be determined as in Experiment 48, taking care that the temperature of the air is the same in both cases, and the musical interval between the forks is to be calculated as in ¶ 134, III.

II. METHOD OF RESONANCE. — Instead of using the method of interference, we may determine the wave-lengths of two forks by the method of resonance, as in Experiment 49, with care as before to avoid changes of temperature. The musical interval should be calculated in the same way (¶ 134, III.).

III. PYTHAGOREAN METHOD. — An instrument which will be found convenient for the determination of musical intervals is represented in Fig. 122. It is

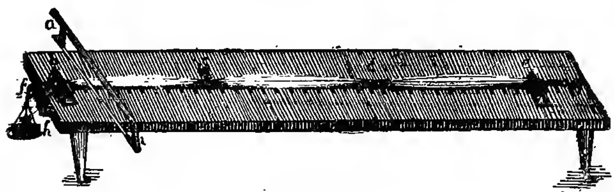


FIG. 122.

called the "*monochord*," and is attributed to Pythagoras. In modern instruments, it consists of a steel wire, *fbcdeg*, fastened to a board at *g*, then passing over two "bridges" (or triangular supports, *e* and *b*) round a pulley (*f*) to a weight (*h*) by which it is kept stretched with a constant force. The positions of the bridges are determined by a graduated scale.

The wire (*be*) is set in vibration by a bow (*ai*), and the distance between the bridges (*b* and *e*) is varied until the note emitted by the wire is in unison with one of the forks. The distance (*be*) is then adjusted so as to produce unison with the other fork. From the two distances in question, the interval between the forks is to be calculated as in ¶ 134 (Formula III.).

Determinations with a monochord should be attempted only by students having a more or less musical ear. The exact adjustment of two notes in unison may be inferred from the cessation of "beats" (Exp. 53).

IV. Harmonic Method. When the musical interval between two forks has been determined by any of the preceding methods, or simply recognized by the ear, the exactness of the interval in question may be tested as follows: The bridges *b* and *e* (Fig. 122) are first placed at a distance which is the *least common multiple* of the two distances giving unison with the two forks. By touching the string lightly with a feather (*c*, Fig. 122) at certain points, it may be made to vibrate in segments as in the figure. The number of segments is first made such that the string is nearly in unison with one of the two forks, and the distance (*de*) adjusted if necessary so that the unison may be perfect. If the wire can be made to divide in such a manner as to sound in unison with the other fork, there must be an exact musical interval between the forks. If, on the other hand, beats are heard, the interval is probably inexact, and by an amount which may be estimated from the frequency of the beats (Exp. 53).

For the practical application of this method, the monochord should be capable of giving a very low note, at least two octaves (¶ 134) below the lower fork; hence the tension of the wire must not be too great. The lowest note which a string can give out under given circumstances is called its "fundamental tone." The other tones are caused by its division into segments, separated by still points or "nodes." These tones are called the "harmonics" of the string. The musical interval between any two harmonics may be calculated from the number of vibrating segments (see ¶ 134, IV.), which must therefore be noted in each case.

¶ 134. **Theory of Musical Intervals.** — If a tuning-fork gives out n waves each l centimetres long in one second, then the furthest wave must be nl centimetres off from the fork at the end of that space of time; and since it travels nl *cm.* in 1 *sec.*, the velocity of sound must be nl *cm. per sec.* The fundamental equation connecting the number (n) of vibrations per second, the wave-length (l), and the velocity of sound (v) is, therefore, —

$$v = nl. \qquad \text{I.}$$

The velocity of sound in air of any temperature may be found from Table 15 *B*. If the humidity is unknown, a mean value (60 per cent) may be assumed; then if the wave-length of a given fork is l, we have, —

$$n = \frac{v}{l}. \qquad \text{II.}$$

When two forks give n' and n'' vibrations per second, with wave-lengths respectively of l' and l'' centimetres, we have from II., —

$$n' = v \div l', \qquad (1)$$

and

$$n'' = v \div l''; \qquad (2)$$

hence, dividing (1) by (2),

$$n' : n'' :: l'' : l'. \qquad \text{III.}$$

The ratio of the rates of vibration is called the musical interval between the forks, and is accordingly in the inverse ratio of their wave-lengths.

Formula III. is applicable to a wire as well as to a tube. When a wire of the length be divides into N segments, the length of each must be $be \div N$; we have accordingly for the lengths l' and l'' of the segments formed by the division of the wire (be) into N' and N'' parts, respectively, —

$$l' = be \div N', \qquad (3)$$

$$l'' = be \div N''; \qquad (4)$$

hence, dividing (4) by (3),

$$l'' : l' :: N' : N'', \qquad (5)$$

which, substituted in III., gives

$$n' : n'' :: N' : N''. \qquad \text{IV.}$$

This shows that the rates of vibration of different harmonics are proportional to the number of vibrating segments in the wire.

It has been stated that the ratio between two rates of vibration, n' and n'', determines the interval between the two notes to which they correspond. The ordinary musical scale consists of a series of notes whose rates of vibration, whether high or low, are always relatively proportional to the following numbers set beneath their names: —

DO	RE	MI	FA	SOL	LA	SI	DO
24	27	30	32	36	40	45	48

The interval between the first and third note of this series is called a "third;" between the first and fourth, a "fourth," etc. The first two are said to be one tone apart; the last two, one semitone apart. The most common musical intervals may be arranged as follows, according to the simplicity of the ratios which they involve when reduced to their lowest terms: —

Name.	Ratio.	Name.	Ratio.	Name.	Ratio.
Unison . . .	1 : 1	Fourth . . .	4 : 3	Minor Third .	6 : 5
Octave . . .	2 : 1	Sixth	5 : 3	Whole Tone .	9 : 8
Fifth	3 : 2	Third	5 : 4	Semitone . .	16 : 15

The sum of two or more intervals is always represented by the product of the ratios in question; thus, when we say that two notes are an octave and a fifth apart, we mean that the higher makes one and one half times as many vibrations per second as the octave of the lower note; or, again, twice as many vibrations as a note a "fifth" above the lower note; that is, in either case, three times as many vibrations as the lower note itself. In the same way an interval of two octaves corresponds to the ratio 4 : 1 between the rates of vibration; an interval of three octaves corresponds to the ratio 8 : 1, etc. It is a fact to be noted that the musical intervals involving the simplest ratios are the most agreeable to the ear.

END OF PART FIRST.

PHYSICAL MEASUREMENT.

Part Second.

MEASUREMENTS IN SOUND, DYNAMICS, MAGNETISM, AND ELECTRICITY.

SOUND — Continued.

EXPERIMENT LI.

VELOCITY OF SOUND.

¶ 135. **Determination of the Velocity of Sound.** — (1) Two data are required for the determination of the velocity with which sound passes from one point to another: 1st, the distance between two stations (see ¶ 136); and 2d, the time occupied in traversing this distance (see ¶ 137). To make use of the results, the temperature of the air must be found at various points between the two stations (see Part I. ¶ 15); and if precision is required, the humidity of the air should also be determined.[1] The velocity of sound is not affected by barometric pressure.

[1] At ordinary summer temperatures (20° to 30°) the effect of humidity upon the velocity of sound may amount to one half of 1 %. See Table 15, B.

(2) If the path traversed by the sound is at right-angles with the direction of the wind, the velocity of sound will not be perceptibly affected by any ordinary atmospheric disturbance. It is, however, increased by the velocity of the wind when the two move in the same direction, or diminished by the same amount when they move in opposite directions.[1] When the directions are oblique, the velocity of sound is always more or less affected. It is therefore best to arrange an experiment so as to find the time occupied by sound in traversing a given distance first in one, then in the opposite direction. In this case, if the velocity of the wind is small and tolerably constant, the *average* result will not be perceptibly affected by it.

(3) Two or more determinations of the velocity of sound should be made between stations at different distances. Any constant error in the estimation either of distance or of time will be shown by a disagreement of the several results. The true velocity of sound is to be calculated in such a case from the *difference in time* required to traverse two given distances (see formula II. below).

(4) Let d be the distance traversed by sound in the time t; then the velocity of sound, v, is to be calculated by the equation

$$v = \frac{d}{t}. \qquad \text{I.}$$

[1] A velocity of the wind amounting to 10 metres per second, or about 22 miles per hour, would affect the velocity of sound by about 3 %.

Distinguishing by subscript numerals 1 and 2 the results in the two cases, we should have

$$v = \frac{d_1}{t_1} = \frac{d_2}{t_2};$$

hence,

$$\frac{d_1}{d_2} = \frac{t_1}{t_2}.$$

Subtracting 1 from both sides of the equation we have

$$\frac{d_1}{d_2} - 1 = \frac{t_1}{t_2} - 1;$$

or, reducing to a common denominator,

$$\frac{d_1 - d_2}{d_2} = \frac{t_1 - t_2}{t_2};$$

whence

$$\frac{d_1 - d_2}{t_1 - t_2} = \frac{d_2}{t_2}.$$

Finally, substituting equals for equals, we find

$$v = \frac{d_1 - d_2}{t_1 - t_2}. \qquad \text{II.}$$

By the use of this formula, constant errors (§ 24) are eliminated.

¶ 136. **Measurement of Long Distances.** — The measurement of long terrestrial distances is in general a problem for which the student must be referred to works on surveying. No particular difficulty will, however, be found in measuring approximately a distance along a moderately straight path; for even variations as great as 8° (nearly 1 foot in 7), either in the direction or in the slope of the path, will introduce an error of less than one per cent in the result.

Distances may also be determined indirectly by means of a sextant. To measure a distance, for example, across a valley, from an observing station, A, (Fig. 123) to an object B, we place (or select) an object C, so that the lines joining B with A and with C

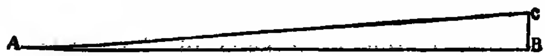


FIG. 123.

may be approximately at right-angles. The distance BC is then measured directly, and the angle CAB is determined from the observing station. Since (by definition)

$$BC \div AB = tangent\ CAB,$$

we have $$AB = \frac{BC}{tan\ CAB}.$$

To obtain with an ordinary sextant (see ¶ 124) results accurate within 1 per cent, the distance BC actually measured should be at least a hundredth part as great as the distance AB to be determined. In regard to the direction of C from B, great accuracy is not required. If the corner of a square be

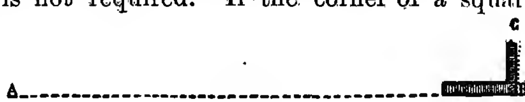


FIG. 124.

placed at B (Fig. 124) with one side directed towards A, any object, C, nearly in range with the other side of the square, will answer for our purpose. An error of 8° in the angle ABC will introduce an error of only 1 % in the result. The object C may be on a level with B or above it, as may be more convenient. The distance BC and the angle CAB must be accurately measured.

In one part of the experiment the distance AB should be as great as possible considering the space at the disposition of the observer, and the distance through which the signals at his command can be seen or heard. If the method of difference is to be employed (¶ 135, 3), it is necessary, in a second part of the experiment, to make use of a much shorter distance. The second distance should be in no case greater than half of the first, and always as small as is consistent with the accurate determination of the time occupied by sound in traversing it. When the time is to be found by an ordinary watch (¶ 137, I.), the smaller distance should be several hundred, the greater several thousand metres. In the pendulum method (¶ 137, IV.), distances of 300, 600, and 900 metres may conveniently be employed. When sound signals are to be sent back and forth between two stations (¶ 137, III.), the minimum distance may be reduced to about 150 metres. The velocity of sound has been determined by the use of echoes (¶ 137, II.) between the Jefferson Physical Laboratory and the Lawrence Scientific School, the walls of which are about 80 metres apart. Long corridors, tunnels, and conduits of various sorts frequently give rise to echoes suitable for the determination of the velocity of sound.

It must be remembered that in the time required for a signal to go from one station to another, then back to the first, the distance traversed is twice that between the stations. When the sound is reflected back to the observer the distance traversed is twice that of the observer from the object causing the reflection. Care must be taken to identify the object in question. In the interval between two successive echoes, sound must obviously traverse twice the distance between two objects which reflect it, as for instance two parallel walls or the two ends of a conduit.

¶ 137. **Measurement of Short Intervals of Time.** — I. One of the oldest methods of estimating the time required for sound to traverse a given distance is to count the ticks of a watch which occur between the flash and the report of a cannon discharged at that distance from the observer (see ¶ 138). When, owing to obstructions in the field of view, it is impossible to see the flash, an electric telegraph may serve in the place of light to inform the observer of the exact moment of the discharge.[1] Instead of counting ticks, a "stop-watch" may be used, or a chronograph may be employed (¶ 266). Amongst various ingenious devices for the measurement of small intervals of time may be mentioned the use of a stream of mercury from a Mariotte's bottle (see Fig. 275, ¶ 250), which may be directed into a receptacle at the beginning of the interval, and diverted at the

[1] The velocity of light is about 30,000,000,000 *cm. per sec.*; hence the time lost in traversing terrestrial distances may generally be disregarded. An electric current is practically instantaneous in its action; but an allowance must be made for the slowness of telegraphic instruments to respond to the current, unless a method of difference be employed. See ¶ 135, 3.

end of the interval. The quantity of mercury collected serves to estimate very precisely the interval of time in question.

II. In certain localities the velocity of sound may be similarly determined by timing the interval between a sound and its echo. When a series of echoes may be heard, the interval between them may be determined by adjusting a pendulum or a metronome so as to keep time with the echoes while they last, then afterward finding the rate of the pendulum or metronome, by timing 100 or more oscillations. Again, a method of multiplication may be used (§ 39). When the last audible echo reaches the observer, a new sound may be made; so that the interval of time to be measured may be indefinitely increased. One of the earliest determinations of the velocity of sound is said to have been made by a monk, who made use of the echo in a cloister caused by clapping his hands. The sounds thus produced were, it is said, so timed as to alternate regularly with the echoes.

III. The effects of an echo may be imitated by a series of sound signals interchanged between two stations. Let us suppose that two observers, each provided with a hammer and a plank, place themselves at suitable distances (see ¶ 136). The first gives a blow with his hammer, then the second returns the signal as soon as the sound reaches him. When the first hears the response, he gives another blow, etc. As in the last method (II.), the interval of time to be measured may be indefinitely multiplied.

With practice, each observer will learn to anticipate the return signal, so that very little time will be lost in the act of repetition. The time thus lost is to be eliminated by making two experiments, as has been suggested above (¶ 135, 3).

IV. Another method[1] is to station two observers let us say 300 or 350 metres apart, and to provide each with a telescope, if necessary, so that he may watch a pendulum, or any other object having a periodic motion, in sight of both observers. Either the length of the pendulum, or the distance between the observers is then varied until a sharp sound made by A, when the pendulum is at the middle point of its swing, is heard by B at the moment when the pendulum, after completing one or more oscillations, again passes the middle point. The distance is then measured, and the time of the pendulum determined. Measurements must also be taken in which sounds made by B are heard by A as the pendulum passes its middle point. The experiment is then repeated with a distance between the observers (¶ 135, 3) two or three times as great as before.

Other methods of measuring short intervals of time will be considered in experiments which follow.

¶ 138. **Proper Methods of Counting.** — In counting the ticks of a watch (which usually occur at intervals of one-fifth of a second), it will be found difficult, if not impossible, to repeat, even mentally, the names of numbers which contain more than one

[1] See Ex. 30, Elementary Physical Experiments published by Harvard University.

syllable.[1] In the following method of counting, this difficulty is avoided: —

1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	2
1	2	3	4	5	6	7	8	9	3

By counting the ticks which actually occur *within* a given interval of time, the length of that interval will on the whole be fairly estimated. There is, however, a tendency in most persons to count one too many ticks. When a given interval contains a whole number of ticks, one occurring at the beginning of the interval should be counted "nought," or not counted at all. Obviously the first and last tick should not both be counted.

With intervals of time (as with intervals of space), care must be taken to distinguish the number of intervals from the number of divisions between which they lie. In the same way that the zero of a scale should not be counted "one," the beginning of an interval of time should not be called one second or one-fifth of a second. A miscount may generally be avoided by pronouncing the word "now" at the beginning of the interval, then beginning the count immediately afterward.

An accurate method of counting is important in a great variety of measurements, especially those which involve rates of vibration or revolution. The student should consider carefully what habits he has formed

[1] The difficulty is greatly lessened by counting every other tick; but on account of the greater inaccuracy, this method of counting is not generally recommended.

in this respect, and if they are not good, whether it is preferable to change them, or to make an allowance for "personal error" in each separate determination.

EXPERIMENT LII.

GRAPHICAL METHOD.

¶ 139. **Determination of Rates of Vibration by the Graphical Method.**[1] — A tuning-fork (*ae*, Fig. 125)

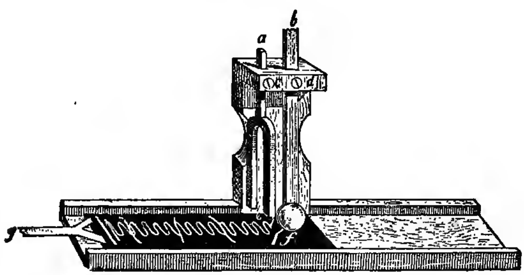


Fig. 125.

making from 100 to 300 vibrations per second, and a pendulum (*bf*), made of an ounce bullet (*f*) and a piece of clock-spring (*b*), are mounted as in the figure, so that when the tuning-fork and pendulum are in vibration, two short and fine brass wires attached one to each may make marks (*h* and *i*, Fig. 126) as close together as possible on a piece of smoked glass.

[1] The experiment here described is essentially the same as that given in Exercise 31, Elementary Physical Experiments, Harvard University. This application of the graphical method is due to Prof. Hall.

The tuning-fork and the spring are then firmly clamped by the screws *c* and *d*.

The smoked glass is now drawn slowly out from under the pendulum and the tuning-fork. The points of the wires *e* and *f* should draw a single line (*h i x*, Fig. 126) upon the surface of the glass. If they do not, the wires should be bent, or their relative position otherwise adjusted. The smoked glass is now to be replaced, and both the pendulum and the tuning-fork are to be set in vibration,—the latter by drawing a violin-bow across one of the prongs. The bow must be drawn slowly at first, and always *in a*

Fig. 126.

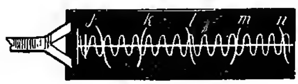


Fig. 127.

direction nearly parallel to the vibration which it is desired to create. That is, the bow should be held at right-angles to the prongs, but nearly parallel to the plane containing them. The smoked glass is again drawn out from under the pendulum and the fork, with a slow but uniform velocity.

The wire attached to the tuning-fork, partaking of its vibration, will trace upon the glass a series of waves. The wire attached to the pendulum would similarly trace a series of much longer waves, were it not that owing to the amplitude of its oscillation, the wire usually leaves the glass at the extreme points of a swing. The result is a series of marks (*j*, *k*, *l*, etc., Fig. 127).

The time required for one complete oscillation of the pendulum is represented by the distance between alternate marks (*j* and *l*, or *k* and *m*, Fig. 127). The number of complete vibrations made by the tuning-fork in the same length of time is to be found by counting the waves executed in the same distance. Thus between *j* and *l* there are (in the figure) about $6\frac{1}{4}$ complete, or $12\frac{1}{2}$ half-waves; and between *l* and *n* there are similarly about 7 waves. In practice, a much greater number would be counted.

If the waves are perceptibly closer together at *k* or at *l* than at *m* or at *n* (or the reverse), the glass has not been drawn with sufficiently uniform velocity. In this case, instead of depending upon the marks (*j*, *k*, *l*, etc.) actually made by the pendulum, it is necessary to draw a line at a distance from each mark equal to that between *h* and *i* (Fig. 126), and at the left or at the right of it, according to whether *h* is at the left or at the right of *i*. The new lines show where the wire attached to the pendulum *would have crossed* the glass, provided that it could have been made absolutely coincident with the wire attached to the pendulum. By the use of lines drawn as above, we may in counting the waves avoid errors due to irregularity in the speed of the glass. The number of whole waves included between two alternate lines should be recorded in each case, together with an estimate of the fractions of a wave left over at each end of the series. This fraction should be expressed in tenths § (26).

To find the rate of vibration of the tuning-fork, the time occupied by one complete oscillation of the pendulum must now be determined. This is done by timing, let us say, one hundred complete oscillations. Having given a signal, one observer begins to count the oscillations of the pendulum, while a second observer, as soon as the signal is perceived, begins to count the ticks of a watch (see ¶ 138). When the pendulum has completed a given number of oscillations, the first observer signals to the second to stop counting.

The number of complete oscillations of the pendulum per second is found from the time required for 100 or 200 oscillations (as the case may be), by simple division, and the result is multiplied by the average number of waves made by the fork during one of these complete oscillations to find the "vibration number," or "pitch" of the fork, — that is, the number of complete vibrations made in one second.

EXPERIMENT LIII.

BEATS.

¶ 140. **Theory of Beats** — When two musical notes, nearly but not quite in unison, are sounded together with about the same degree of loudness, the effect upon the ear is by no means uniform. At regular intervals the sound swells out, and these intervals are separated by moments of comparative silence. Each rise and fall of the sound constitutes a "*beat*."

The increase is due to the mutual re-enforcement of the two sets of vibrations communicated to the air; the decrease is caused by the interference of these vibrations.

Let us suppose that two tuning-forks, one making 256, the other 255 vibrations per second, are started at a given instant by forcing their prongs together and suddenly releasing them. The prongs of both forks will spring apart simultaneously, and each fork will cause a slight condensation of the air on each side of it. This condensation will be followed by a rarefaction when the prongs rebound, then by several alternate condensations and rarefactions, nearly though not quite synchronously performed. The result is that the vibrations reaching the ear at the same distance from both forks are very much greater than if one fork were sounding alone. At the end of half a second, however, the first fork will have made $256 \div 2$, or 128, complete vibrations; so that, as at the start, its prongs will be springing apart; but the second fork will have made only $255 \div 2$ or $127\frac{1}{2}$ vibrations, so that its prongs will be approaching each other. The condensation produced by one fork will tend to offset the rarefaction produced by the other. The effect on the ear will accordingly be less than if one of the forks were sounding alone. This interference of the vibrations will evidently continue as long as the forks are vibrating in opposite ways. At the end of a second, the first fork will have made just 256, the second fork just 255 complete vibrations, and the direction in which the prongs are moving will be in each case the same as at the start, and hence the same for both forks. The sounds will therefore re-enforce each other as at first. It is evident that, with the forks in question, periods of re-enforcement must occur every second, separated by intervals of interference. In other words, two forks making 256 and 255 vibrations per second must give rise to 1 "beat" per second when sounded together.

In the same way it may be shown that two forks differing by *n* vibrations per second give rise to *n* beats per second. In other words, when two musical notes are nearly in unison, *the number of beats per second is equal to the difference between the vibration numbers* corresponding to the two notes in question.

¶ 141. **Determinations of Pitch by the Method of Beats.** — The special apparatus required for this experiment consists of a series of tuning-forks with differences of from three to five vibrations per second, covering an interval of one octave (¶ 134). The first and the last of the series are to be sounded together, to make sure that the musical interval is exact. If the forks are nearly but not quite an octave apart, faint beats may be heard. In this case one of the forks must be loaded with small bits of wax near the end of its prongs until the beats disappear. If the wrong fork is loaded the beats will become more frequent than before. The same effect may be produced if *too much* weight is added to *either fork;* hence care must be taken at first to add very little weight at one time.

The simplest way in general to tell whether a fork is higher or lower than may be required for the purposes of harmony is by the method of loading suggested above. The effect of the additional weight is to lower the rate of vibration of the fork to which it is attached. Whenever by loading a fork it may be brought into harmony with a given musical note, we know that fork to have a higher rate of vibration than the purposes of harmony require.

If, for instance, the first fork in the series gives 61, and the last 120 vibrations per second, the first will have to be loaded until it gives 60 vibrations per second, in order to be in harmony with the other fork. Again, if the second fork gives 64 vibrations per second, it will have to be loaded to bring it in unison with the first fork. We may generally assume that the forks are arranged by the instrument-maker in an ascending series.

The experiment consists in a determination of the number of beats produced in a given length of time by sounding together each pair of consecutive forks in the series, that is, the first and second, the second and third, the third and fourth, etc. The student will do well to begin counting with one of the beats which happens to occur when the second-hand of his watch indicates a round number. The *beginning* of this beat should not be counted (see ¶ 138). One hundred beats should be timed if possible. The time of the last beat should be observed to a fraction of a second. The number of beats per second should be calculated in each case.

The results represent differences between each pair of consecutive forks in the series; hence when added together we have the difference between the first and the last in the series, for the whole difference in question must be equal to the sum of all its parts.

Now two notes an octave apart are to each other, in respect to their vibration numbers, as 2 is to 1 (¶ 134); hence the last number in the series is twice the first. It follows that the difference between the first and last numbers is equal to the first number in the series. The result of adding together the numbers of beats per second is therefore to find the number of vibrations executed by the first fork in one second.

By adding to this number the number of beats per second between the first fork and the second fork we find the pitch of the second fork; and in the same way, successively, the pitch of each fork in the series can be calculated.

EXPERIMENT LIV.

LISSAJOUS' CURVES.

¶ 142. **Theory of Lissajous' Curves.** — We have seen, in Experiment 52, that when a piece of smoked glass is drawn beneath a pointed wire attached to a vibrating tuning-fork, a wave-line is traced upon it. If instead of drawing the glass completely away from the tracer, the motion be suddenly reversed, we shall evidently obtain a double wave which will resemble one of the figures below (Fig. 128, 1, 2, and 3) according to the point (*a*) in the curve at which the reversal takes place. In the first curve the two waves happen very nearly to coincide. We may imagine the reversal to take place so that there should be a perfect coincidence.

Now let us suppose that when the tracer reaches a certain point, *b*, a second reversal takes place, and a third reversal occurs when the tracer returns to the former point, *a*. Evidently, if the reversals are prop-

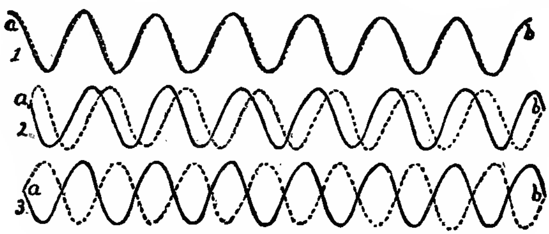


FIG. 128.

erly timed, the tracer will follow the same path over and over.

In practice we obtain a similar result by attaching a small piece of smoked glass to the larger of two tuning-forks. When the larger fork makes one vibration in the same time that the smaller fork makes for instance 8, we obtain tracings as in Fig. 129, 1, 2, or 3, according to the relation which happens to exist between the forks at the start.

These are examples of Lissajous' curves. The reversal of the smoked glass is not sudden, as in the case previously supposed, and its velocity is greatest when the middle of the figure is being drawn. This accounts for the difference in appearance between these curves and those represented in Fig. 128.

It may be shown that whenever two vibrations at right-angles are compounded graphically, as in Fig. 129, unless the times of the vibrations are incommensurate, a Lissajous' curve results. Each musical interval (¶ 134) has, accordingly, its characteristic curves. These curves are in general too complicated to be discussed in an elementary work. We shall confine ourselves to such cases as are represented in

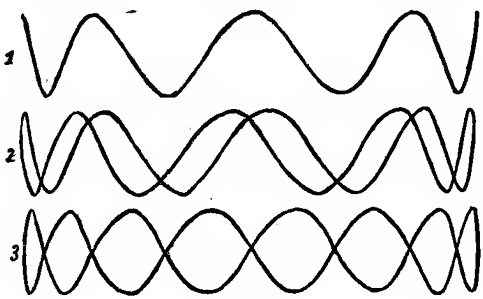


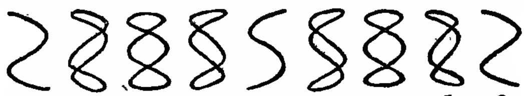
Fig. 129.

Fig. 129, where one fork makes a certain whole number of vibrations while the other makes one.

To find in such cases the musical interval between the forks, we have to experiment until a figure like the third is obtained (Fig. 129, 3). If this figure contains n lobes, then the higher fork makes n times as many vibrations as the lower fork.

It has been so far assumed that the two forks are separated by an exact musical interval, so that at the end of a certain period they find themselves in exactly the same mutual relation as at the start. If this is not the case, it is evident that the tracer will not follow the same path in all cases, but that this path will be continually changing.

Let us suppose that the tracer reaches its highest point, as seen in the figure, when the glass reaches its extreme right-hand or left-hand turning-point. Then the curve traced will be represented as in Fig. 129, 1. If the small fork is a little behind-hand we shall have a tracing as in Fig. 129, 2; and if the small fork has only reached the middle of its course when the glass turns, we shall have a tracing like



1. 2. 3. 4. 5 6 7. 8 9.

FIG. 130.

Fig. 129, 3. Evidently, if the small fork starts as in (1) and falls slowly behind the other, we shall have a series of tracings represented by (1), (2), and (3). It is not until the higher fork has fallen one complete vibration behindhand that the same figure will be repeated.

If the smaller fork is gaining instead of losing, a similar series of changes will be produced. There is in fact no way to tell which fork is too high for the musical interval in question, except as in the last experiment, by loading it and observing the result. A complete cycle of changes in the case of two forks one octave and one fifth apart (¶ 134) is shown in Fig. 130.

The symmetrical lobed figures (3 and 7) appear twice in a cycle; the serpentines appear also twice; but one of them is left-handed (1), the other right-handed (5). The interval between two left-handed (or that between two right-handed) serpentines always represents one complete cycle, and is accordingly equal to the time in which the higher fork makes one whole vibration more or less than would be required to give a perfect musical interval.

Let p be the pitch of the lower fork, that is, the number of vibrations it makes in one second, and let n denote the approximate musical interval between the forks; then the pitch of the higher fork, which we will call P, must be equal to np, nearly. If, however, we observe c cycles per second, the true pitch of the higher fork is $np \pm c$. Here c is positive if by loading the higher fork the musical interval may be made perfect; if on the other hand the lower fork must be loaded, c will be negative. With this understanding we have

$$P = np + c. \qquad \text{I.}$$

and

$$p = \frac{P - c}{n}. \qquad \text{II.}$$

These formulæ apply only to cases in which, as we have supposed, n is a whole number.

¶ 143. **Determination of Pitch by Lissajous' Curves.** —A tuning-fork of known pitch (Exps. 52 and 53) and one approximately an octave above or below it are to be mounted, as in Fig. 131, with their prongs at right-angles. The prongs of one fork (A) are to be coated with lampblack, except at a small point where, by the touch of a pin, the bright metallic surface is made visible. Opposite this point on the other fork (*B*) a lens, *C*, of about 1 inch focus, is to be attached with sealing-wax, at such a distance that a highly magnified image of the point may be seen

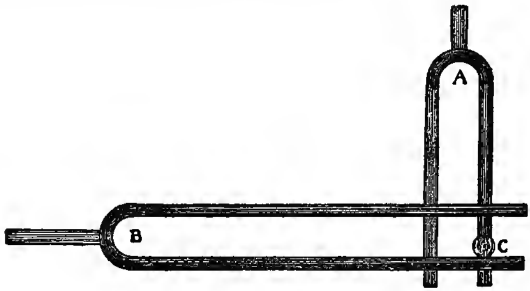


FIG. 131.

through the lens. When a violin-bow[1] is drawn across the fork *A*, the bright spot partaking of the vibration will be apparently extended into a horizontal line, Fig. 132.

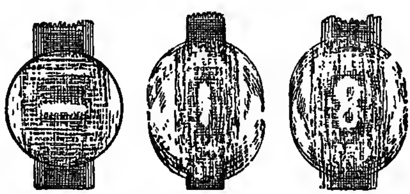


FIG 132. FIG. 133. FIG. 134.

When the fork *B* is set in vibration, the motion of the lens will cause the spot to be apparently elongated into a vertical line, as in Fig. 133. When both

[1] In practice, it will be found convenient that one or both of the forks should be maintained in vibration by electrical means.

forks vibrate simultaneously the vertical and horizontal motions will be combined, and if the forks are separated by an exact octave, one of Lissajous' curves will be formed, as for instance in Fig. 134.

If this curve is permanent in form, the experiment is now finished; but if, as is generally the case, it passes through a series of cycles, as in Fig. 130, ¶ 142, it becomes necessary to count the number of complete cycles which take place in a given length of time. It is also necessary to load one of the forks, as in ¶ 141, until the changes in the cycles become less frequent.[1]

We thus find whether c is positive or negative in the formulæ of ¶ 142. The pitch of one of the forks is finally to be calculated by one of the formulæ in question from the pitch of the other fork, previously determined.

EXPERIMENT LV.

THE TOOTHED WHEEL.

¶ 144. **Construction of a Toothed-Wheel Apparatus.** — A toothed-wheel apparatus capable of giving fairly accurate results is represented in Fig. 135, as seen from above. A vertical cross-section is shown also in Fig. 136. The works (e) of an ordinary eight-day

[1] It is possible to load a fork so that a figure of a certain class (see Fig. 130, 1–9) may preserve its characteristics until the vibration dies away.

spring clock, from which the escapement has been removed, are mounted on a piece of wood, and a disc of cardboard (*a*) is attached to the axle usually carrying the second hand. Two pieces of watch-spring are also attached to this axle at *b*, and bent into loops so that two small loads (*c* and *d*) which they bear may hang quite close together when the wheel is at rest. The friction which the springs exert against the air acts as a governor upon the speed of the machine.

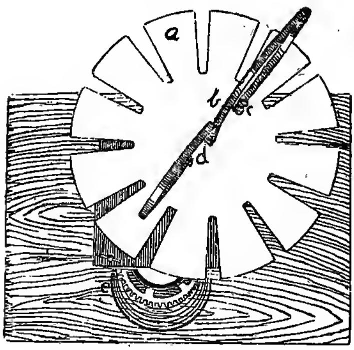


FIG. 135.

The velocity of rotation will be found to vary very little as the force of the main-spring grows less and less. To make the wheel turn faster, the loads (*c* and *d*) may be decreased; or a *slight* change may be produced by winding up the main-spring. To make the wheel go slowly, the load may be increased; or a slight decrease in speed may be had either by waiting for the main-spring to unwind itself, or by applying friction to one of the more slowly moving wheels. The upper surface of the disc, *a*, should be painted black. The number of revolutions which it makes in a given time may be counted by watching a white spot upon it, or still better by listening to the sound

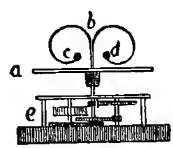


FIG. 136.

made by an object striking lightly against a projection from the wheel or from the axle upon which it is mounted. At equal distances around the circumference of the wheel, narrow radial slits should be cut out. The number of slits must be made with reference to the usual speed of the machine and the number of vibrations per second which the toothed wheel is intended to measure. The wheel represented in Fig. 135 makes about 8 revolutions per second without any load, — the speed being reduced to 4 revolutions per second by a load of a few grams at *c* and *d*. With twelve notches in the disc, this apparatus affords from 48 to 96 nearly instantaneous views of objects seen through the rim of the wheel. The instrument is accordingly suited to the determination of the pitch of tuning-forks making from 48 to 96 vibrations per second. It may also be used for much higher forks, as will be presently explained.

¶ 145. **Theory of the Toothed Wheel.** — By the apparatus just described we are able to obtain at regular intervals a series of instantaneous views of a vibrating object. If the intervals between the views correspond to the period of vibration in question, the same view will evidently repeat itself over and over. If the intervals are sufficiently short, the effect will be a continuous impression upon the eye. Thus when the eye is held close behind the rim of the rotating disc (Fig. 135), the speed of which is properly adjusted, we may obtain a series of views of a tuning-fork, in all of which the prongs are, for instance, at their greatest elongation. The result is that the fork appears to be at rest. To obtain this result the number of slits which pass in front of the eye in one second must be equal to the number of vibrations executed by the fork in the same time. If the wheel is moving a little too fast or too slow, the successive views of the fork will not be exactly the same.

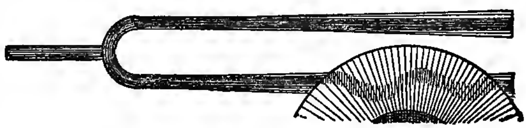


FIG. 137

The position of the prongs will seem to change as if the fork were executing a very slow vibration. When the fork is held close behind the rim of the disc, as in Fig. 137, a different effect is produced.

Let us first consider the effect of a single slit moving along the fork. Let 1, 2, 3, 4, 5, 6, 7, 8, Fig. 138,

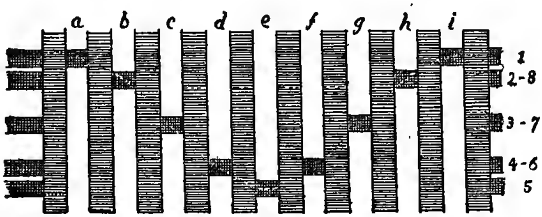


FIG. 138.

be views of the fork seen through such a slit when occupying the successive positions *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, and *i*. These views are evidently situated along the dotted line *ai*. Let us now supply the intermediate views. We shall evidently have the curve shown in

Fig. 137, or in *ab*, Fig. 139. Now let another slit pass along the fork. We shall have similarly a curve, *cd* or *ef* (Fig. 139), which may or may not coincide with *ab*. If it does not coincide with *ab*, we shall probably not see either of the curves, since the light reflected through the slits will hardly have time to affect the eye. If, however, several such curves coincide, the joint effect will be similar to that shown in Fig. 137.

In order that successive curves may coincide, it is necessary that successive slits should reach a given point in the curve (as *a*, Fig. 138) at the same instant that the prong of the tuning-fork reaches that point.

Fig. 139.

In other words, the interval of time between the arrivals of successive slits must correspond with the period of the tuning-fork.

It will be found, if a toothed wheel is adjusted so as to show waves, as in Fig. 137, that when the speed is increased the waves will seem to follow the direction in which the wheel is moving, while if the speed is lessened, the waves will move in the opposite direction. This is the result of a series of wave images (see Fig. 139), each of which is situated in a *slightly* different place from the one preceding it. The direction in which the waves seem to move is a valuable guide in adjusting the speed of the wheel.

It is easy to trace out in a similar manner the appearance of a vibrating fork for any speed of the wheel. Usually it will appear blurred, as if looked at in the ordinary manner. If, however, the wheel is moving twice as fast as it ought, a double wave will be visible, as in Fig. 140. If, again, the fork makes in one second a number of vibrations twice as great as the number of slits which pass a given point, the appearance of the fork will be as in Fig. 141. Care must be taken not to mistake this curve for the double curve of Fig. 140, nor for the regular curve of Fig. 137. We notice that in Fig. 141 there are two complete waves in the distance between two successive slits (*a* and *b*).

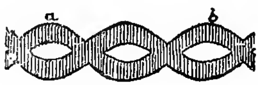


FIG. 140.

FIG. 141.

In the same way this distance will be divided into *n* waves if the fork executes *n* vibrations between successive views from a given point.

By this principle we may find the rate of a fork too high to be measured by the ordinary method.

¶ 146. **Determination of Pitch by means of a Toothed Wheel.** — The experiment consists simply in adjusting the speed of a toothed wheel (Fig. 135, ¶ 144) so that a fork held behind the rim of a wheel (as in Fig. 137, ¶ 145), and making about 64 vibrations per second, will be apparently thrown into simple stationary waves, the lengths of which will be equal to the distance between the teeth of the wheel, then finding how many teeth pass by a given point in one second. We have already considered (¶ 144) the manner in which the speed of the wheel may be adjusted and how the number of revolutions may be counted.[1] The number of revolutions made in one second multiplied by the number of teeth gives the number of teeth per second. This is (see ¶ 139) the "pitch" of the tuning-fork.

[1] If it is found impossible to adjust the speed exactly, or to keep it adjusted, accurate results may still be obtained by counting the number of waves which in one second traverse the field of view. This number is to be added to the number of slits passing a given point in one second if the motion of the waves is opposite to that of the wheel; if both move in the same direction the first number is to be subtracted from the second.

DYNAMICS.

¶ 147. **Different Methods of Measuring Velocity in Dynamics.**—When a body is moving so slowly that it is possible to make a series of observations of its position at different points of time, no particular difficulty is met in the measurement of its velocity. Thus in Exp. 60, to find the average velocity of a ring rotating about its axis, we observe the distance traversed between two ticks of a clock, and divide it by the interval of time in question. Such slow motions are, however, the exception in dynamics. In certain cases instantaneous photography has been employed for the study of rapid motions. The estimation of velocity generally requires, however, special devices, such as have been employed for the velocity of sound (Exp. 51).

(1) In rough measurements, we frequently make use of the sounds produced by a moving body when it strikes different obstacles in its course. A familiar example of this method consists in the determination of the speed of a railway train by counting the number of rails crossed in a given length of time. To find the velocity of a marble rolling in a groove, small tacks may be driven into the groove at such distances that the successive sounds made by the marble in crossing them correspond with the ticks of a clock. The regular increase of velocity caused by a steady

incline is then easily demonstrated by measuring the distances between the tacks.

(2) By substituting for a series of tacks a series of electrical connections which are made or broken by a moving body, we may make use of any of the devices by which time is measured by electrical agency.[1]

The velocity of a rifle bullet has been measured by the interval of time between the rupture of two wires a known distance apart. The time of rupture is usually recorded "*graphically*" by means of a chronograph (see ¶ 266). Curves traced simultaneously by the armature of an electrical sounder and by a tuning-fork (see Exp. 52) enable us to estimate precisely exceedingly small intervals of time.

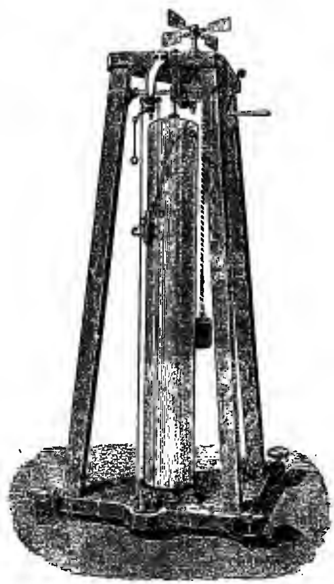


FIG. 142.

(3) There are various devices in which the motion of a body may be directly recorded by the graphical method. Thus, in Morin's Apparatus (Fig. 142), a pencil (*c*) attached to a falling body marks directly upon a revolving cylinder covered with paper. If the rate

[1] See Trowbridge's New Physics, Exp. 71, 72, 73.

of revolution is known, we may obviously infer the position of the body at different points of time from the tracing (ab) made by the pencil.

Another device in which the vibrations of a tuning-fork attached to a falling body may be made to indicate its position, will be found in Trowbridge's New Physics, Exp. 74.

A simple instrument illustrating the graphical method of measuring velocity will be described in the next section.

(4) In studying the motion of fluid streams, the velocity is frequently calculated from the size of a tube or orifice, and from the volume which flows through this tube or orifice in a given time. Thus if a stream

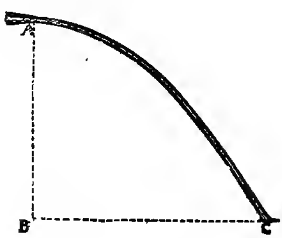


FIG. 143.

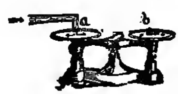


FIG. 144.

of water issues from an orifice $\frac{1}{4}$ *sq. cm.* in cross-section at the rate of 25 *cu. cm. per sec.*, its velocity at the orifice must be 100 *cm. per sec.* This principle has been applied to illustrate the law of falling bodies. A stream of water projected horizontally with a known velocity must traverse a known horizontal distance (BC, Fig. 143) in a known time; hence the time required for gravity to deflect the stream through a known vertical distance (AB) is determined.

(5) The pressure of a stream of gas has been applied to the determination of the mass of the gas when its velocity is known, and conversely for a determination of its velocity when the mass is known. If, for instance, a mass of gas m, impinging with the velocity v, on a scale-pan (a, Fig 144) causes a force, f, to be exerted for a time t, we have from the general formula (§ 106)

$$m = \frac{ft}{v}, \qquad v = \frac{ft}{m}.$$

(6) The laws of falling bodies are frequently made use of for indirect measurements of velocity. Thus since a body is known to fall 4.9 metres in 1 second, the velocity of a stream of water projected horizontally at a distance of 4.9 metres above a certain level will be equal numerically to the horizontal distance traversed before reaching that level, the time in question being 1 second. Again, the velocity of a pendulum when it passes its central point may be estimated by the distance it has fallen in reaching that point, or by the distance it rises after reaching that point (see § 109).

(7) The law of *action and reaction* enables us to make comparisons of velocity. Thus if a bullet of mass m, striking a log of mass M, suspended as in Fig. 145, gives it a velocity V (see § 106), the velocity of the bullet (v) may be found by the equation,

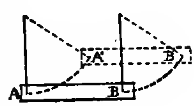


FIG. 145.

$$v = \frac{(m + M)}{m} V.$$

Changes in velocity may be measured by the same principle. If two billiard balls, A and B (Fig. 146), are suspended by cords of equal length so as to just touch each other without pressure, and if the greater, A, is drawn aside to a position A' (Fig. 147) and allowed to strike B while resting at B', the latter will reach a position B'', while the former reaches A''. The velocity acquired by A in falling from A' to A will be proportional to the straight line $A'A$ (§ 109); the velocity after impact will be proportional to AA'' and in the same direction as before; hence the loss

FIG. 146.

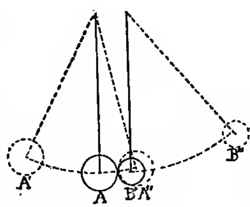


FIG. 147.

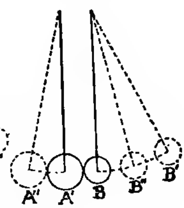


FIG. 148.

will be proportional to $A'A - AA''$. At the same time B gains a velocity represented by $B'B''$.

If on the other hand B strikes A from a position B' (Fig. 148), it will rebound to B'' in the opposite direction; hence its change of velocity will be $B'B + B''B$. The corresponding gain of velocity by A will be represented by $A'A''$.

It is easy to show by experiment that the products of the masses and their respective changes of velocity are equal, whether the balls are elastic or inelastic.[1]

[1] See Ex. 20 of the Descriptive List of Elementary Physical Experiments published by Harvard University.

A comparison of the changes of velocity in question gives a simple means of estimating the relative masses of the balls.

EXPERIMENT LVI.

FALLING BODIES.

¶ 148. **Determination of Distances traversed by Falling Bodies in Different Lengths of Time.** — A wooden rod, *jp* (seen edgewise in Fig. 149), about 25 *cm.* in length, 3 *cm.* in breadth, and 1 *cm.* in thickness, is suspended from the edge, *f*, of a bracket, *ef*, by a strap of paper forked at *h*, so that the rod, when free, may hang in a vertical position. An ounce bullet is next suspended by a thread from the peg, *c*, and lowered to a position, *q*, near the bottom of the rod. The bracket is then moved (by loosening the screws *d* and *g*) so that the rod may barely touch the bullet. Then the bullet is removed, and either the rod is smoked at *j* and at *p*, or pieces of smoked paper are attached to it at these points.

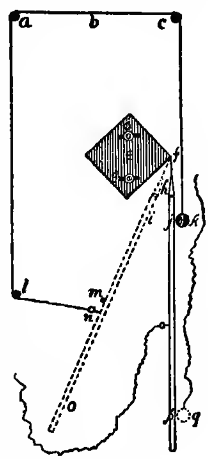


Fig. 149.

The bullet is now suspended at a point, *k*, near the top of the rod, by a thread passing over the smooth round pegs *c*, *a*, and *l*, to a screw-eye, *n*, near the middle of the rod. The rod is drawn one side by the pull on the thread, due to the weight of the bullet. Care must be taken to ease the thread round the pegs, so that the true position of equilibrium may be found. A pin *m* may then be placed so as to mark this position of equilibrium.

To find the height of the bullet a finger is laid upon the thread at *a*, and the thread is slipped off the peg *l*, so that the rod may strike the bullet. A mark will thus be made on the smoked surface at *j*. The thread is now carefully replaced on the peg *l*, so that the tension may be the same as before. When the finger is finally removed from *a*, there should be no slipping of the thread. If there is, the experiment must be repeated, until the bullet, having made a mark on the rod, remains unchanged in position.

Any oscillation of the bullet must now be arrested by lightly pushing the thread, just below *c*, in a direction always opposite to that in which the bullet is swinging, or simply by allowing time enough for it to come to rest. The thread is then burned at *b* by holding a lighted match under it. The rod and the bullet will thus be released at very nearly the same instant. When the rod reaches its vertical position, *jp*, it will strike the bullet at some point, *q*, where the bullet will make a mark on the smoked surface.

The distance between the two marks, one near *j*, the other near *p*, is now to be measured. This distance is equal to that through which the bullet falls while the rod is reaching its vertical position ; that is, in half the time it takes the rod to swing from one side to the other. To determine the time in question, we set the rod once more in oscillation and find how long it takes it to complete 100 or more swings.[1]

To obtain the best results, the oscillations should be timed as will be explained in the next experiment. The time of a single oscillation (either from left to right or from right to left) is then calculated and divided by 2, to find the time occupied by the rod in reaching its vertical position in the middle of one swing. This gives the time occupied by the bullet in falling through the observed distance.

The experiment should be repeated with the same apparatus until results are obtained agreeing within 2 or 3 per cent. The experiment should be then varied by using rods of different lengths. The results should be entered as follows: in the first column, the distance through which the bullet falls; in a second column, the corresponding times of falling; in a third column, the squares of these times, in a fourth column, the ratios of the distances to the squares of the times. Thus: —

1. Distance Fallen.	2 Time Occupied.	3. Square of Time.	4. Ratio of 1 to 3.
19.2 *cm.*	0.20 sec.	0 040	480
80.0	0 40	0.160	500
etc.	etc.	etc.	etc.

It will be seen by the formula $d = \frac{1}{2} gt^2$ (§ 108) that the ratio of the distance to the square of the time must be equal to $\frac{1}{2} g$, which is the distance a body

[1] The student should notice that though the swings grow shorter and shorter in length, there is little or no perceptible change in the rate of oscillation (see § 111). A more exact method of testing this point will be met incidentally in Exp. 58.

falls in one second. The numbers in the fourth column may be considered, therefore, as different estimates of this distance, founded on observations lasting through different intervals of time. These estimates should evidently show an approximate agreement; but the results are modified somewhat by the fact that we are not experimenting with a body which is perfectly free to fall. A device, similar in many respects to that shown in Fig. 149, will be found described in Exp. 20 of the Descriptive List of Experiments in Physics, published July, 1888, by Harvard University. A device in which two electromagnets are used to set free a pendulum and a falling body will be found in Trowbridge's New Physics, Exp. 67.

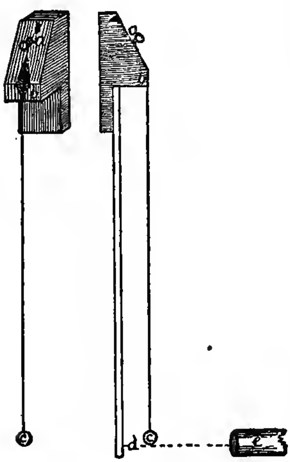


FIG. 150. FIG. 151.

EXPERIMENT LVII.

LAW OF PENDULUM.

¶ 149. **Determination of Times of Oscillation.** — An ounce bullet (*c*, Fig. 150) is to be suspended by a waxed silk thread, passing through a notch (*b*) in the edge of a bracket to and round a pin, *a*, by which the thread can be lengthened or shortened. The lower surface of the bracket must be horizontal (see *b*, Fig. 151), and the groove must be deep enough to reach this surface. It is now required to find the length of the pendulum thus constructed; that is, the distance from its point of suspension, in the surface, *b*, to the middle of the bullet, *c*. This is done by means of a wooden rod, *bd*, graduated in millimetres. The rod is held parallel to the thread (and hence vertical) with its zero at *b*. The height of the centre of the bullet is found from that of the top and bottom by taking the mean. To avoid parallax (§ 25) these heights are sighted through a telescope (*e*), on the same level with them. We thus find the length of the pendulum in question. The time occupied by a hundred or more consecutive[1] oscillations of the pen-

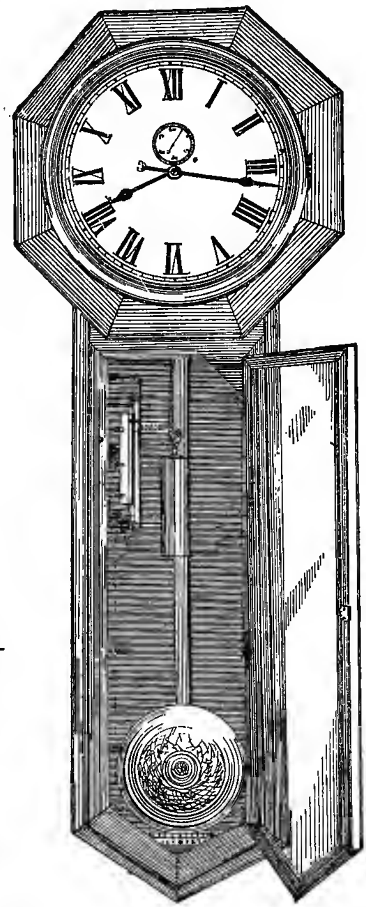


Fig. 152.

[1] The importance of observing long series of consecutive observations must not be overlooked. A student is apt to imagine that 10

dulum is now to be found. The counting is to be begun at a moment when the pendulum begins a swing just as the second hand of a regulator (Fig. 152) indicates some round number of seconds. The time must be written down provisionally in advance. Let us suppose that the observer is ready at 11 h. 8m. He writes down provisionally 11 h. 9m. 0s. When at a given tick of the clock the second-hand indicates 40 sec. he counts that tick 40, the next 41, etc.[1] If on the 60th tick the simple pendulum does not happen to be at the beginning of a swing, a new trial should be made (e. g.) at 11 h. 10 m. 0 sec. In the course of about 10 trials a case should be found in which a swing *seems* to begin exactly on the minute and second provisionally recorded. The observer then begins immediately to count the swings completed by the pendulum (¶ 138). When the pendulum has made, let us say, 100 swings, the time by the clock is again noted. If the clock ticks just as the pendulum completes its 100th swing, the indication of the clock (which will not change for one second) is exact; if, however, the 100th swing is completed half-way between two ticks of the clock, the first indication should be increased by 0.5 seconds. The student should practise in this way the estimation of half-seconds or smaller fractions if possible. The results are invariably to be expressed in tenths.

series of 10 swings each gives an average result as good as one series of 100 swings, whereas in fact, 100 series of 10 each would be required (see § 51).

[1] With a little practice, the student should be able to follow the motion of the second-hand for some time by simply counting ticks, *without looking at the clock.*

The experiment is to be repeated with pendula, the lengths of which are about 10, 20, etc., up to 100 *cm.* The results are to be arranged in two columns, the first showing the length of the pendulum, or the distance from the point of suspension to the centre of the bullet; the second showing the time of vibration, found by pointing off two decimal places from the time in seconds occupied by 100 vibrations. Then a third column is to be calculated, showing the squares of the times of vibration; and a fourth column showing in each case the ratio of the length of the pendulum to the square of the time of vibration. Thus: —

(1) Length of Pendulum.	(2) Time of Swing	(3) Square of Time	(4) Ratio of (1) to (3
8.8 *cm.*	0 30 sec.	0.09	97.8
99.0	1.00	1.00	99.0
etc.	etc.	etc.	etc.

In accordance with the well-known law of the pendulum (§ 110), the squares of the times in column (3) should be proportional to the lengths in column (1), hence the numbers in the fourth column should be (theoretically) the same. In practice variations occur, due not only to errors of observation, but also to the fact that a bullet suspended in air by a silk thread is only an approximation to an ideal simple pendulum.[1]

By comparing the table found in this experiment

[1] A pendulum consisting of a small sphere suspended by a fine thread is sometimes called a simple pendulum. An ideal simple pendulum consists, however, of an infinitely small body suspended in vacuo by a perfectly flexible but inextensible cord entirely devoid of weight. See Deschanel's Natural Philosophy, Chapter VI.

with that obtained for falling bodies in Exp. 56, we discover a curious relation. The length of a pendulum which makes one swing in one second is about 99 cm. The distance a body falls in one second is about 490 cm. The latter is nearly 5 times as great as the former. Again, the length of a half-second pendulum is not quite 25 cm., the distance a body falls in half a second is about 122 cm., that is, nearly 5 times as great as the corresponding length of the pendulum. This proportion will be found to exist in every case.

It is obvious that if this proportion is known,[1] we may calculate the distance through which a body falls in a given time from the length of a pendulum making one swing in the same time. We shall make use of this principle in the next experiment.

EXPERIMENT LVIII.

METHOD OF COINCIDENCES.

¶ 150. **Adjustment of a Pendulum of Peculiar Construction.** — A serviceable device, which conforms approximately to the conditions required of a simple pendulum, is represented in Fig. 153 as seen from in front, and in Fig. 154, in profile. It consists of a cylinder (*gj*) suspended by two vertical loops of silk

[1] The law of falling bodies gives (§ 108) $d = \frac{1}{2} gt^2$; the theory of the pendulum gives (see Appendix) $l = \frac{gt^2}{\pi^2}$; hence we have $d . l :: \pi^2 : 2$ $4.935 : 1$, nearly. This ratio is not affected by the value of g, but is slightly affected by the resistance of the air.

thread passing around the horizontal pins *ab* and *hi*. The diameter of these pins should be exactly the same, and not over 1 *cm*. Their length should be about 10 *cm*. The upper pin (*ab*) is driven through a fixed support; the lower pin should pass as nearly as possible through the centre of gravity of the cylinder. The ends of the thread, after passing over the pin *ab*, are carried each to one of the pins *c*, *d*, *e*, and *f*, by turning which the threads may be lengthened or shortened. A disc is

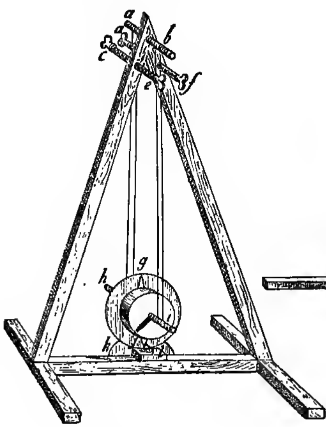


FIG. 153.

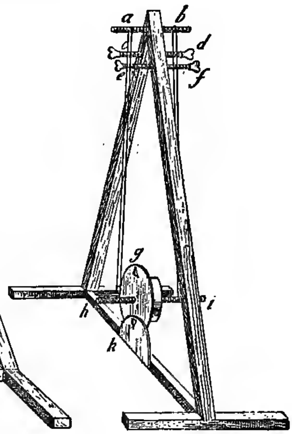


FIG. 154.

also attached to the cylinder, and in this disc are made two *V* shaped holes (*g* and *j*). Opposite the lower hole (*j*) may be placed an opening (*k*), in a shield, through which instantaneous views of objects behind the pendulum may be obtained at regular intervals. A small wire loop may be attached to the pendulum so as to complete an electrical connection between two drops of mercury at *l* when the pendulum is at rest or in the middle of a swing. The length of the pendulum thus constructed is found by measuring the distance between these pins from centre to centre. In the absence of a cathetometer (¶ 262) or other device by which the distance in question may be accurately measured, it is well to adjust it by turning the pins *c*, *d*, *e*, and *f* until a metre rod fits without looseness or pressure between the pins *ab* and *hi*, so as to subtend the vertical distances either between *a* and *h* or between *b* and *i*. The diameters of the pins at *a*, *b*, *h* and *i* are now measured by a vernier gauge (Part I. ¶ 50). The average diameter added to the length of the metre rod gives the distance between the pins from centre to centre.

In regard to the working of this pendulum, it may be pointed out that the cords (*ah* and *bi*) keep the pins (*ab* and *hi*) parallel, hence horizontal, and always the same distance apart. The centre of the pin *hi* swings, therefore, in a vertical plane about the middle point of *ab* as a centre. Now equal parallel forces applied by the cords (*ah* and *bi*) on each side of the pins (*ab* and *hi*) act in all cases like single forces applied at the centres of these pins (see Experiment 61, ¶ 159, 1). If the centre of gravity of the cylinder and disc is in the axis of *hi*, we have, as in the simple pendulum, a weight acting as if it were applied at a single point (in *hi*), and made by forces also applied at the same point (in *hi*) to oscillate about another point (in *ab*) as the centre. There is no rotation either of the cylinder or of the disc to complicate the result, as in the case of an ordinary compound pendulum. Evidently no such rotation can exist, unless the cords (*ah* and *bi*) slip on the pins (*ab* and *hi*). There is, moreover, no tendency to produce such rotation; because forces acting at the centre of gravity of a body (in *hi*) can cause only a linear motion of that centre of gravity. A line in the disc or cylinder which is vertical in one position of the pendulum, remains accordingly vertical in all positions. Here lies an essential distinction between this and other compound pendula.[1]

¶ 151. **Determination of Times of Oscillation by the Method of Coincidences.** — A pendulum between 100 and 101 *cm.* in length, adjusted and measured as in ¶ 150, is placed, let us say, in front of the pendulum of a regulator (Fig. 152, ¶ 149) and set in vibration in an arc not exceeding 10 *cm.* in length (that is, 5 *cm.* on each side of the vertical — see Table 3, *g*). Each swing will occupy a little over a second; hence the first pendulum will fall slowly behind the second. The two pendula will be moving now the same way, now opposite ways. The ticks of the regulator will occur when the first pendulum is now at its furthest right-hand or left-hand point, and now when it is at the middle point of its swing. Every such corres-

[1] The student may notice that the time of oscillation of the stick used in Exp. 56 is considerably greater than that of a simple pendulum (see Table, ¶ 149) equal in length to the distance between the centre of gravity of the stick and its point of suspension. This is owing to the fact that gravity has not only to move the centre of the stick through a certain angle about its point of suspension, but also to turn the stick through the same angle. For a similar reason all ordinary compound pendula are somewhat retarded.

pondence involves a "coincidence" of some sort. The object of this experiment is to find the average interval of time between two coincidences of a given kind. The student will be surprised to find in the reduction of different results (¶ 152) how large an error may be committed in the method of coincidences without introducing any considerable error into the result.

I. OCULAR METHOD. — When the pendula are apparently swinging the same way, the time is to be read by the clock in hours, minutes, and seconds; and again the time is to be noted when the pendula seem to be moving in opposite ways. This should be continued for half an hour or more, according to the length of time that the pendulum may continue to swing perceptibly. The two pendula will probably seem to coincide for a long time in each case. Every effort must be made to determine the middle of such periods of coincidence.

II. EYE AND EAR METHOD (§ 28). — The times may be noted when the ticks of the regulator are heard just as the pendulum under observation reaches its furthest point to the right or to the left; or better, when it reaches the middle point of its swing. In the latter method, the time of coincidence may be generally found within 10 seconds. It may be convenient in some cases to connect an electrical telegraph instrument with a break-circuit in the clock (Fig. 152, *a*) so that the ticks may be re-enforced or reproduced at a distance.

III. OPTICAL METHOD. — Instantaneous views of the pendulum of the regulator may be obtained through the opening, *k*, in a fixed shield (Fig. 153), and an opening, *j*, in the disk of the pendulum. The regulator should be illuminated so that these views may produce a sufficient impression upon the eye. The times are to be noted when the pendulum of the regulator is seen at the middle point of its swing. Times of coincidence may thus be determined within a few seconds.

IV. Electrical Method. — An electrical current is sent first through the break-circuit of the clock (Fig. 152, ¶ 149), then through the break-circuit *lmno* (Fig. 156) attached to the pendulum (see Pick-

Fig. 155.

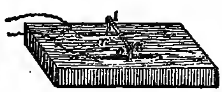


Fig. 156.

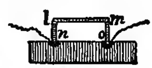


Fig. 157.

ering, Physical Manipulation I. § 41). The ends of these wires should be amalgamated by dipping them first in nitric acid, then in mercury in order to make good electrical connections. The two hollows, *n* and *o* (Fig. 157), must be filled with mercury and raised by thin wedges so that the mercury may touch the wires (*lm*) in the middle point of the swing (*m*, Fig. 155).

When the swings of the two pendula come into a certain mutual relation, an electrical connection will be made by both break-circuits at the same time, and the sounder will respond. After a certain time this relation will cease, and the sounder will become silent. The beginning and end of each period of response should be noted, and the middle of the period found by calculation. This method, though more complicated in detail, requires much less effort than the optical method, and is in general equally accurate.

The experiment is to be repeated with a *hollow* cylinder of sheet zinc, instead of the solid zinc cylinder represented in *gj*, Fig. 153; then again repeated with this hollow cylinder filled with sand or lead shot. The weights of the empty cylinder and its contents should be noted.

¶ 152. **Reduction of Results obtained by the Method of Coincidences.** — The reduction of results obtained by the method of coincidences will be best explained by an example. The times of coincidence should be arranged (see § 61) in three columns of about equal length. These columns should contain an odd number of observations, and should be averaged, thus: —

		min	sec		min.	sec.		min.	sec.
	1st	13	41	6th	24	0	11th	34	32
	2d	15	44	7th	26	3	12th	36	34
	3d	17	51	8th	28	9	13th	38	39
	4th	19	56	9th	30	15	14th	40	46
	5th	21	58	10th	32	23	15th	42	49
Average	3d	17	50	8th	28	10	13th	38	40

The first average corresponds in the example to the time of the 3d observation; the second average corresponds similarly to the 8th observation, and the last average corresponds to the 13th observation. For reasons stated in § 51, these averages are probably more accurate than the single observations to which they correspond. The difference between the first and second averages is 620 seconds; and since between the 3d and 8th observations, to which they correspond, there are 5 intervals, the average for each interval must be 124 seconds. It appears, therefore, that in 124 seconds the first pendulum loses just one swing with respect to the regulator; that is, it makes 123 swings while the regulator makes 124. Assuming that 124 swings of the regulator occupy as many seconds, one swing of the first pendulum must occupy $\frac{1}{123}$ of 124 seconds, or 1.0081 sec. In the same way, between the 8th and 13th observations, we find coincidences on the average 126 seconds apart; hence the average time of one swing is $\frac{1}{125}$ of 126 seconds, or 1.0080 sec. The student should note that the time occupied by one swing (1.0081 sec.) in the first part of the experiment differs very slightly from that (1.0080 sec.) in the last part of the experiment. The difference, due to a decrease in the arc of the pendulum, is in fact only about $\frac{1}{10000}$ of a second (see Table 3, *g*). He should also notice that this small difference in the result corresponds to a comparatively large difference (2 seconds) in the average interval between coincidences. Even with rough methods (¶ 151, I. and II.) such a difference could hardly fail to be observed when sufficiently multiplied by a *long series* of observations. If, conversely, the average interval between coincidences can be found within 2 seconds, the time of oscillation must be accurate within $\frac{1}{10000}$ of a second.

A comparison of results obtained with a solid and with a hollow cylinder of a given size and shape should show that the resistance of the air (which must exert a relatively greater influence in one case than in the other) is slight. A comparison of results obtained with a hollow pendulum filled with *different materials* should show that the time of oscillation of a pendulum of given length is independent of the nature of the substance of which it is composed.

¶ 153. **Relation between the Length and Time of Oscillation of a Pendulum and the Acceleration of Gravity.** — We have already seen (¶ 149) that a relation must exist between the length of a pendulum and the distance traversed by a falling body while the pendulum is making one swing. To find the distance which a body falls in 1.0081 sec. we have only to multiply the length of the pendulum, let us say 100.8 *cm.* by a certain number (4.935) already determined. From the distance which a body falls, and from the time occupied, we may calculate the velocity imparted to the body (see § 108); and from the velocity imparted in a given length of time, we can find that imparted in 1 second (§ 108). This is called the acceleration of gravity, and is denoted by g in the formulæ of § 108. To shorten this calculation, which depends solely on the length and time of oscillation of a pendulum, the following table has been computed for simple pendula between 99 and 101 *cm.* in length: —

TIME OF OSCILLATION.

Length of Pendulum.								
99.0	1.0000	0.9995	0.9990	0.9985	0.9980	0.9975	0.9970	0.9965
99.1	1.0006	1.0000	0.9995	0 9990	0 9985	0.9980	0.9975	0.9970
99.2	1 0011	1.0005	1.0000	0.9995	0.9990	0.9985	0.9980	0.9975
99.3	1 0016	1.0010	1.0005	1.0000	0 9995	0.9990	0.9985	0.9980
99.4	1 0021	1.0016	1.0010	1.0005	1.0000	0.9995	0.9900	0.9985
99.5	1.0026	1.0021	1.0015	1.0010	1.0005	1.0000	0.9995	0.9990
99.6	1.0031	1.0026	1 0020	1.0015	1.0010	1.0005	1.0000	0.9995
99.7	1.0036	1.0031	1.0026	1.0020	1.0015	1 0010	1 0005	1.0000
99.8	1.0041	1.0036	1.0031	1.0025	1.0020	1.0015	1.0010	1.0005
99.9	1.0046	1.0041	1.0036	1.0030	1.0025	1.0020	1.0015	1.0010
100.0	1.0051	1.0046	1.0041	1.0035	1.0030	1.0025	1.0020	1.0015
100.1	1.0056	1.0051	1.0046	1.0040	1.0035	1.0030	1.0025	1.0020
100 2	1.0061	1.0056	1.0051	1.0045	1 0040	1 0035	1.0030	1.0025
100.3	1.0066	1.0061	1.0056	1.0050	1.0045	1 0040	1.0035	1.0030
100.4	1.0071	1.0066	1.0061	1.0056	1.0050	1.0045	1.0040	1.0035
100.5	1 0076	1.0071	1.0066	1.0061	1.0055	1 0050	1.0045	1.0040
100.6	1.0081	1.0076	1.0071	1.0066	1.0060	1.0055	1 0050	1.0045
100.7	1.0086	1.0081	1.0076	1.0071	1.0065	1 0060	1.0055	1.0050
100.8	1.0091	1.0086	1.0081	1.0076	1.0070	1.0065	1.0060	1.0055
100.9	1.0096	1.0091	1.0086	1.0081	1.0075	1.0070	1.0065	1.0060
101.0	1.0101	1.0096	1.0091	1.0086	1.0080	1 0075	1.0070	1.0065
g =	977	978	979	980	981	982	983	984

The length of the pendulum is to be found in the left-hand column; then in line with it the number nearest the time of oscillation is to be selected. Beneath this number, at the bottom of the column will be found the value of g.

EXAMPLE I. Given the length, 100.8 *cm*, and the time, 100.81 sec., required g. We find the time of oscillation, 1.0081, in the 4th column in line with 100.8 in the left-hand column and at the bottom of the 4th column we find the number 979, which represents the acceleration of gravity in question.

EXAMPLE II. Given the length, 100.84, and the time, 100.81, required g. We notice that the times increase by the amount .0005 when the length increases by 0.1 *cm.*; hence 0.04 *cm.* corresponds to .0002 sec.

If, therefore, the length had been 100.8 instead of 1.0084 the time would have been 1.0079 instead of 1.0081. Now 1.0079 comes between two numbers opposite 1.003, namely 1.0081 and 1.0076. Under the first we find 979, under the second we find 980. Since 1.0079 differs from 1.0081 by .0002 sec., and a difference of .0005 sec. makes a difference of 1 unit in g, we must add $.0002 \div .0005$ or $\frac{2}{5}$ of a unit to 979 to find the value of g. We have, therefore, $g =$ 979.4.

The object of this calculation is not so much to determine the value of g, which is already known with sufficient accuracy for all latitudes (see Table 47), and is believed to be the same for all materials, but rather to obtain a check upon the standards and methods hitherto employed for the measurement of length and time.

EXPERIMENT LIX.

INERTIA, I.

¶ 154. **Determinations of Mass by the Method of Oscillations.**—A small glass beaker (d, Fig. 158) is to be suspended from a support, a, by a coiled spring of steel wire, bc, as long and as flexible as may be convenient. A substance whose mass is to be determined is placed in the beaker. The beaker is then pulled downward to a position d', vertically beneath d, then released. It will spring up to a position d'', nearly as far above d as d' is below it. Then it will return nearly to d', and thus make a considerable number of oscillations before it comes to rest. The oscillations should not displace the load in the beaker; if they do, the load must be rearranged, or the oscillations must be diminished in amplitude. The time of oscillation is now to be found as in ¶ 149.

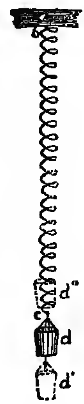


Fig. 158.

The load is next removed from the beaker, and in its stead weights from a set are placed there, sufficient in quantity to stretch the balance to the same point as before. The time of oscillation is again determined. If it is less than before, more weights are added, if greater, weights are removed; and thus by trial (§ 35) the weight is adjusted until the time of oscillation is the same with the weights as with the substance, the mass of which is to be determined.

The student should notice that the time of oscillation is nearly independent of the amplitude of oscillation as in an ordinary gravity pendulum. It should be pointed out, however, that in the vertical oscillation shown in Fig. 158, gravity has nothing to do with the time of oscillation in question, except in so far as it may affect the elasticity of the spring by stretching it to a greater or less extent. When a spring is already loaded the force required to stretch it 1 *cm.* further may be taken as a measure of the stiffness of the spring under the load in question.

The time of oscillation of a load suspended by a spring depends (1st) on the stiffness of the spring and (2d) on the *mass* to be set in oscillation. When two loads give the same time of oscillation under the same circumstances, their masses are necessarily equal.

Having adopted as our standard of mass a certain piece of platinum in the French Archives (§ 6), we should theoretically use platinum weights in this experiment. It has been found, however, that two quantities which have equal masses, estimated as above by the dynamical method, have also equal weights (*in vacuo*); that is, gravity exerts the same acceleration upon them, without regard to the substances of which they are composed (see Exp. 58.) The use of brass weights will not, therefore, in *practice*, introduce any error.

The results of Exp. 59 are to be expressed in grams like results obtained by an ordinary balance. Strictly, however, the word *mass* should be written before or after these results instead of the word *weight* (§§ 152, 153).

¶ 155. **Relation between Weight and Mass.** — The student must not assume that weight and mass are *necessarily* the same. We do not know *why* a body is attracted by the earth, neither do we know *why*, being attracted, it does not move instantly, under that attraction, from one place to another. The former phenomenon we attribute to *gravity* (§ 150), the latter to *inertia* (§ 151).

By the weight in grams of a body we mean the number of grams of platinum to which the body is equal in respect to weight proper (§ 153), or the force exerted upon it by gravity. By the mass in grams of a body we mean the number of grams of platinum to which it is equal in respect to *inertia*, or the necessity of force to set it in motion (§ 152).[1] In the absence of any explanation of gravity and inertia, no reason can be assigned why any proportion should exist between them. There is no proportion between electrical or magnetic forces and the masses upon which they act. The existence of such a proportion between mass and weight is simply an inference from the results of experiment (see Exp. 58). It is possible, so far as we know, that a new substance may be discovered, the mass of which may be disproportional to its weight. It is also possible that if masses could be measured with the same accuracy as weights, slight variations might be discovered which have hitherto escaped observation. We have several instances of physical laws which are approximately but not exactly fulfilled; as for instance the law connecting the molecular weights and specific heats of elementary substances (§ 86, note). At the same time that such variations are *possible*, as far as we know, in the case of gravity and inertia, it is by no means *probable* that any such will ever be discovered. It is much more probable that gravity and inertia are both manifestations of a single principle, according to which, for reasons unknown to us, one must be proportional to the other.

[1] See Hall's Elementary Ideas, published by C. W. Sever, Cambridge, Mass.

EXPERIMENT LX.

INERTIA, II.

¶ 156. **Determination of Force by Observations of Mass, Length, and Time.** — A metallic ring about 20 *cm.* in diameter, and weighing about 500 grams (C D F E, Fig. 159) is suspended horizontally by a spring brass wire *AB*, about 0.25 *mm.* in diameter (No. 31, B W.G.), and at least one metre long. The wire is fastened at the top and held at the bottom by a small vice, *B*. This vice, *B*, is connected by fine iron wires (about No. 31) with four points C, D, E, and F of the ring. A paper millimetre scale is attached to the ring, and the distance through which it revolves is indicated by a fixed marker (*G*).

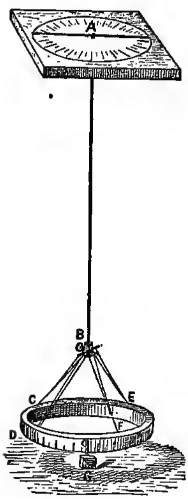


Fig. 159.

The reading of the marker is to be first observed when the ring is at rest. Then the ring is turned through nearly 360°, and released. All pendular vibration must be stopped by touching (if necessary) the wire *AB*. The ring will then have only a rotary movement, due to the "torsion" of the wire. As the ring approaches a turning-point, several readings of the marker are taken at intervals of two seconds. The intervals may be determined by the ticks of a regulator, or by an electrical sounder connected with the regulator.[1]

When the experiment has been repeated a sufficient number of times, the ring is taken down and its weight in grams determined. The vice, B, should not be weighed with the ring. It is better not to weigh the connecting wires with the ring; but their weight (which should not exceed 1 gram) will not in any case introduce a serious error into the result. The material, length, and diameter of the wire AB should be noted. The observations are then to be reduced as in ¶ 157.

¶ 157. **Calculation of Force from Observations of Mass, Length, and Time.** — The rotation of a ring about its axis presents one of the simplest cases in dynamics. The whole mass of the ring is at (nearly) the same distance from the axis in question, and hence acquires (nearly) the same velocity. To find the force exerted upon the ring in the direction of this velocity, we have to find (1) the velocity acquired, (2) the time required to attain this velocity, and (3) the mass acted upon. The force may then be calculated by the general formula (§ 106): —

$$f = \frac{mv}{t}$$

[1] If greater precision is required than can be obtained by the eye, a small bristle attached to the armature of the sounder can be made to mark the seconds on the edge of the ring, which must be previously smoked for this purpose. By employing two such markers on opposite sides of the ring, slight errors due to swinging of the ring can be eliminated.

In practice we make this calculation as in the example below. The observations are numbered and arranged as follows: —

	mm.	Difference in 2 sec.	Mean Velocity.	Difference in 2 sec.	Acceleration
1	552			—	—
		+33	+16.5		
2	585			8.0	4.0
		+15	+ 7.5		
3	600			10.0	5.0
		− 5	− 2.5		
4	595			7.5	3.8
		−20	−10.0		
5	575			10.0	5.0
		−40	−20.0		
6	535			—	—

The differences in the 3d column show the distance passed over in 2 seconds; hence these are divided by 2 to find the distance passed over in 1 second, or the mean velocity for a period of 2 seconds. The velocity is called positive if the ring is turning away from its position of equilibrium, otherwise negative. The 5th column shows the algebraic differences in these velocities; that is, the change of velocity in 2 seconds. To find the acceleration, or change of velocity in one second, the numbers in the 5th column must be divided by 2. This gives the numbers in the 6th column, the average of which is 4.5, nearly. Since we have used *mm.* throughout, the change of velocity in one second amounts to 4.5 *mm. per sec.*, or 0.45 *cm. per sec.*

This is the acceleration strictly of the outer surface of the ring. Let us suppose that the outside diameter is 20.5 *cm.* and the inside 19.5 *cm.*, so that the mean diameter is 20.0 *cm.*; then the average acceleration will be less than 0.45 in the ratio of 20.0 to 20.5. The average acceleration will be, therefore, about 0.44 *cm. per sec.* If now a mass of 500 *g.* receives this acceleration, the force exerted upou it must be $500 \times .44$, or 220 dynes (§ 12). The angle through which the steel wire is twisted is given in circular measure by the ratio of the arc to the radius. Since the latter is 10 *cm.* (nearly), the minimum deflection (53.5 *cm.*) corresponds to 5.35 units of angle. The maximum deflection (60.0 *cm.*) corresponds similarly to 6.00 units of angle. The mean deflection is accordingly not far from 5.7 units of angle. Since one unit of angle in circular measure is equal to 57°.3, nearly, the mean deflection of the ring is about $57°.3 \times 5.7$, or 327°.

We note, therefore, that a piece of steel wire of given length and diameter, when twisted 327°, exerts at a distance of 10 *cm.* from its axis a force of about 220 dynes.

The use which is to be made of this result will be explained in ¶ 165 in connection with a method by which a force similar to the one in question may be directly balanced by gravitation. A more accurate method of reducing results obtained by the "torsion pendulum" will be given in the Appendix (Part IV).

EXPERIMENT LXI.

COMPOSITION OF FORCES.

¶ 158. **Correction of Spring Balances.** — A spring balance consists of a spiral spring, *cd* (Fig. 160), contained in a hollow metallic case, *bh*, to which it is fastened at *c*. The spring is connected by a rod, *di*, with a hook, *ij*, from which weights are hung. A slit, *eg*, is made in the case so that a pointer, *f*, attached to the rod, *di*, may indicate the elongation of the spring on a scale outside of the case. In measuring vertical forces with a spring balance, the instrument is generally suspended by the ring, *a*. When forces in other directions are to be determined, the case (*bh*) should also be supported, so as not to bear against the index, *f*. If this precaution is not observed, large errors from friction may be introduced into the results. Spring balances are usually graduated so as to indicate the weight of a body either in kilograms or in pounds. It must be remembered that such indications are affected by the force of gravity. Thus a spring balance, graduated correctly in England, would give, in Brazil, readings too low by about $\frac{1}{3}$ of 1 %. Obviously spring balances, however sensitive, cannot serve everywhere as standards of *mass* (§ 6). The readings depend, not directly upon the masses suspended, but upon the forces which they exert on the instrument. A spring balance once graduated correctly in *megadynes*[1] should, however, give *forces* correctly (in megadynes) irrespective of locality. A

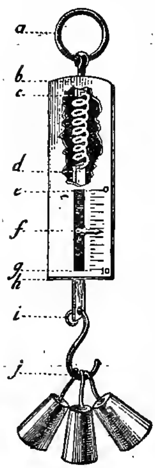


Fig. 160.

[1] The student may be interested to cut a scale of megadynes by the side of the ordinary scale. In latitude 40°–45°, 1 megadyne = 1.02 kilos. = $2\frac{1}{4}$ lbs nearly.

spring balance is essentially an instrument for measuring force, and it is only *in a given latitude* that it may be employed for estimating weights either in kilograms or in pounds. A pair of 10-kilo. (or 24-lb.) spring balances will be suitable for the experiments which follow.

The reading of a spring balance may be corrected by hanging known weights upon it, as in Fig. 160. Weights provided with a ring, a hook, or an eye will be found convenient for this purpose. The reading of the balance should be tested with weights of 1, 2, 3, etc., up to 10 kilos. (or 2, 4, 6, up to 24 lbs.). The zero-reading of the spring balance should also be found, both in a vertical and in a horizontal position. The weights used may be compared by an ordinary balance with standards if it is thought necessary. From these results we are to calculate the corrections to be added to the reading of the spring balance under different loads, in order to find the true load. Thus if the indication with a 4 lb. weight is 3 lbs. 14 oz., the correction is +2 oz. The results should be arranged in tabular form, either in kilos. or in pounds, as follows:—

FIRST TABLE OF CORRECTIONS.

(1) Load in kilos.	Correction in kilos.	(2) Load in lbs.	Correction in oz.
0	−0.10	0	−3
1	−0.05	2	−1
2	+0.08	4	+2
3	+0.25	6	+6
...	...	...	...
10	+0.05	24	+1

One of the weights is now to be attached to the spring balance by a light but strong cord (*ac*, Fig.

161) passing over a pulley (b) made to run as freely as possible. The readings of the balance are to be carefully compared in different positions (a', a'', etc.). To eliminate the effects of the friction of the pulley, the readings are to be made in each case (1) when the weight is being slowly raised, and (2) when it is being slowly lowered. If the two readings differ perceptibly, the mean is to be taken.

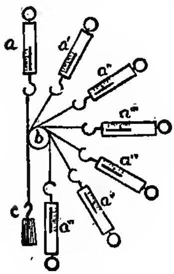


Fig. 161.

The object of testing a spring balance in different positions is to eliminate the effects due to the weight of the hook and spring.[1] From the results we are to calculate the corrections to be added to the readings under different inclinations in order to find the reading in the vertical position. Thus if a 2 lb. weight weighs apparently 2 lbs. 1 oz in the vertical position, and 1 lb. 11 oz. in the horizontal position, the correction for an inclination of 90° is +6 oz. These corrections should be the same for all weights, and should be entered in a second table, as follows: —

SECOND TABLE OF CORRECTIONS.

(1) Inclination of Balance.	Correction in kilos.	(2) Inclination of Balance.	Correction in oz.
30°	+0.02	30°	1
60°	0.08	60°	3
90°	0.16	90°	6
120°	0 24	120°	9
150°	0.30	150°	11
180°	0.32	180°	12

[1] This method was suggested to the author by a similar one employed by Mr. Forbes of the Roxbury Latin School. See also Elementary Physical Experiments, published by Harvard University, page 11, footnote.

¶ 159. **Determinations of Weight by the Composition of Forces.** — It is frequently inconvenient to measure the weight of a body directly, either by ordinary scales, or by a single spring balance, as when the weight of the body exceeds the capacity of such instruments, or when the body forms an inseparable part of a combination. In such cases, we may sometimes make use of principles involved in the composition and resolution of forces.

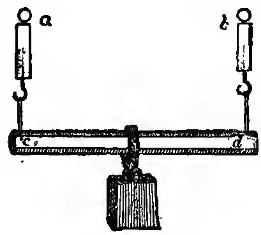


FIG. 162.

(1) To find the force of gravity on a "28-lb." weight with two spring balances, each of 10 kilograms' capacity, we hang the weight (*e*, Fig. 162) at the middle of a stick (*cd*) so that it may bear about equally upon the spring balances (*a* and *b*) while hanging in a vertical position. The reading of each balance is to be noted; then the weight is to be removed, and the readings again taken with the stick alone. The difference between the two readings of a given balance, with and without the weight, corrected if necessary by Table I., ¶ 158, gives the part of the load borne by that balance. The sum of the two parts is of course equal to the whole load.

(2) To find the force of gravity on a "56-lb." weight with a single spring balance of 10 kilograms' capacity, we suspend a lever (*cd*, Fig. 163) as before, except that a cord, *bd*, takes the place of the spring balance (*b*, Fig. 162). The weight is then hung at a

point, *e*, let us say one-fourth the distance from *d* to *c*, and the reading of the spring balance is observed. Care must be taken that the cords *fg* and *hi*, by which the weight is suspended, swing free of the side of the lever as in the cross-section (Fig. 163). A similar precaution should be observed in respect to the cords by which the spring balance, *a*, is attached to the lever at *c*.

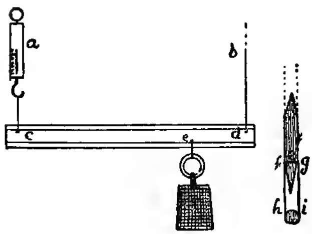


Fig. 163.

The cords should both be vertical. The horizontal distances *cd* and *ed* are to be accurately measured. The weight is now to be removed, and the reading of the spring balance again noted. If F and f are the forces indicated by the spring balance with and without the weight, both being corrected by the first table of ¶ 158, the force (w) exerted by the weight at c is evidently equal to $F - f$. If we call the whole weight W, then since the *couple* (§ 113) produced by W (equal to $W \times de$) is balanced by the couple produced by the spring balance (equal to $w \times cd$), allowing for the weight of the lever, it follows that—

$$W = (F - f) \times cd \div ed.$$

(3) Another method of suspension is represented in Fig. 164. It is assumed that the weight will be able to lift the lever, so that the balance must be applied from under the lever. The reading of the

balance in this position must be corrected both by the first and by the second table of ¶ 158. Thus since the inclination of the balance is 180° (compare Figs. 164 and 161), we must add 0.32 kilos according to the second table (¶ 158), *besides* the ordinary correction for the observed reading from the first table (¶ 158). In addition to the force exerted by the spring balance, we have that part of the weight of the lever which is felt at a, helping to balance the 56-lb. weight.

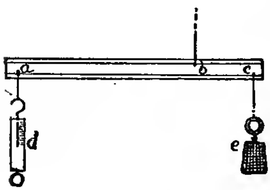


FIG. 164.

To allow for the weight of the lever, we remove the 56-lb. weight, and apply the spring balance as in Fig. 163, so as to sustain the lever at a. The reading of the balance in this position needs to be corrected simply by the first table (¶ 185), and gives the force (f) exerted by the lever at a. This is to be *added* accordingly to the force (F) exerted by the spring balance with the weight (e) to find the total force which balances this weight. Calling this force w, and the load W, we have $w \times ab = W \times bc$, or —

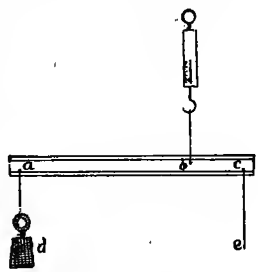


FIG. 165.

$$W = (F + f) \times ab \div bc.$$

(4) To test a 4-lb. weight with a 10-kilogram spring balance, we fasten one end of a lever (c, Fig. 165) to

the ground by means of a vertical cord, *ce*, and suspend the lever from a spring balance by a cord *b*, not far from *c*. The force, *f*, indicated by the balance is to be observed. The weight, *d*, is then hung from the free end of the lever, and the force (F) indicated is again observed. Allowing as before for the weight of the lever we find the force ($F - f = w$) exerted by the spring which balances the load W at *d*. Then since $W \times ac = w \times bc$, we have $W = (F - f) \times bc \div ac$.

If the distance *bc* is one fourth of *ac*, every ounce at *a* will produce an effect at *b* equal to 4 oz. We might therefore weigh a small object to ounces with a balance graduated only to 4 oz. (or $\frac{1}{4}$ lb.).

(5) Another method of weighing small objects is to hang two spring balances, *A* and *B* (Fig. 166), from

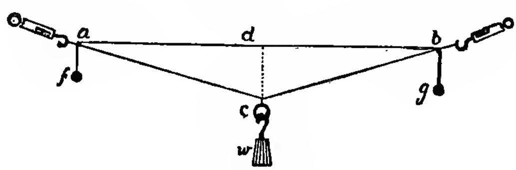


FIG. 166.

nails in the wall, 2 or 3 metres apart, then to connect them by a cord *acb*. At the middle of the cord (*c*) a ring (C) is hung so that the weight, W, may be readily attached. Two pins are driven into the wall opposite points *a* and *b*, on the cords at equal distances (let us say just 1 metre) from *c*. A cord, *ab*, is stretched between them by means of two small weights, *f* and *g*. The perpendicular distance, *cd*, between *c* and *ab* is then measured.

The vertical component of the force A registered by the spring balance near a, is by the triangle of forces (§ 105) equal to $A \times cd \div ac$. The vertical component of the force, B, due to the spring balance near b, is similarly $B \times cd \div bd$. The total sum of these components must balance the combined weight of the ring (C) and of the load (W). That is,

$$W + C = A \times cd \div ac + B \times cd \div bc.$$

To eliminate the weight of the ring, the load (W)

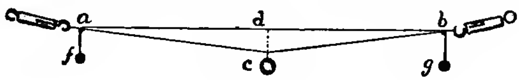


FIG. 167.

is removed, and the experiment is repeated with the ring alone, as in Fig. 167. We have, similarly,

$$C = A \times cd \div ac + B \times cd \div bc.$$

Hence subtracting the last value (C) from the first ($W + C$) we find the weight of the load (W) in question.

We will assume, for simplicity, that a and b are on the same level. A slight difference in level will, however, have no appreciable effect upon the result. The sagging of the cord ab will probably be very small, and will be eliminated in the method of difference by which the result is calculated.

The same method may be employed for the measurement of large weights. If the angle acb is small (see Fig. 168), it will be more accurate to calculate cd from a measurement of ab, than to measure cd directly. Let us suppose that the cords bB and aA have been lengthened or shortened so that the line ab is horizontal. The vertical line cd will then be at right angles with ab; and since $ac = bc$, $ad = bd = \frac{1}{2}ab$. Knowing ad, we may calculate cd by the Pythogorean proposition —

$$cd = \sqrt{(ac)^2 - (ad)^2},$$

and hence find the load C or W as before.

This method would be adopted in practice if for any reason it were inconvenient to obtain a point of

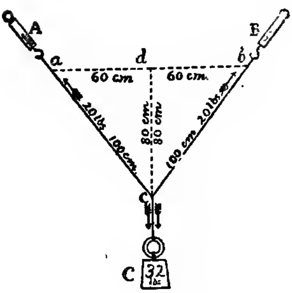


Fig. 168.

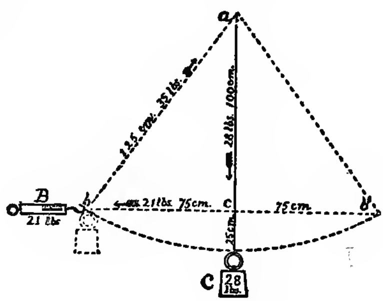


Fig. 169.

suspension directly above the weight. We should prefer, however, to employ a lever long enough to reach, as in (1) or (2), between two available points of suspension, A and B, if it were possible to obtain one of suitable weight and strength.

(6) To measure a weight (C, Fig. 169) when suspended by a cord (ac) we may pull it one side by a spring balance applied horizontally in the direction cb. The reading of the balance (corrected by *both* tables of ¶ 158) gives the force B acting in the di-

rection cb. This with the force of the cord acting in the direction ba produces a resultant which balances the weight of the body C. The direction in which the weight C acts must be parallel to that of the cord ac before the weight was disturbed. Since three forces in equilibrium are proportional (§ 105) to the sides of a triangle to which they are respectively parallel, we have $B : C = bc : ac$, or

$$C = B \times bc \div ac.$$

Instead of measuring bc directly, we may pull the cord ac first one side to a point b, then in the opposite direction to a point b' at a (nearly) equal distance from c. These points may be marked by pins, b and b' driven into the wall or into some other support behind the cord. The distance between b and b' is then measured and divided by 2 to find the distance bc. The point c may be found by a thread stretched between the pins b and b'. In this case the distance ac may be directly measured. Or the distance ab may be found and ac calculated (since ab is known) by the Pythagorean proposition,

$$ac = \sqrt{(ab)^2 - (bc)^2}.$$

By the use of very small deflections, we may measure weights many times exceeding the capacity of the spring balances which we employ.

EXPERIMENT LXII.

CENTRE OF GRAVITY.

¶ 160. **Location of the Centre of Gravity.** — A flat board,[1] *bcde* (Fig. 170), is suspended by a thread *abb'a'* (Fig. 170, 1) passing through a fine hole *bb'* in the board, and over a peg *aa'*. A plumb line, *af*, is also suspended from the same side of this peg, so as to hang as close to the board as possible. A projection of this line upon the board is to be traced in pencil (Fig. 170, 2). The eye must be held in this process

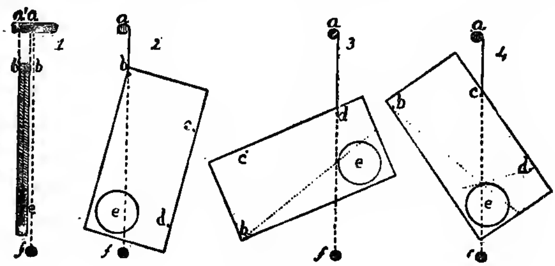


Fig. 170.

so as to look perpendicularly upon the board (§ 25). The board is then to be hung by another point, *d* (Fig. 170, 3), and another line drawn upon it. Then the board is to be suspended from a third point, *c* (Fig. 170, 4), and a third line traced. All three lines

[1] To lend interest to this experiment the board may be made of two thicknesses glued together, with a space (*e*, Fig. 170) between them which has been hollowed out and filled with lead. An irregularly shaped board may also be employed.

should intersect at a point in the surface of the board directly in front of the centre of gravity. If they do not, the experiment must be repeated.

¶ 161. **Determination of Weight by Displacement of the Centre of Gravity.** — A weight (w, Fig. 171) is attached at a to one end of a board whose centre of gravity (c) has been located (¶ 160); and the board is balanced upon a triangular piece of wood (d) or upon a pencil. The line of the support (bb' Fig. 172) is then marked upon the board, and two lines, ab and cb' are drawn from a and c perpendicular to bb'. These lines are then carefully measured. If W is

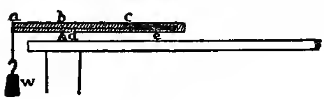


FIG. 171.

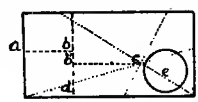


FIG. 172.

the weight of the board, which we may consider as if concentrated at c (§ 112), we have $W \times b'c = w \times ab$;

whence $$W = w \times (ab) \div (b'c).$$

The experiment should be repeated with different weights applied at different parts of the board, and with the line bb' not always at the same place or in the same direction. The different values calculated for the weight of the board should be averaged. From their agreement we may infer the truth of the assumption that the weight of a body acts in all cases as if applied at its centre of gravity.

It is obvious that if W and w are both known, we may calculate the distance ($b'c$) by the formula

$$(b'c) = w \times (ab) \div W.$$

To find the distance of the centre of gravity from an axis (bb') on which a body balances, it is only necessary to know the weight of the body (W), the load (w), and its distance (ab) from this axis. For an experiment (due to Prof. Hall) in which this principle is applied, see Ex. 17 of the Elementary Physical Experiments, published by Harvard University.

EXPERIMENT LXIII.

BENDING BEAMS.

¶ 162. **Determination of the Stiffness of a Beam.** — A square steel rod, ag (Fig. 173), is mounted on two triangular supports with steel edges, i and j, 1 metre

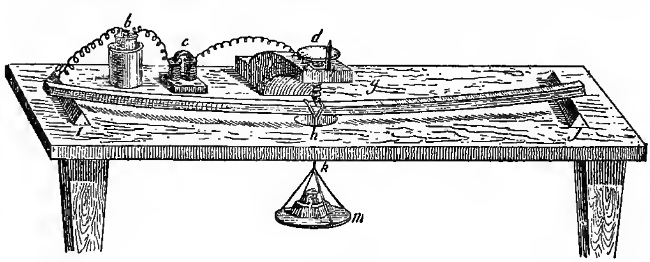


Fig. 173.

apart. A screw with a micrometer head (d) is adjusted so that its point just touches the middle of the beam when a pan, m, is suspended from it by the wires hk. The micrometer is then read. A load, l, is next placed in the pan, and the micrometer is once more adjusted until it touches the beam. The micrometer is again read. Its point is then withdrawn, so as not to be injured by the recoil of the beam when the weight is removed. A new reading is then taken with the pan (m) empty. If this differs greatly from the first, the beam has probably been permanently bent, and the experiment must be repeated with a smaller load. If the reading is the same as before, a larger load may be tried. With a steel beam 100 *cm.* long and not over 1 *cm.* thick, a deflection of several centimetres should be possible without injury to its power of recovery. To discover exactly when the point of the micrometer touches the beam, we may make use of an electrical contact. One pole of a voltaic cell, b, is to be connected with one end of the beam by a wire soldered to it at a. The other pole is connected with one binding post of an electrical sounder c. The other binding post of this sounder is connected by a wire with the metallic nut e, in which the micrometer turns. The point of the micrometer and the surface of the beam beneath it are scraped bright with a file (or better, coated with platinum). When the point of the micrometer touches the beam, the electrical circuit $bceab$ is thus completed, and the armature of the sounder is attracted. A motion of one thousandth of a millimetre is sufficient, under favorable circumstances, to make or break the contact.

Care must be taken to prevent the beam from twisting or rocking under the influence of a load. The load should not bear more heavily on one side of the beam than on the other. Both sides should be supported alike at each end of the beam by the sharp edges *i* and *j*. Various deflections under different loads are now to be determined. Each deflection requires two readings of the micrometer, one with, the other without the load. The distance between the supports *i* and *j* should be measured with a metre rod, and the breadth and thickness of the beams employed should be determined at different points with a micrometer gauge (¶ 50, II.).

(1) The deflection of a beam, let us say 1 *cm.* square, is first to be determined with the supports (*i* and *j*, Fig. 173) exactly 100 *cm.* apart, and with a load causing the greatest deflection which can be employed without permanently bending the beam, or exceeding the reach of the micrometer.

(2) The deflection due to one half this load is next to be found. The student should notice that this deflection is almost exactly half as great as before (see § 115). If it is not, the measurements in (1) and (2) should be repeated. The same should be done if the zero-reading of the micrometer is changed.

(3) To test the stiffness of the middle portion of the beam, the supports *i* and *j* are to be placed 50 *cm.* apart, — that is, with half the original distance between them. The rod is to be mounted upon them as before, but with 25 *cm.* or more at either end projecting beyond the supports. The beam is to be loaded with 4 times the weight used in (1) or 8 times that used in (2). If the beam is equally stiff in all parts, the deflection should now be the same as in (2). (See § 115.)

(4) The experiment is next to be repeated with the supports 100 *cm.* apart, with a beam twice as broad as the one first employed, but having the same thickness and bearing the same load as in (1). If the material of the beam is the same as in (1), the deflection due to a given weight should be the same as in (2), since the breadth and weight have the same relative proportion as in (2).

(5) The beam is now to be turned edgewise, and loaded as in (3). The deflection is to be determined as before. If the depth of the beam is just twice as great as in (2), and the width the same, since the force employed is eight times as great as in (2), the deflection should be the same as in (2).

¶ 163. **Calculations relating to Flexure.** — By five measurements arranged as above, we are able to test (in a single instance in each case) the application of the laws of flexure stated in § 115. These laws may be combined in a single formula. If l is the length of a beam, b its breadth, t its thickness, and d the deflection produced (all in *cm.*) by the force f (in dynes) exerted by the load; and if F is the force necessary to produce a unit deflection in a beam of unit length, breadth, and depth (supposing such a deflection to be possible), we have —

$$F = \frac{fl^3}{bdt^3}.$$

The quantity F is sometimes called the modulus of transverse elasticity. Knowing this modulus, we may evidently compute any one of the five quantities, f, l, b, d, or t, if the other four are known. The student should calculate the value of F from at least one set of measurements. He should also find, by the rule of simple proportion, what force would be required to produce a deflection of 1 *cm.* in the case of each beam which he has employed. Thus if, with a given beam, 1 kilogram produces a deflection of 2 *cm.*, 500 grams would be the force required to produce a deflection of 1 *cm.*

The force (500 grams in this case) producing a unit deflection may be taken as a measure of the *stiffness*[1] of the beam in question. The stiffness of a beam is due to the fact that in order to bend it, the under part must be stretched and the upper part squeezed or compressed. The forces brought into play by stretching will be measured directly in Experiment 65.

EXPERIMENT LXIV.

TWISTING RODS.

¶ 164. **Effect of Couples.** — An instrument serving both to measure and to illustrate the effect of different "couples" (§ 113) is shown in Fig. 174. It con-

[1] Stiffness must not be confounded with breaking strength. A thin beam, though more easily broken than a thick one, is not so in proportion to its flexibility; for by reason of its thinness it can bend much *farther* than a thick beam without breaking. Both the strength and stiffness of a beam are proportional to its breadth; but the former depends upon the square of the ratio which the thickness bears to the length, while the stiffness depends upon the cube of this ratio. (See formula above.)

sists of a rod of ash (*ej*) 1 *cm.* square, driven into a square hole in a block (*j*) which is fastened to the floor. The rod passes through a large hole in a table to a circular disc of wood (*cg*) 20 *cm.* in diameter, at the centre of which is a square hole (*e*), into which the upper end of the rod is tightly fitted. Two markers, *b* and *g*, measure the rotation of the disc by means of a scale of degrees graduated on the edge of the disc. At certain points of the disc (*abc defgh*, Fig. 175), small screw-eyes are placed so that forces may be applied by cords attached to spring

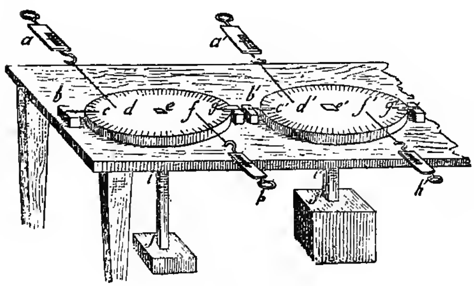


FIG. 174.

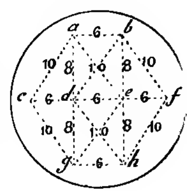


FIG. 175.

balances (*a* and *h*, Fig. 174). It is convenient that four or more of the points (*cdef*, Fig. 175) should be in the same straight line and at equal distances, let us say 6 *cm.* The points *a*, *b*, *g*, and *h* (Fig. 175) may be placed so that *ad*, *dg*, *be*, and *eh* are at right angles to *cf*, and each 8 *cm.* long. This will make the diagonal distances *ac*, *bd*, etc., each 10 *cm.*

A very slight force applied at any point of the disc will cause the rod *cj* (Fig. 174) to bend so as to touch one side of the hole in the table. To keep the rod in the middle of this hole throughout this experiment,[1] equal and opposite forces must be applied to the disc. If these forces are applied at the same point, no effect will be observed. For instance, two equal forces applied at *d* (Fig. 175) in the directions *dc* and *de* (or in the directions *da* and *dg*) will neutralize each other. Again, if the forces and their points of application are all in the same straight line, the effect will be zero. Thus a force applied at *d* in the direction *de* will offset an equal force applied at *e* in the direction *ef*. When, however, the lines in which the two forces act are parallel but not coincident, the couple which results (§ 113) will twist the rod. The angle through which the rod is twisted should be proportional to the magnitude of the couple acting upon the disc. The magnitude of the couple is equal (see § 113) to the product of either of the two forces which constitute it, and the "arm" or perpendicular distance between the lines in which the forces act.

The student should satisfy himself that it makes no difference where the "arm" is situated. Thus two opposite forces of 1 kilogram each applied at *a* and *b* or at *c* and *d*, at right angles to *cf*, will have the same effect as if applied in the same manner at *d* and *e*, respectively. The student will notice, moreover, that the rod is twisted but *never bent* by a pair of equal and opposite forces, whether these be applied at equal

[1] In trying this experiment, several students should work together. One may hold and read one of the spring balances, another the other spring balance, while a third observes the deflection of the disc.

or unequal distances from the *centre* of the disc. He should also satisfy himself that with a given arm (as for instance *de*), the rod is twisted through an angle which is proportional to the forces employed (let us say 1, 2, or 3 kilograms); and that the twists produced by given forces (*e. g*, 1 kilogram each) are proportional to the arms to which they are applied. Arms of the following lengths may be most conveniently employed: 6 *cm.* (*ab*, *cd*, *de*, *ef*, or *gh*); 8 *cm.* (*ad*, *be*, *dg*, or *eh*); 10 *cm.* (*ac*, *ae*, *bd*, *bf*, *gc*, *ge*, *hd*, or *hf*); 12 *cm.* (*ce* or *df*); 16 *cm.* (*ag* or *bh*); and 18 *cm.* (*cf*). Two *equal* forces must be applied in all cases in directions at right-angles to the arms, parallel to the disc, and opposite to each other. They should be made to twist the rod sometimes to the right and sometimes to the left.

To measure accurately the angles through which the disc rotates, both markers (*b* and *g*, Fig. 174) must be observed. It is easy to calculate from a given case by simple proportion what couple would be required to twist the rod through 1°. This gives us a measure of the stiffness of the rod under torsion which may be called its coefficient of torsion.[1]

We next employ a rod, *e'j'*, of half the length of *ej* (Fig 174). This rod must be mounted on a block (*j'*) much higher than *j*. We shall find, if the material and the cross-section are the same, twice the coefficient of torsion. If we use a rod of same length, having, however, twice the diameter, we shall

[1] The coefficient of torsion must not be confounded with the strength of a rod to resist *fracture* by torsion. See note ¶ 163.

find a coefficient of torsion 16 times as great as before (see Laws of Torsion, § 116). It is therefore important to measure and note the length and diameter of the rods employed.

We shall apply the principles illustrated in this section to the determination of the coefficient of torsion of a wire.

¶ 165. **Determination of the Coefficient of Torsion of a Wire by means of a Torsion Balance.** — A hard drawn brass wire about 2 metres long and 0.25 *mm.* diameter (about No. 31, B.W.G.) is stretched horizontally between a knitting-needle (*bd*, Fig. 176) and a fixed support (*k*). The joints should be soldered both at *c* and at *k*, or made equally firm in any other manner.

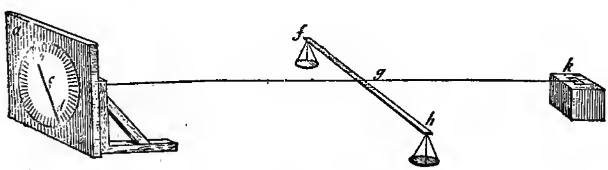


Fig. 176.

The knitting-needle is held in place by a paper protractor fixed on the surface of a board (*ae*). The board and protractor are pierced at the centre (*c*) so that the wire may pass through. A thin strip (*fh*) of some light wood, 20 *cm.* long, is attached at its central point, *g*, to the middle of the wire by sealing-wax. From the ends of this strip two paper scale-pans are suspended by threads. The "torsion" balance thus constructed should not weigh more than one or two grams.

The knitting-needle is first set so that the beam (fh) is horizontal. To do this, the beam must be sighted with reference to the bars of a window, or other horizontal line in the room. The reading of the needle is then found by observing both ends. This is the zero-reading of the instrument. Then a decigram is placed in one of the scale-pans, and the needle is turned until the beam is again horizontal. The decigram is then removed from the scale-pan, and the zero-reading re-determined. If any marked change has occurred, the experiment must be repeated. If the zero-reading is again disturbed, a weight smaller than 1 decigram should be employed.

The weight is to be placed first in one scale pan, then in the other. In each case we note the angle through which the needle must be turned to the right or to the left from its zero position in order that the beam may be made horizontal. It is well to observe the zero-reading after the experiment, since the constancy of this reading is the only safeguard against slipping of the joints or permanent straining of the wires.

Since the balance beam is 20 *cm.* long, the average length of each arm must be 10 *cm.* Since the weight of 1 gram is about 980 dynes, that of 1 decigram will be about 98 dynes; hence the couple exerted by gravity is 98×10 or 980 units. This is balanced by twisting a certain portion of the wire (cg) through an observed number of degrees; hence the couple due to 1° is easily calculated. This couple measures a coefficient of torsion of the wire (see ¶ 164), which will be needed in experiments later on.

We notice that the portion of the wire between g and k is not twisted at the times of making our readings, because the beam fh remains horizontal. The torsion of this part of the wire does not, therefore, affect the result. The only use of the wire between g and k is to keep the balance in place. The length of the wire between c and g should be measured, and its diameter should be found in several places by means of a micrometer gauge (¶ 50, II.). The material should also be noted, in order that we may utilize our results in certain other experiments later on.

EXPERIMENT LXV.

STRETCHING WIRES.

¶ 166. **Young's Modulus of Elasticity.** — A fine steel wire, about 0.25 *mm.* in diameter (No. 31,

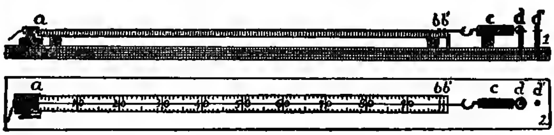


FIG. 177.

B. W. G.) and 1 metre long, may, if made of the best steel, be stretched 1 *cm.* without breaking, or losing its power of recovery. We will suppose such a wire to be held at one end by a small vice (a, Fig. 177) and attached at the other end (b) to a spring balance (c) held in place by a nail (d). Let the reading of this balance be 0. Now let the wire ab be stretched to a point b', by placing the balance over a nail (d), and let the new reading of the balance[1] be F. Then if the length of the wire thus stretched is ab centimetres and the elongation is bb' *cm.*, the stretching of 1 *cm.* will be $bb' \div ab$. This is called the ***strain*** of the wire. When 100 *cm.* are stretched, for instance, 1 *cm.*, we have a strain of 1 per cent or $+ .01$.

Now if the diameter of the wire is measured by a micrometer gauge, and divided by 2, we have its radius, r. From this we can find the cross-section q by the ordinary formula ($q = \pi\ r^2$), or

$$q = 3.1416 \times r^2, \text{ nearly.} \qquad \text{I.}$$

The cross-section can also be determined by finding the weight, w, of a given length (l) of the wire, if its density (d) is known; for since the volume of a wire is equal to $q \times l$, we have by definition (§ 154) $d = w \div ql$, whence —

$$q = \frac{w}{ld} \qquad \text{II.}$$

We will suppose that by either of these formulæ the average cross-section of the wire ab has been found. Now let the force indicated by the spring balance be ***reduced to dynes*** by multiplying by the appropriate factor.[2] Let us call this force in dynes f.

[1] In practice a small force will be required to straighten the wire. In this case the force F, below, must be taken as the difference between the forces exerted by the balance at d and d'.

[2] Thus in latitude 50° 1 kilogram is equal to about 981,000 dynes, 1 lb. avoirdupois to 445,000 dynes, and 1 oz. to 27,800 dynes, nearly.

To find the intensity of the force per square centimetre of cross-section of the wire, we divide it by the cross-section in question. Thus if the wire had a cross-section of one 2,000th of a square centimetre (.0005 cm^2), a force of 5,000,000 dynes would represent an intensity of 10,000,000,000 dynes per square centimetre (since 5,000,000 ÷ 0005 = 10,000,000,000). The result is called the "*stress*" exerted upon the wire (§ 22).

It has been stated (§ 114) that for a given material there is always a certain proportion between the *stress* exerted upon it and the *strain* produced. The ratio of the stress to the strain in the stretching of a rod or wire is called "*Young's Modulus of Elasticity.*" If, for example, a stress of 10,000,000,000 dynes per square centimetre produces in a steel wire an elongation of one half of one per cent, that is, a strain of +.005, the Modulus of Elasticity of the steel is 10,000,000,000 ÷ .005, or 2,000,000,000,000 (two millions of millions) *dynes per square centimetre.* The Modulus of Elasticity has also been defined as the *force* necessary (under Hooke's law, § 114) to produce a unit strain in a rod of unit cross section; that is, *to double the length of the rod.* Evidently, if 10,000,000,000 dynes are required as above to increase the length of a steel rod, 1 *cm.* square, by one part in 200, it would take 200 times as much force to double its length, provided that it kept on stretching at the same rate; hence we find 2×10^{12} for the modulus of elasticity, as before.

Few substances can be stretched one hundredth part of their length without breaking. It is only in the case of exceedingly elastic substances, like India rubber, that the conditions suggested by the last definition can be actually attained. In the case of most substances, we can only calculate by the rules of simple proportion what stress *would* double their length, provided that fracture or other changes did not occur.

The student may notice that steel (see Table 9) has the greatest modulus of elasticity of any known substance, because it requires the greatest force to produce a given amount of stretching; or because, in other words, it yields the least. A substance like India rubber, which is in the ordinary sense particularly elastic, has for this very reason a small *modulus* of elasticity.

¶ 167. **Determination of Young's Modulus of Elasticity.** — The data necessary for a determination of Young's Modulus are, as will be seen from ¶ 166, (1) the length, (2) the cross-section of the wire to be tested, (3) the elongation produced in it by a given force, and (4) the magnitude of this force. The length of a wire may be measured, without any special difficulty, by a tape graduated in millimetres. The cross-section requires much greater care, whether it be determined (as suggested in ¶ 166) by measurements taken with a micrometer gauge at different points, or by its length, weight, and density. The principal difficulty consists, however, in measuring accurately the elongation of the wire, which is usually a very small quantity. To make the elongation as large as possible, long wires are usually employed.

One of the chief sources of error in measuring the elongation of a wire under a given load is due to the yielding of the support to which the wire is attached. Various devices have been suggested by which this effect may be eliminated. The simplest is to measure the distance between two points on the wire. This may be easily done, when a double wire is employed, by means of two micrometers, *a* and *b* (Fig. 178, 1), attached to the wall, and adjusted so as to touch two crossbars borne by the wires in question.[1]

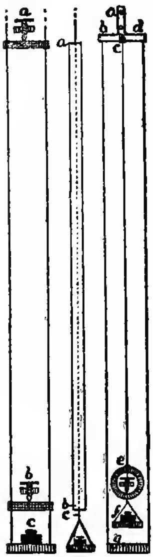


Fig. 178.

To avoid the inconvenience of making observations at a considerable height above the floor, a wire is sometimes surrounded by a tube (*ab*, Fig. 178, 2) attached to it at a point *a*. If the point *a* yields, a point *b* at the base of the tube will yield by an equal amount. The height of this point (*b*) and of a point (*c*) on the wire may be observed (¶ 262) accurately by a cathetometer. The increase of distance between *b* and *c* is evidently equal to the elongation of *ac*. In the Physical Laboratory of Harvard University the effects due to the yielding of the support are avoided by keep-

[1] This device is due to Mr. Forbes, of the Roxbury Latin School.

ing the same weight always upon it. The wires (which are nearly 6 metres long) are attached to a beam by means of a piece of iron (*abd*, Fig. 178, 3) shaped like an inverted T. At the middle of the T a split plug (*c*) driven upwards into a vertical hole firmly grasps the wire. Side wires from the arms of the T hold a small platform (*g*) just above the

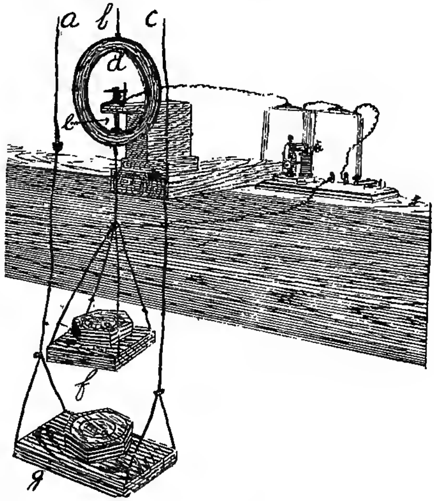


FIG. 179.

floor. The weights to be used in stretching this wire are kept on this platform when not in use. Obviously the beam and the stem of the T are subjected to the same strain whether the load be suspended from the central wire or by the side wires.

A stout ring (*de*, Fig. 179) is attached to the central wire (*b*) by a split plug (*d*). The stretching of the wire is measured by a micrometer, the point of which touches a small level surface on the ring at *e*. The contact is determined by electrical connections, as in ¶ 162. Directly below the point of contact a platform, *f*, is suspended, for the purpose of holding the weights by which the wire is to be stretched. There are many theoretical objections to this form of apparatus, which being of no practical importance have been left out of consideration. It is obviously necessary that the wire should be straight before the stretching forces are applied. For this purpose, a small load is always kept on it. In the apparatus shown in Fig. 179, the weight of the ring (*de*) and platform (*f*) should be sufficient to straighten the wire. In calculating Young's Modulus, we consider only the weight which must be *added to* the load already borne by the wire, in order to produce the observed elongation.

To determine the elongation in question, a reading of the micrometer must be taken with and without the weight. The difference in the readings gives, allowing for the pitch of the screw (see ¶ 52), the distance through which the wire has been stretched by the weight in question.

For a determination of Young's Modulus of Elasticity, a fine steel wire will answer. Care must be taken, however, not to bend the wire sharply over the edge of the vices or split plugs to which it is fastened. If the wire is 0.25 *mm.* in diameter, and free from kinks or bends, it may be made to bear safely a total load of 1, 2, or even 3 kilograms.

If *f* is the force exerted by the weight when reduced to dynes (see ¶ 166), e the resulting elongation of the wire in *cm.*, l the length in *cm.* of that portion of the wire in which the elongation takes place, and q its average cross-section in *sq. cm.*, Young's Modulus of Elasticity (E) is found in *C. G. S.* units (§ 8) by the formula

$$E = \frac{fl}{qe},$$

or by the method of reduction explained in ¶ 166.

EXPERIMENT LXVI.

BREAKING STRENGTH.

¶ 168. **Determination of the Breaking Strength of a Wire.** — A steel or spring-brass wire about $\frac{1}{4}$ *mm.* in diameter (No. 31, B. W. G.), free from kinks or sud-

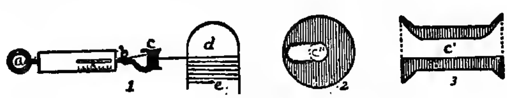


FIG. 180.

den bends, is to be attached at one end to the eye (b, Fig. 180) by which the hook (bc) is attached to the spring balance (abc). The other end is to be fastened to some fixed point, as, for instance, a nail (e) driven into a post (d). A bobbin, c, is to be cut out (as shown in c' and c'' of the cross-sections 2 and 3), so as to fit over the hook of the balance without danger of turning. A few turns of the wire are made about the bobbin; the rest is wound around a post, *d*. The index of the balance is to be watched as a steadily increasing force is applied to the wire.[1] When the wire breaks, the maximum reading is recorded. The position of the break must now be ascertained. If it occurs at *b*, or between *b* and *c*, the result must be thrown out. If the wire breaks at *c* or at *d*, the accuracy of the result is doubtful; because a sharp bend in a wire where it passes round a corner may cause it to break under forces far less than its average breaking strength. If the break occurs between *c* and *d*, the break is probably a fair one. Enough wire will probably remain about the post for several repetitions of the experiment. The results should agree within five or ten per cent. Suspected results, much smaller than the average, may be discarded.

The cross-section of the wire must be found both by measurements with a micrometer gauge and by weighing a known length of the wire, let us say 1 metre, as accurately as possible. (See ¶ 166, formulæ I. and II.) The density of steel may be taken as 7.9, of brass 8.4 in this reduction. The student should compute by simple proportion the force necessary to break a wire one *sq. cm.* in cross-section; he may also calculate what length of the given wire would break under its own weight. Thus if 100 *cm.* of brass weighs 0.42 grams, its cross-section must be 0.42 ÷ 100 ÷ 8.4, or .0005 *sq. cm.* If it takes 2.94 kilograms to break such a wire, a wire 1 *sq. cm.* in

[1] *The hand should be held in such a position as not to be injured by the hook when the spring recoils.*

cross-section would require 2.94 ÷ .0005 or 5,880 kilograms to break it. At 0.42 grams per metre, it would take 2.94 ÷ 0.42 or 7000 metres of the wire to break under its own weight.

Obviously the result of this calculation should be the same whether a large or a fine wire is used, provided that the quality be the same, because both the breaking strength and the weight of a wire increase in proportion to its cross-section.

EXPERIMENT LXVII.

SURFACE TENSION.

¶ 169. **Determination of the Surface Tension of a Liquid.** — I. A piece of fine iron wire is bent as in Fig. 181, so as to form a fork (*fbg*) with parallel prongs (*cf* and *eg*) about 2 *cm.* apart. The fork is then suspended from the hook of a balance (*a*) so as to dip into a beaker of water, as in the hydrostatic method (Exp. 9). The fork must be entirely covered by water when the balance beam is lowered see (¶ 19); but when the latter is raised, the prongs only must dip into the water.

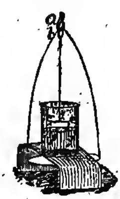


FIG. 181.

The weight of the fork is first balanced as accurately as possible; then the fork is lowered into the water, and suddenly raised out of it. A film of water will probably be found to fill the space between *fcdeg* and the surface of the water. This film will tend to pull the fork back into the water. To balance the pull which it exerts, an additional weight of about 3 decigrams must be placed in the opposite scale-pan. This weight is to be adjusted, by a number of trials, as accurately as possible. As the film gradually evaporates, it becomes lighter and lighter; but as its weight is, in any case, so small that it may be neglected, the change of weight will probably have no visible effect. The student will notice that the tension of the film of water remains sensibly constant as it grows thinner and thinner, until it breaks. This is entirely unlike the tension of solid substances, which depends upon their cross-section. The tension which liquids exert depends simply upon the *breadth of the surface* which tends to contract, not on the cross-section of the solid contents included by that surface. For this reason, the phenomenon is called "surface tension."

In the case under consideration, the film has two surfaces, each let us say 2 *cm.* broad. The total breadth of surface is therefore 4 *cm.* The student is to calculate what force (in dynes) is exerted by a *single* surface 1 *cm.* broad.

The surface tension of liquids depends upon temperature; hence the temperature should be noted. It is greatly affected by impurities in the liquids. An invisible quantity of oil, for instance, produces variations of ten or twenty per cent. Great care must therefore be employed in obtaining the purest distilled water. Both the inside of the beaker and the lower part of the wire should be cleaned with caustic potash, and afterwards rinsed in several changes of distilled water. The parts thus cleaned must not afterwards be touched by the finger.

II. A piece of thermometer tubing with a round bore about $\frac{1}{4}$ to $\frac{1}{2}$ *mm.* in diameter is carefully cleaned with caustic potash, which may be sucked through it with a medicine dropper (of course not by the mouth), then cleaned with distilled water. It is now dried by heat and filled with mercury. The contents are to be placed in a beaker, and weighed. If the quantity of mercury is too small to be weighed accurately, ten tubefuls may be weighed together (§ 39). The length of the tube is to be measured. The tube is now placed in a clean beaker containing pure distilled water (see I.). It should be at first inclined somewhat, so that the water which rises into it through "capillary attraction" may thoroughly wet its inside surface. It is next made vertical (see Fig. 182). The height of the column of water in the tube above the level in the beaker is then measured, both when it barely dips into the water, and when it dips so deep that the water rises nearly (but not quite) to the top of the tube. Other measurements should be taken similarly with the tube turned end for end. All results should agree closely, if the tube is of uniform calibre.

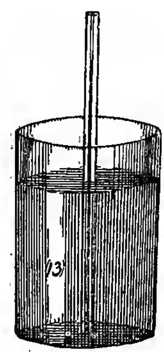


FIG. 182.

¶ 170. **Calculations relating to Capillary Attraction.** — If w is the weight in grams of the mercury which fills a tube, 13.6 the density of the mercury, and l

the length of the tube in *cm.*, the cross-section is (see ¶ 166, formula II.)

$$q = \frac{w}{13.6\,l}.$$

The radius of the tube is connected with the cross-section by the formula

$$q = \pi r^2;$$

hence, solving, we find

$$r = \sqrt{\frac{q}{\pi}} = 0.564\sqrt{q}, \text{ nearly.}$$

If h is the average height of the water in the tube above its level in the beaker, 1.00 the density of water, the volume of water raised is qh, or $\pi r^2 h$; the weight in grams is $1.00 \times qh$, or $1.00 \times \pi r^2 h$, and the weight in dynes (allowing g dynes to the gram) is qhg, or $\pi g r^2 h$. This weight, neglecting the buoyancy of the atmosphere, is sustained by the tension of a film lining the inside of the tube. The breadth of this film is evidently equal to the circumference of the tube ($2\pi r$). If a film $2\pi r$ centimetres broad can sustain a force $\pi g r^2 h$ dynes, a film 1 *cm.* broad would evidently sustain $\pi g r^2 h \div 2\pi r$, or $\frac{1}{2}\,grh$ dynes. That is the "surface tension" of water (S) is given by the formula

$$S = \tfrac{1}{2}\,grh = 490\,rh \textit{ dynes per centimetre (nearly)}.$$

Obviously, if S is constant, the product, $r \times h$, must be constant; that is, the height to which a liquid will rise in a tube is inversely as the radius of that tube.

EXPERIMENT LXVIII.

COEFFICIENT OF FRICTION.

¶ 171. **Determination of Coefficients of Friction.** — I. A piece of planed plank (*b*, Fig. 183) measuring let us say 5 × 20 × 40 *cm.*, is drawn horizontally by a spring balance, *a*, over a planed board *c*. The force necessary to maintain a uniform velocity after the plank is once started, is observed and noted. Then the plank is suspended from the spring balance and weighed. The ratio of the force required to draw a body to the force required to lift it is called a "co-

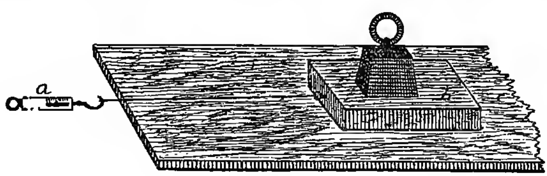


FIG. 183.

efficient of friction." The coefficient of friction in this case is that of wood on wood. If the force of traction varies in different parts of the board, the average should be calculated; and from this the average coefficient of friction may be found. It is instructive to repeat the experiment with the plank edgewise, so as to see whether the diminished area of the surfaces in contact is or is not compensated for by the increased intensity of pressure. For a fair comparison, the side and the edge of the plank should of course be equally smooth, and both parallel to the grain of the wood.

The experiment may also be repeated with the plank flatwise, but with a heavy weight upon it as in the figure. The value of this weight should be found as in ¶ 159, and added to that of the board, in calculating the coefficient of friction in question.

The student will notice that it takes considerably more force to start a body than to drag it after it is once started. This is attributed to the cohesion of

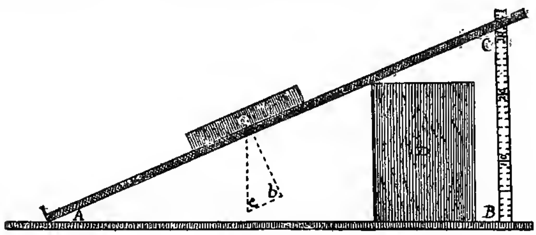


Fig. 184.

particles which takes place at various points, particularly when two surfaces remain long in contact. The ratio of the force required to start a body when resting upon a horizontal surface to the force required to lift it is sometimes called the "coefficient of starting friction." This must not be confounded with the ordinary "coefficient of friction."

II. The board *A C* (Fig. 184) already used in I. is inclined (by means of a nail, *A*, and a block *D*) so that the plank *a*, when once started, slides down it with uniform velocity. A measuring rod *BC* is placed at a point *B*, 1 metre from *A*, and the vertical distance BC to the under side of the board is then measured. The "slope" of the under surface $(BC \div AC)$ is thus found. The slope necessary to maintain a uniform velocity may not be the same from one end of the board to the other. If it is not the same, the average slope should be calculated.

If we resolve the weight of the block ac into two forces, one, ab, perpendicular to the board AC, the other, bc, parallel to it, then by definition (see I.) the coefficient of friction is $bc \div ab$; but, by similar triangles, this is equal to the ratio of BC to AB, which measures the "slope" of the board AC. The average slope which must be given to this board in order that the plank, when once started, may slide down it with uniform velocity, gives accordingly the "coefficient of friction" between the two surfaces in contact. The result should agree closely with that determined as in I.

¶ 172. **Fluid Friction.** — When a well-shaped boat moves through water with a velocity of v *cm. per sec.*, the opposing force (F) which it encounters is approximately equal to the square of this velocity multiplied by the area (a) of the surface wet by the water, measured in *sq. cm.*, and by a certain constant, f (about .003), which is called the *coefficient of friction of water*, that is: —

$$F = fav^2 \textit{ dynes}.$$

Coefficients of fluid friction must not be confounded with coefficients of friction in the case of solids, which are calculated in an entirely different way. The frictional resistance between two solid surfaces depends, as we have seen (¶ 171), upon the pressure between them, but not upon the relative velocity of the surfaces. On the other hand, the resistance offered by a fluid to the motion of a solid does not depend upon the pressure between the surfaces in contact, but does depend upon their relative velocity. The nature of the fluid, the shape and smoothness of the solid, modify the result; but the material of which the solid is composed is generally unimportant. The resistance offered by fluids to the motion of solids or the reverse depends upon disturbances which are wholly confined to the fluid. Every fluid has, therefore, its own coefficient of friction.

When a current of water flows through a *large*[1] tube of the length l and radius r (both in *cm.*), since the area of wetted surface is $2\pi rl$, the force opposing the flow is

$$F = 2\pi r l f v^2 \text{ (dynes).} \qquad (1)$$

This force is supplied by the pressure (p) of the water (measured in dynes per *sq. cm.*) exerted upon an area equal to the cross-section (πr^2) of the tube; that is: —

$$F = \pi r^2 p. \qquad (2)$$

Equating (1) and (2), we find, —

$$p = \frac{2lfv^2}{r} \quad (3), \quad \text{or } f = \frac{pr}{2lv^2} \qquad (4)$$

[1] In capillary tubes, the force encountered is proportional directly to the velocity (see ¶ 250). In tubes from 1 to 5 *mm.* in diameter, for velocities between 10 and 100 *cm. per sec.*, no simple law can be given.

The velocity (v) can be estimated from the cross-section of the tube and from the volume of water which flows through it in a given length of time (¶ 147, 4), the pressure may be found by a pressure-gauge (see Exp. 69) at the point where the water enters the tube, provided that there is a free outlet at the other end, and that both ends of the tube are on the same level. If, as in Fig. 185, one end is higher than the other by an amount ac, equal let us say to h, then if g is the acceleration of gravity and 1.00 the density of water, the hydrostatic pressure is (see § 63)

$$p = 1.00\,gh, \text{ nearly.} \qquad (5)$$

The length (l) of the tube may be directly measured. The capacity (c) may be found by measuring, or (as in ¶ 32), by weighing the quantity of water required to fill it. The cross-section (q) may then be calculated by the equation –

$$q = \frac{c}{l} \qquad (6)$$

Hence the radius (r) is given by the formula —

$$r = \sqrt{\frac{q}{\pi}} = \sqrt{\frac{c}{\pi l}} \qquad (7)$$

The coefficient of friction, f, may now be calculated by formula (4), since all the quantities are known.

The "resistance" of a tube to the flow of a given liquid may be defined as the pressure in *dynes per sq. cm.* required to maintain through that tube a flow of 1 *cu. cm. per sec.* Thus if a rubber tube (*ab*, Fig. 185) 2 metres long and 3 *mm.* in diameter is used as a siphon to conduct water from a cistern, *a*, to a point *b*, it will be found that the outlet (*b*) must be about 10 *cm.* below the level (*a*) in the cistern in order that water may flow through *ab* at the rate of 1 *cu. cm. per sec.* The hydrostatic pressure corresponding to a difference of level of 10 *cm.* is nearly 10 grams per *sq. cm.*, that is, 9800 *dynes per sq. cm.* The "resistance" is therefore about 9800 units.

The resistance of a conduit may also be defined as

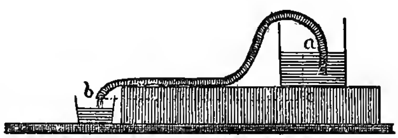


Fig. 185.

the power (in ergs per second) necessary to maintain a unit current (1 *cu. cm. per sec.*) through the conduit in question. This definition bears a strong resemblance to the definition of electrical resistance (§ 136). The fact that power is required to maintain a current through the tubes and valves of a water-motor, together with the friction between the solid parts of the motor, will be found to modify the "efficiency" of the machine. The next experiment relates to determinations of "efficiency."

EXPERIMENT LXIX.

EFFICIENCY.

¶ 173. **Nature of Efficiency.** — Let us suppose that a 20-kilogram weight is suspended by a tackle (Fig. 186) consisting of two double blocks, with four cords passing between them. Let us first suppose that the cords run with absolute freedom round the pulleys which the blocks contain. The force on each cord must evidently be 5 kilograms; and a force of 5 kilograms, applied by a spring balance to the free end of the cord, as in the figure, will just hold the weight in place. If the weight were started upward by any impulse, no matter how small, the force of 5 kilograms constantly applied to the free end of the cord would (in the absence of friction) continue to raise it with a uniform velocity, until the two blocks met together. If the two blocks were 1 metre apart in the beginning, we should have 20 kilograms raised by the tackle through a height of 1 metre. Each of the four cords would be shortened 1 metre in this process, hence there would be 4 metres of slack to be taken up at the free end of the cord. The spring balance must accordingly retreat 4 metres. The work spent upon the machine by a force of 5 kilograms re-

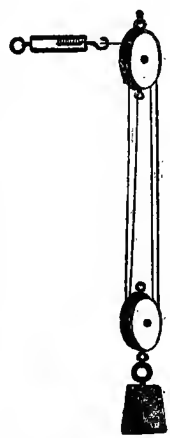


FIG. 186.

treating 4 metres (20 kilogram-metres), would be the same as that utilized by the machine in raising 20 kilograms 1 metre high (see § 14).

Let us now suppose that a slight downward impulse is given to the weight, so that it descends to its original position. The work spent by gravity upon the machine, being 20 kilogram metres as before, is utilized in pulling the spring balance forward through a distance of 4 metres. In the absence of friction, the pull would be 5 kilograms as before. The amount of work utilized (20 kilogram-metres) would be equal, accordingly, to the amount spent upon the machine.

It is not necessary to consider the magnitude of the impulse by which the weight is started upward or downward; for if the weight moves with uniform velocity, it is capable of giving back this impulse, when it has been raised or lowered to any desired point (see § 121), in the act of stopping, when its energy of motion is lost. In the absence of all friction in the pulley-wheels, stiffness in the cords, and resistance in the air, a tackle devoid of weight would constitute a theoretically perfect machine, — that is, all the work spent upon it would be utilized by it. In practice, a considerable part of the work spent upon a machine is always transformed by friction into heat. That *proportion* of the work spent upon a machine which is utilized by it is called the "efficiency" of the machine.

Let us suppose that, instead of 5 kilograms, a force of 10 kilograms is required to raise a 20 kilogram weight by means of the tackle represented in Fig. 186. Then since, in raising 20 kilograms 1 metre, 10 kilograms retreat 4 metres, the work spent is 40 kilogram-metres; but the work utilized is only 20 kilogram-metres. The "efficiency" of the tackle as a machine for raising weights is accordingly $\frac{20}{40}$ or 50%.

Again, let us suppose that a weight of 20 kilograms, descending one metre, exerts a force of only 2 kilograms on the spring balance, which advances 4 metres. Then the work spent by gravity is 20 kilogram-metres, but that utilized is only 8 kilogram-metres; hence the efficiency of the tackle as a machine for utilizing potential energy (§ 122) is $\frac{8}{20}$ or 40%.

Finally, let us consider the tackle as a machine for storing and utilizing energy. A force of 10 kilograms is required to raise the weight, and this force must retreat 4 metres to raise the weight 1 metre. 40 kilogram-metres of work are thus spent upon the machine. The free end of the cord is now attached to some resistance which it is desired to overcome. A force of 2 kilograms is thus applied through a distance of 4 metres. The work utilized by the machine is only eight kilogram-metres. Evidently the efficiency of the tackle as a machine for storing and utilizing energy is only $\frac{8}{40}$ or 20%.

When energy is stored in a machine, part of it is lost. When this energy is utilized, part of what is left is lost. When energy undergoes a series of transformations, a certain proportion is lost in each.

Obviously, in stating the efficiency of a machine, it is necessary to specify where or how the work is spent upon it, and where or how the work is utilized.

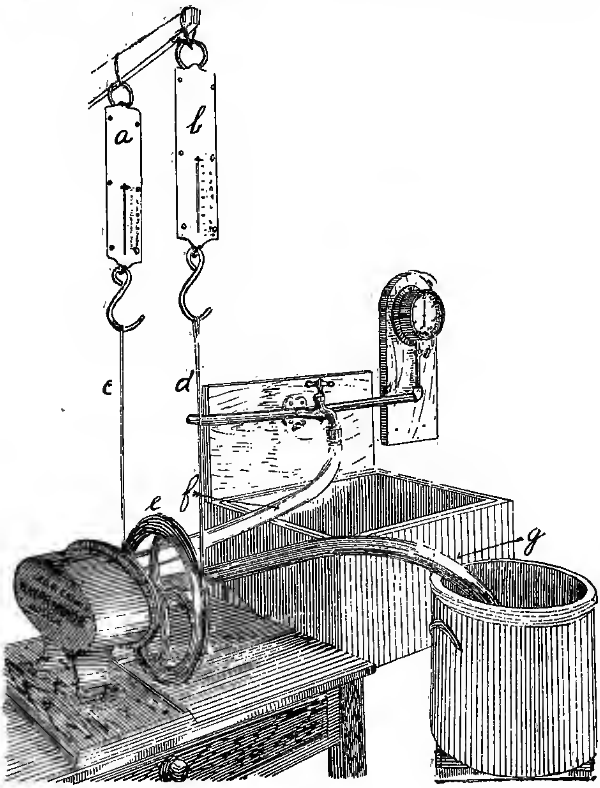


Fig. 187.

¶ 174. **Determination of the Efficiency of a Water-Motor.** — (1) To find the work *utilized by* a water-motor, the circumference of the driving-wheel (*e*, Fig. 187) is first measured, then two spring balances, a and b, are connected by a cord (cd) passing round the wheel. The motor is then started, and the tension of this cord increased until, through the friction which it exerts upon the wheel, the velocity of the latter is reduced to about one-half of its maximum. The speed of the wheel is then determined by counting the number of revolutions made in a given length of time. The reading of each spring balance is also found. If it varies, several observations must be made, and the mean calculated.

The difference between the two readings is equal to the force opposed by friction to the motion of the rim of the wheel, and must be reduced to dynes or megadynes. If the value of this force in dynes is F, if the number of revolutions in one second is n, and if c is the circumference of the wheel in centimetres, then in traversing the distance cn centimetres against the force F dynes, the work done must be cnF ergs. If we suppose that the force reduced to megadynes is equal to f, then cnf represents the work in megergs. Since cnf megergs of work are performed against friction in 1 second, and might be utilized for turning machinery (see ¶ 175), we infer that the work thus utilized would be cnf megergs per second. This measures, therefore, the power of the machine.

(2) To find the work spent in *driving* the motor, we must measure the quantity of water which passes through it in a given length of time. The water may be collected in a stone jar (g, Fig. 187), and weighed on a pair of rough platform-scales (Fig. 188). The

pressure of the water must also be found by means of a pressure-gauge connected with the supply pipe (see Fig. 187). The gauge should be as nearly as possible on a level with the outlet by which water escapes from the motor. The pressure must be reduced to dynes (or megadynes) per square centimetre. If v is the calculated volume in cubic centimetres of the water which flows through the motor in one second, and if P is the pressure of this water in dynes per square centimetre, then the work spent on the motor

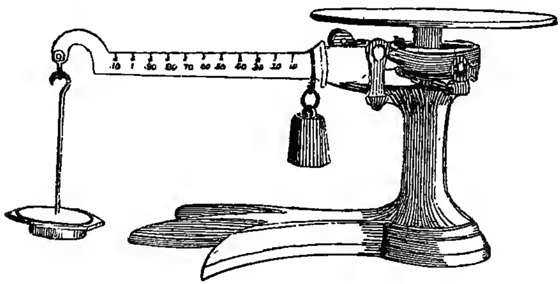


Fig. 188.

is vP ergs per second (see § 118). If p is the value of this pressure when reduced to megadynes per square centimetre,[1] then the work spent on the machine is vp megergs per second.

(3) For the accurate determination of efficiency, it is desirable to make *simultaneous* determinations of the power utilized by the motor, and of the power spent upon the motor. For this purpose, it is well for several students to work together. One may, for in-

[1] The ordinary atmospheric pressure (15 lbs. per sq. in.) is equal very nearly to 1 megadyne per square centimetre. See Table 50.

stance, record the readings of the spring balance, *a*, another those of *b*; a third those of the pressure-gauge; a fourth may attend to turning the stream of water into the stone jar at a given time, and cutting it off at a given time; and a fifth may count the number of revolutions made by the wheel of the motor in the interval in question. When the experiment is performed by a single person, the mean readings of the balances and pressure-gauge must be inferred from observations just before and just after the determinations of velocity.

To calculate the efficiency (e) of the motor, the work *utilized* in one second *by the machine* is to be divided by the work *spent* in one second *on the machine*. We have, accordingly, —

$$e = \frac{cnf}{vp}.$$

In repeating the experiment, the tension of the cord should be increased or diminished. The maximum *power* of a water-motor is usually realized when, by the resistance which it has to overcome, the speed of the motor is reduced to about half its maximum speed. To obtain the maximum *efficiency*, the speed of the motor must be still further reduced.

¶ 175. **The Transmission Dynamometer.** — To measure the power of a motor actually doing useful work, a transmission dynamometer must be employed. One of the simplest forms of this instrument is represented in Fig. 189. Instead of carrying two cords (c and d) from the driving-wheel (g) of the motor to two spring balances (a and b) as in Fig. 187, these cords are made to pass around two pulleys (a and b, Fig. 189) to a second wheel (h), to which the motion is thus transmitted. The pulleys are suspended by two spring balances (A and B). The work done by the motor depends as before upon the difference in tension of the cords c and d; but if the pulleys run freely, the tension of e and f will be the same as that of c and d respectively; hence the forces A and B registered by the spring balances A and B (allowing for the weight of the pulleys) will be $2c$ and $2d$, respectively. It follows that $(c-d) = \frac{1}{2}(A-B)$. The difference between the readings (A and B) must therefore be halved in order to find the difference of tension between the cords[1] c and d.

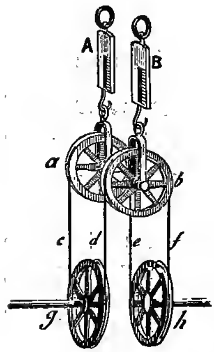


FIG. 189.

When the wheels move so fast that the revolutions cannot be counted, we may find the velocity of the cord, $cdef$, by measuring its length and counting the successive returns of a knot in the cord taking place in a given length of time. In other respects the work utilized is calculated as in ¶ 174, 1.

[1] In practice, if the cord c is approaching g the tension on c will be a little greater than on e; and the tension on d will be a little less than on f, hence the difference of tension between c and d will be greater than the difference between e and f. That is, the work done by g will be a little greater than that received by h. The average between these two quantities is measured by the dynamometer.

EXPERIMENT LXX.

MECHANICAL EQUIVALENTS.

¶ 176. **Different Methods for determining the Mechanical Equivalent of one Unit of Heat.** — (1) If a weight (*d*, Fig. 190) is suspended by a cord passing over a pulley (*a*) and round an axle (*c*), surrounded with water in a calorimeter, and made to descend slowly to a position *d′*, by applying a suitable resistance through a friction-brake, *b*, the work done by gravity in pulling the weight, let us say *w*, through the distance *l* (equal to *dd′*) will nearly all be converted by friction into heat within the calorimeter. Let us suppose that the total thermal capacity of the calorimeter and its contents is *c*, and that its rise in temperature is $t°$; then the quantity of heat developed is *ct*. If gravity exerts a force of *g* dynes on one gram, it will exert *wg* dynes on *w* grams; and a force of *wg* dynes acting through the distance *l*, must perform a quantity of work equal to *wgl* ergs (§ 14). If *wgl* ergs are equivalent to *ct* units of heat (§ 16), one unit of heat must be equivalent to $wgl \div ct$ ergs. To obtain exact results, allowances must be made for the friction of the pulley, *a*, for loss of heat by cooling, etc. By a device similar in principle to the one described above, Joule

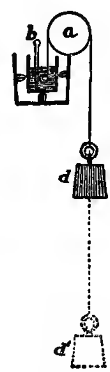


Fig. 190.

found that the mechanical equivalent of one unit of heat is about 41,660,000 ergs.

(2) Two heavy iron bars, A and B, suspended as shown in Fig. 191, may be released simultaneously by burning a cord (see ¶ 148) or by electrical means, so that when the bars meet endwise, a lead bullet (b) may be crushed between them. The work done by gravity in giving velocity to the bars is thus nearly all transformed into heat, through friction of the particles of lead against one another. Most of the heat will accordingly be found in the bullet. If the bullet is immediately lowered into a small calorimeter (c), the quantity of heat may be measured in the ordi-

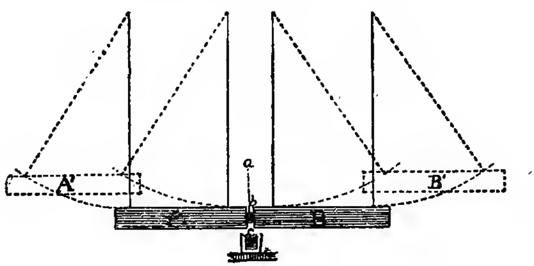


Fig. 191.

nary way (see ¶ 92). To obtain exact results, an allowance must be made for the energy of motion which remains in the bars after impact. If l is the *difference* between the original height of the bars and the height attained by them in their rebound, and w their combined weight, the work done by gravity is, as in (1), wgl. There is no way of allowing accurately for the energy taken up by the bars in the form of vibration, or for the energy of motion directly converted within the bars into heat. It is said that the proportion of energy thus lost is small.[1]

(3) By measuring the temperature of a water-fall above and below the fall, it would be possible to estimate the mechanical equivalent of heat. Thus if the water is 0°.1 warmer at the foot of Niagara Falls than above the falls, where the height is 42.5 metres, we should infer that to cause a difference of 1°, a water-fall must be 425 metres high. Each gram of water falling 425 metres, or 42,500 *cm.* under a force of 980 dynes, nearly, must receive from gravity $980 \times 42{,}500$, or nearly 41,660,000 ergs, in the form of energy of motion. If the conversion of this energy into heat warms it 1°, then the mechanical equivalent of 1 unit of heat must be 41,660,000 ergs.

In practice, the difference of temperature between the top and bottom of a water-fall is generally too slight to be measured accurately with ordinary instruments. Unless, moreover, the volume of a water-fall is very great, evaporation and other causes may affect the result. A rough experiment illustrating this method of determining mechanical equivalents will be described in the next section.

¶ 177. **Determination of Specific Heats by Mechanical Equivalents.** — A kilogram of lead shot is placed in a pasteboard tube (*ac*, Fig. 192) about 5 *cm.* in diameter and 120 *cm.* long, closed by two corks, *a* and *c*.

[1] For an experiment similar in principle, performed by Hirn, see Trowbridge's New Physics, Exp. 105. This modification of Hirn's method is due to Professor Guthrie. The geometrical principles connecting arcs and heights have been already considered in the case of a ballistic pendulum (see § 109).

The free space between the cork, *a*, and the level of the shot, *b*, is to be measured with a metre rod. The cork (*a*) must be removed for this purpose, and its thickness allowed for. A thermometer is now fitted through the cork (*a′*, Fig. 193) so that by inclining the tube the bulb may be completely surrounded by the shot. The temperature of the shot is to be taken; then the thermometer is removed and the hole closed by a wooden plug. The tube is now inverted 100 times in rapid succession. During each inversion the centre of the tube is held at a fixed height. The shot are kept at one end of the tube by centrifugal force until this end comes vertically over the other. Then the rotation should cease, so that the shot may fall through the distance *ab* almost like a solid mass. Care must be taken, however, not to heat the shot through agitation which would result from too suddenly arresting the motion of the tube. The cork, *c*, should be supported by a table or other solid object so as not to yield under the blow given to it by the shot. Under this condition only, the energy of motion of the shot will be converted into heat *within the mass of shot*. The temperature of the shot is again observed in the same manner as before. It should have risen 5 or 6 degrees.

Fig. 192.

Fig. 193.

The experiment is now to be repeated with 1 kilogram of a substance in the form of shot, but of unknown specific heat; for instance, an alloy of zinc and lead. If this substance takes up more space than the lead, the distance fallen through in each reversal of the tube will not be quite so great. In this case more than 100 reversals may be made. The total distance fallen through should be as nearly as possible the same. Thus, if the distance *ab* is 100 *cm.* in the case of the lead shot, and 98 *cm.* in the case of the alloy, the tube should be reversed 102 times in the latter case, instead of 100 times.

¶ 178. **Calculations relating to Mechanical Equivalents.** — If s is the specific heat of the lead shot, w its weight in grams, g the weight of 1 gram in dynes, d the distance in *cm.* fallen through in each reversal, n the number of reversals, and J the mechanical equivalent of 1 unit of heat, then the total work done by gravity is evidently $wg \times nd$ ergs; and the heat into which it is converted is (neglecting all corrections) wst units, which is equivalent to $Jwst$ ergs. We have, therefore, —

$$Jwst = wgnd;$$

whence

$$J = \frac{ndg}{st}.$$

It is interesting to compare the value of J calculated by this formula with that found by Joule (see ¶ 176, 1). On account of many large corrections which have not been considered, the result will probably be too great by some 20 or 30 per cent. The principal source of error usually lies in the cooling of the shot by contact with the sides of the pasteboard tube. This can be avoided by cooling the shot before the experiment to a temperature about 6° below that of the tube. Before repeating the experiment, the tube must be allowed to return to its original temperature. The remaining errors have been found in the long run to balance one another with a probable resultant of about 10 per cent., which may be positive or negative according to the manner in which the manipulations are performed. Instead of computing the mechanical equivalent of heat, we may calculate the specific heat of the lead shot by the formula —

$$s = \frac{ndg}{Jt},$$

where J may be taken as 41,660,000; and if we distinguish by a prime (′) the qualities of an unknown substance, we find similarly, —

$$s' = \frac{n'd'g}{Jt'}.$$

Dividing, we find

$$\frac{s'}{s} = \frac{n'd't}{ndt'}, \text{ or } s' = \frac{sn'd't}{ndt'}.$$

In other words, the specific heats of two substances are to each other as the distances through which they must severally fall in order that each may be raised 1° in temperature. On account of the manner in which the two experiments are performed, the values of s and s' should be affected by constant errors in the same proportion, and hence the ratio between them will be affected only by accidental errors (§ 24). The last formula is therefore less inaccurate than the preceding formulae. To obtain the most accurate results by the aid of mechanical equivalents, as has been described, special devices should be employed to limit the fall of the shot to a given distance. In the absence of due precautions in this respect, the results must be expected to compare unfavorably with those obtained by the ordinary methods (see Exps. 33 and 34). It is nevertheless considered desirable that a student should familiarize himself with a definite example of the conversion of work into heat.

MAGNETIC MEASUREMENTS.

EXPERIMENT LXXI.

MAGNETIC POLES.

¶ 179. **Determination of the Distance between the Poles of a Magnet.** — Compound magnets composed of thin strips of steel bolted together will be found convenient for several experiments in magnetism. Such a magnet, formed of pieces of clockspring, 10 or 15*cm.* long, and 1 or 2*cm.* broad, is represented in Fig. 194.

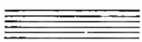


Fig. 194.

In fitting the strips together it may be necessary to soften them by heat; but their temper must be restored (by again heating and *suddenly* cooling them) before they can be thoroughly magnetized. Each strip should be magnetized separately by stroking one end of it ten times from the centre outward with or upon the south pole of a powerful electromagnet. This end will become a north pole (§ 126). The other end is then to be magnetized similarly by the north pole of the electromagnet. The strips are afterward bound together with all the north poles turned carefully in the same direction.

A piece of "ferroprussiate paper"[1] prepared for making "blue prints" is now to be stretched flat over a pane of window-glass, or over a stiff piece of pasteboard, with the sensitive surface uppermost. It is then to be placed over a powerful bar magnet constructed as has been described; and a few iron-filings are to be scattered over it. When the paper is jarred the iron-filings will arrange themselves as in Fig. 195. The sensitive surface is now to be ex-

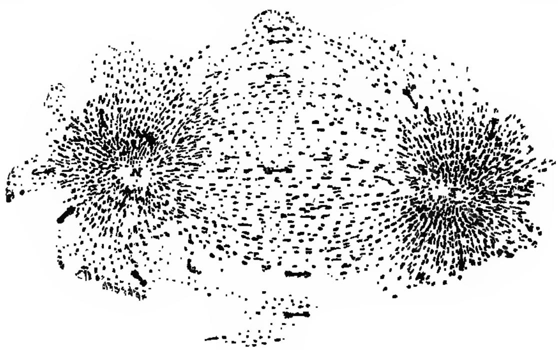


Fig. 195.

posed for about five minutes to direct sunlight, or to the light of the sky for a much longer period, until the surface not covered by the filings becomes quite

[1] To prepare ferroprussiate paper, take I gram citrate of iron and ammonia, 1 gram *red* prussiate of potash, pulverize together and dissolve in 10 grams of water. This quantity should cover 20 or 30 square decimetres of smooth (not porous) paper. It should be applied by lamplight, as rapidly and evenly as possible, with a small sponge, in strokes first lengthwise then crosswise, then dried in the dark. The student is cautioned that all "prussiates" are poisonous. Ferroprussiate paper, already prepared, may be bought of dealers in photographic apparatus.

blue. It is then to be placed in the shade, and the iron-filings removed.

The surface covered by the iron-filings should not have been affected by the light; hence the arrangement shown in Fig. 195 should be represented by a white tracing on a blue ground. To make the print permanent, it is necessary to soak it in water for about ten minutes, after which it may be dried in the sun. To avoid delay in waiting for the print to dry, the student is advised to defer this "fixing process" until the end of the experiment. In the meantime,

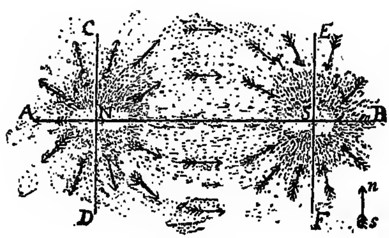


Fig. 196.

the print should be protected from excessive light either from the sun or from the sky. It may be illuminated freely by lamplight or gaslight.

The magnet is now to be placed *over* the print, directly above its former position. A small compass with a needle not more than 1*cm.* long, is to be put beside the magnet at different points in the print. The direction of the north pole of the compass-needle is to be indicated in each case by an arrow drawn in pencil upon the paper (see Fig. 196). The direction of the arrows should agree closely with the lines of iron-filings, although the compass-needle is in a slightly different plane. The results of this experiment will be somewhat affected by the earth's magnetism. It is well, therefore, to note the direction (*sn*) in which the compass points when the magnet is removed to a distance.

A line *AB* is now drawn so as to bisect as nearly as possible the areas *N* and *S*, from which the "lines of force" (§ 127) seem to diverge. The line (*AB*) should agree with the general direction of the lines of force between *N* and *S*, whether indicated by the compass-needle or by the iron-filings. The areas *N* and *S* are again to be bisected by lines (*CD* and *EF*) perpendicular to *AB*. These lines should cut the edge of the areas (*N* and *S*) at a point where the lines of force are also perpendicular to *AB*.

The positions of the poles *N* and *S* are determined by the intersection of the first line (*AB*) with the perpendiculars (*CD* and *EF*.) The distance between the poles is to be measured. The experiment is to be repeated with at least two other magnets as nearly as possible like the first.

The student may be interested to make prints showing the arrangement of iron-filings due to two parallel magnets, both when their north poles are turned in the same direction and when turned in opposite directions.[1]

[1] See Experiment 40 in the Elementary Physical Experiments published by Harvard University.

EXPERIMENT LXXII.

MAGNETIC FORCES.

¶ 180. **Determination of the Strength of Magnetic Poles.** — One of the magnets (*ef*, Fig. 197), used in Experiment 71, is now to be placed horizontally in the pan-holder (*c*) of a balance (the pan being removed), and counterpoised by an observed weight

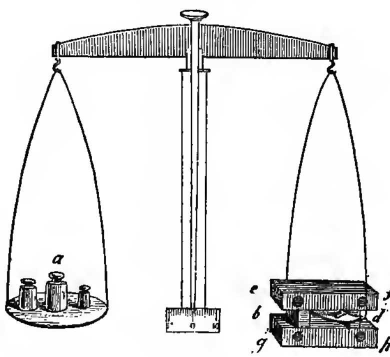


Fig. 197.

in the opposite pan (*a*). A second magnet (*gh*) is to be placed directly under the first, and parallel to it.

The north poles are at first to be turned in opposite directions, so that the magnets may attract each other. Small blocks (*b* and *d*) are now placed between them to keep them apart. The thickness of the blocks should be such that when the balance beam is raised upon its knife-edges, the index (*b*) may point to zero. The weight in the pan *a* is then gradually increased until the magnets are pulled apart. Care must be taken to find the *greatest* weight which the magnets can sustain; for if they be once separated a much smaller weight can hold them apart. In the final adjustment small weights (not over 1*cg.*) should be let fall into the scale-pan from a height not exceeding 1*cm.* The weight necessary to pull the magnets apart is to be noted.

The magnet *gh* is now to be turned end for end, so as to repel *ef*, and the weight in the pan *a* is gradually to be diminished until the magnet *ef* just touches the blocks (*b* and *d*). When a small weight is added to the pan *a* the beam will not turn suddenly as in previous observations; but, being in stable equilibrium, it may balance in any position. Care must therefore be taken to find the *smallest weight* which can cause a separation of the magnets, however slight.

The mean distance between the magnets, from centre to centre, is now to be determined by measuring the thickness of the magnets and the thickness of the blocks with a vernier gauge. In setting the gauge upon a magnet, if the jaws are of iron or steel the blocks of wood (*b* and *d*) should be interposed between the jaws and the surfaces of the magnet, since the strength of the magnet might otherwise be perceptibly affected. The thickness of the blocks may then be found and allowed for. The experiment should be repeated with a third magnet, let us say *ij* in place of *gh*; then with *gh* in place of *ef*. In this way the forces of attraction and repulsion between each pair which can be formed out of the three magnets will be determined.

The student may be interested to prove that it makes no difference which of two magnets is the one suspended. This fact is an illustration of the general principle that action and reaction are equal and opposite. It will be noticed that the attraction between two magnets when close together, is much greater than their repulsion. This is due to the effects of induction (see § 129, footnote).

¶ 181. **Calculations relating to Magnetic Forces.** — If w be the weight in grams necessary to counterpoise a magnet; w_1 the weight of the counterpoise necessary to lift the magnet and at the same time to pull it away from the attraction of a parallel magnet at the distance d; and w_2 the weight similarly required when the two magnets repel each other; then if 1 gram $= g$ dynes, the force of repulsion which we call positive is $+ (wg - w_2 g)$ dynes, and the force of attraction, which we call negative, is $- (w_1 g - wg)$ dynes. The numerical sum, or algebraic difference, Δ, between these forces is accordingly $(w_1 g - w_2 g)$ dynes. Substituting this value in the formula of § 129, we have, if any two of the magnets are equal in respect to the strengths (s and s') of their poles,[1]

$$ss' = s^2 = \frac{\Delta\, d^2}{4}; \text{ or } s = \frac{d}{2}\sqrt{(w_1 - w_2)\, g}.$$

Thus if the attraction between two nearly equal magnets at a distance of 2 *cm.* is 600 dynes, and the repulsion 300 dynes, a force of 900 dynes (0.92 *g.*, nearly) will be required to offset the effect of reversing one of the magnets. the mean strength of their poles is, accordingly, about $\frac{2}{2}\sqrt{.92 \times 980}$, or 30 units each.

The results of this experiment are subject to errors which are sometimes (though rarely) almost as great as the quantities measured. They are nevertheless valuable in enabling us to form an *immediate estimate* of the strength of magnetic poles, which, though rough, may guide us in the less direct but more accurate methods which follow.

[1] If no two of the magnets are equal, we must form three equations from observations made with each pair of magnets; thus —

$$ss' = \frac{\Delta\, d^2}{4}\ (1);\ ss'' = \frac{\Delta'\, d'^2}{4}\ (2);\ \text{and } s's'' = \frac{\Delta''\, d''^2}{4}\ (3).$$

Multiplying (1) and (2) together and dividing by (3) we have —

$$s^2 = \frac{\Delta\ \Delta'}{4\ \Delta''} \times \frac{d^2\, d'^2}{d''^2};\ \text{or } s = \frac{dd'}{2\, d''}\sqrt{\frac{\Delta\ \Delta'}{\Delta''}}$$

EXPERIMENT LXXIII.

MAGNETIC MOMENTS.

¶ 182. **Determination of the Couple exerted by the Earth's Magnetism on a Suspended Magnet.** — A magnet (*gh*, Fig. 198) used in Experiment 72 is to be suspended horizontally by a wire *cf*. The coefficient of torsion of the wire has been found in Exp. 64. The wire is attached at *c* to a knitting-needle (*bd*) revolving on a graduated circle (*ae*) as in the torsion balance (Fig. 176, ¶ 165). The wire is, however, vertical, and the circle horizontal in this experiment. A short piece of wire should be attached vertically by wax to each end of the magnet to serve as a sight. The needle is first turned so that the north pole of the magnet points north, and its reading is taken. Then it is turned until the magnet points east, and the reading again taken. A distant object should now be sighted in the direction indicated by the sights. The needle is then turned so that the magnet points west. The same distant object should be in line with the sights. The reading of the needle is again observed. The experiment should be repeated with the other magnets employed in Experiment 72.

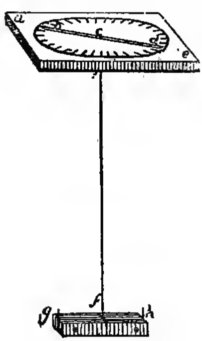


FIG. 198.

If the poles of the magnet are l centimetres apart, if they contain s units of magnetism each, and if the earth exerts on each unit of magnetism a force which has a horizontal component equal to H dynes, then the s units of magnetism in the north pole must be urged northward with a force of Hs dynes, and the south pole will be urged southward with an equal force. The two forces will constitute a couple (§ 113) C, with an arm equal to the distance l, between the poles; since the magnet is at right-angles to the forces in question. We have, therefore,

$$C = Hsl, \text{ or } H = \frac{C}{sl}.$$

This couple must be balanced by an equal and opposite couple due to torsion in the wire. It is obvious that in turning the magnet end for end it must be made to revolve through 180° so as to make an angle of 90° (on the average) with its original (north and south) direction. To produce torsion in the wire the needle must be turned through *more* than 180° in all, or more than 90° from its original setting.

Let us suppose that the needle has revolved through a total angle a, or an average angle of $\frac{1}{2}a$ from its original position; if the magnet had remained pointing to the north the twist in the wire would be $\frac{1}{2}a$; but the revolution of the magnet through 90° causes the wire to untwist through 90° at its lower end. The angle of torsion is therefore $\frac{1}{2}a - 90°$. It is now easy to calculate the couple exerted by the earth. If it requires a couple of t dyne-centimetres to twist the wire through $1°$ (see Experiment 64) it must require $(\frac{1}{2}\,a - 90) \times t$ dyne-centimetres to twist it through the angle in question. Substituting this value for c in the formula above we have —

$$H = \frac{(\frac{1}{2}\,a - 90)\,t}{sl}$$

It is interesting to estimate the value of H by the rough values of s and l already determined in Experiments 71 and 72. If, for instance, the distance between the poles is 10 *cm.*, and the strength of each 30 units, and if the couple produced is 50 dyne-centimetres, then the earth must exert a force of $\frac{1}{6}$ of a dyne on each unit of magnetism when free to move only in a horizontal plane. This is what is meant by the statement that the "horizontal intensity" of the earth's magnetism is $\frac{1}{6}$ or 0.17, nearly. In practice large errors would be committed in estimating the horizontal intensity in this way, on account of the uncertainty of the factor s (see ¶ 181). A much more exact method will be considered in connection with Experiment 74.

The student should note that the couples acting on suspended magnets are proportional to the products of the distance between the poles and the strength of the poles, both of which have been already determined. These products (sl, $s'l'$, $s''l''$) are called the *magnetic moments* of the magnets to which they respectively belong.

EXPERIMENT LXXIV.

MAGNETIC DEFLECTIONS.

¶ 183. **Determination of Magnetic Deflections by means of a Magnetometer.** — A surveying-compass (Fig. 199) is placed in the middle of a wooden table, in the construction of which no iron has been employed even in the form of nails. All iron or steel objects are to be removed from the immediate neighborhood. The directions of the magnetic north, south, east, and west are to be determined by this compass, and marked by pencil lines upon

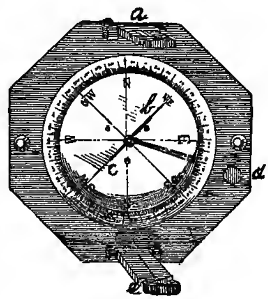


FIG. 199.

the table. In all experiments in magnetism the magnetic points of the compass will be those referred to, unless otherwise stated. A magnet already tested in Experiment 71, considerably longer than the compass needle, is now placed at the east of the compass with its north pole toward the compass (see Fig. 200, 1). The distance of the magnet from the compass must

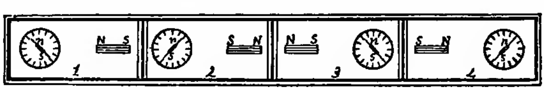


FIG. 200.

be noted. It should be small enough to cause a measurable deflection of the compass, let us say 5 or

10 degrees, but at least twice the length of the magnet.[1] The position of each end of the magnet is then marked in pencil on the table, and the deflection of the compass observed by the reading of two pointers, attached one to each end of the needle.

The magnet is now turned end for end (as in Fig. 200, 2) and the deflection again observed. The experiment is to be repeated with the magnet at an equal distance from the compass, but at the west of it, as in Fig. 200, 3 and 4. There will thus be 8 readings in all, from which the average deflection of the needle may be calculated. The mean distance of the centre of the magnet from the centre of the needle may be found quite accurately by measuring the distance between the outer and between the inner pencil marks on opposite sides of the needle, adding, and dividing by 4. The experiment is to be repeated with the other magnets employed in Experiment 71.

The results of this experiment are to be reduced as will be explained in ¶ 185.

¶ 184. **Theory of the Magnetometer.** — When a magnet is placed near a compass-needle, and at the east or west of it, as in Fig. 200, so that one of its poles is nearer than the other, the needle is deflected under the influence of the nearer pole. The lines of force due to a magnet at any point nearly in line with the two poles are (see Fig. 195) nearly parallel to the magnet; and hence in the case which we have

[1] For very accurate measurements the distance of the magnet from the compass should be at least 4 times the length of the magnet and 12 times the length of the needle.

supposed they are nearly east and west. That is, the magnet tends to make the compass-needle point east and west.

Let us suppose that the magnet is at the east of the compass, and that its south pole is (as in Fig. 200, 2) nearer than the north pole. Then the north pole of the compass-needle (*c*, Fig. 201) will be attracted by the south pole of the magnet more than it is repelled by the north pole. The resultant force will there-

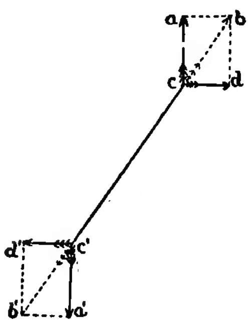


Fig. 201

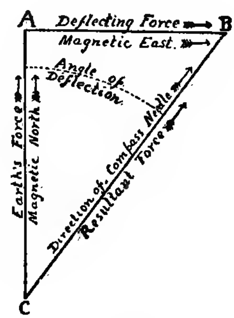


Fig. 202.

fore be an attraction toward the east, which we will represent by the line *cd* (Fig. 201). At the same time the earth pulls the north pole of the compass-needle northward, with a force represented let us say by the line *ca*. The resultant of these two pulls is a force *cb*, easily found by geometrical construction (§ 105).

On the other hand the south pole of the compass-needle (c') will be repelled by the south pole of the magnet more than it is attracted by the north pole. It will accordingly be urged westward with a force $c'd'$. At the same time it is drawn southward by the earth's magnetism with a force $c'a'$. The resultant force, $c'b'$, may be found as before. Assuming that the forces acting upon the south pole of the needle are equal and opposite to those acting upon the north pole, it follows that $c'b'$ must be equal and opposite to cb. If the needle cc' is free to turn, it will obviously take the direction of the two resultants.

The relation between the forces exerted by the earth and by the magnet upon the north pole of the compass-needle is shown in Fig. 202. The magnetic force is represented by AB; the earth's force by by CA; the resultant by CB. The angle BAC is called the angle of deflection. The tangent of this angle is by definition equal to AB ÷ CA; since AB and CA are at right-angles. Obviously, the magnitude of a deflecting force bears to that of a directive force at right-angles to it a ratio equal to the tangent of the angle of deflection produced.

It has been stated that when the two poles of a magnet are at unequal distances from a compass-needle, the nearer pole has the greater effect. Since the two poles are always equal and opposite, the action of a magnet as a whole evidently depends not only upon the strength of its poles, but also upon the difference of their distances from a given point. We must accordingly consider the length of a magnet, as well as the strength of its poles, in calculating the effect which it will produce. It is found, in fact, that the forces produced by different magnets at a given distance are very nearly proportional to the "moments" of the magnets in question, that is (see ¶ 182), to the products of the strength of the poles and the distance between them. The moments of the magnets (sl, $s'l'$, etc.) employed in this experiment have been already determined (¶ 182). If a, a', etc., are the deflections produced, we should have —

$$\frac{sl}{tan\ a} = \frac{s'\,l'}{tan\ a'}, \text{ etc., nearly.}$$

The student should satisfy himself that this is the case before proceeding to the calculations of the next section.

A compass, having on each side of it a pair of revolving supports, capable of holding several magnets, successively at a given distance from the needle, affords one of the most direct and accurate methods of comparing magnetic moments together, and is properly called a magnetometer.

¶ 185. **Calculations relating to Magnetic Deflections.** — Example. Let us suppose that in Fig. 200 the average distance between the centre of the magnet NS and the centre of the needle ns is 25 *cm.*, and that the distance between the poles of the magnet (¶ 179) is 10 *cm.* so that as in (2) the south pole is 20 *cm.* from the needle and the north pole 30 *cm.* from it. Assuming that each pole has a strength of 30 units (see ¶ 181) the attraction of the south pole for a unit of positive magnetism *at the centre of the needle* (see § 129) must be $30 \div (20)^2$ or $\frac{3}{40}$ dyne. The opposite pole must exert a repulsion on the same unit of magnetism equal to $30 \div (30)^2$ or $\frac{1}{30}$ dyne. The resultant of these two forces is evidently $\frac{3}{40} - \frac{1}{30}$ or $\frac{1}{24}$ dyne acting in an easterly direction parallel to AB (Fig. 202). The earth's magnetism acts in a northerly direction parallel to CA (Fig. 202).

Now since $$\frac{AB}{CA} = tan\ CAB,$$

we have $$CA = \frac{ab}{tan\ CAB}$$

If, for example, $CAB = 14°$, the tangent of CAB is .249 (see Table 5) or $\frac{1}{4}$, nearly; then CA is evidently 4 times as great as AB; hence if $AB = \frac{1}{24}$ dyne per unit of magnetism, $CA = \frac{1}{6}$ dyne per unit of magnetism.

In practice an estimate of the earth's magnetism made in this way will be found to differ greatly from that made as in the last experiment, on account of a tendency to underestimate the strength of the magnetic poles in Experiment 71.

Let us suppose that this strength were estimated at 15 units instead of 30 units. Then in the calculation above we should have estimated the earth's field at $\frac{1}{12}$ dyne per unit of magnetism (instead of $\frac{1}{6}$). In ¶ 182, however, we should have estimated the earth's field at $\frac{1}{3}$ dyne per unit of magnetism. That is, our estimate in Experiment 73 would be too great, and that in Experiment 74 too small in proportion to the error originally made in estimating the strength of the poles. Now when one of two estimates is too great, and the other too small in a given proportion, the geometric mean between them must be equal to the quantity which we seek. Hence to find the true value of the horizontal component of the earth's magnetism, we multiply together the estimate of Experiments 73 and 74, and extract the square root of the result. Thus $\sqrt{\frac{1}{3} \times \frac{1}{12}} = \frac{1}{6}$. The result is independent of the value provisionally adopted for the strength of the magnetic poles. If the two estimates agree closely the arithmetic mean may be substituted for the geometric mean (§ 57).

Knowing now the true value of H, we may recalculate the moment (M) of the magnet and the strength of the poles by formulæ derived from ¶ 182:

$$M = sl = \frac{C}{H}; \quad s = \frac{C}{Hl}.$$

EXPERIMENT LXXV.

DISTRIBUTION OF MAGNETISM, I.

¶ 186. **Determination of the Distribution of Magnetism on a Rod by the Method of Vibrations.** — A steel rod (aj, Fig. 203) one metre long, and about 1 *cm.* in

Fig. 203.

diameter, is marked with a file at ten points ($a \ldots j$) 10 *cm.* apart, beginning with a point a, 5 *cm.* from one end of the rod. It is then magnetized by stroking it from *e* to *a* 10 times with the south pole of a powerful electro-magnet, and by stroking it 10 times from *f* to *j* with the north pole of this magnet. A small piece of a sewing-needle (*f*, Fig. 204) about 1 *cm.* long, and highly magnetized is attached horizontally by sealing-wax to a bullet *e*, and suspended by a fine fibre (*cd*) of untwisted silk from a cork (*a*) in a test tube (*bg*).

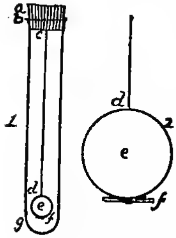


FIG. 204.

The torsion of the fibre (*cd*) should be so slight that the cork (*a*) may be twisted through 360°, without deflecting the needle (*f*) more than a few degrees from the magnetic north, toward which one end should point. The needle is then to be deflected by a magnet; and when the magnet is suddenly taken away the needle should make a series of vibrations in a horizontal plane. The weight of the bullet should be so proportioned to the magnetic strength of the needle that there may be about 10 vibrations completed in one minute. The exact time required for 10 vibrations of the needle is to be determined when it is vibrating in an arc not exceeding 30° or 40° (see Table 3, *g*). The north pole of the needle should be distinctly marked.

The test tube is now to be placed opposite the end of the rod, then held successively on each side of each of the ten points (*a*—*j*, Fig. 203). The direction indicated by the north pole in each position is to be represented by arrows (drawn as in Fig. 203) the direction of which may be compared with that of the lines of force issuing close to the magnet in Fig. 196. In addition, the rate of vibration of the needle is to be determined by counting the number of vibrations completed in 1 minute, or in whatever time may have been required for 10 vibrations under the influence of the earth's magnetism alone. In all cases the arc of vibration should be limited to 30° or 40° (see Table 3, *g*).

The number of vibrations made in the given time on one side of *a* is to be averaged with that made on the other side; and in the same way the average number of vibrations for each of the ten points is to be found. These numbers are then all to be squared (see Table 2). The results are to be plotted on co-ordinate paper (see § 59). Distances in centimetres are represented by a horizontal scale at the top of the figure, and the square of the number of oscillations is shown by the vertical scale at the left of the figure. Thus, if opposite the point *b*, 15 *cm.* from the end of the magnet, the needle makes 60 vibrations per minute, we place a cross at the right of the square of 60 (3600) and under 15 *cm.* The vertical distances are measured upward if the north pole of the needle is repelled by the bar, and downward if it is attracted by it. In the same way other points may be found through which

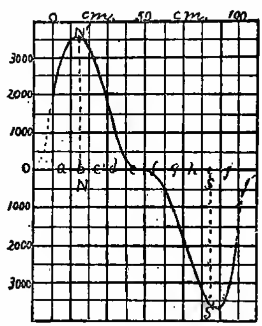


FIG. 205.

a curve is to be drawn as in Fig. 205. Evidently, in this figure, N represents the "positive" or "north" end of the magnet.

This method of representing the distribution of magnetism depends upon the general principle that forces are proportional to the squares of the rates of oscillation which they produce (see § 110). The curve represents accordingly the strength of the magnet at different points as compared with the strength of the earth's magnetism. We should strictly allow for the effect of the earth on all the rates of oscillation; but as it is represented only by 100 units on the vertical scale, this effect would be hardly perceptible.[1]

The student should draw by the eye two vertical lines NN' and SS', dividing each area enclosed by the curve as nearly as possible into two equal parts. The distance between these lines indicates approximately the distance between the poles of the magnets. This latter may therefore be found by the scale at the top of the paper.

EXPERIMENT LXXVI.

DISTRIBUTION OF MAGNETISM, II.

¶ 187. **Magneto-Electric Induction.** We have seen that when iron-filings are brought into the neighbor-

[1] The effects of "induced magnetism" may introduce errors of 5 or 10 per cent in this experiment (see ¶ 207). The shape of the curve in Fig. 208 will not, however, be materially altered.

hood of a powerful magnet, they tend to arrange themselves along certain lines called "lines of force." These lines of force are not, like the meridians upon the surface of the globe, purely geometrical conceptions. According to Tyndall, the apparently empty space between the poles of a powerful electro-magnet "cuts like cheese." The most surprising fact connected with this phenomenon is that a knife with which such a magnetic field is cut becomes temporarily electrified. The point and the handle of the knife resemble, for the time being, the two poles of a voltaic cell, from which a current of electricity can be derived by making the proper connections. It is not necessary to use a knife; any piece of metal, a wire for instance, will do as well. All tendency to produce a current ceases when the knife or wire stops moving, or as soon as all the lines of force have been cut. The effect of a sudden motion upon a galvanometer may accordingly be almost instantaneous. In such cases it is measured by the "throw" of the needle (§ 109). It is found that the "throw" is proportional, other things being equal, to the intensity and extent of that part of the magnetic field which has been cut through, or, according to a system of representation universally adopted, it is proportional to the *number of lines of force* which have been cut.

If a loop of wire is placed around the middle of a long bar-magnet (Fig. 206) and suddenly made to slip off one end of the magnet, it will evidently cut nearly all the lines of force on that end of the magnet.

A delicate galvanometer connected with the ends of the loop will be affected. This affords a convenient method of comparing the strengths of different magnetic poles. In practice we employ a coil of wire instead of a simple loop; for when each turn cuts all the lines of force, the effect is found to be proportional to the number of turns which the wire makes about the magnet. It is not necessary to slide the coil completely off the magnet. A motion of a few centimetres may affect the galvanometer. When the motion is confined to one end of the magnet it will be found to deflect the needle in opposite ways according to which way the coil is moved. In other words the direction of the electrical current depends

FIG. 206.

upon the direction of the motion. Let us suppose the direction of the motion to be always the same, that is, from left to right, or from the north toward the south end of the magnet. Then the galvanometer will be deflected one way when the motion of the coil takes place near one end of the magnet, and the other way when it takes place near the other end of the magnet. That is, the direction of the electrical current depends on the direction of the lines of force. Near the middle of the magnet a neutral point will generally be found. If the coil be moved from this neutral point toward either end of the magnet, it follows from the statements made above that the direction of the current will always be the same. This direction is with the hands of a watch, as seen from the south pole of the magnet.

The throw of the needle is proportional, other things being equal, to the distance through which the coil is moved; hence it is important in comparing results that this distance should be always the same. If the coil is moved always through a given distance, the effect will be found to be greatest when the motion takes place near the ends of the magnet, where the lines of force are the thickest. In other words the magnitude of the electrical current depends upon the closeness of the lines of force. The effect is very nearly the same whether the coil moves *more* or *less swiftly*[1] through a given distance. In the first case we have a rapid motion, and hence a comparatively strong current lasting for a short time; in the second case we have a weaker current lasting for a proportionately long time. The forces exerted upon the galvanometer needle are proportional to the current; hence, by the fundamental law of motion (§ 106),

$$ft = mv,$$

since the product (ft) of the force and the time of its action is the same in both cases, the momentum given to the needle must be the same.

We shall make use of these facts to estimate the relative strength of the magnetism of a rod in differ-

[1] In order that this may be true, the duration of the motion must be several times less than the time occupied by one vibration of the galvanometer needle.

ent parts, and to distinguish positive from negative magnetism.

¶ 188. **Construction of an Astatic Galvanometer.** — A delicate galvanometer, such as has been already employed for the detection of currents created by a thermopile (Exp. 39), is represented in Fig. 207, and may be constructed as follows: —

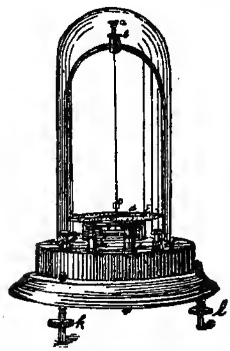


Fig. 207.

Two magnetized needles, *c* and *h* (Fig. 208), of nearly equal strength are connected by a vertical piece of wire, with their north poles *in opposite directions*, and suspended horizontally, by a fine thread (*bc*) of untwisted silk, from a screw *a*. This screw is held by a nut *b*, itself capable of rotation, so that the thread may be raised or twisted at pleasure. The two needles *c* and *h* should form a nearly "astatic" combination (*a* privative and ἵστημι, to stand); that is, one which, owing to the

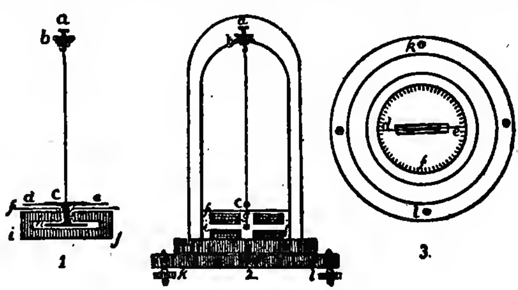


Fig. 208.

equal and opposite forces exerted upon it by the earth, has no strong tendency to stand in any particular position.

The strength of either magnet may generally be increased by stroking one of the poles, as in ¶ 179, with the dissimilar pole of a powerful magnet, or diminished by touching similar poles together. A very light touch is usually sufficient to produce a perceptible change in a magnet. The delicacy of the instrument depends upon the delicacy of the balance which can be established between the two needles. It is generally possible to make the combination point permanently east and west. In practice, however, the needles are magnetized so that the time occupied by one oscillation is 5 or 10 times as great as that of either needle by itself. The needle is then sufficiently astatic for most purposes. It may be remarked that the rate of oscillation of an astatic needle is the best test of its adjustment (see ¶ 193, 4).

100 metres of insulated copper wire about $\frac{1}{2}$ *mm.* in diameter are now to be wound on the two rectangular bobbins *f* and *i* (Fig. 208, 1 and 2).[1] The bobbins are shaped so that the lower needle (*h*) may hang inside of them, and the upper needle (*c*) just above

[1] If it is desired to use the instrument later on (Exp. 86, II. and Exp. 95) as a differential galvanometer, the 100 metres of wire should be cut in two, and the two parts twisted together before winding them on the bobbins. The galvanometer will thus have four terminals instead of two. If two of the terminals are temporarily joined together, the other two may be connected with binding-posts in the ordinary manner.

them. Two indices of aluminum wire, *d* and *e* (Fig. 208, 1 and 3), are then attached to the upper needle, and a cardboard protractor (*f*) is set beneath them. The instrument is usually mounted on wooden supports, with levelling screws *k* and *l*, and covered with a glass shade to cut off currents of air. The galvanometer thus constructed should be sensitive to a few millionths of an ampère.

¶ 189. **Determination of the Distribution of Magnetism on a Rod by the Method of Induction.** — A coil (*b*, Fig. 209) consisting of about 100 turns of No. 20 insulated copper wire, wound on a brass bobbin, is fitted to a brass tube *ad* so as to slide freely between

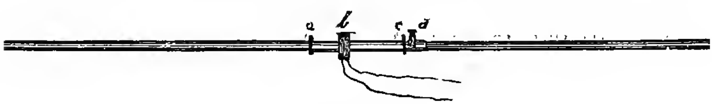


FIG. 209.

the stops *a* and *c*, through a distance of about 10 centimetres. The tube must be large enough to admit the long magnet employed in Experiment 75. It is first to be fastened near one end of this magnet by means of the clamp *d*, so that a point (*a*, Fig. 203) 5 *cm.* from the end of the magnet may come half-way between the stops *a* and *c* (Fig. 209).

The needle of a delicate galvanometer (Fig. 207), such as has been already employed for the detection of electrical currents (Exp. 39), is now to be loaded, if necessary, by attaching small bits of lead with sealing-wax to each end of the needle, so that its time of oscillation may be at least 10 seconds. The instrument is to be set up with the plane of its coils approximately north and south. The nut *b* is then turned so that, by the torsion of the thread *bc*, the needle of the galvanometer is made to point to 0°. The terminals of the coil *b* (Fig. 209), are then to be connected with the terminals of the galvanometer.

The coil (*b*) is then suddenly made to slide from *a* to *c* (Fig. 209), and the throw of the galvanometer is noted. When the oscillation of the needle has ceased[1] the coil is made to slide back suddenly from *c* to *a*, and the throw of the galvanometer is again noted.

The experiment is to be repeated with the tube clamped so that other points (*b*, *c*, *d*, *e*, etc., Fig. 203) may come successively half-way between the stops *a* and *c* (Fig. 209).

In each case two throws of the galvanometer are to be observed. The direction of each throw is to be noted, and the average deflection calculated.

The positions of the centre of the tube with respect to the magnet are also to be noted. The results are to be plotted on co-ordinate paper as in Fig. 205,

[1] The student should learn to stop the vibrations of a magnetic needle. If a magnet is directed toward a needle as in Fig. 200, ¶ 183, a deflection in either direction may be produced. If the magnet be turned so as to tend to cause a deflection at every instant opposite to the motions of the needle, the latter will come very quickly to rest. To stop a wide oscillation, the magnet must be brought near the needle, but when the oscillation becomes feeble, the process should be continued from a greater distance. To affect an ordinary astatic needle, the magnet should be held not only at right-angles with it, but also considerably above or below it. A perfectly astatic needle should not be affected by a magnet in the same horizontal plane.

¶ 186, except that the vertical distances are to represent throws[1] of the galvanometer needle, instead of squares of the rates of oscillation. If the throw in a given case is in the same direction as at the north end of the magnet when the coil is stopped in a given direction, the distances are to be measured upward; otherwise downward. From the curves thus obtained the poles of the magnet are to be located as in ¶ 186, and the distance between them is to be estimated. The result should agree closely with that obtained in the last experiment.

EXPERIMENT LXXVII.

MAGNETIC DIP.

¶ 190. **The Earth's Magnetism.** — If fine iron-filings are sprinkled over a horizontal pane of glass, they will show a slight tendency to arrange themselves in lines parallel to the magnetic meridian, particularly if the glass be jarred. One might infer that the lines of force due to the earth's magnetism are horizontal. This is not, however, the case; the direction in which the lines are inclined is from north to south, according to the compass, but the lines make any angle with the horizon (§ 128); 70° or 80° for instance in the United States. We have already made use of

[1] If the throws exceed 30° the student should plot the *chords* of the angles in question (Table 3), instead of the angles themselves (see § 109).

the surveying-compass to find the magnetic meridian (¶ 183). The compass affords, however, little or no idea of the angle which the lines of force make with the horizon, because a compass-needle is suspended so as to move approximately in a horizontal plane.[1] To find the magnetic dip (§ 128), we may make use of an instrument known as the "dipping-needle." A simple form of this instrument consists of a knitting-needle *ad* (Fig. 210), with an axis *bc* soldered to it a right-angles and resting on two glass surfaces *b* and *c*, attached by sealing-wax to wooden supports (*be* and *cf*), and made horizontal by means of a spirit level.

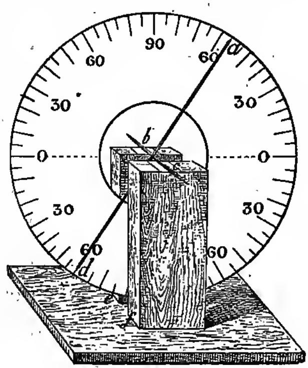


Fig. 210.

In practice the needle must be balanced by bending the axis *bc*, or by adding bits of sealing-wax or solder to it, so that it will stay, *when unmagnetized*, in any position, as *ad*. Then the needle is magnetized by stroking the end *a* ten times from the centre outward with the north pole of a powerful magnet, and by stroking the end *d* similarly with the south pole of the magnet. The needle will no longer balance in any position; but the north pole will, in north lati-

[1] The needles of surveying compasses intended for use in widely different latitudes are frequently provided with a small sliding weight by which variations in the magnetic dip and intensity may be counterpoised.

tudes, dip downward as in Fig. 210. To measure the angle of the dip, a cardboard protractor, cut out at the centre so as not to interfere with the axis of the needle *bc*, is attached vertically to one of the wooden supports (*be*), and turned round so as to be north and south according to the compass. The axis *bc* is made to point horizontally east and west, and to coincide as nearly as possible with the axis of the graduated circle. The mean reading of the two ends (*a* and *d*) of the needle should then give correctly the angle of the dip. Errors of parallax must of course be guarded against (§ 25). Various other sources of error may be eliminated by a series of experiments. In some of these the axis *bc* should be turned end for end, in some the whole instrument should be turned end for end, and in some the magnetism of the needle should be reversed by stroking the end *d* upon the north pole, and the end *a* upon the south pole of a magnet. By averaging the various results, the angle of the magnetic dip may be determined within a few degrees.

¶ 191. **The Earth Inductor.** — If a hollow square of wire *CDEF* is laid upon the floor with the side *CD* magnetically east and west, and rotated about *CD* as an axis into the position *ABCD*, it is evident that the wire *EF* must cut all the lines of force due to the earth's magnetism which pass through the areas *ABCD* and *CDEF*. The line *CD* will cut no lines of force, because it is stationary; and the wires *CE* and *DF* will cut none, because their motion is in a plane parallel to the lines in question. All the lines cut will therefore be included in the area *ABEF*.

If the square is now held against the west wall of the room, in the position *C′D′E′F′*, and rotated as before about an axis (*C′D′*) perpendicular to the lines of force, into the position *A′B′C′D′*, the number of lines cut will be as before included in the area *A′B′E′F′*; and similarly if the square is rotated about an axis *C″D″*, in the north wall of the room

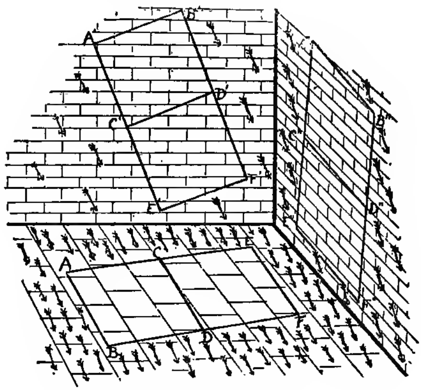


Fig. 211.

perpendicular to the lines of force, the lines cut will all be included in the area *A″B″E″F″*. Now the areas (*ABEF*, *A′B′E′F′*, *A″B″E″F″*) are all equal,—each being twice the area included by the square. If, therefore, we connect the terminals of the square with a galvanometer, and observe the throws of the needle which take place when the square is suddenly turned over, we shall have a means of comparing the relative numbers of the lines of force which pass through the square in its three different positions.

From these data we may infer the direction of the lines of magnetic force. If, for instance, the throw of the needle is much greater when the square is turned over on the north wall of the room than on the west wall, we may infer that more lines of force pass through the square in the former position; and that, accordingly, these lines are more northerly than westerly. If, again, the throw is much greater when

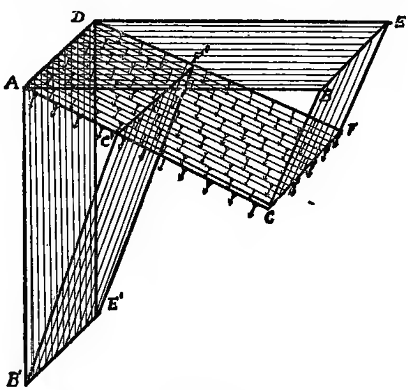


Fig. 212.

the square is turned over on the floor than on either wall, we may infer that the lines of force are more nearly vertical than horizontal. We will suppose, for simplicity, that the walls of the room face exactly north and west by the compass, so that no lines of force pass through the loop when held against the west wall of the room.

Let *ABED* and *AB′E′D* (Fig. 212) represent respectively the square in its horizontal and in its vertical position, AD being magnetically east and west; let the plane $ADF'FCC'A$ be drawn perpendicular to the lines of force, and the planes $BEFC$ and $B'E'F'C'$ parallel to the lines. Then the areas $ADFC$ and $ADF'C'$ include respectively the lines which pass through the square in its two positions. Since the lines are equally spaced, their numbers are as the areas which include them. These areas are to each other as $AC : AC'$, or since by construction $BC = AC'$, they are to each other as $AC : BC$. This ratio $(AC : BC)$ is by definition the tangent of the angle ABC, which measures the magnetic dip.

Now if a' is the angle through which the needle is thrown when a loop of wire is turned over on the floor, and if a'' is the same for the north wall of a room, the impulses given to the needle are to each other as the chord of a' is to the chord of a'' (see § 109), or approximately as a' is to a''. It follows that the angle of the dip a is given by the formula —

$$tan\ a = \frac{chord\ a'}{chord\ a''} = \frac{a'}{a''}\ \text{nearly.}$$

The same proportion will be found to hold for a round loop of wire. In practice we employ a coil of wire, containing, let us say 100 turns, since the effect upon the galvanometer increases with the number of turns.

The student should note that a sliding motion given to such a coil either along the floor or along the wall causes no deflection of the galvanometer. This is because the lines of force are cut by the two halves of the coil in opposite ways. It will be found to make no difference whether the coil is rotated about an axis passing through its centre, or on one side of it. We need to consider only the angle through which rotation has taken place. A coil capable of being thus rotated 180° about a horizontal and about a vertical axis constitutes what is called an "earth inductor," because of the currents of electricity which by the action of the earth's magnetism, may be "induced" in it.

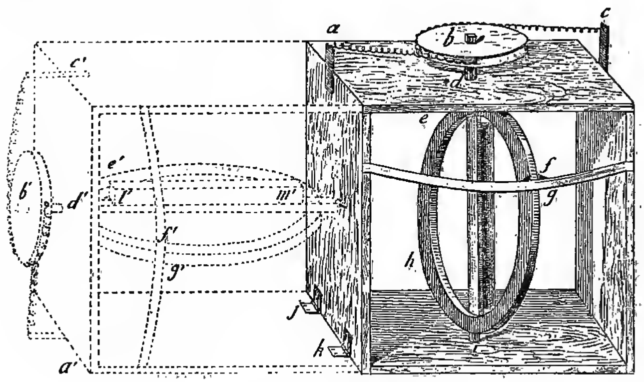


FIG. 213.

¶ 192. **Determination of the Magnetic Dip by means of an Earth Inductor.** — A convenient form of earth inductor is represented in Fig. 213.[1] It consists of a coil of wire *h*, mounted on a wooden axle *di*, with a head *b*, through which the coil may be set in rotation

[1] The instrument may be greatly simplified if it is intended only to be turned by hand. This generally requires the co-operation of two students, one to turn the earth inductor properly, the other to observe the throws of the galvanometer.

by the spring *cbd*. An auxiliary spring *ad* may also be employed to hasten the rotation through the first right-angle, and to slacken it in the second right-angle, so that the coil may be arrested by the catch *f*, when it has rotated through exactly 180°. By winding the spring *abd* round the head of the axle in the other direction, the coil may be made to return to its original position. The apparatus is permanently attached to the floor by means of two hinges *j* and *k*, the axes of which are east and west. If the coil is properly counterpoised, it will operate also when the whole instrument is tipped on its side, as represented by the dotted lines in Fig. 213.

Wedges are to be placed beneath the frame so that the axis of the coil may be exactly vertical in one position, and exactly horizontal in the other position. The catch *f* must be adjusted if necessary, so that the coil may be horizontal in the second position. If the hinges are properly placed the plane of the coil will be at right-angles to the magnetic meridian in both positions.

The axis of the coil is first to be made horizontal, and the terminals of the coil are to be connected (see ¶ 193, 11) with a galvanometer (Fig. 207, ¶ 188), placed at a considerable distance from the earth inductor so as to avoid jarring, and adjusted as in ¶ 189. The catch *f* is then to be lifted by pulling a string attached at *g*. The throw of the needle is to be noted. When the needle has come to rest (see ¶ 189, foot-note) the coil is made to return suddenly to its original position by the same mechanism. The throw of the needle is again observed, and the mean throw (a') calculated.

The experiment is to be repeated with the axis of the coil vertical. The mean throw (a'') is to be found. The angle of the dip (a) is then to be calculated by the formula (see ¶ 191),

$$tan\ a = \frac{a'}{a''}, \text{ nearly.}$$

ELECTRICAL MEASUREMENTS.

CURRENT STRENGTH.

¶ 193. **General Precautions in the Measurement of Electric Currents.** — Nearly all measurements of electric currents involve the use of galvanometers depending upon the deflection of a magnetic needle. The same precautions must accordingly be observed in electrical as in magnetic measurements.

(1) DELICACY OF SUSPENSIONS. A needle weighing less than 10 grams may be safely suspended by a single fibre of the best cocoon silk. When several fibres are employed they should be fastened together with wax, but *not twisted together*. If great delicacy is desired, the finest possible thread should be employed.

When a needle is hung on a pivot, as in an ordinary compass, great care must be taken to preserve the sharpness of the steel point upon which it turns. A lever should be arranged so as to lift the needle from the pivot when the instrument is not in use; and when in use, care should be taken not to jar the compass. A slight jarring may be used as a last resort to relieve the friction between the needle and its pivot when the latter has been already dulled. It is preferable, when possible, to observe the turning-points of the needle while oscillating in a small arc, and from these to infer its position of equilibrium (see ¶ 20).

(2) PRESERVATION OF MAGNETISM. The needle of a galvanometer should be carefully protected from strong magnetic forces, whether due to permanent magnets or to electric currents, since such forces are apt to affect the magnetism of the needle. This precaution is especially important in the case of "astatic" needles (¶ 188), since the slightest change in either of the two parts of which such needles are composed may completely destroy the balance between them, and thus seriously injure the delicacy of the combination.

Strong currents should never be sent through delicate galvanometers. The terminals of such galvanometers (*a* and *b*, Fig. 214) should be joined together with a wire or "shunt" (*c*), forming a cross-connection between the wires (*d* and *e*) which convey the current to and from the galvanometer. An electric current of unknown strength should be first tested by the galvanometer *with the shunt*. If the galvanometer shows little or no deflection, the shunt may be safely removed.

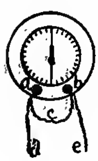


FIG. 214.

(3) MAGNETIC SURROUNDINGS. All iron, steel, or other magnetic substances should be removed, if possible, from the neighborhood in which magnetic measurements are to be performed. The positions of magnetic bodies which cannot be moved should be accurately noted. Especial care must be taken to guard against *changes* in the position of magnetic

bodies in a course of experiments.[1] The position of a galvanometer should be accurately located, since considerable variations, both in the direction and in the strength of the earth's magnetism, often occur in different parts of the same building, unless special care has been taken to avoid the use of iron in its construction. When there is no simpler way of describing the place of an instrument, its distances may be found from the floor and from two walls of the room.

(4) Rate of Oscillation. Any change in the strength of the magnetic forces acting upon a needle, in the magnetism of the needle itself, or in the freedom of its suspension will be found to affect its rate of oscillation. It is well, therefore, to determine this rate before and after every experiment in which such changes are likely to occur. This precaution is particularly important in the case of astatic needles and in the method of vibrations (Exp. 82).

(5) Excentricity. When a compass-needle is suspended at a point not exactly in the centre of the graduated circle by which its position is determined, errors due to "excentricity" may be introduced. Such errors are avoided by *reading both ends of the needle.*

(6) Zero-Reading. A galvanometer is always to be adjusted (except in the method of vibrations, Exp. 82) with the plane of its coil vertical, and parallel to the needle in its zero position, — that is, the position which the needle takes when no current is flowing

[1] Students should be cautioned against carrying small objects made of iron or steel about their person.

through the coil. In the case of a galvanometer provided with an ordinary compass-needle, the plane of the coil is accordingly to be made parallel to the magnetic meridian. In this position the reading of the needle should be zero. It is well to make sure (§ 32) that the zero-reading is not disturbed in the course of an experiment, either by dislocation of the galvanometer or by changes in the position of magnetic bodies in the vicinity (see 3).

(7) MUTUAL INDUCTION. To prevent the coils of one instrument from affecting the needle of another instrument, these instruments should be separated as widely as may be practicable. In certain

FIG. 215.

delicate experiments the effects of magnetism produced in one building are measured by electrical wires carried to an entirely separate building. Coils of wire are in general made horizontal if possible; magnets vertical; since in these positions minimum magnetic effects are usually produced on galvanometers in their vicinity.

(8) CONNECTING WIRES. The wires conveying an electric current to and from an instrument should be parallel and close together, so that the equal and opposite currents in these wires may neutralize each other as far as magnetic effects are concerned. A typical case is represented in Fig. 215, where by the parallel wires *bc*, *de*, and *af*, a battery *B* is connected

through a rheostat R with a galvanometer G (see Exp. 92). It will be found convenient in practice to twist the wires together. In rheostats the wires are wound double (see Fig. 240, Exp. 86) to avoid magnetic effects.

(9) Reversal of Currents (§ 44). Every instrument capable of being affected by magnetic influences from outside should be provided with means of reversing the current through it, without changing its direction in other parts of the circuit. Any such instrument is called a "commutator." A convenient form of "commutator" is represented in Fig. 216.[1]

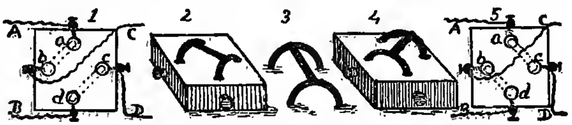


Fig. 216.

(10.) Waste of Power. The commutator may be made also to serve as a "key," — that is, to cut off

[1] This commutator consists of a square block of mahogany or ebonite, with four holes *abcd* (Fig. 216) bored half-way through it. The screws of four binding-posts are driven horizontally into these holes, which are then filled with mercury. Two copper rods (Fig. 216, 3), bound together by a handle of mahogany or ebonite, are bent so as to reach respectively either from a to b and from c to d, or from a to c and from b to d (see Figs. 216, 2 and 4). The wires (A and B) from the positive and negative poles of a battery are connected with two *opposite* mercury cups, as a and d; the wires C and D, leading to the instrument in which the current is to be reversed, are connected with the other pair of opposite cups (as b and c). It will be seen that in one position of the commutator (Fig. 216, 1 and 2), the wire A is connected with C, while B is connected with D; in the other position (Fig. 216, 4 and 5) A is connected with D, while B is connected with C.

the current from the battery. This is done by simply removing the rods (Fig. 216, 3) from the mercury cups. In the absence of a commutator or key, one of the battery wires should be disconnected when the battery is not in use, not only to prevent unnecessary waste of power, but also to avoid serious errors which may result either from the deterioration of the battery or from heating the wires.

When a battery is not required for several days it is well to empty out the fluids which it contains, each into a separate vessel, in which it may be preserved for future use, if not already exhausted. The zincs and coppers or carbons should be placed in pure water; the porous cups left to soak in a solution of dilute sulphuric acid so as to be ready for immediate use; the clamps, being disconnected from the poles of the battery, should be carefully cleaned and dried.[1]

(11) Electrical Connections. All electrical connections depending upon metallic contact should be carefully examined. The metallic surfaces should be scraped bright and bound together with considerable pressure. A good electrical connection between two copper wires may generally be made by twisting them together. A soldered joint is to be preferred if the connection must remain good for an indefinite length of time. A liberal supply of binding-posts, screw-cups, and couplings, will be found of value in electrical measurements.

[1] These remarks apply particularly to cells of the Daniell or Bunsen type (Figs. 234 and 235, Exp. 84). With a Leclanché cell (Fig. 236), these precautions are unnecessary.

The best temporary connection is undoubtedly made by dipping copper into mercury (see 9). The surface of the copper should first be amalgamated by dipping it into nitrate of mercury and rubbing it with a cloth.

(12) INSULATION. Care must be taken that electrical connections are not made when they are not wanted. The student should carefully examine the insulating material with which his wires are wound, particularly when the wires are to be twisted together. He should make sure that there is no current between any two of the binding-posts of a commutator or rheostat which can be detected by a galvanometer when the metallic connections are broken. The outside of battery cells should be dry for if they are not, electrical leakage is apt to take place. There is in fact more or less leakage in all experiments; but if the apparatus be perfectly dry this will probably not be enough to affect the accuracy of any of the measurements which follow.

EXPERIMENT LXXVIII.

CONSTANTS OF GALVANOMETERS.

¶ 194. **Construction of a Single-Ring Tangent Galvanometer.** — A form of galvanometer frequently employed, because of its simplicity of construction, is represented in Fig. 217. A horizontal cross section is given also in Fig. 218. The instrument consists of a compass (*a*, Fig. 217, and *dgif*, Fig. 218) mounted on a wooden support in the middle of a coil of insulated wire. The compass needle (*eh*) is made very short[1] so that the *whole* of it may be virtually at the centre of the coil. To assist in reading the deflections of the needle, two long light pointers (*f* and *g*)

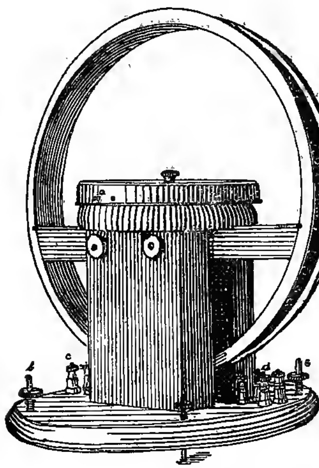


Fig. 217.

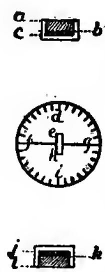


Fig. 218.

are attached to it at right angles. The wire is wound on a grooved brass ring in a single layer. The ends of the wire are carried to binding-posts (*c*, Fig. 217) at the base of the instrument as close together as possible. Levelling screws (*b* and *e*, Fig. 217) are usually added. In the construction of the instrument

[1] The length of the needle should not exceed $\frac{1}{12}$ the diameter of the coil. Kohlrausch, Physical Measurement, Art 63.

neither iron nor steel must be used (¶ 214, 3) except in the magnet itself, and in the steel pivot upon which it turns. The compass should have a lever to lift the needle from the pivot when the instrument is not in use (¶ 214, 1).[1]

¶ 195. **Law of Tangents.** — When an electrical current of sufficient strength is sent through the coils of a galvanometer, lines of magnetic force due to the current may be recognized by the aid of iron-filings scattered upon a horizontal piece of glass. We will suppose that the plane of the coil is parallel to the magnetic meridian (that is, vertical, and magnetically north and south ¶ 214, 6), and that the glass passes through the centre of the coil. Lines of force will then be formed in a direction which, if the current is sufficiently powerful, may differ imperceptibly from east and west near the centre of the coil.

FIG. 219.

When a compass-needle is placed at the centre of the coil, it takes a direction, as might be expected, parallel to the lines of force passing through that point. If we suppose the current to be ascending on

[1] Single-ring galvanometers in the Jefferson Physical Laboratory have been constructed with 10 turns of No. 16 insulated copper wire, wound on a brass ring 36 *cm.* in diameter. The supports are made of wood. The needle is $2\frac{1}{2}$ *cm.* long. The pointers are of aluminum, and each about 5 *cm.* long. The circle is divided into degrees and half-degrees. The coil is arranged in sections of 1, 2, 3, and 4 turns, with connections so that any number of turns can be employed from 1 to 10. By sending the current through these sections in different directions the sections may be tested against one another.

the north side of the coil, and descending on the south side, the north pole of the needle will point nearly to the east. The electric current *tends* in fact to deflect the compass-needle due east and west, but the earth's magnetism combined with it always gives to the needle a more or less northerly direction.

The actual direction of the compass-needle is determined (see ¶ 184) by two forces: one, H, due to the horizontal component of the earth's magnetism acting in a northerly direction; the other, F, due in this case, not (as in ¶ 184) to a magnet, but to the magnetic effect of the electrical current acting in an easterly or westerly direction. The angle (a) of deflection is given accordingly, as in ¶ 184, by the formula,

$$\frac{F}{H} = \tan a. \qquad (1)$$

The units of current now in use have been defined (§ 132) with reference to the magnetic field which a current produces in a coil of wire. If L is the length of the wire, R its mean radius, and c the current in absolute units, we have

$$F = \frac{cL}{R^2}. \qquad (2)$$

Or if C is the current in ampères (§ 19), we have—

$$F = \frac{1}{10}\frac{CL}{R^2}. \qquad (3)$$

Substituting this value in (1) we have—

$$\frac{CL}{10\,R^2H} = \tan a. \qquad (4)$$

Let us suppose that two currents C and C' produce the deflections a and a' respectively; then

$$\frac{C\,L}{10\,R^2\,H} = tan\ a; \tag{5}$$

and

$$\frac{C'\,L}{10\,R^2\,H} = tan\ a'. \tag{6}$$

Dividing (5) by (6) we find —

$$C : C' :: tan\ a : tan\ a'; \tag{7}$$

that is, in a given galvanometer two currents are proportional to the tangents of the angles of deflection which they respectively produce. This is known as the *Law of Tangents.*

¶ 196. **Calibration of a Tangent Galvanometer.** — The single-ring galvanometer described in ¶ 194

Fig. 220.

may approximate more or less closely to the conditions required of a perfect tangent galvanometer. To test the accuracy with which the "Law of Tangents" (¶ 195) is fulfilled, a battery of six small Daniell cells may be employed. The cells should be as nearly as possible of the same size and composition.

The plane of the galvanometer coil is to be made parallel to the magnet meridian (¶ 193, 6) so that the compass-needle points to 0° at both ends; then the two terminals are to be connected, with the poles of the battery arranged in series, as in Fig. 220, and in

Fig. 221, 1, so that the cells may all act together. The connecting wires should be well insulated (¶ 193, 12) and twisted together (¶ 193, 8). The deflection of the galvanometer is to be found by reading both ends of the needle (¶ 193, 5).

The connections of the poles of the first cell (A) are now to be interchanged (Fig. 221, 2) so that it acts against the other five. The deflection is to be found as before. Then the original connections of A are to be restored, but those of the second cell (B) reversed (as in 3), and the deflection again noted;

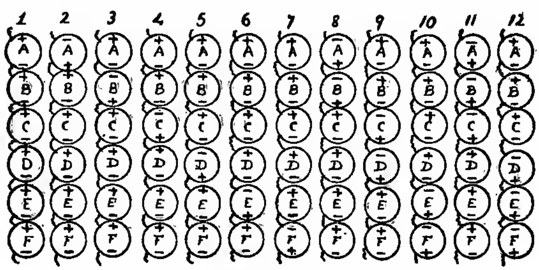


FIG. 221.

and so in turn each cell is to be opposed to the rest (as in 4, 5, 6, and 7). Then A and B are both to be reversed (as in 8), then C and D (as in 9), then E and F (as in 10). The student may be interested to test the equality of the cells by opposing A, B, and C against D, E, and F (as in 11, or as in 12). In repeating the measurements, the connections of the galvanometer should be interchanged (¶ 193, 9), and the measurements should be repeated in the inverse order, to eliminate variations in the strength of the cells. The results are to be reduced as in ¶ 197, below.

¶ 197. **Reduction of Results of Calibrating a Tangent Galvanometer.** — In (1) we have six cells in series; in (2), (3), (4), (5), (6), and (7), we have in each case one cell opposed to five others or the equivalent of four cells. The *average* deflection gives, therefore, the effect of four cells of the same *average* strength as the six cells in (1). In (8), (9), and (10), we have in each case two cells opposed to four others, or the equivalent of two cells in all; the average deflection corresponds accordingly to two cells of the average strength.

In 11 and 12 there should be little or no deflection. Since the galvanometer is sensitive to the direction as well as to the magnitude of the current, the deflections in 11 and 12 should be equal and opposite.

The results are arranged in tabular form below:

1. No. of cells acting.	2. Average deflection.	3. Tangent of deflection.	4. Ratio of 3 to 1.
6	56°.5	1.511	.252
4	45°.3	1.011	.253
2	27°.1	.512	.256

We notice that the path of the electrical current is the same in all the arrangements, except that in some cases it passes through a given cell in one direction, in other cases in the opposite direction. It is stated that the electrical resistance of a cell is the same, regardless of the direction of the current.[1]

[1] Work is required to drive a current backward through a cell, whereas if a current passes through it in the ordinary direction, the cell is a source of power (see § 137). In calculating the *electrical re-*

The total electrical resistance is accordingly the same in each of the twelve arrangements shown in Fig. 221. It is also stated that the electro-motive force of a battery is proportional to the number of cells acting, hence by Ohm's law (§ 138) the ratio of the numbers in the third column to those in the second column should be nearly constant. If it is not, the galvanometer should be discarded for accurate purposes. The experiment should be repeated with a galvanometer in which the Law of Tangents is at least approximately fulfilled.

¶ 198. **Determination of the Constant of a Single-Ring Galvanometer.** — It is evident from formula 4, ¶ 195, that the deflection of a galvanometer depends

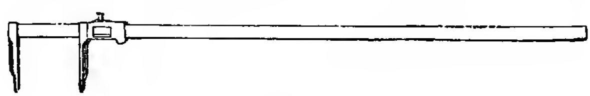


FIG. 222.

not only upon the electrical current, but also upon the length and radius of the coil of wire through which it flows. In order to measure currents with a galvanometer, it is therefore necessary to determine

sistance of a cell we do not consider the gain or loss of power due to chemical agency, but only the loss of power due to conversion into heat. The statement that the resistance of a cell is the same without regard to the direction of the current does not mean, therefore, that it is as easy to drive a current backward through it as to drive it forward, but that the cell would be *equally heated* in both cases. The truth of this statement has recently been called into question, but the method of calibration described above has been found practically to yield accurate results.

accurately the dimensions of the coil of wire. To find the diameter of a coil, we measure with a long vernier gauge (Fig. 222) the distance between the flanges of a bobbin (*al*, Fig. 223) upon which the coil is wound. Then we find the thickness of two blocks *ab* and *kl* which fill the space between the wires and the edges of the flanges. Subtracting *ab* and *kl* from *al* we have the outside diameter (*bk*) of the coil. We now measure the width of the bobbin and the width of the flanges.

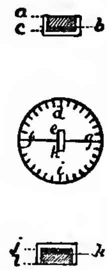


FIG. 223.

Subtracting the latter from the former, we have the width of the coil of wire. The whole number of turns of wire is now to be counted. Usually the groove is broad enough for one more turn of wire than that actually wound upon it, since this amount of space is necessary for turning the wire. The width of the groove is to be divided by the number of turns which would fill it, to find the average diameter (*bc*, or *jk*) of the wire. Subtracting this from the outside diameter (*bk*) we have the mean diameter (*bj*, or *ce*) of the coil. Dividing by 2 we have the mean radius of the coil.

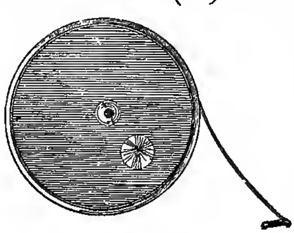


FIG. 224

Instead of measuring the diameter of the coil, we may find its circumference by passing a thin steel tape graduated in *mm.* around the outside of the coil. If *c* is the circumference, the outside diameter is

$c \div \pi$. From this the mean diameter and radius may be calculated as before. The results are to be still further reduced as in ¶ 199.

¶ 199. **Calculation of the Constant and Reduction Factor of a Tangent Galvanometer.** — The constant (K) of a coil of wire is equal to the ratio of its length to the square of its radius (§ 133). That is, in the notation of ¶ 195,

$$K = \frac{L}{R^2}. \tag{1}$$

Substituting this value in formula 4, ¶ 195, we have

$$\frac{CK}{10\,H} = \tan a, \tag{2}$$

or solving for C,

$$C = 10\,\frac{H}{K} \tan a. \tag{3}$$

The constant, K, of a given galvanometer is therefore an important factor in the calculation of a current from the deflection which it produces in that galvanometer.

If n is the number of turns in the coil,[1] we have

$$L = 2\,\pi\,n\,R, \tag{4}$$

which substituted in (1) gives

$$K = \frac{2\,\pi\,n\,R}{R^2} = \frac{2\,\pi\,n}{R}. \tag{5}$$

[1] The student must remember that when a coil is made in two parts, so that half the current flows through each, the effect is the same as if the whole current flowed through one half. The total number of turns must therefore be halved in order to find the effective number n.

By this formula the constant of the tangent galvanometer is to be calculated. Thus for 6 turns of radius 18 *cm.* we have a constant $2 \times 3\frac{1}{7} \times 5 \div 18$, or 1.75, nearly. With such a galvanometer, assuming that the horizontal intensity of the earth's magnetism is 0.175, nearly, we should have from (3) —

$$C = 10 \times \frac{.175}{1.75}\, tan\ a = tan\ a \text{ (nearly)};$$

that is, the current in ampères would be numerically equal to the tangent of the angle of deflection produced.

In most galvanometers this is not the case. To find the current, we have to multiply the tangent of the angle of deflection by some factor, which may be greater or less than unity. This is called the *reduction factor* of the galvanometer.[1]

Denoting it by I, we have from (3) —

$$I = 10\,\frac{H}{K}. \tag{6}$$

It is important to find the reduction factor of a galvanometer which is to be used often, since it greatly shortens the reduction of results.

Substituting from (6) in (3) we have simply —

$$C = I\, tan\ a. \tag{7}$$

It may be observed that if $a = 45°$, so that *tan a*

[1] Some writers call the reduction factor "the constant" of a galvanometer. Since the reduction factor depends upon the earth's magnetism (see 6), it is evidently not constant. The effect of changes in the earth's magnetism in a short course of experiments may, however, generally be disregarded.

$= 1$, we have $C = I$. The reduction factor of a galvanometer is therefore numerically equal to the current which deflects it 45°; that is, the current which produces a field of force at the centre of the coil equal to the horizontal component of the earth's magnetism.

EXPERIMENT LXXIX.

COMPARISON OF GALVANOMETERS.

¶ 200. **Construction of a Double-Ring Tangent Galvanometer.** — A "double-ring" tangent galvanometer is represented in Fig. 225, also in horizontal section in

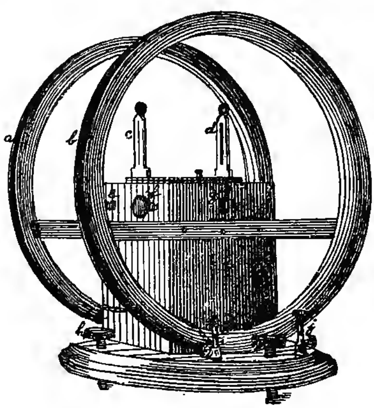


Fig. 225.

Fig. 226. It consists of two parallel coils of wire wound on brass or wooden rings *a* and *b*, with a surveying-

compass *cd* between them (see also Fig. 199, ¶ 183). In the case of a single-ring galvanometer, it has been stated that the length of the needle should not exceed $\frac{1}{12}$ the diameter of the coils. In the double-ring galvanometer, it may be $\frac{1}{4}$ of this diameter without introducing any serious error into the results (Kohlrausch, Art. 93). For measuring battery currents, each coil should contain about six turns of No. 12 insulated copper wire. It is recommended that the average diameter of the coils should be 32 *cm.* and the mean distance between them 16 *cm.*[1] The needle of the surveying-compass should be not more than 8 *cm.* long. When a current is made to divide in such an instrument into two parts, so that half flows through each coil, it is found that the tangent of the angle of deflection is approximately equal to the magnitude of the current in ampères.

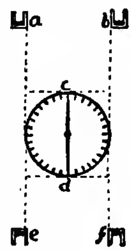


FIG. 226.

¶ 201. **Determination of the Reduction Factor of a Galvanometer by the Method of Comparison.** — The single-ring galvanometer (Fig. 217) is to be adjusted with its coil north and south (¶ 193, 6), as near as possible to the place (¶ 193, 3) where the horizontal intensity of the earth's magnetism was determined (¶ 183). The double ring galvanometer (Fig. 225) is to be similarly adjusted in some position conven-

[1] These dimensions have been calculated for places where the horizontal component of the earth's magnetism is .169 or .17 nearly. In places where this horizontal component is nearly .18 the dimensions should be 30 and 15 *cm.* respectively.

ient for future measurements. This position should be accurately noted. The two instruments (A and C, Fig. 227) are then to be connected in series with a constant battery (B) capable of yielding a current of one or two ampères. The deflection of each galvanometer is to be found by reading both ends of each needle (¶ 193, 5). The connections of C are then reversed (see ¶ 193, 9), and both deflections again noted. The connections of A are next reversed and new readings taken. Finally the connec-

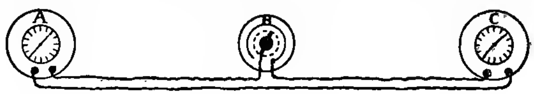


Fig. 227.

tions of C are again reversed, so as to be the same as at the start, — the needles being read as before.

The observations of the two galvanometers should be made at the same time, as nearly as possible. Let a be the average angle through which A is deflected; a' that through which C is deflected; then if the reduction factors (¶ 199) of A and C are I and I' respectively, the current C which traverses both galvanometers must be (see ¶ 199, formula 7) —

$$C = I \tan a = I' \tan a';$$

hence the reduction factor (I') of C may be found by the equation —

$$I' = I \frac{\tan a}{\tan a'}.$$

We notice that the reduction factors of two galvanometers are to each other *inversely* as the tangents of the angles of deflection produced by a given current.

The student should be cautioned *not* to connect the two galvanometers in multiple arc (§ 140); for in this case the current divides into two parts, which may or may not be equal. Not knowing the ratio between the two parts, we can draw no conclusion as to the relative sensitiveness of the two galvanometers.

When the instruments are connected as above *in series*, the same current (if there is no leakage) must traverse the coils of both.

EXPERIMENT LXXX.

THE DYNAMOMETER.

¶ 202. **Construction of a Dynamometer.** — A form of dynamometer useful for measuring battery currents is represented in Fig. 228. It consists of a wooden bobbin, *fgpn*, with two grooves, in each of which are wound 50 turns of No. 16 insulated copper wire. Small holes are bored through the bobbin at *f*, *g*, *n*, and *p*, so that it is possible to measure directly the inner and outer diameters of the coil. The average diameter is about 25 *cm.* A small hollow wooden cube (*ijkl*), measuring 5 *cm.* each way, is now wound with $80\frac{1}{2}$ turns of No. 24 copper wire, the ends of which

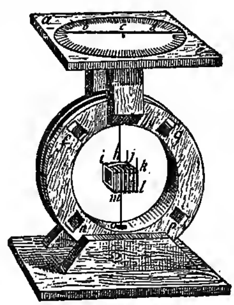


FIG. 228.

are connected by No. 31 spring brass wires (*ch* and *mo*) to a fixed point beneath, *o*, and to the centre (*c*) of a knitting needle (*bd*), as in the torsion balance (see Fig. 176, ¶ 165). The length of the wire should be taken so that the coefficient of torsion of the wire *ch* may be some round number, let us say 10 dyne-centimetres, per degree (see ¶ 165). Thus if 100 *cm.* of the wire has been found (Exp. 64) to have a co-efficient of torsion of 2 dyne-centimetres per degree, we may make *ch* just 20 *cm.* long, so that it may exert a couple of $\frac{100}{20} \times 2 = 10$ units per degree.

It will be observed that the constant of the large coil, having in all 100 turns, and a mean radius of 12·5 *cm.*, is (see ¶ 133) —

$$K = \frac{2 \times 3.1416 \times 100}{12.5} = 50, \text{ nearly,} \qquad (1)$$

while the magnetic area of the smaller coil is (see § 134) —

$$A = 80\tfrac{1}{2} \times 5 \times 5 = 2000, \text{ nearly.} \qquad (2)$$

The constant of the dynamometer is accordingly (§ 135) —

$$D = 50 \times 2{,}000 = 100{,}000 \text{ absolute units, nearly.} \quad (3)$$

In other words, a current of 1 *absolute unit* would create a couple of 100,000 units, tending to twist the wire. A current of 1 ampère (being $\frac{1}{10}$ of the absolute unit) will have $\frac{1}{10}$ the effect, not only in the cube (*ijkl*), but also in the large coil (*fgpn*). The couple produced, depending upon the product of these two effects (see §§ 133, 134), will be accordingly less than D (in formula 3), in the proportion of 100 to 1. It follows that 1 ampère will exert in this instrument a couple of about 1000 dyne-centimetres; and that it will require a twist of 100° in the wire *ch* to balance it if, as has been supposed, 1° corresponds to 10 dyne-centimetres. Since the couple produced is proportional to the square of the current (§ 135), the current must be proportional to the square root of the angle of torsion which is required to balance this couple.

The proportions of the dynamometer have been chosen above so that the square root of the number of degrees indicated by the needle *bd* may give at once (approximately at least) the current in tenths of an ampère.

¶ 203. **Determination of the Constants of a Dynamometer.** — Before making use of a dynamometer to measure electrical currents, it is necessary to find (1) the constant of the large coil (*fgpn*, Fig. 228), (2) the magnetic area (§ 134) of the small coil (*ijkl*), and (3) the coefficient of torsion of the wire.

(1) The diameter of the large coil may be determined as in ¶ 198; but as the coils of the dynamometer contain several layers of wire, it is more accurate to measure directly the outside and inside diameters. For this purpose holes are made at f, g, n, and p, in the side of the bobbin. The number of turns, if unknown, may be estimated by counting the layers and the number of turns in each. From the whole number of turns and from the mean diameter of the coil, the constant (K) is to be calculated as in ¶ 199.

(2) To find the mean diameter of the square coil, the outside diameters *jk* and *kl* are to be measured by a Vernier gauge. The diameter of the wire is to be found by measuring the width of the 80 or more turns between *i* and *j*, then dividing by the number of turns. Subtracting this diameter from the outside diameters *jk* and *kl*, we have the mean diameter of the coil. Unless a wire passes through the middle of the cube in the direction *co*, it is obvious that there must be a whole number of turns plus one half turn on the cube *ijkl*. To avoid making a mistake, the turns should be counted on both sides of the cube. The magnetic area, *A*, of the square coil is then calculated as in § 134.

(3) The instrument is now to be laid upon its side, and a light balance-arm is to be attached to the cube (see Fig. 176, ¶ 165). The wire *ch* will probably have to be supported near *h* to prevent it from sagging under the weight of the cube. The wire should, however, rest freely upon the support, so as not to affect the torsion. The coefficient of torsion of the wire *ch* is then to be found as in ¶ 165.

¶ 204. **Determination of Reduction Factors by means of a Dynamometer.** — The Dynamometer is now to be set upright with the plane of the large ring north and south, and adjusted by twisting the needle *bd* so that the planes of the large and small coils are at right-angles. A fixed mark should be placed on the wall of the room so as to be in line with two sights *jk* on the small coil, when the coil is at right-angles to the large coil. The reading of the needle is to be observed. The instrument is then to be connected (as in ¶ 201) in series with a single-ring tangent galvanometer, and with a battery of several Bunsen cells, capable of sending a current of about 1 ampère through the circuit. The needle *bd* is to be turned until the sights *j* and *k* on the small coil come in line with the same mark as before. The reading of the needle is to be again observed, and also that of the tangent galvanometer.

The current is now to be reversed in the large coil, but not in the small coil of the dynamometer; then reversed in the battery; then the original connections of the dynamometer are to be restored. In each case readings of the dynamometer and of the galvanometer are to be made.[1]

If t is the coefficient and a the angle of torsion of the wire, the couple is ta. If K is the constant of the large coil, A the magnetic area of the small coil, we have for the current c, by § 135 —

$$c = \sqrt{\frac{ta}{KA}}, \text{ in absolute units;}$$

or in ampères,
$$C = 10\sqrt{\frac{ta}{KA}},$$

since an ampère is one tenth of an absolute unit.

From the current, C, and the mean deflection, d, which it produces in the tangent galvanometer, we

[1] The couple produced by a current may also be measured by turning the instrument on its side as in ¶ 203, 3, and directly counterpoising the current with weights placed in one pan of the balance.

may find the reduction factor of the latter by the formula —

$$I = \frac{C}{tan\ d}.$$

We may also find the horizontal component (H) of the earth's magnetism by the formula —

$$H = \frac{IK}{10},$$

derived from ¶ 199, 6, using the new value of I.

If the values of I and H found by means of the dynamometer differ from those previously determined (Exps. 74 and 78) by more than 5 or 10 %,[1] the student should repeat all the measurements upon which these values depend.

EXPERIMENT LXXXI.

ELECTRO-CHEMICAL METHOD.

¶ 205. **Determination of the Reduction Factor of a Galvanometer by the Electro-Chemical Method.** — The galvanometer is to be adjusted with the plane of its coil parallel to the magnetic needle (¶ 193, 5), and its exact position noted (¶ 193, 3). The terminals

[1] The use of a small square coil in a dynamometer is simply for convenience in the explanation of the instrument to students. For accurate measurements, a round coil is to be preferred. In any case there are certain corrections to be applied to the dynamometer on account of the size and shape of its coils (unless these be carefully proportioned) which if neglected may account for errors of 3 or 4 %.

of the galvanometer (h and i) are to be connected with the poles of a Daniell cell, a and b (Fig. 229, 2), through a commutator *defg* (see ¶ 193, 9). The ordinary copper (or positive) pole is replaced by a spiral of copper wire (b, Fig. 229, 1 and 2) with a coupling c, provided for convenience in weighing. The spiral should have been cleaned with nitric acid before the experiment. The solution of sulphate of copper with which it is surrounded should be saturated and free from all impurities, especially acid, ammoniacal, and oxidizing or reducing agents. The deflection of the galvanometer should be about 45,

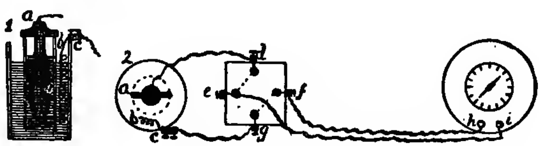


Fig. 229.

— more rather than less. If it is less than 30° the porous cup should be changed, or another cell substituted. When the spiral has been freshly coated with copper by the action of the battery, it should be disconnected from the coupling (c), dipped in three changes of fresh water, then in alcohol, and dried in a temperature not exceeding 100°, to avoid oxidation of the copper. Its weight is then to be found within a milligram, if possible, by a series of double weighings (Exp. 8).

The spiral is now to be replaced in the cell, and connected with the galvanometer as before. The time when the connection is made must be accurately noted. The deflection of the galvanometer is to be recorded at intervals of one minute. Each end of the needle should be alternately observed (¶ 193, 5). At the end of $25\frac{1}{2}$ minutes the commutator *defg* is to be suddenly turned (see ¶ 193, 9) so that the current through the galvanometer may be reversed. Observations of the galvanometer needle are to be continued, at intervals of one minute, for another 25 minutes. There will thus be 50 observations in all. At the end of 50 minutes and 50 seconds, exactly, the current is to be suddenly cut off. The copper spiral is to be cleansed in three changes of water, with care not to dislodge any of the fresh deposit, then dipped in alcohol, dried, and reweighed accurately as before. The results are to be reduced as in ¶ 230.

¶ 206. **Theory of the Electro-Chemical Method.** — It has been found that a current of 1 ampère deposits 1 gram of copper in the course of 50 minutes and about 50 seconds (the total duration of the experiment). The strength of the solution has little or no effect upon the result, always provided that *enough* copper is present in it (§§ 142, 143). The amount of copper deposited varies only with the strength and duration of the current.

If C is the strength of the current in ampères, t the time in seconds, and w the weight of copper deposited, we have accordingly —

$$w = \frac{Ct}{3050}, \text{ nearly,} \qquad (1)$$

and
$$C = \frac{3050\,w}{t}, \text{ nearly.} \tag{2}$$

If, as in the experiment, $t = 50$ minutes and 50 seconds, that is, 3050 seconds, we find simply —

$$C = w. \tag{3}$$

That is, the average value of a current in ampères is numerically equal to the weight in grams of copper deposited by it in 3050 seconds.

Now from ¶ 199, 7, we have, at any point of time,

$$C = I \tan a, \tag{4}$$

where a is the angle of deflection produced by the current in a tangent galvanometer, and I is the reduction factor of the galvanometer. Hence, averaging the different results from the 50 observations of the needle, we find, comparing (3) and (4) —

$$w = \text{average of } I \tan a. \tag{4}$$

In practice, if the angles do not differ by more than 10 %, the same result (nearly) may be obtained much more easily by averaging the angles themselves, then finding the tangent of this average. That is, if A is the average angle of deflection —

$$w = I \tan A, \text{ nearly.} \tag{6}$$

The reduction factor may now be calculated by the formula —

$$I = \frac{w}{\tan A}. \tag{7}$$

Having found the constant, K, of the galvanometer (¶ 199, 1), we may calculate the horizontal component (H) of the earth's magnetism, as in ¶ 204, by the formula (derived from ¶ 199, 6) —

$$H = \frac{IK}{10}. \qquad (8)$$

If the value of H obtained by the electro-chemical method does not agree with previous determinations (Exps. 74, and 80), the last experiment (Exp. 81) should be repeated until at least 3 results, obtained either by the same or by different methods, agree within let us say 5 %. All previous measurements leading to a different result should now be repeated.[1]

EXPERIMENT LXXXII.

METHOD OF VIBRATIONS.

¶ 207. **Construction of a Vibration Galvanometer.** — A form of galvanometer easily constructed is represented in Fig. 230. It consists of a coil *cfg* (made by winding 14 turns of No. 18 insulated copper wire upon a hoop of wood, brass, or pasteboard, 10 *cm.* in diameter) with a short magnetized needle *e*, attached to a bullet *d* and suspended at the centre of the coil by a fine waxed fibre (*cd*) of untwisted silk (see ¶ 186). The strength of the magnet and the weight of the bullet should be proportioned so that the

[1] The student will do well to examine his *calculations* before repeating the measurements upon which they depend. A common error is a miscount or misconception of the number of turns of wire utilized in the coil of a galvanometer or dynamometer, particularly when the coils are connected in multiple arc. See footnote, ¶ 199.

needle may complete 10 vibrations in about 1 minute. A short test-tube may be employed to cut off currents of air (see Fig. 204, ¶ 186).

The ends of the coil may be carried to binding-posts, *f* and *g*. Connections at *f* and *g* may also be made by simply twisting the wires together (¶ 193, 11).

When an ordinary battery current is sent through the coil, the magnetic field of force created by the current will greatly increase the rate of vibration of the needle. We have seen (¶ 186 and § 110) that a field of magnetic force is proportional to the *square*

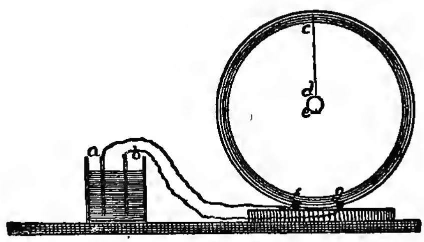


FIG. 230.

of the number of vibrations which it produces in a magnetic needle. In accordance with this law, the dimensions of the instrument have been chosen so that the square of the number of vibrations completed in 1 minute may represent approximately the strength of the current in thousandths of an ampère.

In calculating these proportions, it was assumed that the needle made exactly 10 vibrations per minute under the influence of the earth's magnetism, the strength of which was taken as 0.176 dynes per unit of magnetism (see Exps. 72, 73, 74, 80, and 81). No allowance was made for the effects of magnetism induced in the needle, which (unless the needle be of the best steel and highly magnetized) may account for errors of 5 or 10 per cent with currents of 1 or 2 ampères. To obtain accurate results with a vibration galvanometer, it would be necessary both to calibrate it (see ¶ 196) and to compare it (as in Exp. 79) with a galvanometer of known reduction factor. When, however, as in this experiment, the instrument is to be used for rough work and for relative indications only, such tests need hardly be applied.

The influence of the earth's magnetism upon the vibration galvanometer must be allowed for, as will be explained in ¶ 209.

¶ 208. **Determination of the Relative Strength of Battery Currents by means of a Vibration Galvanometer.** — A vibration galvanometer (¶ 207) is to be set up with the plane of its coil vertical, but (contrary to the usual custom, ¶ 193, 5) at right-angles with the magnetic meridian. The time required for 10 vibrations of the needle (which should be about 1 minute) is now to be accurately determined. The needle may be set in vibration by bringing a magnet near it, then suddenly taking the magnet away. The arc of vibration should not exceed 30 or 40 degrees (see Table 3, *g*).

The terminals of the galvanometer, *f* and *g*, are now to be connected respectively with the poles, *a* and *b*, of a battery constructed as will be described below. The student must notice carefully whether the needle points in the same direction as before, or whether the needle is reversed. In the latter case the connections of the galvanometer with the battery should be interchanged; that is, *f* should be connected with *b*, and *g* with *a*.

The number of vibrations made in 1 minute (or whatever time was required for 10 vibrations under the earth's magnetism) is now to be accurately determined. In no case should the arc of vibration exceed 30 or 40 degrees.

The battery to be employed in this experiment consists of a glass tumbler, half-filled with dilute sulphuric acid[1] (10 % by weight), a porous cup with an internal diameter not less than 5 *cm.*, containing a solution of sulphate of copper, and two strips, one of sheet zinc, the other of sheet copper, each 5 by 10 *cm.* Connecting wires should be soldered to both strips. The current from this battery is to be tested under the following conditions:

(1) When the zinc and copper strips are placed side by side in the sulphuric acid, but not touching each other.

(2) The same after the zinc has been amalgamated by rubbing it with mercury.

(3) (4) (5) The same after the current has been allowed to flow for five, ten, and fifteen minutes respectively.

(6) The same except that the bubbles gathered

[1] To avoid accidents in mixing sulphuric acid with water, the acid should be poured in a fine stream into the water, so that the heat generated may be quickly dissipated.

on the copper strip have been removed by a camel's-hair brush, without exposing the copper to the air.

(7) The same, except that the copper has been exposed for a few minutes to the air.

(8) The same except that the copper has been amalgamated by being rubbed with nitrate of mercury.[1]

(9) The zinc and copper strips are now to be carefully weighed; the zinc is to be replaced in the sulphuric acid, but the copper is to be immersed in the solution of sulphate of copper contained in the porous cup, and the latter is to be placed in the tumbler containing the acid.[2]

(10) (11) (12) The same after the current has been allowed to run for five, ten, and fifteen minutes respectively. The zinc and copper strips are now to be reweighed. The results are to be reduced as will be explained in the next section.

¶ 209. **Reduction of Results obtained with the Vibration Galvanometer.** — It has been stated that the square of the number of vibrations completed in one minute by a vibration galvanometer constructed as in ¶ 207, gives approximately the current to which these vibrations are due in thousandths of an ampère. To find, accordingly, the current in ampères, we square the number of vibrations produced in the given length of time, and divide by 1000.

[1] Copper may also be amalgamated by dipping it into nitric acid, then rubbing it with mercury by means of a cloth. Care must be taken not to let nitric acid come in contact with the hand.

[2] This combination constitutes a Daniell cell. See also Fig. 235, ¶ 211.

It must not, however, be forgotten that the earth's magnetism alone accounts for about 10 vibrations per minute. The earth's field is accordingly equivalent to that produced in the vibration galvanometer by $\frac{100}{1000}$ or 0.1 ampère. Care should have been taken in the experiment to have the earth's magnetism and the current acting always in the same direction. In this case all the results will be too great by 0.1 ampère. By subtracting this amount in each case, the effect of the earth's magnetism will be eliminated.

The strength of each current in ¶ 208, (1) to (12), should be calculated roughly in this way.

The student will notice that the visible action of the sulphuric acid on the zinc is arrested by amalgamating the zinc with mercury; that the action begins again when the zinc is connected with the copper strip, but that the bubbles of gas are then set free from the copper instead of from the zinc; that the amalgamation of the zinc does not impair the usefulness of the battery; that the current steadily decreases when both strips are in sulphuric acid, though it is temporarily increased by removing the bubbles from the copper, and by exposing the copper to the air; that amalgamation of the copper does not prevent the formation of bubbles upon it, nor improve in any way the action of the battery; that the formation of bubbles is arrested by placing the copper in the solution of sulphate of copper, and that in this case the battery furnishes a steady current; that the zinc plate loses in weight, but that the copper plate gains in weight by a nearly equal amount,[1] owing to fresh copper deposited upon it. We have already made use (in Exp. 81) of the quantity of copper thus deposited to measure an electrical current.

EXPERIMENT LXXXIII.

THE AMMETER, I.

¶ 210. **Testing an Ammeter.** — The name "ammeter" (an abbreviation of ampère-meter) is given to any instrument indicating directly the strength of electrical currents in ampères. Ammeters are manufactured in various forms. Most of them depend upon the attraction which an electrical current, circulating in a coil of wire (*b*, Fig. 231), exerts upon a permanent magnet or upon a core of soft iron. In some instruments this electro-magnetic attraction is balanced by a spring, in others by gravity; in others again it is balanced by the attraction of a permanent magnet (*c*).

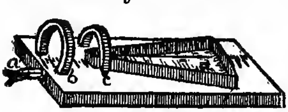


Fig. 231.

Such instruments depend for their accuracy upon the constancy of the magnet, and even if correctly graduated at the start, are subject to errors which may be indefinitely great. Recently instruments have been

[1] If we assume that there is no wasteful action of the battery, the quantities of zinc dissolved and of copper deposited should be to each other as the atomic weights of zinc and copper, 64.9 and 63.1 respectively.

manufactured in which currents are measured by the attraction between two coils of wire traversed by the same electrical current. Such instruments are properly called electro-dynamometers (see Exp. 84). If carefully graduated, they may serve as standards for the determination of electrical currents.

Ammeters are usually intended to measure currents of at least 10 ampères, and being generally sen-

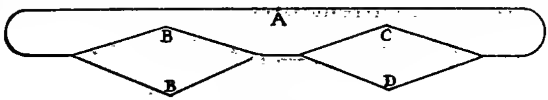


FIG. 232.

sitive only to about $\frac{1}{10}$ ampère, they cannot measure small currents very precisely. On the other hand, the tangent galvanometers described in ¶ 194 and ¶ 200 are intended to measure currents of a few ampères only. To compare an ammeter with such instruments, it must be connected with two or more of them in *multiple arc* (§ 140). A powerful battery

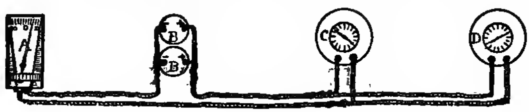


FIG. 233.

of three or four Bunsen cells is then included in the circuit. A diagram of connections is given in Fig. 232, where A represents the ammeter, BB the battery, C and D two galvanometers. To avoid the influence of the connecting wires upon the instrument (¶ 193, 8), the arrangement would practically be made as in

Fig. 233. The battery cells are represented in both diagrams (Figs. 232 and 233) as being connected in multiple arc (§ 140), since in this way they usually yield the greatest current through instruments of low resistance (§ 146).

If a, a', &c., are the deflections of the galvanometers; I, I', &c., their reduction factors, the currents through them are respectively $I \tan a$, $I' \tan a'$, &c. Hence the total current C is —

$$C = I \tan a + I' \tan a' + \&c.$$

The experiment should be repeated with batteries containing different numbers of cells, or the same number differently arranged, so as to produce currents of from 1 to 10 ampères.

The results should be tabulated in the ordinary manner, in three columns, containing respectively, (1) the current calculated from the galvanometer deflections; (2) the current indicated by the ammeter, and (3) the corresponding correction of the ammeter.

EXPERIMENT LXXXIV.

THE AMMETER, II.

¶ 211. **Determination of Battery Currents by means of an Ammeter.** — The electrical resistance (§ 136) of ammeters is usually so slight that it may be neglected. To measure the maximum current which a battery can produce, the screw-cups of the ammeter are to be connected by short thick copper wires with the pole-cups of the battery in question. The wires should be parallel or twisted together, as in the last experiment (see Fig. 233), and scraped bright at both ends (¶ 193, 11). The indication of the instrument is to be noted.

With any instrument of the class known as ammeters, the student is to determine the maximum current which can be derived from various well-known forms of voltaic battery, as, for instance, the Bunsen

FIG. 234.

FIG. 235.

FIG. 236.

cell (Fig. 234), the Daniell cell (Fig. 235) and the Leclanché cell (Fig. 236). The observations may be continued in each case at intervals of five minutes for half an hour.[1] The material employed in each cell, and the dimensions of every part,[2] should be

[1] An old Leclanché cell may be employed for this experiment. It may serve subsequently for experiments with Wheatstone's Bridge, but for other purposes it will be rendered nearly useless.

[2] If a sufficient current cannot be obtained from a single cell of a given sort, two or more cells should be employed. The student should notice that with instruments like the ammeter having a very low resistance, it is more effective to arrange batteries in multiple arc than in series. See § 136, also Figs. 232 and 233, ¶ 210.

carefully noted. The corrections for various currents indicated by the ammeter have been found in the last experiment. The proper correction should be applied to each reading. The results are to be represented by a series of curves (Fig. 237) plotted on the same sheet of coordinate paper. A scale at the top of the paper indicates the time in minutes, and a scale at the left of the paper represents the current in ampères. Each curve should be marked with the name of the cell or battery to which it belongs.

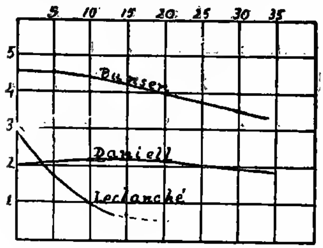


Fig. 237.

ELECTRICAL RESISTANCE.

EXPERIMENT LXXXV.

METHOD OF HEATING.

¶ 212. **Determination of Resistances by the Method of Heating.** — A short spiral (*a*, Fig. 238) of fine German silver wire, .01 *cm.* diameter (about No. 36) and 15 *cm.* long, is soldered to the two terminals *b* and *c* of two insulated copper wires, *d* and *e*, passing through a cork fitting the inner cup of a calorimeter (*B*, Fig. 239). The wires (*bd* and *ce*) should be so thick that their electrical resistance may be neglected in comparison with that of the spiral. The cork and wires are then inverted and placed in the calorimeter (*B*, Fig. 239) containing a sufficient quantity of distilled water to cover the spiral. The temperature of the water, which should be slightly below that of the room,

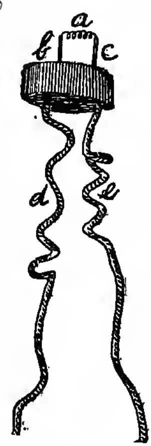


Fig. 238.

is found by a series of observations (¶ 92, 10) made with a thermometer passing through the cork as in Fig. 239. The thermometer is provided with a stirrer (see ¶ 65, Fig. 50) so that a uniform temperature may be maintained.

The instrument thus constructed (*B*, Fig. 239) is to be connected in series with a Bunsen cell (A) and with a tangent galvanometer (C) adjusted in the same place and manner as in Exp. 83.

The time when the connection is made must be accurately noted. The tangent galvanometer is to be observed at intervals of one minute. Between the observations, the water in the calorimeter is to be stirred by twisting the stem of the thermometer. When the temperature reaches that of the room, the direction of the electrical current is to be suddenly reversed by interchanging the battery connections (see ¶ 193, 9). The observations of the galvanometer are

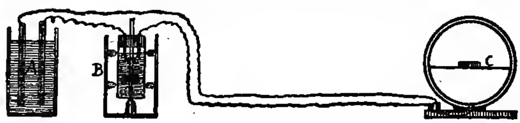


FIG. 239.

to be continued until the temperature of the water rises as high above that of the room as it was originally below it. Then the circuit is to be broken. The time when the current is interrupted must be accurately recorded. Several more observations of the temperature within the calorimeter are to be made at intervals of one minute, so that the resulting temperature may be accurately determined.

The weight of the calorimeter and of the water which it contains are finally to be found by weighing the calorimeter with and without the water.

¶ 213. **Calculation of Resistance by the Method of Heating.** — Let w be the weight of water, and W that of the calorimeter from which its thermal capacity

c is to be calculated,[1] and let t_1, and t_2 be the temperatures of the water at the moment when the circuit was first made and finally broken. These temperatures are to be inferred from the observations made before and after the experiment (see ¶ 93, 2). Since the average temperature of the water agrees with that of the room, no allowance need be made for cooling in the mean time (¶ 93, 3). The quantity of heat, H, generated by the electrical current is therefore —

$$H = (w + c) \times (t_2 - t_1).$$

Now let T be the time in seconds during which this heat was generated; then the average rate at which the heat was generated must have been $\frac{H}{T}$ units per second. Since 1 unit of heat per second corresponds to a power of 4.166 watts (§ 15), the power, P, spent by the electrical current, in watts, is —

$$P = 4.166 \frac{H}{T} = \frac{4.166\ (w + c) \quad (t_2 - t_1)}{T}. \qquad \text{I.}$$

We now calculate the average current, C, in ampères, from the angles of deflection (a) averaged as in ¶ 206, and from the reduction factor of the galvanometer, I, already determined (Exps. 78–81) by the formula —

$$C = I \tan a. \qquad \text{II.}$$

We have finally, by Joule's Law (§ 136) for the resistance, R, of the conductor in ohms —

$$R = \frac{P}{C^2}. \qquad \text{III.}$$

[1] If the calorimeter is of brass, its thermal capacity is .094 W. nearly. To this should be added about 0.5 units for the thermal capacity of the thermometer and stirrer. See ¶ 90 (2).

If the experiment were varied so as to make the current just 1 ampère, then, since $C = I$, R would be equal to P. This is in accordance with the definition of resistance (§ 136). The student should bear in mind that the resistance of a conductor in ohms is nothing more or less than the power in watts required to maintain in that conductor a current of 1 ampère.

EXPERIMENT LXXXVI.

COMPARISON OF RESISTANCES.

¶ 214. **Construction of a Rheostat.** — A rheostat may be constructed as in Fig. 240. A series of brass blocks (IJ) is firmly attached to a plate of ebonite

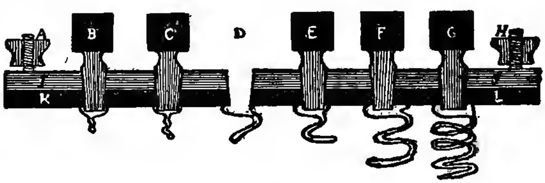


FIG. 240.

(KL), which is a non-conductor of electricity. The brass blocks are connected by coils of German-silver wire, which should be well insulated with silk. Each wire should be doubled in the middle (see Fig. 240), and the double wire should be coiled up or wound on a bobbin. The equal and opposite currents in any part of the coil thus neutralize each other as far as external magnetic effects are concerned.[1] Brass plugs B, C, &c., are fitted into hollows between the blocks, so as to make good electrical connections. When all the plugs are in place, a current flowing through the blocks in series from the binding-post A to the binding-post H, should meet with a hardly appreciable resistance. If, however, one of the plugs (as D) is removed, the current is obliged to pass through one of the coils. It meets therefore, with a certain electrical resistance.

The resistance of the first coil in the series is usually 1 ohm (§ 20); that of the second is 2 ohms; the third and fourth are either 2 and 5 or 3 and 4 ohms. It is thus possible, by taking out one or more plugs at the same time, to introduce resistances from 1 to 10 ohms into the path of a current. The series of resistances may be extended by adding three new coils of 20, 20, and 50 ohms' resistance. With seven coils, we may thus obtain any resistance from 1 to 100 ohms. With three more coils of 200, 200, and 500 ohms resistance, we may extend the limit to 1000 ohms. With additional coils of 0.1, 0.2, 0.2, and 0.5 ohms, the resistance may be adjusted to a tenth of an ohm, &c. For convenience, extra coils of 1, 10, 100, and 1000 ohms are usually provided. The same results may be obtained by the series 1, 2, 3, 4, 10, 20, 30, &c. The line of resistances is usually bent, as in Fig. 241, so as to occupy as little space as possible. Connections with the two ends of the series are made

[1] The effects of "self induction" should also be to a great extent eliminated by this method of winding the coils.

by means of the binding-posts *a* and *d*. It is convenient for many purposes to include an entirely separate line of resistances, *befc*, in the arrangement. In the first part of this experiment the inner line will not be required. It should therefore be entirely disconnected from the outer line by the removal of the plugs which join the two lines together.

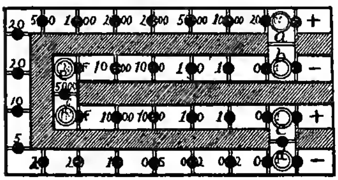


Fig. 241.

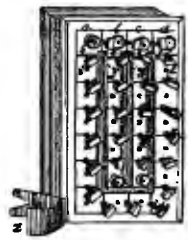


Fig. 242.

Both series of resistances are usually packed in a box (Fig. 242), variously called a "box of coils," a "resistance box," or simply a "rheostat."

¶ 215. **Determination of Resistances by the Method of Substitution.** — To find the electrical resistance of any conductor, as for instance the coil of the dynamometer employed in the last experiment, the coil (*C*, Fig. 243) is to be connected in series with a battery (*B*) and a tangent galvanometer (*G*). The deflection of the galvanometer is to be carefully observed. The dynamometer is now to be disconnected,

Fig. 243.

and in its place a rheostat, *R* (Fig. 244), is to be introduced into the circuit by means of the binding-posts *c* and *d*. The plugs connecting the inner and outer lines of resistance are to be removed, so that the current can circulate only through the outer line. The plugs along this line should all be driven lightly into place, and turned round in their sockets, so as to make good electrical connections. Enough plugs are now to be removed to reduce the deflection of the galvanometer to its former magnitude.

The resistance in ohms brought into play by the removal of each plug is indicated by the number opposite its socket (Fig. 241). If the first resistance tried is too small, that is, if it fails to reduce the current sufficiently, one about twice as great is tried; if the first resistance is too large, we try one about half as great. In fact we use with a set of resistances the same method of approximation as with a set of weights (¶ 2).

Fig. 244.

In the process of trying the several resistances, the current from the battery is liable to change. It is well, therefore, to replace the dynamometer in the circuit, and having observed the galvanometer, to substitute immediately the box of resistances (as previously adjusted) for the dynamometer. When two conductors can be thus substituted one for the other in an electrical circuit without affecting the current, their electrical resistances are evidently equal according to the general principle of substitution (see § 43). We have only, therefore, to add together the resistances of those coils in the box through which the current flows, in order to find the resistance of the dynamometer.

To save time in making connections, the terminals of the coil *C* may be carried to the binding-posts *a* and *e* of the rheostat (Fig. 245). One of the battery wires is then carried to *d*, the other to the galvanometer *G*, and back to *f*. Plugs connecting *b* with *c*,

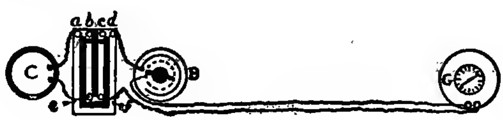


Fig. 245.

b with *e*, and *c* with *f*, are to be removed; the others are to remain. The binding-posts *e* and *f* are thus insulated from the rest of the instrument. The battery current then flows from *d* to *a* through the outer line of resistances, then from *a* to *e* through the coil *C*, then through *f* to the galvanometer *G* and back to the battery. If *b* and *c* be now connected by the insertion of a plug, the current will flow directly from *d* to *a*, and thus the rheostat resistance will be "cut out of the circuit." If the plug connecting *b* and *c* be removed and inserted between *b* and *e*, the current, after flowing through the outer line of resistances, will make a short circuit from b to e, instead of passing through the coil C. The coil will therefore be "cut out of the circuit." By moving a single plug, accordingly, from one place to another, the rheostat may be substituted in the circuit for the dynamometer, and *vice versa*. The accuracy of the units indicated by the box of resistances may be provisionally taken for granted.

¶ 216. **Determination of Resistances by the Method of Interchange.** — A battery, B, (Fig. 246), is to be connected with a coil, C, of unknown resistance, and with a rheostat, R, of variable resistance in multiple arc (§ 140). The wires from the coil and from the rheo-

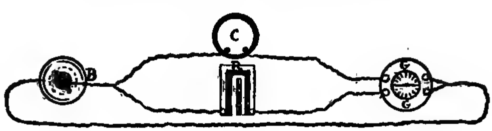


FIG. 246.

stat are to be carried back to the battery, each through one half of a differential galvanometer, $G\,G$. The resistance of the rheostat is to be adjusted if possible, by the removal of plugs, so that the deflection of the galvanometer may be reduced to zero. Since this occurs when the currents through the two halves of the galvanometer are equal, the total resistance in the two branches of the circuit containing C and R must be equal. Assuming therefore that the two halves of the galvanometer and the connecting wires have equal resistances, the resistance of the coil C must be equal to that of the rheostat R.

To make sure that the two halves of the galvanometer are exactly alike, the positions of the coil (C) and rheostat (R) should now be interchanged, and the resistance of the rheostat readjusted if necessary.

In the absence of a set of resistances by which the rheostat may be adjusted within, let us say, $\frac{1}{10}$ of an ohm, two adjustments must be made. In one, the resistance (R_1) of the rheostat will be too small, and the galvanometer will be deflected $x°$ in one direction. In the other adjustment the resistance (R_2) of the rheostat will be too great, and the galvanometer will be deflected $y°$ in the opposite direction.

The resistance (R) sought can evidently be found by the ordinary method of interpolation (§ 41, ¶ 26), that is —

$$R = R_1 + \frac{x}{x + y}(R_2 - R_1), \text{ nearly.}$$

In the absence of a differential galvanometer, the student should make by the method of substitution (¶ 215) as many determinations of resistance as time will allow. Other methods of comparison will be considered in experiments which follow.

EXPERIMENT LXXXVII.

WHEATSTONE'S BRIDGE.

¶ 217. **Determination of Electrical Resistances by a Wheatstone's Bridge.** — A form of Wheatstone's Bridge used by the British Association and ordinarily known as the "B. A. Bridge," is represented, with slight modifications, in Fig. 247, which gives a view of the apparatus from above. Three strips of copper, *ab*, *ce*, and *fg*, are arranged in a line on a piece of wood, with small spaces between them. A fine German-silver or platinum wire *hj*, often called the "Bridge wire"[1] is stretched over a rail 1 metre long, graduated in *mm*. The wire is soldered at both ends to corners of the strips (*ab* and *fg*), which are turned up so as to be on a level with the wire. A cross-wire is attached to a slider (*i*, Fig. 248) so that it may be made to touch the wire *hj* at any point. Binding-posts are usually added at *a*, *b*, *c*, *d*, *e*, *f*, *g*, and *i*. The latter serves to connect any conductor (as *Gi*) with the cross-wire, and thus to make an electrical connection between it and any point of the wire *ij*.

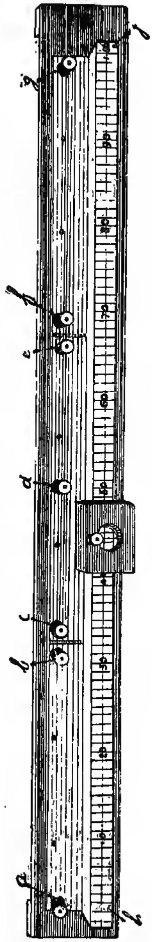


Fig. 247.

The terminals of a delicate galvanometer *G*, (see also ¶ 188, Fig. 207) are to be connected with the binding-posts *d* and *i*. The resistance coil *C*, tested in Exp. 85, is to connect *b* and *c*. Two binding-posts (*a* and *d*, Fig. 242)

[1] To avoid misconceptions arising from this name, it may be well to point out to the student at the start that the "Bridge wire" is *not* the "*Wheatstone's Bridge*" (§ 141).

of the rheostat used in Exp. 86 (R, Fig. 248) are to be connected by thick copper wires with e and f (Fig. 248). One of the plugs is to be removed from the rheostat, so as to give a resistance of 1 ohm. The poles of a battery (B) are then to be connected with the binding-posts, a and g.

The current from the battery is thus made to divide into two parts. One part flows from a to d through the coil C, then from d to g through the resistance R (or the reverse); the other part flows from a to i, through the resistance of the wire hi; then from i to g through the resistance of the wire

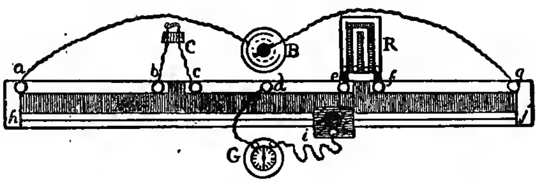


FIG. 248.

ij (or the reverse). The resistance of all other conductors may be neglected. The galvanometer circuit forms a cross-connection or "Wheatstone's Bridge" (§ 141) between the points d and i of the parallel circuits adg and aig. The points a, d, g, and i correspond accordingly to A, B, C, and D in Fig. 18, § 141. The slider i is to be moved from one end of hj to the other until a point i is found having the same potential as d (§ 141), so that the galvanometer shows no deflection. The distances hi and ij are to be carefully measured. The poles of the battery are next to be interchanged and the experiment repeated.

The average of the distances hi and ij is to be found. Assuming that the wire is uniform, the resistance of these portions A and B will be to each other as their lengths, hi and ij. That is —

$$\frac{A}{B} = \frac{hi}{ij}.$$

The resistance C is now calculated from the resistance R in the box of coils (1 ohm in this case) by the formula (§ 141) —

$$C = R \times \frac{hi}{ij}. \qquad \text{I.}$$

The experiment is to be repeated with the places of C and R interchanged. In this case the formula will become —

$$C = R \times \frac{ij}{hi}. \qquad \text{II.}$$

By removing from the box of coils different plugs, other measurements of the resistance C may be made. The student should satisfy himself that with various values of R, the same value of C is always obtained. The most accurate value is usually that which is found when R is nearly equal to C.

If the value of C thus determined differs by more than 10 % from that found in the last experiment, the latter should be repeated. By this means gross errors in the box of coils may be found out. It should be remembered that the British Association Unit which is copied in many boxes of coils is only about 987 thousandths of a true ohm.

EXPERIMENT LXXXVIII.

SPECIFIC RESISTANCE.

¶ 218. **Specific Resistance.** — The specific electrical resistance of a given material may be defined as the resistance of a conductor made of that material, 1 *cm.* long and 1 *sq. cm.* in cross-section. In the practical units of the volt-ohm-ampère series, the specific resistance, S, is equal accordingly to the electromotive force in volts (see § 138) required to maintain a current of 1 ampère between two opposite faces of a centimetre cube cut out of a given substance; or again, it is equal to the *power in watts* (see § 137) required to do the same thing. The power required to maintain a current of 1 ampère through L centimetre-cubes of the substance, arranged in series, so that the same current traverses each, is obviously LS watts. If we place Q rows of centimetre-cubes side by side, each row containing L of the cubes, it is obvious that to maintain a current of 1 ampère in each row will require LS watts; hence the total power required for all the rows will be QLS watts.

Since each row is traversed by a current of 1 ampère, the compound conductor, consisting of Q rows, must carry a current of Q ampères.

The resistance of this conductor may now be calculated by Joule's Law ($P = C^2\ R$, see § 136);

for substituting QLS for P, and Q for C, we have —

$$R = \frac{P}{C^2} = \frac{QLS}{Q^2} = \frac{LS}{Q}. \qquad \text{I.}$$

We notice that in the formula L represents the length and Q the cross-section of the compound conductor. The resistance of any conductor is accordingly proportional to its length, and inversely as its cross-section. To find it, we multiply the specific resistance by the length and divide the product by the cross-section. Obviously, specific resistances of different materials are important factors in calculations relating to electrical resistance.

To calculate specific resistance (S), we must first find the actual resistance (R) of a conductor of known length (L) and cross-section (Q): we then have, from I., —

$$S = \frac{RQ}{L}. \qquad \text{II.}$$

It will be found convenient to express the result in terms of microhms (§ 2) instead of ohms. This is done by moving the decimal point six places to the right (*i. e.*, multiplying by 1,000,000).

¶ 219. **Determination of Specific Resistance.** — A fine German-silver wire (not insulated), about 1 metre long, is soldered (near *a* and *b*, Fig. 249) to two copper strips. These strips are to be so thick that their electrical resistance may be neglected. They are to be scraped bright (¶ 193, 11), and connected with the binding-posts *b* and *c* of a Wheatstone's bridge apparatus, in place of the coil used in the last experiment (see Fig. 248, ¶ 217). To prevent the wire from crossing itself at any point, it may be looped round a glass jar *a* (Fig. 249). The resistance (R) of the wire is to be found as in the last experiment.

The wire is now to be straightened, and the distance *between* the copper strips accurately determined. This gives the length (L) of the conductor spoken

Fig. 249.

of in the last section. The diameter (d) of the wire is to be measured at let us say ten different points with a micrometer gauge (¶ 50, II.), and the results averaged. The cross-section (Q) of the wire is then calculated by the ordinary formula —

$$Q = \tfrac{1}{4}\,\pi\,d^2.$$

The specific resistance of the German silver of which the wire is composed is finally to be calculated by formula II. of the last section.

The experiment may be repeated with wires of different lengths, diameters and materials.

EXPERIMENT LXXXIX.

THOMSON'S METHOD.

¶ 220. **Determination of the Resistance of a Galvanometer by Thomson's Method.** — The terminals of a galvanometer, G (Fig. 250), and of a rheostat, R, are to be connected with a Wheatstone's Bridge apparatus in the same manner as any other resistances would be connected, when it is desired to compare them

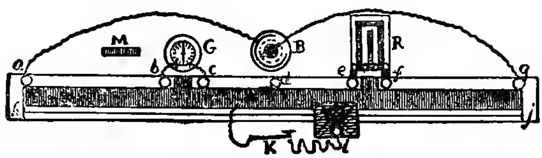


FIG. 250.

together (see Exp. 87). A battery, B, is also to be connected in the same manner. Instead, however, of putting a second galvanometer in the circuit di, to tell when the current in that circuit is reduced to zero, a simple key, K, is placed there.

The galvanometer needle will probably be strongly deflected by the current passing through the instrument. It must be brought back nearly to zero by a powerful magnet, M, properly placed. If the battery is too strong for the magnet, a weaker battery may be substituted, or the same result may be obtained by connecting the poles of the battery with a cross-wire or shunt of sufficiently low resistance. The key is now to be closed. If the effect is to increase the deflection of the needle, the slider (i) is to be moved toward that end of the "Bridge wire" (hj) nearest the galvanometer. If the effect is to diminish the deflection, the slider is to be moved toward the rheostat. Finally a point (i) is found where the closing of the key has no effect upon the galvanometer. The resistance of the latter is then calculated as in the last experiment.

The experiment is to be repeated with a rheostat resistance as nearly as possible equal to that of the galvanometer. The current should be reversed, and the resistances interchanged as in Experiment 87.

The resistance of the galvanometer is to be calculated by one of the formulæ of ¶ 217.

¶ 221. **Explanation of Thomson's Method.** — Thomson's method of measuring the resistance of a galvanometer depends upon the fact that when the circuit di (Fig. 250) is closed through K, more or less current will ordinarily pass from i to d, or the reverse.

The electrical potential (§ 139) of the point d will therefore be affected, just as the pressure at a given point in a water pipe would be affected by connecting that point with one in another pipe where the pressure was different. Since the current from a to d depends (according to Ohm's Law, § 138) upon the difference of potential between those points, it is evident that if a retains the same potential as before, any change in the potential at d must affect the current. The deflection of the galvanometer is accordingly increased or diminished. The object of nearly neutralizing the deflection is that any change in it may be made perceptible; for if the needle were already deflected for instance 89°, since 90° is the maximum possible deflection, it would be hard to detect an increase in the current. We have seen that the electrical potential at *d* is changed when it is connected with a point *c* at a different potential; obviously if *d* and *i* are at the *same potential,* there will be no change in the potential of *d*, and hence no change in the deflection of the galvanometer. The student should note that we may find a point *i*, having the same potential as a point *d*, either (1) by observing the deflection of a galvanometer in the circuit *di* (see Exp. 87), or (2) by observing the *change* in the deflection of a galvanometer in any other branch of the compound circuit.

The chief difficulty in this experiment lies in the arrangement of a permanent magnet so as to neutralize the deflection of a galvanometer needle without destroying temporarily the sensitiveness of the instrument. The advantage of this method, aside from its theoretical interest, is chiefly in cases where it is impossible to obtain a second galvanometer sufficiently sensitive to measure the resistance of the first.

EXPERIMENT XC.

MANCE'S METHOD.

¶ 222. **Determination of the Internal Resistance of a Battery by Mance's Method.** — A rheostat (R, Fig. 251) and a galvanometer (G) are to be connected with a Wheatstone's Bridge apparatus as in Experiment 87; and a battery cell (B) is to be put in place of the unknown resistance (C, Fig. 248). Instead, however, of placing a second battery in the circuit ag, a simple key (K) is put there.

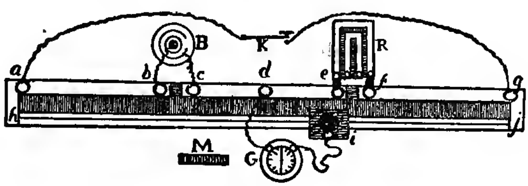


Fig. 251.

The needle of the galvanometer will probably be strongly deflected by the current passing from d to i, or the reverse. As in the last experiment, this deflection must be nearly reduced to zero, by bringing a powerful magnet (M) near the galvanometer. A shunt may be introduced if necessary between the terminals of the galvanometer (see ¶ 193, 2). The key is now to be closed. If the deflection of the galvanometer is increased, the slider (i) is to be moved toward the battery. If the deflection is diminished, it should be moved toward the rheostat. The change in the position of the slider will probably throw the galvanometer and magnet out of adjustment. The position of the magnet must therefore be changed. After a series of trials the slider may be placed at a point i, where no *sudden* effect is produced upon the galvanometer by closing the key.

If the galvanometer is affected one way when the key is first closed, then the other way, the first effect is the one by which the adjustment of the slider is to be made.

The experiment is to be repeated with a resistance in the rheostat as nearly as possible equal to that of the battery; but the methods of reversal and interchange employed in Exp. 87 will hardly be justified by the accuracy of the experiment. The resistance of the battery is to be calculated by one of the formulæ of ¶ 217.

¶ 222 a. **Explanation of Mance's Method.** — The effect in Mance's method of the battery current upon the galvanometer has generally to be diminished by shunting the galvanometer. The opposite difficulty however, sometimes arises. When it is desired to measure the resistance of a battery composed of two nearly equal cells, opposed to one another, the current from these cells may be insufficient to affect the galvanometer. In this case an auxiliary battery must be introduced into the circuit *akg*. We will first suppose that such an auxiliary battery is employed. If the two cells of which the resistance is to be measured *exactly* neutralize each other, the case differs from that of an ordinary Wheatstone's Bridge only in the nature of the resistance which is to be measured. The theory is therefore the same.

If, however, one of the two cells is stronger than the other, an allowance must be made for the current which flows from the battery (B) through the galvanometer, whether the auxiliary battery is connected or not. This is done by neutralizing the deflection of the galvanometer due to the battery B.

The fundamental principle upon which Mance's method depends is that two batteries in any system of conductors, however complicated, produce each the same effect as if the other were not present. The current in any part of the circuit is in fact the algebraic sum of the two currents which the batteries would separately produce. We have seen that a battery in the circuit *akg* affects a galvanometer in the circuit *di*, unless the resistances *ai* and *ij* are proportional to *ad* and *dg* respectively. If a current already exists in the galvanometer a *change* in that current must be produced by a battery in the circuit *akg*, unless the proportion above is fulfilled.

Let us now suppose that the battery in the circuit *akg* is just strong enough to neutralize the current from the battery B, which would naturally flow through the circuit *akg*. Then the effect of introducing this battery into the circuit may be simply to arrest the current in *akg*. The same effect is produced by breaking the circuit by means of the key K. Evidently the act of opening or closing the key in a circuit is equivalent to connecting or disconnecting a battery of considerable strength.

When the circuit is made the resistance between the poles of the battery is much less than when the circuit is broken. The result is an increased current from the battery, and in a very short time a change in its electromotive force. The observations should, therefore, be taken the moment that the circuit is closed. The galvanometer needle sometimes first jumps in one direction, then slowly changes to the other direction. The slow movement in the needle may be explained as the result of a gradual change in the electromotive force of the battery. The first effect indicates which of the resistances is too great or too small.

The chief advantage of Mance's method is that it enables us to measure the resistance of batteries at a given instant while furnishing a current. Concordant results must not be expected between Mance's and other methods. It is now thought that there is something not yet understood in the nature of battery resistances which causes these resistances to appear to be greater or less according to the manner in which they are determined.

EXPERIMENT XCI.

USE OF A SHUNT.

¶ 223. **Determination of the Resistance of a Galvanometer by means of a Shunt.** — I. Two tangent galvanometers (*ab* and *gh*, Fig. 252) already employed in Exp. 79, are to be set up in the same places as in that experiment, and connected in series with a battery (B) capable of causing deflections of from 50° to 60°. The connecting wires *bcdeg* and *afh* are to be made bare at a point between the two galvanometers and at a point (e) between the galvanometer (gh) and the battery. The wires are to be clamped at these points by the binding-posts of a rheostat (R). All the plugs are now to be put into their places. The galvanometer *gh* will then be short circuited through the rheostat (R). The deflection of the galvanometer should accordingly fall to 0°. If it does not, the plugs in the rheostat should be turned

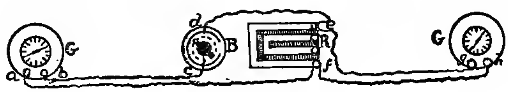


FIG. 252.

round in their sockets with light pressure until at least a minimum deflection is obtained.[1]

When plugs are removed from the box of coils, a part only of the current will flow through the rheostat. The galvanometer (gh) will then be deflected. Plugs are to be removed from the box until the deflection of the galvanometer (gh) reaches about 30° or a little more than half the deflection of *ab*. The resistance of the rheostat is to be noted, and the deflections of the two galvanometers are to be simultaneously determined as in Exp. 82. This method

[1] The plugs should be carefully cleaned if necessary by rubbing them with paper.

is applicable to galvanometers of low resistance. The results are to be reduced by ¶ 224, I., formula (5).

II. Instead of the galvanometer *ab*, a second rheostat resistance may be introduced into the circuit *edcbaf*. The value of this resistance is to be noted. The deflections of the galvanometer *gh* must be observed (as in I.) with and without the shunt *ef*. The resistance of the shunt must also be noted.

This method requires a constant battery (see Exp. 84), with an internal resistance which is either known (see Exps. 92 and 93) or so small that it may be neglected in comparison with the resistance in the circuit *edcbaf*. The method is used in practice only in the case of high-resistance galvanometers. On account of the extreme sensitiveness of such instruments, the current from an ordinary voltaic cell must be reduced by the use of a very large resistance in the circuit *edcbaf*. In comparison with this resistance, that of the voltaic cell may usually be neglected. The resistance of the shunt should be such that when connections are made through it, the deflection of the galvanometer may be about half as great as when these connections are broken. The results are to be reduced by ¶ 224, II., formula (12).

¶ 224. **Calculations of Resistance depending upon the Use of a Shunt.** — I. If I and i are the reduction factors of the two galvanometers, A and a their deflections, then since the whole current, C, passes through the first galvanometer (*ab*, Fig. 252), it must be given by the equation (see formula 7, ¶ 199) —

$$C = I \tan A. \qquad (1)$$

Only a portion (c) of this current passes through the second galvanometer (gh); this portion is —

$$c = i \tan a. \qquad (2)$$

The remainder (c') of the current flows through the rheostat. Evidently —

$$c' = C - c = I \tan A - i \tan a. \qquad (3)$$

Now the current (c) through the galvanometer (gh) must be to that (c') through the shunt inversely as the resistances (let us say G and S) in question (§ 140). That is —

$$c : c' :: S : G. \qquad (4)$$

The resistance of the galvanometer (G) may therefore be found by the formula —

$$G = \frac{c' S}{c} = S \frac{I \tan A - i \tan a}{i \tan a}. \qquad (5)$$

It should be remembered that the resistance of the galvanometer (gh, Fig. 252), calculated by this formula, includes that of the wires, eg and fh, connecting it with the rheostat. The result is rendered inaccurate by any bad connection within the rheostat. A minimum deflection of 1° in the galvanometer (gh), produced with all the plugs in place in the rheostat (R), indicates an under estimate of both the galvanometer and rheostat resistances not far from 1 or 2 %.

II. If E is the electromotive force of the battery (B, Fig. 252), R the resistance in the circuit $edcbaf$ (including strictly the internal resistance of the battery), and if G is the resistance of the galvanometer, the current, C, produced (when the connection between e and f is broken) must be (see § 138) —

$$C = \frac{E}{R+G}. \tag{1}$$

If now a connection is made between e and f through a shunt of the resistance S, so that the current flows partly through G and partly through S, the resistance (r) of this multiple circuit will be (solving the equation in § 140) —

$$r = \frac{GS}{G+S}. \tag{2}$$

The current C' now becomes —

$$C' = \frac{E}{R+r}, \tag{3}$$

or, substituting the value of r and reducing, —

$$C' = \frac{E\,(G+S)}{RG+RS+GS}. \tag{4}$$

The portion (c) of this current which flows through the galvanometer is to the whole current (C') as S is to $G+S$ (§ 140); that is —

$$c = C' \frac{S}{G+S}. \tag{5}$$

Substituting the value of C' from (4) we have —

$$c = \frac{E\,(G+S)}{RG+RS+GS} \times \frac{S}{G+S} \text{ or}$$

$$c = \frac{ES}{RG+RS+GS}; \tag{6}$$

hence $$E = \frac{cRG + cRS + cGS}{S}. \qquad (7)$$

But from (1) $$E = CR + CG;$$

hence $$\frac{cRG + cRS + cGS}{S} = CR + CG, \qquad (8)$$

$$cRG + cRS + cGS = CRS + CGS, \qquad (9)$$

$$cRG + cGS - CGS = CRS - cRS, \qquad (10)$$

and $$G\,(cR + cS - CS) = RS\,(C - c), \qquad (11)$$

whence, finally, $$G = \frac{RS\,(C - c)}{cR + cS - CS}. \qquad (12)$$

In the use of this formula it is necessary to know only the relative values of the currents C and c. With nearly all instruments, when the deflections are small, the currents are proportional to these deflections. We may accordingly substitute the deflections produced in such cases for the currents which they represent.

EXPERIMENT XCII.

OHM'S METHOD.

¶ 225. **Determination of the Resistance of a Battery by Ohm's Method.** — A tangent galvanometer (G, Fig. 253) and a rheostat (R) are to be connected in series by the wires bc, de, and af, with a Daniell cell (B) capable of deflecting the galvanometer needle 50° or 60° when all the plugs of the rheostat are in their places. The deflection of the galvanometer is to be accurately observed. The 1-ohm plug is now to be removed from the rheostat, and the deflection again noted. The resistance of the rheostat is then gradually increased until the deflection of the galvanometer is reduced to less than half of its original magnitude. In each case, the deflection is to be carefully observed, and the resistance noted.

The connections at *b* and *f* being now interchanged (¶ 193, 9) so that the direction of the current through the galvanometer is reversed, the experiment is to be repeated. If any differences are observed in the deflections corresponding to a given resistance, the mean angle of deflection is to be calculated in each case.

Fig. 253.

If a_1 and a_2 are the mean angles of deflection in any two cases, R_1 and R_2 the corresponding rheostat resistances, C_1 and C_2 the currents through the galvanometer, I the reduction factor of the galvanometer (Exps. 78, 80, 81), B the resistance of the battery, galvanometer, and connecting wires, then we have (see ¶ 199, 7) —

$$C_1 = I \tan a_1 \quad (1); \quad C_2 = I \tan a_2. \qquad (2)$$

Now by Ohm's law (§ 138) these currents are inversely as the corresponding resistances, that is —

$$C_1 : C_2 :: R_2 + B : R_1 + B, \tag{3}$$

hence we find —

$$\frac{R_2 + B}{R_1 + B} = \frac{C_1}{C_2}, \tag{4}$$

$$R_1\ C_1 + BC_1 = R_2\ C_2 + BC_2, \tag{5}$$

$$BC_1 - BC_2 = R_2\ C_2 - R_1\ C_1, \tag{6}$$

$$B\ (C_1 - C_2) = R_2\ C_2 - R_1\ C_1, \tag{7}$$

$$B = \frac{R_2\ C_2 - R_1\ C_1}{C_1 - C_2}, \tag{8}$$

and finally, substituting the value of C_1 and C_2, and cancelling I, we have —

$$B = \frac{R_2 \tan a_2 - R_1 \tan a_1}{\tan a_1 - \tan a_2}. \tag{9}$$

The student may thus calculate several values of B. The best value for R_1 is 0; that is, we obtain the most accurate results by utilizing the observation of the galvanometer when all the plugs are in place. Evidently if $R_1 = 0$, the value of B becomes simply

$$B = \frac{R_2 \tan a_2}{\tan a_1 - \tan a_2}. \tag{10}$$

The best value for R_2 is one nearly equal to B. The simplest way to find this value is to calculate the value of B from any two of the observations. It must be remembered that the battery resistance thus calculated includes that of the galvanometer and connecting wires. Having found the resistance of the galvanometer, &c. from the last experiment, we may find by subtraction the *internal* resistance of the battery. The results with a tolerably constant battery should agree with those obtained by Mance's Method (Exp. 90) within 5 or 10 %.

The calculation of the electromotive force of a battery from the results of Ohm's Method will be considered in ¶ 230. It may be remarked that if this electromotive force is not constant, formula (3) is not justified. In this case the succeeding formulæ which depend upon (3) may give false or even absurd results.

EXPERIMENT XCIII.

BEETZ' METHOD.

¶ 226. **Explanation of Beetz' Method.** — In Beetz' method two batteries, B' and B'' (Fig. 254) are placed in the same circuit ($abcda$) but so as to be op-

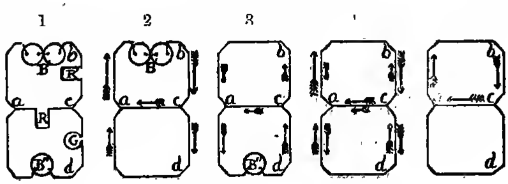


Fig. 254.

posed to each other; and the circuit is divided into two lobes, like a figure 8, by means of a wire ac, acting as a shunt to both batteries. A known resistance R' is placed between b and c; another known resistance (R) is introduced between a and c; a delicate galvanometer (G) is placed between c and d. We will suppose that the two positive poles of the batteries are connected at c.

Let us now consider what effect the battery B' would produce if B'' were not acting. The current descending in the branch bc would divide into two parts (Fig. 254, 2); one flowing directly from c to a, the other indirectly from c to a through d. These two parts would unite at a, and thence return to the battery.

Let us next consider what effect B'' would produce if B' were not acting. The current ascending in dc (Fig. 254, 3) would divide into two parts; one flowing directly from c to a, the other indirectly from c to a through b. Both parts uniting at a would return to the battery.

When both batteries act together, each may be considered to produce the same effect as if the other were not acting. The result is represented in Fig. 254, 4. We notice that in the diagrams the *portion* of the current from B' which flows through d is as great as the *whole* current from B''. To produce this effect it is evident that the battery B' must be stronger than B''. It is also evident that two equal and opposite currents through d must neutralize each other; hence the result of combining two batteries as in Fig. 254 may be such as is represented in Fig. 254, 5; namely, a current entirely confined to the circuit bc, containing the stronger battery, *no current whatever flowing through the weaker battery.*

In practice we employ a battery, B', *more than suffi-*

cient to reverse B''; then we weaken the current which it sends through the circuit d, either by increasing the resistance R', so that the whole current from B' is reduced, or by diminishing the resistance R, so that a greater portion of the current may flow directly from c to a, without passing through the battery B''. The use of the galvanometer, G, is simply to tell when an exact balance has been established between the two opposing currents through d (see Fig. 254, 4). No current is then indicated by the galvanometer.

It is possible to calculate by Ohm's Law (§ 138) and by the principle of divided circuits (§ 140) the magnitude of each of the currents represented in Fig. 254, 4, and thus to find under what conditions the currents through d are equal and opposite. The expressions become, however, more or less complicated. The final solution, which is simple, may be obtained much more easily by the method which follows.

¶ 227. **Principle of Electromotive Forces in Equilibrium.** — Let E' be the electromotive force, and B' the resistance of the first battery; let E'' be the electromotive force of the second battery (B''), and let C be the current through the rheostat R. Then if, according to the diagram (Fig. 254, 5) the current through B'' has been reduced to zero, the current C, having no choice of circuits must flow through B' and R' as well as through R. The result is the same as if the circuit through B'' did not exist. We have accordingly an electromotive force E', causing a current C through a total resistance $R + B' + R'$. Hence, by Ohm's Law (§ 138), —

$$E' = C\,(R + B' + R'). \qquad (1)$$

The power of the battery is spent in heating the several resistances R, B', and R'. We need to consider only the power (P) spent in heating the resistance R. We have (see § 136) —

$$P = C^2\,R. \qquad (2)$$

The ratio of this power (P) to the current (C) determines that part (E) of the whole electromotive force (E') which is required to maintain the current (C) through the resistance (R) in question. Since in passing through the resistance R the loss of potential is E, we have (see §§ 137, 138, and 139) —

$$E = \frac{P}{C} = \frac{C^2\,R}{C} = CR. \qquad (3)$$

The power spent by the battery B'' upon a small current C'' flowing through it in the ordinary direction (from a to c) will be $C''\,E''$ (§ 137); but the power required to take electricity from a point a to a point c, where the electrical potential is higher than at a by the amount E, is $C''\,E$. Evidently such a current through the battery can exist only on condition that E'' is greater than E.

On the other hand, a current C'' flowing from c to a would represent an expenditure of power equal to $C''\,E$. The power required to drive the current backward through the battery B'' is, however, $C''\,E''$.

Evidently a reversed current can exist only if E is greater than E''. It follows that if E and E'' are equal, the current through B'' will be reduced to zero. It is evident, conversely, that if the galvanometer in the diagram (Fig. 254, 1) shows no deflection, E and E'' must be equal; that is (from 3), —

$$E'' = CR; \tag{4}$$

from which we find —

$$C = \frac{E''}{R}, \tag{5}$$

a formula by which we may calculate the current from a battery (B') which, flowing through a known resistance, R, neutralizes a known electromotive force, E''.

¶ 228. **Calculation of Battery Resistances in Beetz' Method.** — For the determination of the resistance of a battery by Beetz' method, two experiments are necessary. Let r_1 and r_1' be the values of R and R' (¶ 226) in the first experiment, and let r_2 and r_2' be the corresponding values in the second experiment. Then from ¶ 227 we have, dividing (1) by (4), —

$$\frac{E'}{E''} = \frac{B + r_1 + r_1'}{r_1}, \tag{1}$$

and

$$\frac{E'}{E''} = \frac{B + r_2 + r_2'}{r_2}. \tag{2}$$

Assuming that the proportion between E' and E'' is the same in both experiments, we have, equating (1) and (2), —

$$\frac{B + r_1 + r_1'}{r_1} = \frac{B + r_2 + r_2'}{r_2} \qquad (3)$$

$$Br_2 + r_1 r_2 + r_1' r_2 = Br_1 + r_1 r_2 + r_1 r_2' \qquad (4)$$

$$Br_2 - Br_1 = r_1 r_2' - r_1' r_2 \qquad (5)$$

$$B = \frac{r_1 r_2' - r_1' r_2}{r_2 - r_1}. \qquad (6)$$

The same result may be obtained from formula (8), ¶ 225, namely, —

$$B = \frac{R_2 C_2 - R_1 C_1}{C_1 - C_2}, \qquad (7)$$

by substituting for the total external resistances R_1 and R_2 their values, $r_1 + r_1'$ and $r_2 + r_2'$ respectively, and also substituting for the two corresponding currents C_1 and C_2 their values (from ¶ 227, formula 5) $\frac{E''}{r_1}$ and $\frac{E''}{r_2}$ respectively. The factor E'' is cancelled in the reduction.

Beetz' method differs from Ohm's method chiefly in the manner in which we estimate the relative strength of two currents. In Ohm's method the ratio between the currents is determined by the angles of deflection produced in a tangent galvanometer. In Beetz' method, it is determined by the resistance between the poles of a constant battery, enabling the current to neutralize the effect of that battery. Beetz' method is essentially a null method (§ 42).

Beetz' method may be used not only to measure the resistance of a battery (see 6), but also, when that resistance has been found, to determine the relative magnitude[1] of two electromotive forces (see 1 and 2, also ¶ 230, 8).

When the electromotive force of a battery is known, it furnishes us with the means of measuring currents with great precision (see formula 5, ¶ 227).

¶ 229. **Determination of Battery Resistances by Beetz' Method.** — The copper or positive pole (P, Fig. 255) of a battery (B), consisting of two Daniell cells in series, is to be connected by a wire ($PKK'P'$) with the positive pole (P') of a weaker battery (B'). The circuit is to be completed between the negative poles (N' and N) of the batteries through a delicate galvanometer (G) provided with a shunt (S) to pre-

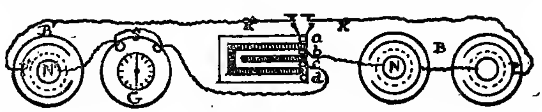


FIG. 255.

vent it from being injured by the battery currents (¶ 193, 2) and through the *inner* line of resistances, *bc*, of a box of coils. The inner and outer lines, *bc* and *da*, are to be connected with a plug between *c* and *d*, but separated at *a* and *b* throughout the experiment. The wire $PKK'P'$ is to be made bare at *a* and connected at that point with the binding-post of

[1] If a tangent galvanometer be introduced into the circuit of the stronger battery (B'), for instance between *a* and *b* (Fig. 254), so that the current C becomes known, we may calculate also the absolute values of the electromotive forces by formulæ (1) and (4) of ¶ 227. This important modification of Beetz' method is due to Poggendorff. See ¶ 230, 3, and Exp. 99.

the outer line of resistances. Keys (K and K') are to be placed one on each side of a. When all the plugs are in place, and the keys closed, the circuit of the battery (B) is completed through the lines of resistance bc and da, the course of the current being $PKadcbN$. The circuit of B' is also completed through the outer line da, thus: $P'K'adcGN'$. The student should note the direction in which the galvanometer is deflected.

When the connection between a and d is broken by removing the "infinity plug,"[1] both of the circuits named above are interrupted. If the keys K and K' are closed, the batteries will be opposed to one another. Neither battery can furnish a current unless it is strong enough to force it backward against the other battery. If the battery B is stronger than B', the current will follow the course $PKaK'P'N'GcbN$. Since the current in B' is reversed, the galvanometer will be deflected in the opposite direction. The student should make sure that this is the case. If it is not, there is probably some error in the connections, which must be corrected.

The infinity plug is now to be returned to its place, and other plugs removed between a and d.

It will be seen that when the resistance of the

[1] Two of the brass blocks in each chain of resistances should have no metallic connection between them, except that furnished by the plug. When the plug is removed there should be no perceptible current from one block to the other. In other words, the resistance between the blocks should be practically infinite. The plug in question is called accordingly the "infinity plug." It is usually marked ∞ or INF.

outer line *ad*, common to the two battery circuits, is very small, the galvanometer is deflected one way; when the resistance is very large the galvanometer is deflected the other way. The next step is to find, by gradually increasing the resistance, at what point the change in the deflection takes place.

To avoid using up the batteries (¶ 193, 10), the keys K and K' should be left open, except at the moment when it is desired to test the deflection of the galvanometer. The key K in the circuit of the stronger battery is always to be closed first, then the other key, K', immediately after it. As soon as the direction of the deflection has been recognized, the keys are opened in the inverse order.[1]

If the galvanometer is deflected in the same way as when all the plugs are in place, the resistance of the outer line (*ad*) is to be increased; if it is deflected as when the connection in *ad* is broken, the resistance is to be diminished. The sensitiveness of the galvanometer may be increased if necessary by removing the shunt (S) but the student must not forget to replace the shunt before proceeding to the second part of the experiment. The resistance of the outer line (*ad*) causing the deflection of the galvanometer to disappear is to be recorded. If no such resistance can be found, the two nearest resistances should be noted, and the deflections (one in one direction, the other in the other direction) caused by each should be observed. From these results the desired

[1] A "double key" or other mechanical contrivance for closing two circuits one after the other will be found useful in this experiment.

resistance is to be calculated as in ¶ 216, by interpolation (§ 41).

So far the resistance in the inner line *bc* has been zero. This resistance is now to be increased by removing the 10-ohm plug. If the keys be closed, the galvanometer will be deflected. To reduce the deflection to zero, it will be necessary to increase the resistance of the outer line (*ad*). The resistances of both parts of the rheostat (*bc* and *ad*), causing equilibrium in the galvanometer are to be noted.

The battery resistance is to be calculated by formula 6, ¶ 228; remembering that the values of *ad* correspond to the resistances r_1 and r_2, common to the two circuits, while the values of *bc* correspond to the resistances r_1' and r_2', in the circuit of the stronger battery.

ELECTROMOTIVE FORCE.

¶ 230. **Different Methods for the Determination of Electromotive Forces.**

I. ABSOLUTE METHODS. Electromotive force (see § 137) is defined as the ratio of the power spent by any source of electricity to the current which it produces. We must distinguish between methods (1–4) in which the power thus expended is absolutely measured and those (5–12) in which comparative results only are obtained.

(1) METHOD OF HEATING. The power spent by an electric current may be measured in the same way as electrical resistance (Exp. 85), by passing a current from a battery through a coil of wire surrounded with water, and calculating from the rise of temperature of the water how much energy has been spent by the current in a given length of time.[1] If the strength of the current be known, the loss of potential may be found by the general formula (§ 137) —

$$E = \frac{P}{C}.$$

Thus if a current of 2 ampères is found to heat the equivalent of 100 grams of water 15° in 1000 seconds, so that it generates $1\frac{1}{2}$ units of heat in one second,

[1] See Glazebrook and Shaw, Practical Physics, § 74.

since 1 unit of heat per second is equivalent to 4.166 watts (§ 15), $1\frac{1}{2}$ units per second would be equivalent to 6.249 watts, or $6.249 \div 2 = 3.124$ watts per ampère. We know, therefore, that the difference in potential (§ 139) between the two ends of the coil of wire must be 3·124 volts. It will not do, however, to assume that this is equal to the electromotive force of the battery; for we have left out of account the heat generated by the electrical current in the connecting wires and in the interior of the battery. Unless the electrical resistance of the battery be unusually small in comparison with that of the coil, a considerable portion of the electrical energy will be thus wasted.

At the same time that the method of heating can not in practice be employed to determine *directly* the electromotive force of a battery, it must be remembered that all determinations of electromotive force which involve a measurement of current and resistance may depend *indirectly* upon the method of heating, since this is one of the fundamental methods by which resistances are measured (Exp. 85).

(2) Ohm's Method. Having once determined a standard of resistance by the Method of Heating (Exp. 85), we have seen how by various methods of comparison (Exp. 86–93) the resistance of any part of an electrical circuit may be found. In Ohm's method, we find the current (C) in a simple circuit, and calculate the resistance (R) of this circuit by adding together the resistances of its separate parts.

Then, by Ohm's Law, we have for the electromotive force (E) the general equation (§ 138) —

$$E = CR.$$

Substituting in this formula the value of R, which in the absence of any resistance except that of the battery, galvanometer, and connecting wires, is given by formula 10, ¶ 225, namely —

$$B = \frac{R_2 \tan a_2}{\tan a_1 - \tan a_2},$$

and substituting also the corresponding value of C, namely, $I \tan a_1$, we have —

$$E = \frac{IR_2 \tan a_1 \tan a_2}{\tan a_1 - \tan a_2}.$$

The student may show that the same formula is obtained if we multiply the total resistance ($B + R_2$) in the second part of the experiment by the current ($C_2 = I \tan a_2$) which flows through it. The agreement of the two results must not be taken as an indication that the electromotive force is the same in both parts of the experiment, but as the necessary consequence of the formulæ of ¶ 225, in framing which we have *assumed* that the electromotive force of the battery is constant.

(3) Poggendorff's Method. It has already been shown in Beetz' method (Exp. 93) that the current from a battery may be neutralized by meeting a counter current caused by division of a current from a more powerful battery into two parts. This is

the principle of Poggendorff's absolute method (see Exp. 99), which differs from Beetz' method simply in the fact that a tangent galvanometer is introduced into the circuit of the more powerful battery (B', Fig. 254) as a means of measuring the current (see note, ¶ 228). Given the current, C, and the resistance, R, the electromotive force (E) is calculated by the ordinary formula (§ 138) —

$$E = CR.$$

(4.) ELECTROSTATIC METHODS. The electromotive force of a powerful battery may be measured by the repulsion between two pith-balls charged by the battery under certain conditions (see ¶ 258). Electrostatic forces are also measured in absolute electrometers of various kinds (see ¶ 270). It should, however, be remembered that results obtained by such instruments are strictly in the electrostatic system. Since the relation between the electrostatic and the ordinary (electromagnetic) systems are not known with any great degree of accuracy, the use of electrometers, as far as the latter system is concerned, is practically confined to the comparison of electromotive forces (see ¶ 230, 11, also ¶ 270).

II. COMPARISON OF ELECTROMOTIVE FORCES. The absolute measurement of electromotive force is, like the absolute measurement of resistance upon which it depends, a more or less difficult problem. The *comparison* of two electromotive forces may, however, be made with a considerable degree of precision.

(5) THE VOLT-METER. Two electromotive forces may be compared by the currents separately produced by them through equal resistances. When the resistance of a battery is unknown, it is evident that this method cannot in general be applied; for the battery resistance may be a considerable part of the resistance of a circuit. In practice, few batteries have a resistance of more than 10 ohms; in fact 1 ohm would be much nearer the average battery resistance. Hence if a galvanometer has a resistance of several thousand ohms, the battery resistance may usually be disregarded. This is the principle on which volt-meters are constructed (Exps. 96 and 97).

(6) WIEDEMANN'S METHOD. In Wiedemann's Method (Exp. 94), two batteries are joined in series with a tangent galvanometer of low resistance. Whether the batteries act in the same or in opposite ways, the total resistance in the circuit is the same (see note ¶ 197). It follows, therefore, from Ohm's law (§ 138), that the current is proportional in one case to the sum, in the other case to the difference of the electromotive forces E and e; hence the sum $(E + e)$ is to the difference $(E - e)$ as the currents C and c produced, that is —

$$E + e : E - e :: C : c.$$

(7) METHOD OF OPPOSITION. Let us now suppose that N cells of the electromotive force E being opposed to N' cells of the electromotive force E' reduce the current to zero, then obviously the electromotive force $NE = N'E'$; or, $E' : E :: N : N'$.

This is a fundamental method of comparing electromotive forces, the usefulness of which is limited only by the difficulty of obtaining enough cells of each kind to make an exact balance. We note that, in this method, we compare the electromotive forces of two batteries when at rest, and not (as in previous methods) when in action. The method of opposition is essentially a "null method" (§ 42) for the comparison of electromotive forces.

(8) BEETZ' METHOD. When, as in Experiment 93, a battery current is neutralized by *part* of the current from a more powerful battery, we cannot find the electromotive force of either battery absolutely, unless, as in (3), the whole current from the stronger battery is measured, as well as the resistance which it traverses between the poles of the weaker battery. We may, however, find the relative electromotive forces from formulæ 1 and 2, ¶ 228. Hence if the electromotive force of one battery is known, that of the other may be determined. It may be remarked that by this method we compare the electromotive force of one battery *when at rest* with that of another *when in action.*[1]

(9) CLARK'S POTENTIOMETER. Again, if a current (C) flowing through a resistance R neutralizes one battery (as in Exp. 93), while the *same current* flowing through a resistance r neutralizes another

[1] By substituting one battery, B, for another, B' (Fig. 254), as the active source in Beetz' Method (Exp. 93) we may compare the two successively with a third electromotive force, B''. This gives us a null method by which we may compare the electromotive forces of two batteries (B and B') *when in action.*

battery (in the same manner), the electromotive forces of these batteries, being CR and Cr respectively, are to each other as R is to r. The proportion between them may therefore be found, independently of any measurement of electrical current. This is the principle of Clark's Potentiometer (Exp. 98), and is undoubtedly the best method of comparing the electromotive forces of two constant batteries *when not in action.*

(10) USE OF CONDENSERS. The relative strength of two batteries may be found by charging a condenser (see ¶ 257) first by one battery, then by the other. The quantity of electricity stored in the condenser is found to be proportional to the electromotive forces in question. It is estimated by discharging the condenser through a ballistic galvanometer, and observing, as in Experiments 76 and 77, the throw of the needle.

(11.) USE OF ELECTROMETERS. The electromotive force of a battery may be determined by connecting the poles with an electrometer (¶ 270); but in order to interpret the indications of the instrument, it must first be calibrated by a series of electromotive forces of known strength. The chief advantage of the use of an electrometer over that of a volt-meter is in the case of inconstant electromotive forces, especially those which disappear as soon as a current begins. The use of a condenser has the same advantage, and is frequently preferable on account of the liability of electrometers to be out of order. Neither instrument is suitable for an elementary class of students.

(12) Use of an Electric Spark. Electromotive forces may be estimated roughly by the distance which an electric spark can be made to jump (see Table 36). This method is particularly suited for experiments with a Ruhmkorff coil, or other instrument in which large differences of potential exist for an instant only.

EXPERIMENT XCIV.

WIEDEMANN'S METHOD.

¶ 231. **Determination of Electromotive Forces by Wiedemann's Method.** — (1) Two Daniell cells, *A* and *B*, one of which (*A*) has been used in Ohm's method (Exp. 92), are to be connected in series with a tangent galvanometer (*C*, Fig. 256, 1). The connections are to be such that the cells act together. The deflection of the galvanometer is to be observed. (2) Then the connections of *B* are to be reversed (Fig. 256, 2), and the deflection again noted. (3) The galvanometer connections are then to be interchanged, and the deflection observed (Fig. 256, 3). (4) Finally the connections of *B* are to be interchanged, so that the two cells may act together as at first (Fig. 256, 4), and the deflection of the galvanometer determined.

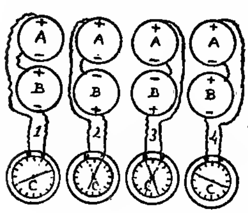


Fig. 256.

Let E be the electromotive force of the stronger cell, and e that of the weaker cell; let A be the average deflection caused by the joint action of the two cells, and C the corresponding current; let a be the average deflection, and c the current produced by the two cells when in opposition; then by formula 7, ¶ 199—

$$C = I \tan A, \tag{1}$$

$$c = I \tan a. \tag{2}$$

Now by Ohm's law (§ 138), as has been explained in ¶ 230, 6, we have—

$$\frac{E + e}{E - e} = \frac{C}{c}, \tag{3}$$

or
$$Ec + ec = EC - eC, \tag{4}$$

whence
$$eC + ec = EC - Ec, \tag{5}$$

or
$$e\,(C + c) = E\,(C - c)\,; \tag{6}$$

from which we find—

$$e = E\,\frac{C - c}{C + c}. \tag{7}$$

Substituting the values of C and c from (1) and (2) and cancelling the factor I, we have—

$$e = E\,\frac{\tan A - \tan a}{\tan A + \tan a}, \tag{8}$$

or
$$E = e\,\frac{\tan A + \tan a}{\tan A - \tan a}. \tag{9}$$

It should be noted that if the reversal of the cell B does not affect the direction of the current,—that is, if the deflections in Fig. 256, 2 and 3, are in the same direction as in 1 and 4 respectively, — the electromotive force of the cell *B*, being less than that of *A*, is to be calculated by formula 8; but if the reversal of *B* causes a reversal of the current, the electromotive force of *B* is greater than that of *A*, and is hence to be calculated by formula 9. The electromotive force of *A*, already computed, may be found from the results of Ohm's method by the formulæ of ¶ 230, 2. The electromotive force of the two cells combined is now to be calculated by adding *E* and *e* together.

II. The experiment is to be repeated with the battery composed of the two cells just employed and a Bunsen cell. The cells are first to be set up in series with the Bunsen cell and the galvanometer, then both of the Daniell cells are to be reversed.

The deflections are to be observed and the electromotive force of the Bunsen cell is to be calculated.

EXPERIMENT XCV.

THE THERMO-ELECTRIC JUNCTION.

¶ 232. **Determination of the Electromotive Force of a Thermo-electric Junction** — An iron wire (*ab*, Fig. 257) and a German-silver wire (*ac*), insulated by surrounding them with India-rubber tubes, are soldered together at *a*; and the junction (*a*) is enclosed in a steam heater. The other ends, *b* and *c*, are soldered to insulated copper wires, *bd* and *ce*. The junctions *b* and *c* are placed in a beaker and covered with melting ice. A thermo-element is thus formed with an electromotive force of about 3 thousandths of a volt. The object of this experiment is to measure the electromotive force in question.

I. The terminals of the thermo-element (*d* and *e*) are to be connected with two pole-cups of a differential galvanometer (*dg*) so that the current from the thermo-element circulates in one half of the coil of the galvanometer.

The other half of the galvanometer is to be connected through a rheostat (*hi*) with the poles (*j* and

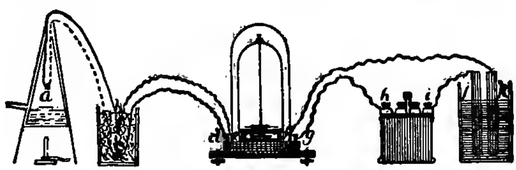


FIG. 257.

k) of a voltaic cell of known electromotive force (¶ 230, 2). There should be at first, let us say, 1000 ohms' resistance in the rheostat. The connections are to be made so that the current from the Daniell cell may produce upon the needle an effect opposite to that due to the thermo-element. The resistance of the rheostat is now to be increased or diminished until the two currents exactly neutralize each other. The rheostat resistance (R_1) is then noted.

An additional resistance (r) of known amount, about equal to that of the galvanometer (see Exp. 89), is now to be introduced between *b* and *d*, or between c and e, and the resistance of the rheostat (hi) again adjusted so as to produce equilibrium. The new value of the resistance (R_2) is also to be noted.

II. If a differential galvanometer cannot be obtained, the thermo-electric junction is first to be connected with the galvanometer, and the deflection (D) noted; then the resistance (r) is to be introduced, and the deflection (d) again noted. The Daniell cell is then to be connected with the galvanometer through a resistance (R_1), such that the deflection of the needle is the same as D. Then the rheostat resistance is increased to a value R_2 which produces a deflection equal to d. The results of I. and II. are to be reduced by formula (10), ¶ 233.

¶ 233. **Calculation of the Electromotive Force of a Thermo-electric Junction.** — If in the thermo-electric circuit (*abdeca*, Fig. 257), e is the electromotive force, and b the electrical resistance of the thermo-element, g the resistance of the galvanometer, or that part of it which is included in the circuit in question, c_1 the current in the first part of the experiment, c_2 the current in the second part of the experiment, and r the resistance added; if, furthermore, in the voltaic circuit (*fghijkf*, Fig. 257), E is the electromotive force, B the battery resistance, G the galvanometer resistance, R_1 and R_2 the two rheostat resistances, and C_1 and C_2 the corresponding currents, we have (§ 138), since the currents c_1 and C_1' are equal, —

$$c_1 = \frac{e}{b+g} = C_1 = \frac{E}{B+G+R_1}; \qquad (1)$$

and since the currents c_2 and C_2 are equal —

$$c_2 = \frac{e}{b+g+r} = C_2 = \frac{E}{B+G+R_2}. \quad (2)$$

From (1) and from (2) we find —

$$e = E\frac{b+g}{B+G+R_1}, \quad (3)$$

and

$$e = E\frac{b+g+r}{B+G+R_2}. \quad (4)$$

By either of these formulæ (3 or 4) we may calculate the value of e from the observed values of r, R_1, and R_2, if b, g, B, G, and E, are known (Exps. 87–92). The student should bear in mind that the resistance of each part of the galvanometer in this experiment is about twice that of the two parts in multiple arc (§ 140), and half that of the two parts in series. A result independent of the battery and galvanometer resistances may be obtained by combining the observations obtained in the first and second parts of the experiment. Dividing (2) by (1) we have —

$$\frac{b+g}{b+g+r} = \frac{B+G+R_1}{B+G+R_2}, \quad (5)$$

whence $(b+g)\,B+(b+g)\,G+(b+g)\,R_2$

$$= (b+g)\,B+(b+g)\,G+(b+g)\,R_1 + r\,(B+G+R_1), \quad (6)$$

that is, —

$$(b+g)\,R_2 - (b+g)\,R_1 = r\,(B+G+R_1), \quad (7)$$

or $$(b+g)\,(R_2-R_1) = r\,(B+G+R_1); \quad (8)$$

from which we find —

$$b + g = \frac{r\,(B + G + R_1)}{R_2 - R_1}. \qquad (9)$$

Substituting this value in (3) and cancelling $(B + G + R_1)$, we have finally —

$$e = E\,\frac{r}{R_2 - R_1}. \qquad (10)$$

EXPERIMENT XCVI.

THE VOLT-METER, I.

¶ 234. **Calibration of a Volt-Meter.** — The name volt-meter is given to any instrument capable of indicating directly the value of an electromotive force in volts. One of the forms ordinarily employed (Fig. 258) is similar in external appearance to the ammeter shown in Fig. 231, ¶ 210. There is, however, an essential distinction between these instruments.

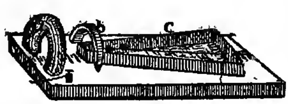


Fig 258.

In the ammeter, the coil a is made so as to have the smallest possible electrical resistance, in order that this resistance may be neglected. In the volt-meter, the finest possible wire is employed in this coil, so that the current which flows through it may be neglected. The simplest way to calibrate a volt-meter is to connect it with a battery containing different numbers of voltaic cells in series (see Fig. 220, ¶ 196). Having found the electromotive force of each cell (see

¶ 230), we may calculate that of the whole battery by adding these electromotive forces together. The difference between this calculated value and the observed reading of the volt-meter gives the correction of the volt-meter for the reading in question. A delicate galvanometer (*G*, Fig. 259) connected in series with a rheostat (*R*) is a convenient substitute for a volt-meter in the measurements relating to the electromotive force of batteries. The resistance in the galvanometer circuit should be so great that we may entirely neglect the current which flows through the instrument in comparison with the other currents used in this experiment. To test such a combination, it is to be connected with a battery of known electromotive force, as for instance, the Daniell cell employed in Experiment 92. If a common astatic galvanometer is employed (Fig. 207, ¶ 188), the resistance of the rheostat should be such as to give a deflection of about 45°. This resistance should be noted, and should remain unchanged through all the experiments with the instrument of which it now constitutes an essential part.

An ordinary astatic galvanometer does not obey the law of tangents (¶ 195) closely enough even for rough determinations. It is necessary, accordingly, to test the reading of the instrument with a series of electromotive forces bearing known ratios to one another.

A simple device by which this object may be attained consists of a uniform straight wire, traversed by a current from a constant battery. The "bridge-wire" of the Wheatstone's apparatus (*hj*, Fig. 259) may be employed. A battery (*B*) of two Bunsen cells in series will probably be required to give the necessary current. The poles should be connected with the ends of the wire by means of screw cups (*b* and *f*) provided for that purpose.

Contact is now to be made between this wire and the terminals of the volt-meter (*GR*) at points 10 *cm.* apart. This may be done by the aid of two sliders, similar to the one used in Experiment 87. Pressure must be exerted upon the sliders to insure a good electrical contact (¶ 193, 11). The deflection

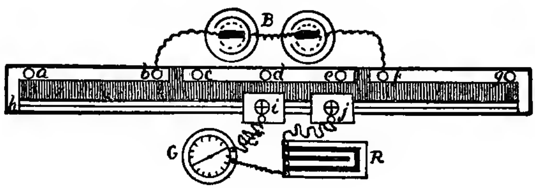


Fig. 259.

of the galvanometer is to be noted. The experiment is to be repeated with contact at two other points the same distance apart, but in a different part of the wire.[1]

The sliders are now to be interchanged and the deflections determined as before.

The direction of each deflection, whether between north and east or between north and west should be noted.

[1] A record of the reading of each slider corresponding to a given deflection should be preserved, since it may be useful in comparing the resistances of different parts of the wire.

The experiment is now to be repeated with contacts at two points 20 *cm.* apart, then 30 *cm.*, 40 *cm.*, &c., up to 80 or 100 *cm.* (the length of the wire). The observations should be repeated in the inverse order to eliminate variations in the strength of the battery.

The average deflections, corresponding respectively to 10, 20, . . . 80, or 100 *cm.*, are now to be calculated, and the results are to be plotted on co ordinate paper as is Fig. 260. The distance between the sliders is here represented by a scale at the top of the figure, and the deflections by a scale at the left. The deflection produced by the Daniell cell is also to be plotted, and the number of centimetres corresponding to this deflection found (see § 59). If the electromotive force of the Daniell cell is E volts (¶ 230), and if D is the distance between the sliders which produces an equal current, the distance d corresponding to 1 volt is —

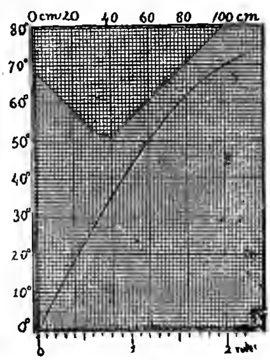


Fig. 260.

$$d = \frac{D}{E}.$$

This distance is to be indicated on the diagram and is to be divided into tenths or smaller parts. The division may be extended across the base of the figure. The theory and uses to be made of the diagram will be explained in the next experiment.

EXPERIMENT XCVII.

THE VOLT-METER, II.

¶ 235. **Determination of Electromotive Forces by means of a Volt-meter.** — A volt-meter, calibrated as in ¶ 234, is to be connected with various cells or batteries, one at a time. The deflection caused by each is to be noted. The electromotive force of each is then to be found (see § 59) by means of the curve already plotted (Fig. 260, ¶ 234). A point a is first located in the scale of degrees corresponding to the deflection in question. Then a point b is found on the curve at the right of a, and below b a point c is found in the scale of *electromotive force* into which the base of the figure has been divided.

The student is to determine rapidly in this way the electromotive forces of all the cells which he has employed.

The principle upon which this method depends is that the difference of potential between two points on a wire of *uniform resistance* is proportional to the distance between those points represented by the scale at the top of Fig. 260. For if R is the resistance of 1 *cm.* of the wire, the resistance of d centimetres will be Rd. Hence from the general formula of § 139 —

$$e = cr = cRd, \tag{1}$$

$$\frac{e'}{e''} = \frac{crd'}{cRd''} = \frac{d'}{d''}. \tag{2}$$

If the scale at the bottom of Fig. 260 is constructed so as to give one electromotive force correctly, all electromotive forces should be correctly represented.

EXPERIMENT XCVIII.

CLARK'S POTENTIOMETER.

¶ 236. **Comparison of Electromotive Forces by means of Clark's Potentiometer.** — The positive or carbon pole of a battery (*B*, Fig. 261), consisting of two

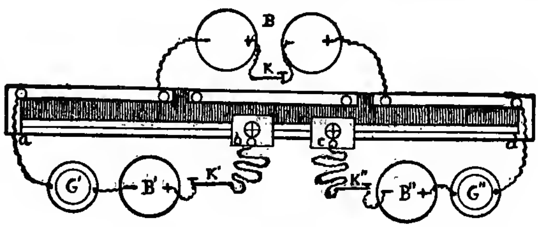


Fig. 261.

Bunsen cells in series, is to be connected with one end, *d*, of a Wheatstone's Bridge wire. The negative or zinc pole is to be connected with the other end (*a*) of the wire. A key, *K*, is to be included in the circuit. The negative (or zinc) pole of a Daniell cell (*B'*) is to be connected with *a*. The positive (or copper) pole is to be joined through a key, *K'*, and a delicate galvanometer, *G'*, to a slider (*b*), by which an electrical connection may be made at any point of the wire. The positive or carbon pole of a Leclanché cell is to be connected similarly with *d*, while the negative (or zinc) pole is to be connected through a key, K'', and a galvanometer, G'', with a second slider at c.

The key K' is first pressed for an instant, and the direction of the deflection noted. Then K and K' are both pressed, the connection being completed first in K then in K'.

If the deflection is in the same direction as before, the distance ab is to be increased; if it is in the opposite direction the distance is to be diminished. The experiment is now repeated until a point b is found such that in pressing both K and K', no deflection is observed. In this case the point b has the same potential as the positive pole of the battery B'.

In the same way a second slider is to be placed at a point c, where the potential is the same as that of the negative pole of the Leclanché cell.

The key K being now closed, the keys K' and K'' are to be pressed simultaneously. If the adjustments have been accurately made, neither galvanometer will be deflected. If this is not the case, the adjustments must be repeated.

By the principle explained in ¶ 235, if the wire ad is of uniform resistance, so that the resistances of ab and cd are proportional to their lengths, the difference of potential between a and b must be to that between c and d as ab is to cd. We have, therefore, —

$$\frac{E''}{E'} = \frac{cd}{ab}, \text{ or } E'' = E'\,\frac{cd}{ab},$$

where E' and E'' represent the electromotive forces, respectively, of the batteries B' and B''. By this formula, knowing the electromotive force of the Daniell cell (¶ 230), we may calculate that of the Leclanché cell. In repeating the experiment, the places of the Daniell and Leclanché elements should be interchanged. If the two sliders should interfere with each other, either 1 or 3 Bunsen cells should be used (in *B*) instead of 2. The experiment may also be repeated with other batteries. Clark's Potentiometer is especially adapted to the determination of the electromotive forces of *inconstant* elements.

EXPERIMENT XCIX.

POGGENDORFF'S METHOD.

¶ 237. **Determination of Electromotive Forces by Poggendorff's Absolute Method.** — The zinc pole *d* (Fig. 262) of a Bunsen battery is to be connected with one

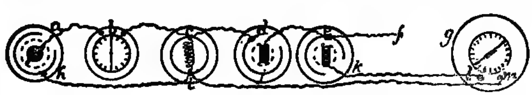


Fig. 262.

terminal (*c*) of the resistance-coil used in the Method of Heating (Exp. 85.) The zinc pole (*a*) of a Daniell cell is to be connected with the same terminal through a delicate galvanometer, *b*. The copper pole (*h*) of the Daniell cell is to be connected with the terminal (*i*) of the rheostat, and the carbon pole (*k*) of the Bunsen cell is to be connected through a tangent galvanometer (*glm*) with the same terminal (*i*).

A portion (*de*) of a German-silver wire (*def*) having in all a resistance about equal to that of the resistance-coil (*ci*), let us say 1 ohm, is to be included in the circuit of the Bunsen battery.

The wire *def* is to be disconnected for a moment, and the direction of the galvanometer deflection noted. Then the extreme end (*f*) of the wire (*def*) is to be bound in the clamp *e*. If the deflection is in the same direction as before, a longer wire must be employed, and if the two Bunsen cells are still unable to reverse the Daniell cell,[1] other cells must be added to the first, either in series or in multiple arc (§ 140).

We will suppose that a battery (*de*) and a wire (*def*) have been found such that when the wire is clamped at *f*, the current in the Daniell cell is reversed; but when clamped at *d*, the current flows in its natural direction.

The wire (*def*) is next to be clamped at a point (*e*), found by trial, so that the current in the Daniell circuit may be reduced to zero. The galvanometer (*b*) will then show no deflection.

In practice, we clamp the wire at a point (*e*) so that the Daniell cell is barely reversed, and wait for a condition of equilibrium to come about through the gradual weakening of the Bunsen cell. At the moment when the astatic galvanometer (*b*) points to 0° the reading of the tangent galvanometer (*g*) is to be taken.

[1] The student may be reminded that unless similar poles meet at *c* and at *i*, it will be impossible in any case to produce a reversal of the current.

The experiment is to be repeated with the connections of the galvanometers reversed one at a time, as in Experiment 79.

If a is the mean angle of deflection of the tangent galvanometer and I its reduction factor, the current C is (see ¶ 199, 7) —

$$C = I \tan a \text{ ampères.} \qquad (1)$$

If R is the resistance of the coil (ci) in ohms (Exp. 85) we have a difference of potential (e) between its terminals c and d (see § 139) equal to —

$$e = CR = RI \tan a \text{ volts.} \qquad (2)$$

This is equal to the electromotive force of the *Daniell cell* (see ¶ 130, 3).

For a simplified diagram of Poggendorff's Method, see Fig. 254, 1, ¶ 226. The only change to be made in this diagram is the introduction of a tangent galvanometer in the upper circuit (abc).

EXPERIMENT C.

ELECTRICAL EFFICIENCY.

¶ 238. **Determination of the Efficiency of an Electric Motor.** — A small electric motor, such for instance as is represented in Fig. 263, is to be connected through an ammeter (Fig. 231, ¶ 210) or through a tangent galvanometer (A, Fig. 264), with a voltaic battery (BB) containing at least twice as many cells as are required to keep the motor (M) in motion. Thus if the motor can be started with 2, but not with 1 Bunsen cell, a battery of 4 Bunsen cells should be em-

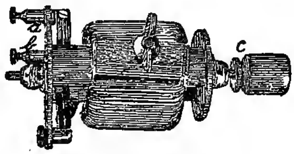


Fig. 263.

ployed. The poles of the battery are to be connected through a volt-meter or its equivalent (see Exp. 96) consisting of an astatic galvanometer (G) and a rheostat (R). The work done by the motor (M) is to be determined as in Experiment 70, by observing the readings of a pair of spring balances (SS') con-

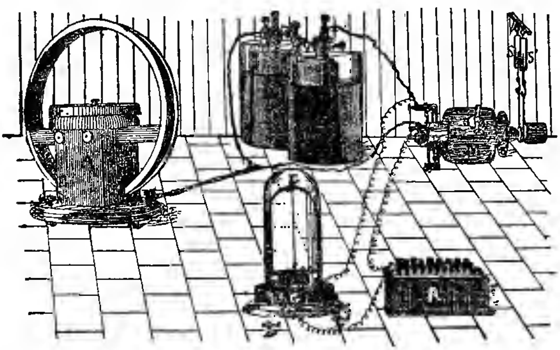


Fig. 264.

nected by a cord passing round the pulley of the motor. Ordinary letter-balances will probably answer for this experiment. The tension of the cord should be such as to reduce the speed of the motor to about one half its maximum; but different experiments should be made with different tensions. The number of revolutions made by the wheel of the motor in a given length of time may be determined by an instrument called a "revolution counter" especially devised for this purpose. This consists of a shaft *ab* (Fig. 265) which can be easily connected with the axle of the motor, and a toothed wheel (*c*) with teeth fitting into a thread cut on the shaft at *b*, The revolutions of the shaft are indicated on a dial (*d*) by a pointer (*e*) attached to a wheel (*c*). The circumference of the pulley is to be measured.

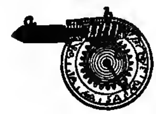


FIG. 265.

Instead of a revolution counter, we may make a band of thread 60 *cm.* long, passing from the pulley of the motor over a second pulley-wheel. Every time that the knot in this band passes a given point shows that the pulley-wheel has advanced 60 *cm.* The velocity of the circumference of the pulley-wheel can be found by this method by counting the number of times that the knot passes a given point in 1 minute. If the band is just 60 *cm.* long, this number represents the velocity in *cm. per sec.* without any reduction.

The power in ergs per sec. utilized by the motor is to be calculated from these data as in ¶ 174, 1, and reduced to watts (§ 15) by dividing by 10,000,000; that is, by pointing off 7 places of decimals. The power in watts spent upon the motor is found by multiplying together the current in ampères indicated by the ammeter (or its equivalent) and the electromotive force in volts indicated by the volt-meter, or its equivalent (see § 137).

The efficiency of the motor is to be found by dividing the power utilized by the power spent (see ¶ 174, 3).

II. Instead of an electric motor, we may employ a small dynamo-machine, driven by a water-motor. The work spent by the water is to be calculated as in Experiment 69. The work utilized is to be found as above by multiplying together the current in ampères and the electromotive force in volts. The former is to be measured by an ammeter in the main circuit of the dynamo-machine; the latter by a volt-meter connected with the poles of the dynamo-machine. The experiment should be repeated with greater or less resistance interposed in the main circuit.

The student can hardly fail to notice the similarity of the method by which we calculate the work of an electrical current to that used in the case of a current of water (§ 118). The same general method is employed in all measurements of electrical efficiency.

EXPERIMENTS FOR ADVANCED STUDENTS.

The principal methods by which physical quantities are measured have been considered in the course of the 100 experiments which have been described. Various modifications of these methods have already been alluded to. On account, however, of either the practical or the theoretical difficulties involved, and the expense of the necessary apparatus, measurements of certain physical quantities have been hitherto entirely omitted. This course would, however, be incomplete without an outline, at least, of the methods by which some of these quantities may be determined. Most of the experiments about to be mentioned are suitable only for advanced students. For this reason it has been been thought unnecessary to describe them in detail, or to include in the text proofs of the formulæ involved, except when these proofs are necessary to an understanding of the methods employed. The Proofs of other formulæ will be considered separately in Parts III. and IV.

¶ 239. **The Piezometer.** — To measure the compressibility of a liquid, we place it in a glass bulb (*C*, Fig. 266) with a narrow neck or stem (*D*) containing a small mercury index. The bulb is to be placed in a stout glass cylinder filled with water. A considerable hydrostatic pressure is then generated by means of the thumb-screw, A, and measured by a small air manometer, E (see ¶ 77). The contraction of the liquid in the stem is observed. Since the bulb is at the same pressure inside and out, there is no tendency to stretch or to crush it. An allowance must, however, be made for the compression of the sides of the bulb. It can be shown geometrically that the capacity of a bulb decreases, when thus subjected to a uniform pressure, in the same proportion as the volume of a solid would decrease under the same circumstances. The ratio of the pressure in dynes per square centimetre to the decrease in volume of 1 cubic centimetre is called the "Coefficient of Resilience of Volume."[1] It is usually calculated from "Young's Modulus" (Y), determined as in Experiment 65, or as in ¶ 248, I., and from the "Simple Rigidity" (S) of a solid. The simple rigidity may be found from the coefficient of torsion, T, (*i. e.*, the couple necessary to twist a wire 1°, see Exp. 64), and from the length, l, and radius, r, of the wire, by the formula —

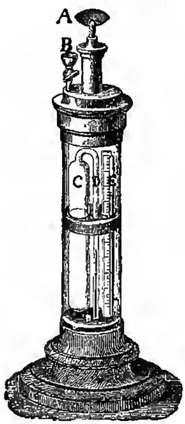


FIG. 266.

$$S = \frac{360\ Tl}{\pi^2\ r^4}.$$

It may also be found as in ¶ 248, II. Denoting by M the "coefficient of resilience of the solid," or

[1] Everett, Units and Physical Constants, Arts. 63–65.

"modulus of volume elasticity," as it is sometimes called, we find —

$$M = \frac{SY}{9S - 3Y}.$$

A mean value of M for glass may be taken as 400, 000,000,000 dynes per square centimetre. The quantities S, M, and Y are (in the case of glass and many other substances) related to each other in about the proportion of the numbers 6, 10, and 15 respectively.

If C is the capacity in *cu. cm.* of the bulb (Exp. 11), and P the pressure to which it is subjected, measured in *dynes per sq. cm.*, the contraction of the interior volume of the bulb (V) in *cu. cm.* is —

$$V = \frac{CP}{M}.$$

If V' is the apparent contraction in *cu. cm.* of the liquid, its real contraction is $V + V'$, and the Coefficient of Resilience of volume (M') of the liquid is —

$$M' = \frac{PC}{V + V'}.$$

By making the bulb in two parts, a solid may be introduced into it and surrounded with liquid. The Coefficient of Resilience of the solid may be deduced from its effect on the apparent contraction of the liquid in question.

¶ 240. **Use of a Weight Thermometer.** — If a bulb similar to that employed in ¶ 239, be filled with mercury at an observed temperature t_1, then warmed to the temperature t_2, a certain quantity of mercury will be driven out of it. Let the weight of this mercury be w, and let the whole original weight of the mercury be W_1, both weights being reduced to vacuo (§ 67), then the weight, W_2, remaining in the bulb is $W_1 - w$. If v_1 and v_2 are the specific volumes of mercury at the temperatures t_1 and t_2 (see Table 23, *A* and *B*), then the capacities of the bulb (c_1 and c_2) at these temperatures must be —

$$c_1 = W_1 v_1 \text{ and } c_2 = W_2 v_2.$$

It may be shown by geometry that when a vessel is expanded uniformly by heat, its capacity is increased in the same proportion as the volume of a solid would increase under the same circumstances. The cubical expansion, e, of glass is accordingly (see ¶ 63) —

$$e = \frac{c_2 - c_1}{c_1 (t_2 - t_1)};$$

hence the linear coefficient, ϵ, is (see § 83) —

$$\epsilon = \frac{1}{3} \frac{c_2 - c_1}{c_1 (t_2 - t_1)}.$$

This is considered to be one of the most accurate methods of obtaining the coefficient of expansion of various kinds of glass.

By collecting and weighing the mercury which is driven out of a bulb or *weight thermometer*, we may estimate the relative rise of temperature in different cases. The instrument is useful in determining precisely the maximum rise of temperature within an enclosure which has to be kept closed at the time when the temperature is taken.

The weight thermometer has also been employed to measure the cubical expansion of solids *enclosed in the bulb.* If c_1 is the capacity of the bulb at the temperature t_1, and if W_1 is the weight of mercury required to fill the space between the solid and the bulb, the volume of the solid V_1 is evidently $c_1 - W_1\ v_1$. If when heated to the temperature t_2, at which the capacity of the bulb is c_2, w grams of mercury are driven out, so that W_2 (or $W_1 - w$) grams remain, then the volume V_2 of the solid is $c_2 - W_2\ v_2$; hence we may find the cubical coefficient of expansion (e) by substituting these values of V_1 and V_2 in the ordinary formula (see ¶ 63) —

$$e = \frac{V_2 - V_1}{V_1\ (t_2 - t_1)}.$$

¶ 241. **Conduction of Heat.** — (I.) The conductivity of various insulating materials may be found approximately by filling the space between the inner and outer cups of a calorimeter (¶ 85) with these materials, and finding the rate at which heat is lost. If A is the mean area of the surfaces between which conduction takes place, L the distance between them, t the difference of temperature, and T the time in which Q units of heat pass from one surface to the other, the specific conductivity (c) of the material is —

$$c = \frac{QL}{tTA}.$$

II. A metallic rod (AD, Fig. 267) is surrounded, one end by steam, the other by melting ice. The central portion is covered with insulating material. Two thermometers, B and C, are inserted in holes in the rod, partly filled with mercury. If L is the length of the rod between B and C, A the area of its cross-section, t the difference of temperature between the points (B and C), and w the weight of ice melted in the time T, after a steady flow of heat has been established, *less the quantity melted in the same time when the rod is replaced by insulating material*, then since the latent heat of liquefaction of water is 79, the specific conductivity (c) of the rod is given by the formula—

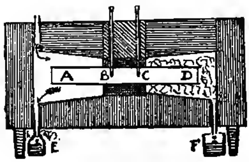


FIG. 267.

$$c = \frac{79\, wL}{tTA}.$$

The specific conductivity of a given material represents the quantity of heat which would flow in one second from one side of a unit cube made of that material to the opposite side of the cube when the difference of temperature between the two sides is 1°.

The results of this experiment will be slightly modified by the manner in which heat flows through the insulating material which surrounds it. To avoid errors from this source, the distance between the thermometers should be as small as, or smaller than the diameter of the rod. This method should be applied only to metals or to substances which are good conductors of heat.

¶ 242. **Latitude.** — The latitude of a place is usually determined by an observation of the "altitude" of the sun at "apparent noon;" that is, the time when it attains its greatest "altitude," or angular distance from the horizon. The true altitude (a) of the sun is defined as the angle which a line drawn from the centre of the earth to the centre of the sun makes with a plane passing through the centre of the earth and parallel to the horizon of the place in question. The declination (d) of the sun is defined as the angle which the same line makes with the earth's equator. The sun's declination (see Tables 44) may be found in nautical almanacs calculated in advance for every day of each year. The difference between local and Greenwich time, and the hourly change in declination must generally be allowed for. The latitude (l) of a place is by definition equal to the complement of the angle between the horizontal and equatorial planes. We have, accordingly, —

$$l = 90° - a \pm d. \qquad \text{I.}$$

If the sun is (as in summer) above the equator, the sign of d is to be taken as positive; if the sun is below the equator, d is to be called negative.

I. In nautical observations, the apparent altitude of the sun is determined by means of a sextant (see Exp. 44). The lower "limb" (or edge) of the sun is made to coincide with the sea-horizon. The observed altitude (A) must be corrected as follows: —

(1) For Semi-diameter. The apparent semi-diameter (s) of the sun (not far from 16′), given exactly in the nautical almanac for every day in the

year (see also Tables 44), is to be *added* to the observed altitude of the lower limb of the sun, since the altitude of the sun's centre is wanted.

(2) DIP OF THE SEA-HORIZON. A line drawn from the eye of the observer to the sea-horizon makes a certain angle with a true horizontal plane. This is called the "dip of the sea horizon." It may be calculated by the formula —

$$h = \sqrt{m} \times 1\tfrac{3}{4}' \text{ (nearly)},$$

where m is the height in metres of the eye above the sea-level. The dip (h) must be subtracted from the observed altitude.

(3) FOR REFRACTION. Atmospheric refraction tends to make heavenly bodies appear higher than they really are. The correction (r) is accordingly to be subtracted from the observed altitude. It is given by the equation —

$$r = cotan\ A \times 1' \text{ (nearly)}.$$

(4) FOR PARALLAX. The apparent altitude of a body as seen from the earth's surface is obviously less than if it could be observed at the earth's centre. In the case of the stars, on account of their enormous distance, the difference is imperceptible. The correction for parallax (p) is given in general by the equation —

$$p = P\, cos\ A \text{ (nearly)},$$

where P is the "horizontal parallax" of the body in question; that is, its correction for parallax when seen on the horizon. In observations of the sun with an ordinary sextant, since P is less than 9″, all corrections for parallax may usually be neglected. It is only in the case of the moon, where P is in the neighborhood of 1°, that the correction for parallax becomes important.

The true altitude (a) of a heavenly body is found in general from the observed altitude (A) by applying the corrections for semi-diameter (s), dip of the horizon (h), refraction (r), and parallax (p) as follows:

$$a = A + s - h - r + p. \qquad \text{II.}$$

II. Observations of latitude taken on land are usually made with an "artificial horizon." This may consist of a plate-glass mirror (made horizontal by two spirit-levels and levelling-screws) or simply a dish of mercury (B, Fig. 268). The lower limb of the sun is made to coincide with its own reflection in the horizontal surface. The observed angle (D) between the direct and reflected rays reaching the sextant (A, Fig. 268) is measured, and *halved*, to find the apparent altitude of the sun.

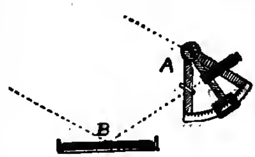


FIG. 268.

The result is corrected as above for semi-diameter, refraction, and (if sufficiently accurate) for parallax.[1] We have —

$$a = \tfrac{1}{2} D + s - r + p. \qquad \text{III.}$$

[1] The correction for "dip" is obviously to be omitted in the case of an "artificial horizon," since the plane of the reflecting surface should be perfectly horizontal.

The latitude is finally calculated by formula I. above.

¶ 243. **Longitude.** — The longitude of a place may be determined by a sextant observation of the altitude of the sun (see ¶ 242) an hour or two after sunrise or before sunset. For the reduction of the results, in general, the student is referred to works on navigation. A simple (though not very accurate) method of finding the longitude of a place is to measure the altitude of the sun at an observed time t' (about an hour before noon), then to determine exactly the time t'' (about an hour after noon) when the sun descends to the *same altitude*. Obviously the time of "apparent noon," t, (neglecting the change in the sun's declination), is half-way between t' and t'', that is —

$$t = \tfrac{1}{2}(t' + t''), \text{ nearly.} \qquad \text{I.}$$

If e is the "equation of time" (see Tables 44), given in all nautical almanacs, the time (T) of "mean noon" is (by definition) given by the formula —

$$T = t \pm e. \qquad \text{II.}$$

The sign of the quantity e is positive if the sun is fast, but negative if the sun is slow.

It is assumed that the chronometer employed in this experiment has been set so as to indicate correctly the time of a given meridian, as for instance that of Greenwich, from which it is desired to measure longitude. If it does not indicate this time correctly, an allowance must be made for the error of the chronometer. At sea, several chronometers are frequently carried. In certain cases a chronometer may have to be set by a lunar observation. For the reduction of such results (which is exceedingly complicated), the student is referred to works on navigation. On land, the standard time of a given meridian is usually obtainable by means of the electric telegraph.

It may be remarked that the longitude of a place is given by formula II. in hours, minutes and seconds.

¶ 244. **Indices of Refraction.** — I. If A is the angle of a prism (Exp. 45), and D the angle of minimum deviation (Exp. 46) of a ray of light of a given wave-length, the index of refraction (μ) of the material of which the prism is composed is (for light of that wave-length) —

$$\mu = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2} A}.$$

Certain "doubly refracting" substances have two indices of refraction instead of one. To determine them we employ a prism cut so as to produce the maximum separation of the two rays into which a single ray of monochromatic light can be decomposed by the given prism angle. The minimum deviation of *each* ray is then measured, and the two indices of refraction are calculated separately by the ordinary formula.

II. If R is a mean radius of curvature of the two surfaces of a double convex lens (Exp. 21), and F its principal focal length (Exps. 41–43), the index of refraction of the material of which the lens is made may be found by the formula —

$$\mu = 1 + \tfrac{1}{2}\,\frac{R}{F}.$$

If the same lens (B, Fig. 269) be enclosed between two flat glass plates (A and C), and the space be filled with a liquid, with the index of refraction μ', then if F' is the principal focal length of the combination, we have —

$$\mu' = \mu - \tfrac{1}{2}\,\frac{R}{F'}.$$

If R_1 and R_2 are the two radii of curvature of the two sides of the lens, the mean radius of curvature should strictly be calculated by the formula —

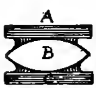


Fig. 269.

$$R = \frac{2\,R_1\,R_2}{R_1 + R_2}.$$

¶ 245. **Polarization.** — The vibrations which constitute ordinary light are, according to modern theories (§§ 92, 93), at right-angles with the direction in which the light is propagated. In a vertical beam of light, for instance, the vibrations are supposed to be confined to a horizontal plane. The vibrations appear in general to be distributed uniformly in every possible direction perpendicular to the path of the ray. Certain substances and certain optical combinations have, however, the property of stopping all the vibrations — or rather all their *components* (§ 105) — except those in a certain direction, as for instance north and south. The light transmitted is then said to be polarized.

In many optical instruments, light passes successively through two such combinations. The first is called the "polarizer" (*e*, Fig. 270), the second is called the "analyzer" (*a*). If the polarizer and analyzer are placed so that the direction of the vibrations transmitted is the same in both cases, the light which has passed through one will also pass freely

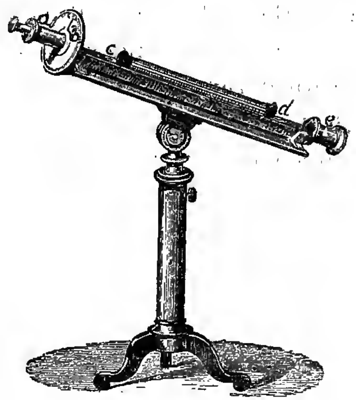


Fig. 270.

through the other; but since the polarizer transmits only vibrations in a given direction, if the analyzer is placed so as to stop all vibrations in this direction, a beam of light which has passed through the polarizer will be completely cut off by the analyzer. The position of the analyzer when this occurs is indicated by a pointer attached to it. The reading of the pointer with respect to the graduated circle *b* determines the zero-reading of the instrument.

Certain substances have the property of changing the direction of the vibrations in a beam of polarized light which they transmit. Thus in passing *upward* through a solution of cane sugar, north and south vibrations are gradually changed into a northeast and southwest direction.[1]

When a substance producing "rotation of the plane of polarization" is placed between the polarizer and the analyzer in its zero-position, the analyzer will no longer cut off all the light transmitted by the polarizer. To produce perfect darkness, the analyzer must obviously be turned through an angle equal to that through which the plane of polarization has revolved. The instrument shown in Fig. 270 affords, accordingly, a means of measuring the rotation of the plane of polarization.

To test the strength of a solution of sugar with this instrument, we pour the solution into a tube *cd* with glass ends, and interpose the tube in the path of the beam *ea* of polarized light. The analyzer is then turned to the right from its zero-position, until the light which it transmits is reduced to a minimum.

[1] When light is polarized by reflection, it is said to be polarized in a plane perpendicular to the reflecting surface, and containing both the incident and the reflected rays. According to Fresnel's theory the vibrations in a beam of polarized light take place at right-angles with the "plane of polarization." The action of a solution of sugar upon a beam of polarized light *approaching the eye* is to rotate the plane of polarization (and hence also the direction of the vibration) *with the hands of a watch.* The student should note that this is called a right-handed rotation in optics; but that it is opposite to the motion of an ordinary right-handed screw, which when turned to the right moves *away from* the eye.

Let a be the angle in degrees through which it is turned when ***sodium light*** is employed, and let d be the depth of the sugar solution, equal to the distance between the glass ends of the tube cd; then experiments show that the strength of the solution (s) in grams per *cu. cm.* is given by the equation (Kohlrausch, § 46), —

$$s = .15 \frac{a}{d} \text{ (nearly).}$$

The rotation varies considerably with lights of different colors (see Table 31 *E*). For this reason, when ordinary white light is employed perfect darkness can never be attained.

There are various optical effects (besides the darkness produced by an analyzer) which depend upon the plane in which light is polarized. Many of these have been applied to the determination of angles of rotation of the plane of polarization. The method described above has been chosen because of its simplicity.

¶ 246. **Color.** A piece of colored paper (c, Fig. 271) may be mounted in front of a white screen (d) and illuminated by a candle (a) through a piece of ruby glass (b), all other light being cut off. The distances ac and ad must be adjusted so that c and d appear equally bright when viewed from a point near b. The "relative luminosity" of the surface c is then equal to $(ac)^2 \div (ad)^2$ as far as reflected red rays are concerned.

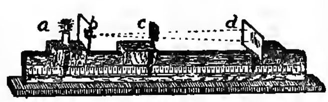


Fig. 271.

A transparent gelatine plate stained with an emerald green mixture of common green and yellow inks is now substituted for the ruby glass (b), and the relative luminosity is again determined. Finally, a gelatine plate stained with a violet mixture (Hofmann's violet containing a trace of soluble Prussian blue) is employed.

The three relative luminosities of the surface e, obtained as above by means of red, green, and violet rays, completely determine the color of the surface in question (see ¶ 115).

¶ 247. **Velocity of Light.** — The velocity of light was determined by Fitzeau in 1849.[1] A beam of light made intermittent by passing between the teeth of a revolving wheel, was sent to a distant mirror, then reflected back to the eye through the same wheel. When the wheel (which had 720 teeth) made 12.6 revolutions per second, the flashes of light, in traversing a total distance of 17,326 metres, were retarded so as to strike a tooth instead of the space between two teeth; hence the light was cut off. When the speed of the wheel was doubled, so that 18,144 teeth passed a given point in one second, the light reappeared; when trebled it disappeared, &c. It was inferred from this experiment that a beam of light required $\frac{1}{18144}$ of a second to traverse 17,326 metres; whence the velocity of light would be about $18{,}144 \times 17{,}326$ metres per second, or nearly thirty thousand million *cm. per sec.*

[1] See Deschanel's Natural Philosophy, § 686; Ganot's Physics, § 507.

Foucault has measured the time required by light to traverse short distances (a few metres only) by the use of a revolving mirror.[1] A beam of light (*AB*, Fig. 272) striking the mirror (*B*) was reflected to a fixed concave mirror (*CC′*) *with its centre of curvature in the axis of the revolving mirror* (*B*), then back on its course to the revolving mirror (*B*), and thence to the eye. The beam strikes the eye only for a very short time during each revolution of the mirror, but on account of the rapidity of rotation a continuous effect is produced. When the speed of rotation reaches several hundred revolutions per second, the mirror turns through a perceptible angle while the light is passing from *B* to *C* or to *C′* and back again.

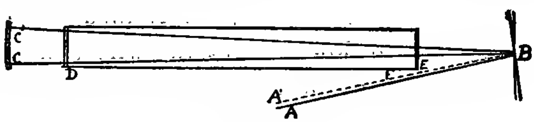


FIG. 272.

Hence the return path *BA′* differs slightly from the original path *AB*.

With a distance *BC* equal to about 4 metres, and with from 600 to 800 revolutions per second, divergences of about 40″ or 50″ were observed. The velocity of light was found to be 29·8 (or nearly 30) thousand million *cm. per sec.*

By passing the beam of light through a tube of water (*DE*, Fig. 272) it was found that the velocity of light in water is about $\frac{3}{4}$ that in air.

[1] Deschanel's Natural Philosophy, § 687; Ganot's Physics, § 506.

¶ 248. **Velocity of Sound in Wires.** — I. If a wire stretched between two vises be stroked horizontally near one end by a piece of resined cloth, a musical note may result from the *longitudinal vibrations* into which the wire is thrown. The pitch of the note is to be determined by a "pitch pipe" (Fig. 273) or any instrument serving a similar purpose. The number of vibrations corresponding to the note may be found by reference to Table 43. If l is the length of the wire between the vises, and n the number of vibrations per second, the velocity of sound (v) is —

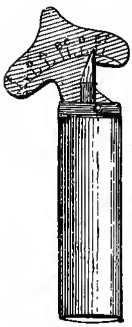


FIG. 273.

$$v = 2nl.$$

II. If a strip of resined cloth be drawn slowly *round* the wire (like a belt round a pulley) a musical note may result from *torsional vibrations* set up in the wire. The velocity of these torsional vibations may be found by the same formula as above. The note due to longitudinal vibrations is usually about a "sixth" (¶ 134) above that due to torsional vibrations. Hence the two velocities of sound are to each other as 5 to 3, nearly.

If d is the density of the wire, Y Young's Modulus of Elasticity (¶ 166) and S the simple rigidity of the wire (¶ 239) v_1 and v_2 the velocities of longitudinal and torsional vibrations, we find —

$$Y = v_1^2 d. \qquad \text{I.}$$

$$S = v_2^2 d. \qquad \text{II.}$$

¶ 249. **Reversible Pendulum.** — A reversible pendulum (Fig. 274) may be made of cast iron,[1] so that although the two knife-edges A and B are at very unequal distances from the centre of gravity (C) the time of oscillation on both knife-edges is nearly the same. The position of C must be found approximately (Exp. 62), and the distances AC and BC measured. The distance AB must be accurately determined (by measuring DE, DA, and BE with a vernier gauge, and subtracting DA and BE from DE). If t' is the time of oscillation on the knife-edge A, and t'' that on B (see Exp. 58), the time t of oscillation of a simple pendulum of the length AB is —

Fig. 274.

$$t = t' + \frac{BC}{AC - BC}(t' - t'').$$

Denoting by l the distance AB, the acceleration of gravity (g) may now be calculated by the ordinary formula —

$$g = \frac{\pi^2 l}{t^2}.$$

[1] For a half-seconds pendulum, the following dimensions are suggested: extreme length of the shaft (DE), 45 *cm.*, breadth $3\frac{1}{2}$ *cm.*, thickness 1 *cm.*; ends sharpened to an angle of about 70°; triangular knife-edges (steel better than cast iron) 2 *cm.* long, sides 1 *cm.* broad; distance of each knife-edge from nearest extremity, 10 *cm.*; holes 1 × 2 *cm.*; disc 14 *cm.* in diameter, 2 *cm.* thick; centre of disc 24 *cm.* from one knife-edge, 1 *cm.* from the other. This pendulum should weigh about 3 kilograms. The centre of gravity should be about 5 *cm.* from one knife-edge, and 20 *cm.* from the other. In observations of its time of oscillation, the knife-edges may rest upon the upper surface of a short steel rod, 7 *mm.* square, driven horizontally into the wall.

¶ 250. **Coefficient of Viscosity.** — A liquid contained in a Mariotte's bottle (*a*, Fig. 275) is fed through a rubber tube (*be*) into a capillary tube (*cd*), and collected in a small vessel (*e*). The weight (w) which passes through the tube in a given length of time (t) is found, and the height (h) of the inlet (b) above the orifice (d) is determined. The length (l) of the tube (*cd*) is measured, and its radius (r) is found (see ¶ 170). Then if d is the density of the liquid (Exp. 14), and g the acceleration of gravity (Exp. 58), the coefficient of viscosity of the liquid is given by the formula, —

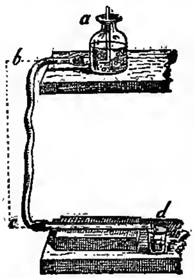


FIG. 275.

$$\eta = \frac{\pi g d^2 h r^4 t}{8\, wl}.$$

This coefficient of viscosity is the force in dynes necessary to maintain a difference of velocity equal to 1 *cm. per sec.* between two opposite faces of a centimetre cube.

The ordinary coefficient of liquid friction (see ¶ 172) depends upon the *square* of the velocity, and has no relation to the coefficient of viscosity.

¶ 251. **Electro-chemical Equivalents.** — If, in Experiment 81, I is the reduction factor of the galvanometer, determined as in Experiment 83, w the weight of copper deposited by the current C in the time t, and a the average angle of deflection, we have for the electro-chemical equivalent (q) of copper —

$$q = \frac{w}{Ct} = \frac{w}{t\, I \tan a}.$$

By the same formula we may find the electro-chemical equivalent of any other substance acted upon by the current C_0 whether that action be to deposit the substance in question, or to cause it to go into solution. In the case of a gas set free at one of the electrodes of a voltameter (C or D, Fig. 276), we find the weight indirectly from the volumes collected in graduated tubes (A and B). originally filled with the liquid (E) which is decomposed by the current. A battery of two or three Bunsen cells should be used with a gas voltameter.

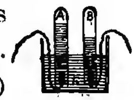


FIG. 276.

If w', w'', w''', &c., are the weights of different substances acted upon by a given current traversing a series of voltameters for a given time, the electro-chemical equivalents q', q'', q''', &c., may be found (if any one is known) from the proportion —

$$w' : q' :: w'' : q'' :: w''' : q''', \&c.$$

¶ 252. **Correction of Rheostats.** — An arrangement of a set of resistances, convenient for the purposes of correction, is represented in Fig. 277. The outer horse-shoe (ad) contains 18 coils capable of being combined so as to give any resistance under 4,000 ohms, within one tenth of an ohm. The inner horse-shoe ($befc$) contains resistances arranged in pairs of 1, 10, 100, and 1000 ohms each. Opposite a and c are two extra blocks. These are permanently

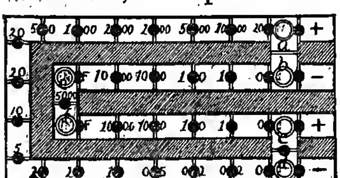


FIG. 277.

connected together, underneath, by a thick copper rod. One of them is joined to the positive pole of a battery. Two blocks opposite *b* and *d* are similarly joined together, and one of them is connected with the negative pole of the battery.

One terminal of the galvanometer is now carried to *e* (or to *f*). The other terminal is to be connected with one of the blocks in the outer line of resistances *between two coils*, or sets of coils, which are to be compared. A pair of resistances about as great as the coils in question is now introduced into the inner horse-shoe. When the battery is connected with *a* and *d*, the rheostat assumes the form of a Wheatstone's Bridge (§ 141). The inner horse-shoe furnishes two of the arms *be* and *cf*. The connections of these arms may be interchanged by breaking the battery connections at *a* and *d*, and making them at *b* and *c*. The arrangement of blocks furnishes in fact a commutator within the box of coils. By the use of this commutator, errors due to inequality in a given pair of resistances may be eliminated (§ 44).

The 1-ohm coil is first to be tested against the smaller coils, together equal to 1 ohm; then joined in series with the smaller coils, and tested against each of the 2-ohm coils; then the 5-ohm coil, the 10-ohm coil, &c., are to be tested each against its equivalent in terms of the coils below it in the line of resistances. If differences are observed, the sensitiveness of the galvanometer to a change of 1 ohm (or 0.1 ohms) in the outer line of resistances must be determined. The differences in question may then be estimated by interpolation (see ¶ 216). The results are to be reduced as in ¶ 217. When the ratios of the different coils in the outer series have been found, that of any pair of coils in the inner horse-shoe may be determined by comparison.

¶ 253. **Resistance of Electrolytes.** — We may substitute in Exp. 87 an alternating current for a common battery current; in this case the galvanometer must be replaced by some instrument like the dynamometer, sensitive to alternating currents. A telephone is sometimes found to give satisfactory results with a rapidly alternating current. Usually a loud note is heard in the telephone; but when the Wheatstone's bridge is in adjustment, the sound either completely ceases or reaches a minimum.

The advantage of using alternating currents is that, in the short time during which they last, the effects of polarization are so small as to be almost inappreciable. The method is especially valuable in the determination of the resistances of batteries and electrolytes. It is not, however, always successful, on account of various causes tending to destroy the minima of sound. To obtain satisfactory results, the resistance to be measured should be not less than 10 or 15 ohms. The electrodes should consist of platinum strips, at least 10 *sq. cm.* in area, and freshly coated with platinum through electrolytic action (Kohlrausch, 6th ed. 72 II.),

¶ 254. **Measurement of Electrical Capacity.** — A "condenser" consists of two sets of thin metallic plates, arranged alternately, as in Fig. 278, so that although the plates are very close together, there is no metallic connection between the two sets. The plates are generally separated by thin layers of glass, mica, or paper dipped in paraffine. The plates of one set are all connected with one binding-post (A); those of the other set with another binding-post (B). A condenser is charged by connecting A and B each with one pole of a battery. It may then be disconnected from the battery, and discharged through a galvanometer by carrying the terminals to A and B. Care must be taken not to touch both terminals at the same time.

FIG. 278.

The capacity of a condenser is defined as the quantity of electricity which can thus be stored in it by a battery having an electromotive force equal to 1 unit *in absolute measure.* The "farad" is a thousand millionth part of the electro-magnetic unit of capacity. The distance between the plates of a condenser is usually very small in comparison with the area of the separate plates. To calculate the electrical capacity of such a condenser, we measure the thickness (t) and total area (A) of the insulating layers, then if s is the "specific inductive capacity" of the insulating material (¶ 256), the capacity (C) of the condenser is given in electrostatic units by the equation —

$$C = \frac{As}{4\pi t}, \qquad \text{I.}$$

or, since it has been found by experiment that 1 microfarad is equivalent to about 900,000 electrostatic units,[1] the capacity (c) in microfarads may be calculated by the formula —

$$c = \frac{As}{36{,}000{,}000\,\pi t} \text{ microfarads (nearly).} \qquad \text{II.}$$

The specific inductive capacity (s) of the insulating material must in general be found as in ¶ 256; but when the plates of a condenser are separated by air spaces, since the specific inductive capacity of air is taken as 1, the capacity of a condenser may be calculated from direct measurements of the area and thickness of the insulating material.

The capacity of any condenser may be determined by measuring the quantity of electricity stored in it by a battery of known electromotive force. With the aid of clockwork, a condenser is to be charged by a battery and discharged through a galvanometer n times a second; the deflection of the galvanometer being noted. Then if R is the resistance in ohms through which the same battery produces the same deflection (see Exp. 95, II.) we have —

$$c = \frac{1{,}000{,}000}{nR} \text{ microfarads.} \qquad \text{III.}$$

In practice we must employ a very sensitive galvanometer capable of measuring currents at least in millionths of an ampère. The time of oscillation of the needle should be 10 seconds or more, in order that the intermittent discharge through the instrument may produce a sensibly constant effect. An ordinary condenser of 1 microfarad capacity cannot

[1] Everett, Units and Physical Constants, Arts. 177, 185.

be charged and discharged satisfactorily more than 10 or 100 times per second.[1] To avoid large errors due to this cause, the speed of the mechanism should be reduced until an approximate agreement is obtained between two or more results.

The experiment may be performed with an ordinary astatic galvanometer, but only by the use of a condenser of great capacity and a battery of high electromotive force.

¶ 255. **Comparison of Condensers.** — The capacities of two condensers may be compared by charging them, successively, by a given battery, then discharging them successively through a ballistic galvanometer (see ¶ 187). The capacities will then be approximately as the chords of the throws (§ 109).

The capacities of two condensers may be compared with great precision by including the condensers in two adjacent arms of a Wheatstone's bridge (see Exp. 87). One pole of the battery must be applied between the two condensers. The resistances in the other two arms of the bridge should be great, and adjusted so that a sudden *reversal* of the battery current causes no sudden deflection of the galvanometer.[2] If C_1 and C_2 are the capacities of the two

[1] Owing to effects of "electrical absorption" and "residual charge," the quantity of electricity stored in or obtained from a condenser depends somewhat upon the time during which connections are made. See Ganot's Physics, § 773. When a condenser is rapidly charged and discharged, these phenomena almost entirely disappear; but the resistance of the various conductors may reduce the quantity of electricity which can flow in and out of the condenser to an indefinitely small amount.

[2] See Glazebrook and Shaw, Practical Physics, §§ 81, 82.

condensers, R_1 and R_2 the resistances adjacent to them, respectively, we have —

$$C_1 : C_2 :: R_1 : R_2.$$

We have seen (¶ 254) that the capacity of a condenser with air spaces between its plates may be measured. The capacity of such condensers is generally so small that comparisons cannot be made by ordinary methods. By substituting an alternating current for the battery and a telephone for the galvanometer (see ¶ 253) in the combination described above, comparisons of these and even smaller capacities should be possible.

¶ 256. **Specific Inductive Capacity.** — When two condensers are similar in every respect except the nature of the insulating materials used in their construction, their capacities (c and c') are to each other as the "specific inductive capacities" (s and s') of these materials. Since the specific inductive capacity of air may be taken as 1, we have in general, from ¶ 254, I., —

$$s = \frac{4\,\pi\,ct}{A}.$$

The specific inductive capacity of a given insulating material may accordingly be found by constructing a condenser with that material between its plates, measuring the area of and distance between these plates, and determining as in ¶ 254 or as in ¶ 255 the capacity of the condenser.

Winkelmann's method for testing specific inductive capacities consists in the use of three parallel plates,

A, B, and C (Figs. 279 and 280), equal in area, and 15 or 20 *cm.* in diameter. A and B are separated by an air space of the thickness a, while B and C are separated by an air space of the thickness b, and by a thickness c of the material whose specific inductive capacity is to be determined. The outer plates A and C are connected either through a telephone (T, Fig. 279) with each other, or through a differential telephone (DT, Fig. 270), and through a metallic conductor (G) with the ground. The central plate

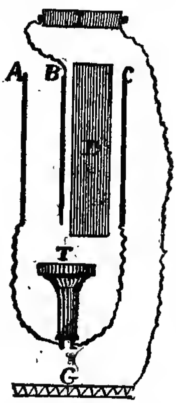


Fig. 279.

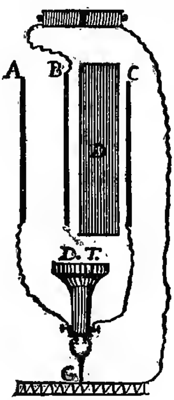


Fig. 280.

(B) is joined to one pole of an induction coil, the other pole of which is connected through G with the ground. The distances a and b are then adjusted so that the sound heard in the telephone is reduced to a minimum. The specific inductive capacity (s) is then given by the formula—

$$s = \frac{c}{a - b}.$$

In Winkelmann's method we may consider that the plates A and B form one condenser, while the plates B and C form another condenser. When the capacities of these two condensers are equal, a given charge of electricity on B must raise A and C to the same potential; hence *if the effect be simultaneous* no current will flow through the telephone. In practice, most dielectrics cause a slight retardation in the charging of a condenser, so that although the telephone gives a minimum of sound, it never becomes perfectly silent.

¶ 257. **Comparison of Electromotive Forces by means of a Condenser.** — The pole cups of a condenser (A and B, Fig. 278) are to be connected as in ¶ 254 with the poles of a battery, then disconnected from the battery, and connected with the terminals of a ballistic galvanometer, the throw of which is to be observed. The experiment is to be repeated with a second battery. If a' and a'' are the throws, E' and E'' the electromotive forces, we have (see § 109), if the angles are small, —

$$\frac{E'}{E''} = \frac{\text{chord } a'}{\text{chord } a''} = \frac{a'}{a''}, \text{ nearly.}$$

In this experiment it is important that the duration of charging, discharging, and changing connections should be exactly the same in the two cases.

¶ 258. **Electrostatic System.** — Two gilt pith-balls (b and c, Fig. 281), of equal weight (w) and diameter (d) are both to be suspended from an insulated point a, by fine cotton threads of equal length (l).

The threads may be blackened with a lead-pencil to make sure that they will conduct electricity. One pole of a battery (de), of several hundred volts, is to be connected with the point (a) of suspension; the other pole with the ground.

The balls b and c, being similarly charged, will now repel each other. A considerable divergence should be observed. The distance (s) between the centres of the two balls is to be found by a sextant placed at a fixed distance (see ¶ 124). The electro-

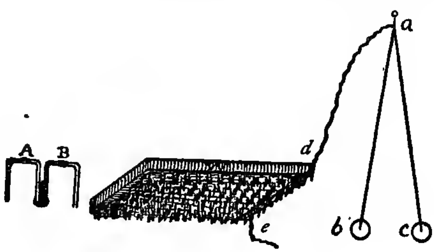


FIG. 281.

motive force (e) of the battery in electrostatic units is then (roughly) —

$$e = \sqrt{\frac{2\,wgs^3}{ld^2}}.$$

The pith-balls should be about 1 *cm.* in diameter, and not over .05 *g.* in weight. The cords *ab* and *ac* should be at least 100 *cm.* long, but not over 0.01 *g.* in weight. All electrical conductors should be removed as far as possible from the neighborhood of the balls b and c.

A water battery (*de*, Fig. 281) will be found convenient for this experiment. It may be constructed of alternate strips of zinc and copper soldered together in pairs and attached with pitch to the under side of a board so that drops of water or dilute sulphuric acid may be taken up between adjacent pairs (as *A* and *B*).

It has been found by experiment that one unit of electromotive force in the electrostatic system is equal to about 300 volts, or 30 thousand million absolute units in the electromagnetic system. It is an interesting fact that the ratio between the absolute units of the two systems is equal, within the limits of errors of observation, to the velocity of light (see § 93).

INSTRUMENTS OF PRECISION.

The apparatus employed in the course of experiments which has been described is of the simplest possible form. The most accurate results can be obtained only by the use of instruments especially designed for a given purpose. The following sections

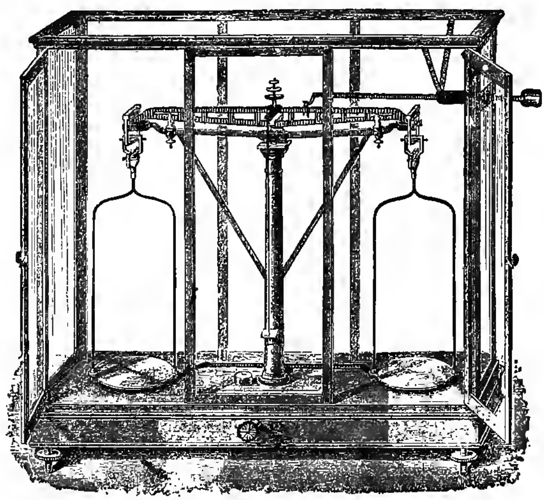


FIG 282.

contain a brief description of the construction and adjustments of certain instruments of precision, which though unsuitable for an elementary class of students, might be advantageously employed by advanced students in place of the ordinary apparatus.

¶ 259. **Analytical Balances.** — The adjustments of an analytical balance (Fig. 282) and the precautions in using it are essentially the same as those described in Experiment 6. In addition to the mechanism, operated from outside the case, by which in a fine balance *all* weight may be removed from the knife-edges, there is often a pan-arrester, which has to be moved before two weights can be exactly balanced. A preliminary adjustment of the weights should be carried as far as centigrams on an ordinary balance. The weights may then be transferred to the analytical balance, and a finer adjustment made by means of a rider (*e*, Fig. 283) made of platinum wire. The rider can be placed

FIG. 283.

at any point (*e*) of a graduated scale on the balance-beam by means of a hook (*d*) attached to a rod (*ac*) passing through a tube (*b*) in the side of the balance-case. The necessary motion is given to the hook by pushing, pulling, or twisting the rod (*ac*).

The indication of the pointer is always found while it is in oscillation (¶ 20); but since the weights may be adjusted by means of the rider with any degree of precision, the method of interpolation (¶ 20), though generally quicker, need not be employed.

In finding the position of the rider necessary for an exact balance, the same method of approximation should be employed, at first, as in the adjustment of weights; that is, the rider should be placed midway between two distances on the scale, one too great the other too small, until the deflection of the pointer and the sensitiveness of the balance indicate directly where it should be placed. When finally observations of the swings of the pointer show that it would come to rest at its zero-position, the position of the rider is noted.

The accuracy of the rider is tested by weighing a small weight with it. To obtain results accurate to a tenth of a milligram, the set of weights employed (even the best) should be most carefully tested (¶ 25).

The advantage of weighing with a rider is that the final adjustment of two weights may be made with the balance-case closed. The air within the case should always be kept perfectly dry with chloride of calcium (or with concentrated sulphuric acid), which must be renewed from time to time. Neither arm of the balance should be exposed to the heat of a fire or lamp, or to the cold glass of a window. The method of double weighings should if possible be employed. If it is not employed, care must be taken that the pans are equal in weight, and that in the zero-position, the balance-beam is horizontal and the pointer vertical.[1]

[1] When the greatest accuracy is desired, arrangements must be made to carry on the ordinary processes of weighing from a distance. Thus at the International Bureau of Weights and Measures at St. Cloud, not only the suspension of weights from the balance-beams, but also the interchange of the contents of the scale-pans is accomplished by a series of shafts leading from each instrument nearly to the centre of a large room in which the finest balances are contained. Mechanical contrivances are also employed for the final adjustment of weights *in vacuo*.

¶ 260. **Comparators.** — A simple form of comparator is represented in Fig. 284. It consists of two reading microscopes (*A* and *B*) mounted on supports (*E* and *F*) which slide along a rail (*GH*). The sliding supports may be clamped at any point of the rail by thumb-screws (*C* and *D*). A small scale of tenths of millimetres (*b* and *b′*, Fig. 284) is placed in the tube of each microscope at a distance from the object glass (*c*) equal to twice its focal length. The eye-

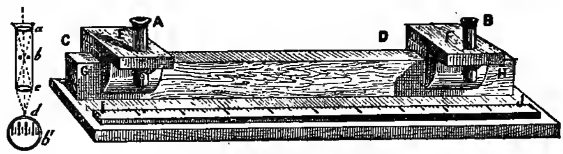


Fig. 284.

piece (*a*) is first focussed upon this scale, then raised or lowered until a given object is in focus. Let us suppose that the two microscopes are thus set, one upon each end of a scale. It is obvious that if a standard scale be now substituted any difference between the two will be not only readily detected, but easily measured in tenths of a millimetre and such fractions of a tenth as may be estimated by the eye (§ 26).

Care must be taken to have the *upper* surfaces of the two scales on the same level, so that both scales may be in focus, and to have the microscopes firmly clamped, and not subjected to any strain between observations.

¶ 261. **The Dividing-Engine.** — A dividing-engine (Fig. 285) consists essentially of a micrometer (*c*) with a long screw (DG) fixed in position, so that when the micrometer is turned, a nut (EF) gives a slow motion to a slide (B) to which a reading microscope (A) is usually attached. The length of an object parallel to the screw is determined by the number of turns of the micrometer necessary to make the microscope travel from one end of the object to the other. The microscope is of course provided with cross-hairs, so that it may be set exactly on a given point. The screw is always to be turned in a given direction in measuring a given distance; otherwise an error due to looseness of the screw ("backlash") may be made. The pitch of the screw in different parts is

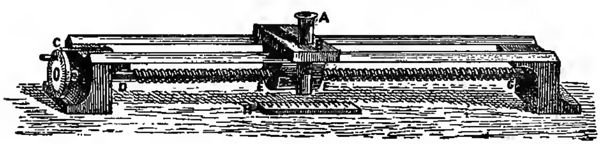


Fig. 285.

found by measuring with it a standard scale of known length (see ¶ 52). If the nut is long and fits equally well in all parts of the screw, no *great* variations of pitch can occur.

The dividing-engine is especially useful in measuring distances between the lines of a scale, or lengths of columns of mercury in the calibration of a tube (see ¶ 71). The results may be more precise than those obtained with any other instrument for the measurement of length.

¶ 262. **The Cathetometer.** — (κατὰ, down, τίθημι, to place, and μέτρον, measure) is an instrument for measuring vertical distances (Fig. 286). It consists of a horizontal telescope or reading microscope (*b*) sliding on a vertical shaft (*ah*), which is capable of rotating about its own axis. Sometimes the shaft is graduated, the carriage to which the telescope is attached being provided with a vernier, so that the height of the telescope may be read. Slow motion may also be given by a micrometer screw (*ef*). The cathetometer may then be used for measuring small vertical distances, just as the dividing-engine (¶ 261) is used for horizontal distances. The micrometer is useful in measuring precisely, for instance, the distance through which a wire is stretched (Exp. 65). For ordinary purposes, neither the micrometer nor the vernier is required. The shaft is first adjusted by the eye so as to be as nearly perpendicular as possible, by means of the levelling-screws (*h*, *i*, and *l*) at the base of the instrument, then the telescope is made horizontal according to a spirit-level (*c*) with which it is provided. Then the shaft is rotated about its axis. If the axis is not vertical, the bubble in the spirit-level will tend to move in a given direction. The

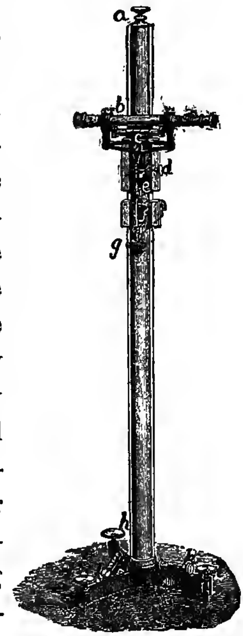


FIG. 286.

top of the shaft is to be inclined slightly in this direction. After a series of trials the axis may in this way be made perfectly vertical.

The object to be measured is to be set up with the aid of a plumb-line, beside a vertical scale, so as to be at the same distance from the cathetometer as the scale is, both at the top and at the bottom. The telescope of the cathetometer, accurately levelled, is to be focussed by means of the cross-hairs upon one end of the object (¶ 116, 3), then rotated so as to bear upon the scale, and the reading of the scale noted. If the spirit-level is disturbed, the cathetometer must be readjusted and the reading redetermined. The reading of the lower end of the object is to be found in the same way. By putting a graduated scale in place of the cross-hairs, the divisions of a scale may be divided into very small parts. This method is not so precise as that depending upon the use of a vernier or micrometer attached to the cathetometer, but may, in unskilled hands, give fully as accurate results.

¶ 263. **Micrometer Eye-Pieces.** — Instead of moving a telescope or a reading microscope bodily, as in ¶¶ 261 and 262, it is sometimes convenient to mount the cross-hairs upon a small slide within the eye-piece of an instrument, and to give a slow motion to the slide by means of a micrometer screw. The value of the micrometer divisions must be found for each instrument. A micrometer eye-piece gives indications much more precise than a fixed scale; but care must be taken not to alter the setting of an instrument by pressure upon the eye-piece in adjusting the micrometer, and, as in the dividing-engine (¶ 261), to turn the instrument always in a given direction up to a setting. If the micrometer is turned too far, it must be turned backward a considerable way, then forward to the desired point.[1]

In the best optical circles two microscopes with micrometer eye-pieces are usually provided. These are placed on opposite sides of the circle, in order that errors due to excentricity may be avoided.

¶ 264. **Regulators.** — For experiments involving the accurate measurement of time, a clock with a compensating pendulum, or a chronometer with a compensating balance is indispensable. The clock or chronometer should be provided with an electric break-circuit, and must be rated by observations with either a sextant (¶ 243) or a transit (see Pickering's Physical Manipulation, § 178), or by comparison with time signals from some observatory.

In the Physical Laboratory of Harvard College, the regulator employed is a common seconds-clock with a wooden pendulum-rod controlled by an electrical time circuit. The control consists simply of a fine spiral spring connecting the pendulum with the armature of a telegraph instrument in the circuit. Electrical signals, sent from the Astronomical Observatory at intervals of two seconds, are thus made to act mechanically upon the pendulum. When the latter

[1] The "backlash" should be taken up, in so far as possible, by the action of a spring. Errors due to "backlash" may be thus greatly diminished, but not completely eliminated.

has been carefully rated without the control, very small impulses are sufficient to prevent it from gaining or losing.

¶ 265. **Kater's Pendulum** (Fig. 287). — In Kater's form of reversible pendulum (see 249) the rod (*de*) is usually made of brass, a little over a metre long, 2 or 3 *cm.* wide and about 5 *mm.* thick. Two steel knife-edges, *bc* and *fg*, are attached firmly to this rod with a distance of about 1 metre between them. They are supported when the pendulum is in use, by agate planes, *b* and *c*. The bob (*h*) is a brass cylinder, weighing 1 or 2 kilograms. Movable counterpoises, *d* and *e*, serve to adjust the centre of oscillation. Two light and firm metallic pointers (*a* and *i*) may be used to magnify the oscillations.

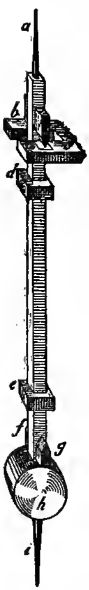


FIG. 287.

In addition to these adjustments, clamps with tangent-screws may be employed to obtain a slow motion of the counterpoises. The knife-edges *bc* and *fg* are sometimes made movable (one or both of them). In this case, verniers are usually attached, so that the distance between the knife-edges may be read by a scale on the shaft *de*. The zero-reading of the vernier is found by bringing the knife-edges together against a pressure equal to the whole weight of the pendulum. The accuracy of the main scale is tested by a comparator (¶ 260) at the ordinary temperature of the experiments, and under a strain equal to the average weight which the shaft sustains.

¶ 266. **Chronographs.** — A chronograph consists generally of a cylindrical drum (*A*, Fig. 288) rotated uniformly by clock-work. The surface of the drum is coated with lampblack, so that a style (*B*), attached to the armature (*c*) of a telegraph instrument may make a mark upon it. The line *AB* represents the trace caused by an ordinary seconds break-circuit. At the point *D* there is an extra break due to a signal given by hand. If the drum revolves uniformly, the exact time of such a break can evidently be determined by measuring the distance from it to the nearest second-mark, and comparing this with the distance between two second-marks.

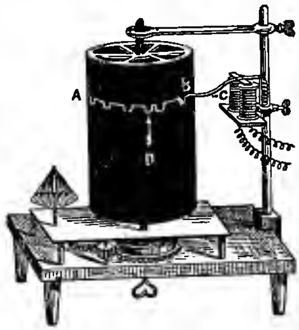


Fig. 288.

The pitch of a tuning-fork may be determined very exactly by the trace made on the surface of a chronograph (see ¶ 139).

It may be said in general that the chronograph is valuable as a means of determining precisely the interval of time between any two phenomena which, with or without the agency of electricity, are capable of affecting the motion of a style.

¶ 267. **The Siren.** — The siren (Fig. 289) is an instrument for producing a musical note of any pitch, and at the same time registering the number of vibrations constituting that note. It is operated by a constant air pressure from a bellows, specially constructed for this purpose. The air enters the wind-chest of the instrument at (F), issues obliquely from a series of holes (of which E is one) in the top of the wind-chest, and strikes obliquely against the sides of a series of holes (of which D is one) in a disc (C), which is thereby set in motion. When the two series of holes come opposite, the air escapes freely from the wind-chest; when they are not opposite, the current of air is nearly cut off. The irregular flow of the air sets the atmosphere in vibration. The number of vibrations in a given length of time is indicated by the dials A and B.

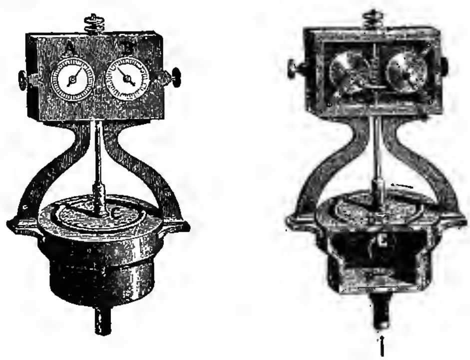


FIG. 289.

In practice the speed of the siren is regulated by pressure on the top of the bellows used to drive it. The note is slowly raised until it agrees with one whose pitch is to be determined. When the two notes are nearly in unison beats will be heard (¶ 140). By a slight change of air pressure, perfect unison may generally be obtained. This will be shown by a cessation of beats. The unison is maintained for a given length of time during which the number of vibrations made by the siren is registered. In some instruments the dials may be thrown in and out of gear at a given moment. This facilitates the observations of the dials, but care must be taken that the speed of the siren is not affected.

It must be remembered that beats occur not only when two notes are in unison, but also when they are nearly an octave apart, and to a somewhat less extent, when they are separated by any other musical interval (¶ 134). A musical ear is therefore almost a necessity in the adjustment of a siren. The chief advantage of the siren is that it enables us to find the pitch of notes not easily determined (as is Exps. 52, 54, and 55), by either optical or graphical methods.

¶ 268. **Mirror Galvanometers.** — A very sensitive galvanometer is made by suspending a small mirror (*F*, Fig. 290) in the middle of a coil *E* of insulated wire, by means of a single fibre of cocoon silk (*DE*). Small bits of "hair-spring" (used in watches) highly magnetized, all in the same manner, are fastened with the smallest possible quantity of wax to the back of the mirror. A large curved magnet (*BC*) capable of sliding up and down the tube (*A*) or turning round it, is adjusted so as to nearly neutralize the effect of the earth's magnetism on the magnets attached to the

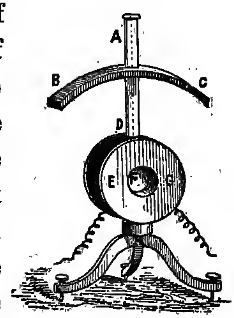


Fig. 290.

mirror. The sensitiveness of this instrument when accurately adjusted, though less permanent than that of an astatic combination, is for the time being fully as great.

In some galvanometers a converging mirror is used, so that a spot of light may be projected on a transparent screen. The existence of a current is indicated by the motion of the spot of light with respect to a scale graduated on the screen.

In other instruments a plane mirror is employed, with a long-focus lens mounted permanently in front

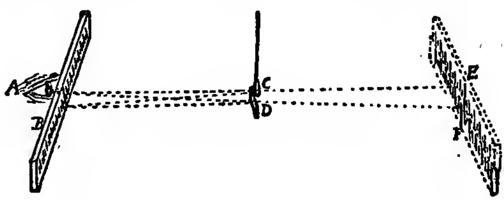


FIG. 291.

of it. The deflection of the mirror is frequently observed by means of the reflection (*E*, Fig. 291) of a scale (*B*) in the mirror (*C*), seen from a point (*A*), where either the eye or a telescope may be placed.[1]

¶ 269. **Electrical Standards.** — Copies of "standard ohms" may be obtained from most dealers in electrical apparatus. The terminals should be thick copper

[1] Prof. B. O. Peirce has shown that excellent results may be obtained without any telescope (*A*), by placing beneath the mirror *C* a fixed mirror *D*, so that the two reflections (*E* and *F*) of the scale (*B*) very nearly coincide. When the two mirrors are parallel, the zeros of the two scales are opposite, *no matter where the eye may be placed.* The slightest deflection of the mirror causes an apparent motion of the scale reflected in it.

rods, capable of being amalgamated with mercury and connected by mercury cups with a Wheatstone's Bridge Apparatus. Unless special care be taken in making these connections, the most accurate standards of resistance may lead to very erroneous results.

Standard cells of Latimer Clark's pattern may easily be obtained. Their electromotive force is about 1·435 volts at 15°. The decrease is about ·00077 volts for a rise of temperature of 1° Centigrade. The uses of a constant cell have been alluded to in ¶¶ 228, 230.

"Standard ampères" are now being made by some dealers. When the attraction of a coil of wire for

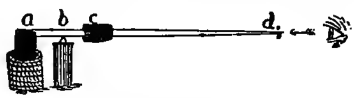


FIG. 292.

a piece of soft iron is balanced by gravity (Fig. 292), an allowance must be made for variations in gravity when the instrument is transported from one latitude to another. A standard ampère depending upon the action of a spring, though subject to many theoretical objections, would be practically useful as a check upon results obtained by other methods. Let us suppose that such an instrument is connected in series with a rheostat and a tangent galvanometer, that a current, sent through both, is increased until the instrument indicates 1 ampère, and that the galvanometer is then read. The reciprocal of the tangent of the angle of deflection should agree closely with the reduction factor already found (Exp. 83).

¶ 270. **Electrometers.** — Various forms of quadrant electrometer may now be obtained from manufacturers. The theory of these instruments is exceedingly complicated, and the results are more or less uncertain. The principal use of the instrument is in the case of inconstant cells, to confirm results obtained by the use of a condenser. Such instruments in general have to be calibrated by means of cells of known electromotive force.

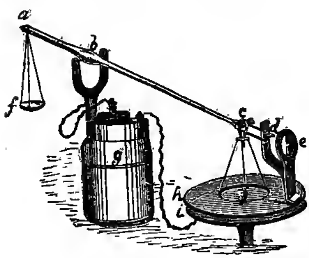


Fig 293.

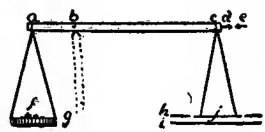


Fig. 294.

Thomson's absolute electrometer (Figs. 293 and 294) depends upon the attraction between two plates *j* and *i*, when charged oppositely with electricity. The plate *j* is suspended from one end (*c*) of a balance-beam (*ac*). The force exerted upon it is counterpoised by weights in a pan (*f*) suspended from the other end of the beam (*a*.) The deflection of the beam is observed by means of a sight (*d*) and a lens (*e*). The plate *i* is very much larger than *j*, which is surrounded by a ring (*h*) charged to the same potential as the movable disk (j), to equalize the distribution of electricity upon the latter.

If w is the weight required to balance the attraction of the two plates, d the distance between them, and a the area of the suspended plate (j), then the difference of potential (e) between the plates is given in electrostatic measure by the formula—

$$e = d \sqrt{\frac{8 \pi g w}{a}}.$$

It is said that an absolute electrometer may be made sensitive to the difference in potential between the two poles of a Daniell cell. It is especially valuable for the calibration of other forms of electrometer better suited for actual use, and for determinations of the fundamental relations between the electrostatic and electro-magnetic systems.

END OF PART II.

PHYSICAL MEASUREMENT.

Part Third.

PRINCIPLES AND METHODS.

INTRODUCTION.

THE first step in all scientific progress consists in a classification of different objects based upon similarities and differences. The distinguishing characteristics of solids and liquids, minerals, metals, crystals, &c., were undoubtedly observed long before history began. The necessity for shelter and clothing must have drawn attention to the difference between insulating substances and conductors of heat; and in the same way all physical properties of importance to mankind cannot have failed to receive early recognition. The manner in which different branches of science have been developed is perhaps best illustrated in the case of electricity, the phenomena of which were virtually unknown[1] before the

[1] The development of electricity from amber was known to Thales several years before Christ. It would appear, however, that at this time little or nothing else was known about electricity. Ganot's Physics, § 723.

end of the sixteenth century. We find in very early writings tables like the following:—

CONDUCTORS OF ELECTRICITY.

Metals.	Animal Substances.	Sea Water.
Charcoal.	Vegetable Substances.	Vinegar, &c.

NON-CONDUCTORS.

Resins.	Glass.	Wax.
Sulphur.	Silk.	Oils, &c.

A division of substances into two classes may in certain cases be exceedingly useful. The reactions which take place in chemical solutions are, for instance, frequently determined by the solubility or insolubility of the compounds which may be formed. It is rarely necessary to make fine distinctions in the statement of chemical solubilities.[1] The term "sparingly soluble" must occasionally be employed; and, again, comparisons must be made between different solubilities. Most substances, however, are either very soluble, or else very insoluble, in a given liquid; and a single word, "soluble" or "insoluble," conveys to the chemist a valuable piece of information.

In the construction of electrical instruments, on the other hand, it became important to distinguish both good conductors and good non-conductors from a large class of substances called "semi-conductors" (Ganot's Physics, § 725); and with the growing importance of electricity came the necessity of still further distinctions. Substances were finally ar-

[1] See Storer's Dictionary of Solubilities

ranged in a list in the order of their power to conduct or to insulate electricity (Deschanel's Natural Philosophy, § 409). In the same way certain bodies, at first classed simply as positive or negative with repect to the charges of electricity which they receive when rubbed together, are in later works arranged as follows (Deschanel, § 411) : —

Fur of Cat.	Feathers.	Silk.
Polished Glass.	Wood	Shellac.
Wooden Stuffs.	Paper.	Rough Glass.

If any of the substances in this list be rubbed with one following it, it will generally become "positively electrified;" but if rubbed with one preceding it, it will be "negatively electrified." Such an arrangement is evidently more useful than a simple division into two classes.

Mohs' scale of hardness consists of 10 substances :[1]

1. Talc.	3. Calc Spar	5 Apatite.	7 Quartz.	9. Sapphire.
2 Gypsum.	4. Fluor Spar	6 Feldspar.	8 Topaz.	10. Diamond.

Each substance contained in this list will scratch the one above it. If, accordingly, a piece of steel which will scratch feldspar is scratched by quartz, its hardness must be represented by a number between 6 and 7 (let us say 6.5) on this arbitrary scale.

The distinction between any two substances in such a list is purely qualitative ; that is, we know only that each possesses a certain quality or property *more than* the one below it. We do not know whether the

[1] Cooke's Chemical Physics, p 209.

gaps in the list are great or small, equal or unequal. We have no idea even of the relative values which the numbers (1–10) represent. Still, the assignment of numbers to the different substances may be considered as a first attempt to obtain precise results; and in the case of physical quantities which admit of no more exact estimation, the value of an arbitrary scale like that of Mohs must not be overlooked.

The next step in the accurate representation of results is to make the intervals between different scale-numbers equal, — or, at least, to make them follow in regular progression. Among the earliest applications of this principle may be mentioned the arbitrary hydrometer scales of Beaumé, Beck, Cartier and Twaddell. A mark was made upon a hydrometer (see Fig. *a*) to show how deep it sank in water; and this mark was numbered 0 or 10, as the case might be. Then the hydrometer was floated in some other liquid of known composition, and another mark was made to show how deep it sank in that liquid. The second mark was also numbered arbitrarily — 60 or 80, for instance (see Table 40). The distance between the two marks was then subdivided. The scale of an ordinary thermometer (see Fig. *b*) is constructed in a similar way. A mark is made to show where the mercury stands when surrounded with melting ice, and another

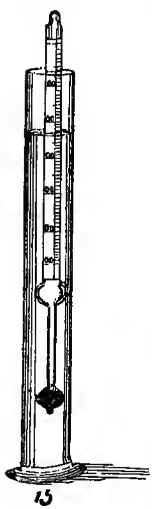


FIG. *a*.

mark is made to show where it stands in steam (see Exp. 25). The distance between the two marks is divided by Fahrenheit into 180 parts; by Celsius, into 100 parts; by Réaumur, into 80 parts. Fahrenheit called the freezing-point of water 32°, without any scientific reason; Celsius and Réaumur called it 0°. Their scales are accordingly simpler than Fahrenheit's, but none the less arbitrary. The Celsius scale is still in use in the ordinary centigrade thermometer (§ 4); the other scales, together with the hydrometer scales of Baumé, Beck, Cartier, and Twaddell, are going out of use. The gradual disap-

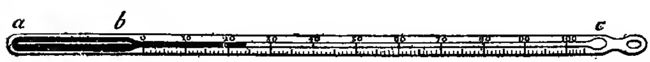


FIG. *b*.

pearance of arbitrary scales is in general an indication of scientific progress.

It is obviously desirable that the numbers in a scale should be proportional to the quantities which they represent. With the advance of science in the early part of the present century, we find an abundance of physical tables showing the relative values of different quantities (§ 3). Specific gravities of solids and liquids compared with water, specific gravities of gases and vapors compared with air or with hydrogen, specific heats compared with water, &c., were all more or less accurately determined.

At the same time that the physical properties of different bodies were compared together, the changes which take place in a given substance under varying conditions were carefully studied. The expansion of solids, liquids, and gases due to heat were, for instance, observed and tabulated. We find in Biot's "Physique' (1821, vol. i., page 320) a table showing the relative densities of water at different temperatures, some of which are compared below with the best results of modern observers, as given by Everett in § 34 of his "Units and Physical Constants." Calling the density of water at 4° equal to 1, these results become[1] —

	Biot.	Everett.	Difference.		Biot.	Everett.	Difference.
0°	.99993	.99987	+ 6	50°	.98778	.98820	− 42
4°	1.00000	1.00000		60°	.98251	.98338	− 87
10°	.99973	.99975	− 2	70°	.97652	.97794	− 142
20°	.99832	.99826	+ 6	80°	.96998	.97194	− 196
30°	.99579	.99577	+ 2	90°	.96285	.96556	− 271
40°	.99225	.99235	− 10	100°	.95537	.95865	− 328

This is but one of the many fairly accurate determinations dating back even into the last century. Most of our modern physical laws and principles were known in the early part of the nineteenth century, and a great number of physical properties had been investigated. The results of this early period are, however, characterized by the absence of all data by which it is possible to find anything more than the relative values of different quantities. The powers

[1] The results quoted by Biot, though creditable for his time, were generally inaccurate in the fourth and sometimes even in the third place of decimals. They were, nevertheless, carried out, according to the custom of early observers, to 7 and 8 decimal places.

of different metals to conduct heat were, for instance, given by Despretz as follows, counting gold as 1,000 (Ganot's Physics, § 404): —

	Despretz.	Wiedemann and Franz
Platinum	981	158
Silver	973	1880
Copper	897	1384
Iron	374	202
Zinc	363	374
Tin	304	273
Lead	179	160

That these results were not particularly accurate may be inferred by comparing them with those of Wiedemann and Franz (1853), reduced in the right-hand column to the same system.[1] Thus platinum, which is the best conductor of heat according to Despretz, is the worst according to Wiedemann and Franz. Even, however, if we assume the accuracy of either set of results, it is still impossible to apply them unless we know, in a single case, how much heat flows from one place to another through a bar or plate of given length, breadth, thickness, and material, and the difference of temperature to which this flow of heat corresponds.

The determination of relative values (such as are contained in the table above) is in general a much easier task than the determination of absolute values (see Table 8, *et seq.*); and has the advantage that gross errors are not so likely to be made.

Relative measurements are, however, to a certain

[1] Wiedemann and Franz counted silver as 100 See Deschanel's Natural Philosophy, § 333.

extent non-committal, and hence justly unpopular with scientific men. The highest end of physical measurement is not attained unless every quantity with which it has to deal is compared directly or indirectly with the so-called *absolute units* (§ 8) which lie at the base of the system. Quantities subjected to such comparisons are said to be *determined in absolute measure.*

We have seen that, historically, in various branches of science, the absolute system of physical measurement has been approached by a series of stages. The first stage may be called classification; the second, ordination; the third, numbering; the fourth, graduation; the fifth, comparison; the sixth and last, determination. The first two stages deal with qualities, and involve only qualitative experiments. Physical measurement is properly confined to the last two stages. It deals exclusively with the numerical relations between different physical quantities. Measurements are, accordingly, quantitative in their nature.

It is unnecessary to distinguish physical measurement from measurement in general, as the term is usually employed. It is only physical quantities which are capable of being measured. Measurement implies observation; exact measurement implies accurate observation. The observation required in physical measurement is, it is true, exceedingly limited in its character (see § 23). In the natural sciences, the powers of observation have their widest application. In physical measurement the *sharpest* use of this faculty is required. The student is apt to imagine that an increase of precision in the instruments at his disposal would relieve the continual tax which he feels upon his power of observation. Quite the reverse is generally true. The better the instrument, the harder it is to do justice to it. One must learn to obtain the best possible results with rough instruments before one is fitted to use instruments of precision. The habit of accurate observation is an important object to be gained by a course of physical measurement.

The most accurate results in physical measurement often require practice, not only in observation, but also in manipulation. The skill acquired in a course of quantitative determinations is an advantage by no means to be overlooked.

The principal benefit to be expected from a course of laboratory instruction is, however, familiarity with the *experimental method* and the processes of inductive reasoning which it involves. Certain of these processes belong especially to quantitative determinations. The results of physical measurement frequently depend, not only upon a long series of observations, but also upon a more or less complicated chain of reasoning, including the mathematical calculations by which the observations are reduced. A single error in any one of the data, or in any step in the process of reduction, will in most cases entirely change the result. The student is not, however, in physics as in philosophy, necessarily misled by such an error. Physical measurement abounds in what are called "check methods" (§ 45), by which errors either in observation or in reasoning may generally be detected. Having once discovered the sources of error into which he has fallen, the student is less likely to commit the same errors in the future. The result of a course of physical measurement should be to give him a just confidence in what he has seen with his own eyes, and in what he has reasoned out in his own mind.

The student should learn, as early as possible, to distinguish between real and apparent accuracy. A kilogram of wood may, for instance, be weighed to a milligram on a good balance. Such a weighing would be called *precise.* The true weight would, however, be very inaccurately determined, if no account were taken of the buoyancy of the atmosphere, which may amount to several thousand milligrams.

A given degree of accuracy implies an equal degree of precision; but precision does not necessarily imply accuracy. Exact results are those which are both accurate and precise.

When a measurement, however inaccurate, is repeated several times *in exactly the same manner*, more or less concordant results are usually obtained. The object of the scientific observer is not to make his determinations *look* more accurate than they really are, but, on the contrary, to bring to light the errors by which they are affected. He seeks accordingly every possible variation of the conditions under which an experiment is tried, in order *to bring out discordances*, if possible, between methods which ought (as far as he knows) to give exactly the same result. The simplest changes — the manner, for instance, of supporting an instrument — have frequently a most unexpected effect, and lead to the disclosure of unknown sources of error.

The student must not be discouraged by the discovery that his results are less accurate than he expected. He will find by comparing together the determinations of distinguished scientific men, that great discrepancies frequently exist between them. He must not be deceived by the number of decimal places to which their work is carried out. According to a custom prevalent, especially in the early part of this century, 3, 4, and even 5 figures, having little or no significance (§ 55) are often appended to results (see footnote, page 590). Within the last twenty years, the physical constants have acquired certain *conventional values.* There is an undoubted tendency to publish determinations by which these values are confirmed, and to suppress others equally good, leading to different results. The concordance of modern determinations is therefore, to a certain extent, apparent rather than real.

From time to time (as every one knows who follows scientific proceedings) inaccuracies in the accepted values of the physical constants force themselves upon our attention. In view of these facts, the student should return with increased confidence to his own determinations. When an investigation has been completed, and all sources of error, in so far as possible, allowed for, the facts should be made known, no matter who has arrived at a different result.

The student should learn to value different determinations for what they are worth. It is a very rough weighing that is not accurate within one part in a thousand; but some of the best electrical measurements are subject to much greater errors.

The results of some observers in determining the conductivity of different substances for heat are twice as great as the results of others; these results are however, useful. They show, for instance, that it would be impracticable to heat a house by a system of conducting rods radiating from a common centre; but that the thin metallic coatings of a furnace offer a comparatively slight resistance to the passage of heat. A knowledge even of the *number of ciphers* necessary to express the magnitude of certain quantities,— as, for instance, the weight of molecules,— may be useful in certain calculations. The fact that some measurements are necessarily inexact should not prevent the student from doing his best where accurate work is possible.

The results of physical measurement can, from their nature, never be, like those of mathematics, perfectly exact. Errors of greater or less magnitude are not only possible, but we may say almost certain to occur. Herein lies an important distinction between mathematical and physical problems. A mathematical solution is either right or wrong. In regard to the results of physical investigations, we have to consider *how far* each is likely to be in error. The quantitative methods which characterize physical measurement are extended even to the errors committed in these measurements. The treatment of such problems forms an important branch of the mathematical theory of probability, upon which all inductive methods are founded. It is not easy, from a philosophical standpoint, to regard the probable accuracy of results obtained by observation in *exactly the right attitude.* One cannot strictly affirm the accuracy of any figure in a result; but, as concerns some figures, it is difficult if not impossible to formulate the slightest doubt without enormously exaggerating the real uncertainty. Discussions of "probable error" (§§ 50–52) are characteristic of physical measurement, and teach a species of reasoning which, in problems of insurance, has assumed great practical importance.

One of the principal advantages derived from a course of physical measurement is, as has been said, the acquisition of habits of accurate thinking. When two quantities have been compared together, it is evident that, if the magnitude of one is known, that of the other must be determined. It is not, however, always clear *what* is determined by a given observation. It must be borne in mind that a physical determination consists, essentially, in the comparison of a quantity with one *better known than itself.* At the beginning of this century, the density of water at high temperatures was known only within a few tenths of 1 %. To-day, the density of water is one of the best known physical constants. The same experiment

(Exp. 19) which one hundred years ago constituted a determination of the density of water, now furnishes data only for calculating the volume of a solid, or the rate of expansion of the material of which it is composed. Great care must be taken to make a proper use of the results of physical measurement. One may, for instance, measure the circumference and radius of a circle, and from the results calculate the ratio which one bears to the other. It would, however, be incorrect to speak of this experiment as a determination of the ratio in question, since this ratio, being capable of exact mathematical calculation, is better known than the scale readings upon which the result depends. Physical measurement may be occasionally employed as a check upon mathematical calculations, particularly when (as in certain applications to physics) there is any doubt as to the validity of the assumptions upon which the calculations depend. Any attempt, however, to establish mathematical principles by data obtained from observation is an obvious abuse of the experimental method.

The so-called "proofs" of well-known physical laws and principles founded upon rough and insufficient data are hardly less objectionable.[1] The use of the experimental method as an illustration of such laws is not denied. One of the objects, however, of a course of physical measurement is to teach a stu-

[1] It may be remarked that the Law of Boyle and Mariotte (§ 79) was thus taught and implicitly believed in for more than a century, before more exact observation showed that this law is only approximately fulfilled.

dent how to make the best use of the tools at his command. The laws and principles which have been most carefully studied by scientific men should be made the instruments, not the objects of elementary research. The teacher should avoid, in so far as possible, experiments whose ostensible object is to establish well-known facts, — like the conservation of energy,— the truth of which is not really in question.

Among the habits of accurate thinking which it is the object of physical measurement to teach, may be mentioned those involved in a diligent and methodical search after the errors which are likely to be committed in one's work. It is hoped that the classification of errors in Chapter II. may be of assistance to the student who is thrown more or less upon his own responsibility. It is of course impossible to anticipate in any such classification all errors which may arise; but there are certain kinds of errors of such frequent occurrence that one must always be on one's guard against them. The student should ask himself, for instance, in respect to every scale reading, Have errors of parallax been guarded against (§ 25)? Have errors been committed in the estimation of tenths (§ 26)? Are there mechanical devices by which such errors could be diminished (§ 27)? Has the zero of the scale been carefully adjusted (§ 32)? Has the scale been carefully tested (§§ 31, 37)?

In addition to these considerations, by which errors may be frequently avoided, there are certain general methods, considered in Chapter III., by which (when they can be applied) the accuracy of a result is *always increased.* The student who is planning for himself the details of a physical measurement should consider these general methods one by one. He should ask himself, for instance, Is the method proposed the most direct (§ 36)? Could not more accurate results be obtained by dealing with larger quantities (§§ 38, 39)? or quantities which happen to be more nearly coincident (§ 40)? Could not precision be gained by the use of differential instruments (§§ 41, 42)? or accuracy by the check methods (§§ 43-45)? Would it be possible to reverse or interchange the quantities compared (§ 44)? or to obtain and average results from several determinations (§ 46)? These and similar questions must occur habitually to every successful observer.

A course in physical measurement is not especially suited to students who wish to become acquainted with a wide range of physical phenomena. Dealing, however, with quantities of nearly every description, and with the numerical relations which exist between them, it affords numerous examples of the application of physical laws and principles. It is only through the aid of definite examples that most persons can arrive at an understanding of physics. It has been assumed in the experimental course described in Parts I. and II. of this book, that the student is already familiar with the *statements* of physical phenomena contained in ordinary text-books. If this is the case, he must expect to gain definiteness rather than scope in his conceptions from a course of quantitative determinations.

It would be impossible, in the limited space which can be devoted to the subject in the present volume to describe or explain in full more than a very small part of the principles which underlie physical measurement. The brief notes contained in Chapters V.-X. are intended simply to recall to the student (who has already taken a course in general physics) the laws and principles which he has to employ, and the proofs upon which they rest. They may also be useful to the instructor as a basis for his lectures, or to the student who is just beginning the study of physics as a "syllabus" of what he should read in order to follow intelligently the course of physical measurement described in Parts I. and II. For a full explanation of the physical principles involved in this course, the student is referred to the standard works of Daniell, Deschanel, and Ganot.

The advantages of a course in physical measurement have been considered chiefly from an educational standpoint. It is hardly necessary to point out that Physical Measurement is a science of great practical importance. The nice adjustments of the different parts of a machine would, for instance, be impossible without accurate measurements. Success in Chemistry, in Astronomy, in Surveying, in fact in all branches of Civil and Electrical Engineering, depends to a great extent upon a thorough understanding of the Principles and Methods of Physical Measurement.

CHAPTER I.

GENERAL DEFINITIONS.

§ 1. **Nature of Measurement.** — Measurement consists in finding out by observation how many things of one sort correspond in magnitude to a given number of another sort. When 10 spaces on a measure divided into inches are found to reach through the same distance as 254 spaces on a millimetre scale, the length of the inch is said to be measured in millimetres, and conversely the millimetre may be said to be measured in inches. Either the millimetre or the inch may be used as a standard of comparison. When a quantity of known magnitude is compared with one of unknown magnitude, the latter is said to be measured in terms of the former. Thus, if a load is found to be equal in weight to a given number of grams, its weight in grams is said to be measured. It is obviously impossible to compare, in general, magnitudes of different sorts, — as, for instance, length and volume; but under certain circumstances, correspondences or relations exist between such quantities. When a stream of water, for instance, striking an obstacle with a velocity between 2 and 3 miles per minute is found to warm itself 1 Fahrenheit degree, a certain relation between temperature and velocity is said to be established. Such relations are properly objects of physical measurement. Measurements are either relative or absolute (§ 8), and may be classed, accordingly, as comparisons or determinations.[1]

§ 2. **The Metric System.** — The metric system is now generally adopted in scientific work. It is so called from the metre, or standard of length upon which it is founded (§ 5). The metre is equal to about 39.37 English inches. A cubic metre of ice-water weighs 1 "tonne" (1,000,000 grams) or 2205 lbs. nearly. There are, accordingly, 15.432 grains, or about 15 drops of water in one gram (§ 6). In the metric, as in other systems, the unit of time is the second (§ 7). The chief advantage of the metric system consists in the simplicity of the relations which exist between the standards of length and mass, and in the use of units each of which is some decimal multiple or sub-multiple of the others in the same series.

These units are distinguished, in the metric system by the aid of prefixes, which have the following significations: *mega*, one million, *kilo*, one thousand; *hecto*, one hundred; *deka*, ten, *deci*, one tenth; *centi*, one hundredth; *milli*, one thousandth, and *micro*

[1] The word "absolute" must not be confounded with the word "exact." Measurements are said to be "absolute" only when *fundamental* standards or units are employed (see § 8). We speak of the measurement rather than the determination of *variable* quantities, as for instance the strength of an electric current We speak also of the measurement of *accidental* quantities, like the length or weight of a body, especially when, as in measurements of length, *direct* methods can be employed. (See Chap. III.) On the other hand, a magnitude is said to be "determined" rather than "measured" by an *arbitrary* scale, and measurements of *invariable* quantities, like the physical constants. are customarily called "determinations"

one millionth. Thus a kilometre means a thousand metres; a microvolt a millionth part of a volt. When the unit begins with a vowel, the last vowel of the prefix is generally omitted; thus a million ohms is called a megohm.

§ 3. **Relative magnitudes.** — There are certain quantities which can be defined without reference to any particular system of measurement, such for instance as include simply a ratio between two things. Thus specific gravity is the proportion which the weight of a substance bears to that of an equal bulk of water; specific heat the proportion of heat it absorbs as compared to that absorbed by an equal weight of water; and specific electrical resistance is sometimes, though not generally, used in a similar sense.[1] Again, strains are defined as the proportion of the distortion which is produced to the whole quantity acted upon. Thus if a body has been stretched or sheared by an amount equal to $\frac{1}{100}$ of its length, or compressed by $\frac{1}{100}$ of its volume, it is said to have suffered a strain of $\frac{1}{100}$. Angles too are determined[2] by the ratio of the arc which they subtend to the radius; and the sine, cosine, or tangent of any angle[3] is simply the ratio between two of the three sides of a right-angled triangle in which the given angle occurs. Another instance is the index of refraction, or ratio of the velocity of a wave outside of a medium to its velocity in it. It

[1] See Experiment 88; also Trowbridge, New Physics, Experiment 120.

[2] See Table 3, columns *a* and *c*.

[3] See Table 3, columns *b*, *e*, and *f*.

is clear that when only a ratio is concerned, the results from all systems must agree.

§ 4. **Scale of Temperature.** Our present scale of temperature, though recently introduced, is equally independent of any particular system of units by which other physical quantities are measured.

The temperature of melting ice is defined as 0° on the centigrade scale; that of condensing steam as 100° under a standard atmospheric pressure, or that which sustains at Paris a column of mercury 76 *cm.* long, and at 0°.[1] At other points temperature is measured provisionally by the indications of a mercurial thermometer made of ordinary glass, the tube being divided into 100 parts of equal capacity between 0° and 100.°

It is assumed that a thermometer reaches, after a time, the same temperature as the bodies with which it is in contact.[2]

§ 5. **Unit of Length.** — The unit of length adopted in nearly all scientific work is the centimetre, or hundredth part of the length, at 0° centigrade, of a standard metre still preserved in the French Archives. This metre was intended to be the ten-millionth part of the distance along a meridian from the equator to the poles, but it was made about $\frac{3}{4}$ of a millimetre too short, the earth's quadrant being now supposed to lie between 10,007 and 10,008 kilometres; being, moreover, subject to shrinkage, though the amount has never been measured. The only absolute determination of the centimetre which we possess is in

[1] See § 5 below; also Table 14.

[2] For a further discussion of temperature see § 74.

wave-lengths of light. It contains, for instance, 16,972 waves of sodium light in air.

§ 6. **Unit of Mass.** — Our unit of mass is the gram, or thousandth part of the standard kilogram of the French Archives, which was intended to be equal to the weight in a vacuum of a cubic decimetre of distilled water at its temperature of maximum density (very near 4° centigrade). In addition to the error in the metre already noticed, the standard kilogram was made about 13 milligrams too light; but if this is taken into account, the gram can easily be reproduced from a given standard of length which has been compared either with the original metre or with wave-lengths of light. (See § 152.)

§ 7. **Unit of Time.** — The unit of time which we use is the second, of which there are 86,400 in a mean solar day. The second depends therefore on the rotation of the earth with respect to the sun. As no change has been detected in the rotation of the earth by comparing it with other astronomical motions, the second would seem to be practically constant. In one second, sound passes through 33,220 centimetres of dry air at 0° centigrade; light through 30 thousand million centimetres of empty space, as nearly as we can tell. From any of these data the second could be reproduced independently of the rotation of the earth.

§ 8. **Absolute System.** — The system followed in this work is that recommended by the British Association, and is known from its fundamental units as the centimetre-gram-second system, often abbreviated C. G. S.

The three units of length, mass, and time are called fundamental, because all other units of this system are derived from them; and they may be called absolute, because they can be reproduced (without the use of any standard) from the general properties of such universal substances as salt, water, and air. It is in this sense only that any system of measurement may be called absolute.

§ 9. **Surface, Volume, and Density.** — Surface or area is measured in square centimetres; volume or capacity in cubic centimetres; density in grams per cubic centimetre. Density in general is defined as the ratio of mass to volume. (See § 154.)

§ 10. **Velocity.** — Velocity is expressed in centimetres per second. It is well to remember that a velocity of one hundred centimetres per second or one metre per second corresponds to a very slow walk, only a little over two miles per hour. It is incorrect to speak of a velocity of so many centimetres, or of so many miles. A railway train may move at the rate of one mile per minute, while a steam roller makes only one mile per hour. Both the distance traversed and the time occupied in so doing are necessary to specify a velocity.

§ 11. **Acceleration.** — Acceleration is defined as the rate of change of velocity,[1] or the change of velocity per unit of time. If a steamer starting from a wharf acquires in one minute a velocity of three miles per hour, in two minutes a velocity of six miles per hour,

[1] For a discussion of what is meant by a change of velocity, see § 105.

in three minutes a velocity of nine miles per hour, etc., increasing its velocity every minute by three miles per hour, we should say that its acceleration amounts to three miles per hour per minute. It would be incorrect to speak of its acceleration as three miles per hour, for a horse and carriage might acquire the same velocity in one second.

It is necessary to state not only the magnitude of the velocity acquired but also the time it takes to acquire it. Since velocity is measured in centimetres per second, and time in seconds, acceleration is expressed in centimetres per second per second. The repetition of the words "per second" in scientific works is not therefore, as is commonly supposed, simply a printer's favorite mistake.

§ 12. **Force.** — The dyne or unit of force is defined as that force which acting on a gram for a second would give it a velocity of one centimetre per second.

A dyne is almost too small a force to be felt. It may be thought of as the weight of a piece of very thin tissue-paper a centimetre square; meaning by weight the force with which, for instance, it presses against the hand. In the same sense a drop of water weighs from 50 to 100 dynes; a man from 50 to 100 millions of dynes.

The dyne can be best represented by means of a delicate spring-balance. The weight of a gram in latitude 40°–45° is shown by such an instrument to be about 980 dynes; at the equator, however, it is only 973 dynes, and at the poles nearly 984. The weight at the centre of the earth would be nothing.

On the other hand a given number of dynes as above defined always stretches the balance to a given mark, whether at the equator or at the poles. Hence we say that the weight of a gram varies,[1] but the dyne, in terms of which we measure it, remains always the same. Force in general is measured as the product of mass and acceleration. (See § 106 and § 153.)

§ 13. **Couple.** — The unit couple is a force of 1 dyne acting on an arm 1 centimetre long, at right angles to it, with an equal and opposite force at the other end of the arm. A couple consists in general of two equal forces acting in opposite directions, not in the same straight line but in two parallel lines, and is measured by multiplying together *either* force in dynes by the arm, or perpendicular distance between the two lines of action. Anything which can twist a body or make it spin contains a couple; anything which can push it or pull it or shove it to one side contains a force. All motions originate either in forces or in couples or in combinations of forces and couples. (See § 113.)

§ 14. **Work.** — The unit of work is the erg, defined as the amount of work done in moving through a distance of one centimetre against a resistance of one dyne. It makes no difference how long it takes to complete the motion; but we assume that there has been no gain or loss of velocity on the part of the

[1] By the weight of a gram is here meant the varying force with which gravity attracts it. This is the proper signification of weight. Some writers, however, use weight in the sense of mass, or quantity of matter. The mass of a gram is by definition constant. See "Elementary Ideas, etc.," by E. H. Hall (published by Sever, Cambridge).

moving body, since that would also have to be taken into account. (See § 121.) Work in general is measured as the product of the force in dynes, and the motion in centimetres; considering of course only the effect or component of the force in the direction of the motion. (See § 119.) When the force acts on a body in the direction in which it is moving, it is said to do work upon the body; when the force opposes the motion, the body is said to do work against the force.

Those who have been accustomed to measure work in foot-pounds (multiplying the motion in feet by the number of pounds which have been raised), may notice that the erg or dyne-centimetre naturally replaces the foot-pound in a system in which all forces are measured in dynes and all distances in centimetres.

While three hundred foot-pounds in England are the same thing as three hundred and one foot-pounds in Brazil, the erg has one great advantage in that it is the same all the world over. Ten million ergs are sometimes called a joule.

§ 15. **Power.** — The practical unit of power is the watt, or ten million ergs per second. A man can easily do the work of 100 watts. One horse-power is rated at 746 watts. It takes about 4.166 watts to generate, through friction, one unit of heat per second. (See below.) A common paraffine candle is equivalent in heating power to 60 or 70 watts; 10 or 12 candles represent a horse-power.

§ 16. **Unit of Heat.** — The unit of heat is the quantity required to raise a gram of water from 0° to 1°

centigrade. It takes about forty-two million ergs to bring this about; more exactly, 41,660,000; hence this number is said to represent the mechanical equivalent of heat. Other substances take more or less (generally less) heat than water to raise 1 gram of them 1° in temperature, and more or less work in proportion. This proportion determines the specific heat of the substance in question. (See also § 86.) Specific heat is strictly defined as the number of units of heat necessary to raise 1 gram of a given substance 1° in temperature.

§ 17. **Unit of Magnetism.** — A unit quantity of magnetism is one which attracts or repels an equal quantity at a centimetre's distance with the force of 1 dyne. There are two kinds of magnetism, positive and negative. Two positives or two negatives repel each other, while positives and negatives attract.

§ 18. **Unit of Electrical Current.** — The absolute C. G. S. unit of electrical current is one which in flowing through a centimetre of wire acts with a force of 1 dyne upon a unit of magnetism, distant 1 *cm.* from every point of the wire.

§ 19. **The Ampère.** The practical unit of current is the ampère or tenth of an absolute unit. A common quart Daniell cell will give a current of about 1 ampère under favorable conditions.

§ 20. **The Ohm.** — The practical unit of resistance is the ohm. It was intended to be the electrical resistance of a wire in which a current of 1 ampère would generate in one second an amount of heat equivalent to 10,000,000 ergs. That is, an engine of 1 watt power would keep up a current of 1 ampère through such a resistance. In point of fact the standard ohm prepared by the British Association is a little more than 1% too small, and as this error has been kept in our copies, we have to allow for it in our calculations.

The ohm may be remembered as the resistance of about fifty metres of copper wire 1 *mm.* in diameter, or as that of a column of mercury 106 *cm.* long and 1 *sq. mm.* in cross section. The value of the latter resistance at 0° is adopted in France and elsewhere as the legal definition of the ohm. The liquids of a quart Daniell cell usually offer a resistance of about 1 ohm.

The resistance of a conductor in general is numerically equal to the power necessary to maintain a unit of current through it.

§ 21. **The Volt.** — The practical unit of electromotive force is the volt, or that which is required to maintain a current of 1 ampère through a resistance of 1 ohm. A Daniell cell has an electromotive force of about 1 volt.

Electromotive force in general is defined as the ratio of the power (§ 15) to the current. We have seen that it takes one watt to maintain a current of 1 ampère through a resistance of 1 ohm; and that it takes 1 volt to do the same. It will not do to conclude that one volt is the same thing as one watt; two volts will keep up a current of two ampères through one ohm, but four watts will be required. Electromotive force corresponds not to power but to hydrostatic pressure. (See §§ 137–139.)

§ 22. **Intensity.** — There are various other terms a definition of which might be useful here, but it has been thought better to explain each as the necessity arises. The use of the word "intensity" in the sense of concentration is, however, important. By intensity is meant the proportion of one quantity per unit of some different quantity. The force in dynes (about 980) with which gravity attracts each gram of matter is sometimes called the intensity of gravity. Intensity of pressure, generally called simply *pressure*, is expressed in dynes per square centimetre, corresponding to the ordinary use of pounds per square inch. The pressure of the atmosphere is, for instance, about one megadyne per *sq. cm.*, averaging in this latitude about 1.3% more than this. Intensity of stress, or simply *stress* is measured in the same units; as when we say that steel bars break under a stress of eight thousand megadynes per *sq. cm.* In the same way intensity of illumination ought to be expressed, not as it often is, in candle power, but in candle power *per square centimetre* of surface illuminated. Intensity should always be distinguished from quantity in this way. Like *rate* with respect to time, or the word *per*[1] with respect to quantities in general, intensity signifies a ratio or proportion.

[1] Everett's Units and Physical Constants, page 10.

CHAPTER II.

OBSERVATION AND ERROR.

§ 23. **Coincidence.** — Almost every physical measurement involves the reading of a scale of some sort, by means of what may be called an index or pointer. Temperature, for instance, is measured by a thermometer, consisting of a tube of glass with a scale marked upon it, let us say in degrees, and an index of mercury or some other liquid moving up and down the tube. Aneroid barometers, pressure-gauges, clocks, compasses, and galvanometers are read by a hand or pointer of some sort moving over a dial. An ordinary balance has an index, and a small scale behind it to show, when the weights are nearly adjusted, which pan is the heavier, and how much. Spring balances are read by the position of a small index. When the length of a body is measured by the scale on a metre rod, one end of the body is used as the index; or, again, a mark on a sliding scale is used as an index with respect to a fixed scale, and conversely. The above list contains a small part of the various instruments used in physical measurement; but a great part of those from which numerical results are actually obtained. Most observations therefore consist in reading scales of various sorts, by noticing the point with which the index apparently coincides.

The coincidence of two objects *in position* may be determined with great delicacy by the touch, or the coincidence of two sounds *in time* by the ear; but most observations relate to the coincidence or agreement of two phenomena *both in space and in time*, and can be made conveniently only by the eye.

§ 24. **Classification of Errors.** — It is obvious that mistakes are likely to arise in observation, as when we take a figure 3 for a figure 8; but mistakes of this sort should be distinguished from errors proper. A reasonably small error is more likely than a large one; but a mistake in the thousands is as probable as in the units. (See § 156.)

Errors may be divided into two classes: constant errors, or those which always tend to increase or to diminish a result by a definite amount; and accidental errors, or those which tend sometimes to increase it and sometimes to diminish it. Constant errors can be allowed for if we have sufficient information about them; but no correction can be applied for accidental errors.

For instance, in measuring length, the temperature of a tape, the moisture which it may have absorbed, the strain upon it, and the curvature of the surface measured, all affect the result. It is impossible to predict whether the temperature will be higher or lower, the dampness greater or less, the strain more or less intense than when the tape was graduated. We study accidental errors as we would combinations of "heads and tails" in tossing coins. No result is entirely free from them. Their influence may be indefinitely reduced (§ 46), but never completely eliminated.

Errors may further be distinguished into three classes: first, errors of observation (§§ 25–30); second, instrumental errors (§§ 31, 32); and third, errors of inference (§§ 33, 34). The various methods of avoiding errors of observation are considered below in connection with the sources from which they arise, the commonest of which are as follows: uncertainty in a point of view (§ 25), the coarseness of a scale (§ 26), the minuteness of the object observed (§ 27), the necessity of observing two different things at the same time (§ 28), the unequal rates at which different sensations are transmitted (§ 29), and the effect of mental impressions (§ 30).

§ 25. **Parallax.** — In many scales where the index is between the graduation and the eye, the apparent position of the pointer is affected by the point of view. The index seems to *slide along* the scale as the eye moves from one end to the other. This phenomenon is called *parallax* (from *παρά*, along, and *ἀλλάσσω*, to alter). Clearly to avoid errors from parallax, the eye must be held in a fixed position so as, for instance, to look perpendicularly upon the scale. To this end one of the simplest devices is to use a mirror parallel to the scale and behind it if possible. The eye is placed so as to see its own reflection in the mirror in the direction of the pointer; in this case the line of sight must be perpendicular to the scale.

§ 26. **Estimation of Tenths.** — One may readily distinguish in most cases whether the pointer apparently coincides with a certain mark on a scale, or with the space between two marks; but this is by no means the limit of the eye's accuracy. If the pointer falls between two marks, it is generally possible to decide whether it is half-way between them, or nearer to one than to the other. In other words, the eye is accurate to fourths. It is, in fact, possible to imagine the space between two marks in an ordinary scale divided into at least ten parts, and to decide correctly in the majority of cases in which of these parts the pointer lies.

Fig. 1.

The ten diagrams in Fig. 1 show the relative positions of a pointer dividing the space between two marks into various proportions, the figures indicating the number of tenths to the left of the pointer in each case. A close study of such diagrams will in a short time justify the division of spaces into tenths by the eye. It is assumed henceforth that in the case of any index and scale under favorable conditions, the reading is expressed in tenths of the smallest divisions. The estimation of tenths is not confined to the eye. It will be found that the ear is equally reliable. Thus the time between two ticks of a clock can be divided into tenths, so that the occurrence of a sound can be determined with practice to a tenth of a second.

§ 27. **Mechanical Devices.** — When a space or line is too small to be seen we generally resort to a lens or microscope, as in Experiment 19; but there are various other devices to measure small distances. One of the most delicate tests of the adjustment of the four points of a spherometer to the same plane is the noise made by rocking the instrument from side to side, (see Experiment 20), and an electrical contact is sensitive to a change of distance which the eye fails to see (see Experiment 65). The motion of the top of a vacuum chamber in an aneroid barometer is magnified by a system of levers, and finally by a chain passing round a small axle so as to render the smallest motion perceptible. When a motion is too rapid to be seen by the naked eye, we may still often observe it through some optical device. An instantaneous view, for instance, will show the body as if at rest, and in the case of periodic motion a series of instantaneous views may give it an apparent motion so slow that it is easily observed (see Experiment 51). Again motion may be made to record itself by marking on a moving surface. The vertical motion of a barometer is thus recorded by means of a pen on a piece of paper moving by clockwork horizontally beneath it. This method is called graphical. Any instrument which moves uniformly so that time can be accurately recorded in this way is called a chronograph, literally a *time-writer* (from χρόνος, *time*, and γράφω, *to write*). A chronograph can be used to record the vibrations of a tuning-fork, even one which emits the highest or fastest audible note.

Similar results can be obtained when the pen is not moved directly by the tuning-fork or moving body, (see Trowbridge, New Physics, Experiment 155), but indirectly through the aid of electricity, and various electrical devices may be employed to magnify the effects of small intervals of time, and thus detect the smallest variation from coincidence (see ¶ 147). Optical, Graphical, and Electrical Devices include the principal methods of aiding observation.

§ 28. **Use of Two Senses.** — When we wish to observe two things in different places at the same time we often resort to the use of two senses. The Eye and Ear method[1] consists, for instance, in the use of the eye to watch one moving body while the ear listens for the occurrence of a sound defining the motion of another.

This is the method by which one ordinarily compares his watch with a striking clock or with a noon gong. The sense of touch is used by the engineer to help him count correctly the revolutions of a wheel without looking off his watch, and a variety of methods can be devised by which two or more senses bring together from different sources a knowledge of what is taking place at different places at a given time. The use of two senses often obviates the necessity of employing complicated mechanical devices.

§ 29. **Personal Equation.** — It is generally found that the eye is quicker than the ear to report what is taking place, but the difference is greater in some persons than in others. Thus if two persons were

[1] See Pickering's Physical Manipulation, § 15.

to estimate at what time the report of a cannon is heard, one would tend always to return figures greater than the other, let us say by several hundredths of a second. Such a difference, however small it may seem, might seriously affect a determination like that of the velocity of sound, and is a perpetual source of annoyance in astronomy. The allowance which each *person* must make to produce results *equal* to the true or average result is called his *personal equation.* It is not specially considered in this course of measurement, being eliminated together with what is called "zero error," as explained in § 32.

§ 30. **Effects of Anticipation.** — One of the most dangerous sources of error in observation lies in the habit of anticipating results. Experience shows that under the influence of a strong expectation, the eye is not only incapable of estimating fractions correctly, but that it becomes blinded to gross errors, — pronounces weights, for instance, equal when the balance-beam is not free to move; reads sixty-odd centimetres instead of seventy-odd, several times in succession. It is sometimes necessary to prepare one's self by calculating beforehand — particularly in astronomy — the values which one expects to observe; but independence of observation is obtainable only in ignorance of the meaning of the indications which one records, and particularly in ignorance of the fact whether the values obtained are likely to be too great or too small.[1]

[1] The teacher may amuse himself at the expense of his class by determining the effects of "gravitation" towards various values which he may choose to suggest.

For these reasons the following rule will be found useful: Take your observations first; second, give a copy to some one else; third, reduce them; fourth, report the result; and fifth, inquire what values others have found.[1]

§ 31. **Instrumental Errors.** — Without any fault on the part of the observer, errors often arise through the imperfections of the instruments which he employs. These may be divided into two classes: first, errors of adjustment, as when two parts are not exactly parallel or perpendicular; and second, scale errors, for instance, irregularities in a graduated rod or in a set of weights.

The various tests which have been devised to correct errors of adjustment will be described in connection with the several instruments to which they belong. Scale errors may arise either from a change in, or from the original misplacement of, certain fixed points; like the "freezing" and "boiling" points of a thermometer, or from inaccurate calibration. They are avoided in general as explained in § 36. The commonest error of this sort is a misplacement of the zero of a scale.

§ 32. **Zero Error.** — When the greatest care has been taken to read one end of a scale correctly, an error often arises because the other end is out of adjustment. The graduation of a tape measure seldom begins at the ring, and yet it is common to see

[1] The examination of substances whose composition is known only to the teacher — or to the apothecary — will afford a sufficient opportunity to test the application of this rule.

distances measured by professional mechanics as if this were the case. It is always well, even when no error of this sort is suspected, to confirm an observation by taking two others, the difference between which should agree with a previous result. Thus the length of a pencil might be found by laying it along the middle portion of a metre-rod instead of making one end of it even with the rod, and in this manner, even if the end of the rod were worn away or broken off, the true length of the pencil would be discovered. This is called the method of difference.

The error due to the inaccuracy of the beginning or zero of a scale is called zero error, and it is necessary to guard against such errors in general. It should be borne in mind that every measurement, like that of length, depends upon at least *two observations*, or their equivalent; and that the accuracy of one is just as important as that of the other. However evident it may seem to be that if the quantity which is being measured were taken away, the index would point to zero, it is continually necessary to test the truth of this fact. The balance when both pans are empty, from a slight dislocation of one of the knife-edges, often tends to one side; springs do not always return to their original length after stretching, owing to a permanent set; galvanometer-needles do not always point north and south when the current is cut off, — a bunch of keys may perhaps account for the variation.

§ 33. **Errors of Inference.** — One must distinguish carefully between what he sees and what he infers.

It would be impossible to state any general principle by which errors of inference may be avoided; but in order to correct them, it is often necessary to refer to the original observations from which the inferences have been drawn. Hence the necessity of preserving the records, however rough in form, made at the instant when a given phenomenon occurs. The turning-points of an index should for instance be recorded, and not simply the position where it is *inferred* that the pointer will come to rest; or, if at rest, its actual position should be noted, not the weight which one *infers* would produce an exact adjustment. Again, the reading of a standard English barometer should be written down first in inches, and afterwards reduced to centimetres.

In addition to the observations necessary to a given measurement, every circumstance should be noted which may have a possible influence on the result. The appearance of air-bubbles, in hydrostatics, may, for instance, determine the relative accuracy of different weighings. The *time* of an experiment enables us to supply the barometric pressure, roughly, at a later date, by consulting a weather report. An exact description of *place* may furnish a subsequent clue to the magnetic deviation. We must also be able to *identify the instruments* which we have used, if we would confirm the inferences drawn from their indications. In fact, the severest test of a laboratory note-book must occasionally be applied, namely, one's ability to repeat with it a measurement from beginning to end.

It is important to the clearness of one's notes to enter actual observations in one place and calculations in another. Errors in reasoning are almost always due to confusion in regard to the nature of the quantities dealt with. The student should learn from the first to *write opposite each number what that number represents.* Every figure necessary to the calculation of a result should be preserved for future reference, — even those which enter, for instance, into ordinary multiplication or division. In calculation, as in observation, corrections are most easily made in those records which are most complete.

§ 34. **Logical Analysis.** — The use of logical analysis for the purpose of discovering unknown sources of error is seldom dwelt upon by writers on physical measurement. It is, however, obvious that the reduction of results may be thrown into the form of a demonstration; and after errors of observation have been allowed for, if the reasoning is correct, unknown errors must lie in the assumptions. It is, therefore, important to determine what these assumptions are.

Thus in the case of a Nicholson's hydrometer we reason that since the weight required to sink it to a given mark is, let us say, 30 grams at 10 o'clock without a load, and 10 grams at 11 o'clock with a load, *assuming that a given weight always produces a given result*, the apparent weight of the load must have been equivalent to that of 20 grams, according to the set of weights.

Both theory and experiment show that the assumption is true only when the temperature of the water is constant and when various other conditions are fulfilled. Changes in quantities which we unconsciously assume to be constant are a frequent source of error in physical measurement.

CHAPTER III.

GENERAL METHODS.

§ 35. **Methods of Trial and Approximation.** — The ordinary method used in the arts for testing the diameter of a wire is to fit it into a series of slits, each narrower than the one before it, until one is found which the wire cannot be made to enter. A series of trials, *systematically arranged*, leads very quickly to the desired result. The trials are of course limited in practice to a set of slits of *about* the same width as the wire. The first trial should be made with one near the *middle* of such a set; for if this slit be two small, little time is lost, while, if it be too great, only half of the set remains to be tried. In any case, we find out which half contains the slit fitting the wire. The second trial should be made about the middle of this half. A quarter of the original set then remains to be tried. A third trial is made near the middle of this quarter, &c.[1] By thus continually halving the limits between which an unknown quantity has been found to lie, its precise value may be determined with the smallest possible number of trials.

In certain cases, we have no clew whatever to the magnitude of the quantity which we desire to meas-

[1] 10 halvings reduce a quantity in the proportion 1024 : 1; 20 halvings reduce it in the proportion 1,048,576 to 1.

ure.[1] A bad electrical connection may, for instance, amount to a small fraction of an ohm (§ 20), or to several million ohms. We begin, therefore, by comparing it with a standard which comes in the *order* of its magnitude, as expressed in the decimal system, about half-way between the extreme limits within which measurement is possible. With an apparatus capable of measuring resistances from 1 to 1,000,000 ohms, we should first try, for instance, 1000 ohms. If 1,000 were too great, we should next try 10 ohms; and if this were too small, 100 ohms. Very few trials are usually required to determine the order of magnitude to which any measurable quantity belongs.

When the result of a given trial can be anticipated, this trial is needless, and should be omitted from the series which would otherwise be made. We begin, for instance, by comparing an unknown weight with a standard as nearly equal to it as possible. Then a second standard or combination of standards is tried. A good practical rule is to try weights in their *order of magnitude*,[2] each weight in a set being generally about half or twice as great as the one next above or below it. If the first estimate be reasonably close, the result of following this rule will be *probably* to turn the balance. It is evidently useless to make

[1] If there is any doubt whether the apparatus which we employ is capable of measuring the unknown quantity, it is well to compare this quantity at the start (1) with the smallest and (2) with the largest available standard. A reversal of the indication of an instrument obtained in this way is valuable, because it shows that the instrument is in working order and that a measurement can probably be made.

[2] See Pickering's Physical Manipulation, vol. i., page 48.

changes in weight which are *certain* to turn the scales. If, accordingly, two weights appear by any chance to be nearly balanced, a much smaller change should be made.

The method of trial employed in weighing is essentially the same as that used in finding the diameter of a wire. When an unknown weight has been found to lie between two limits, in the absence of any indication which limit is the nearer, we try a weight as nearly half-way between these limits as convenience will allow. To avoid, however, complicated combinations of a set of weights, we follow this rule only in so far as may be possible by the addition of one weight at one time or by the substitution of one weight for another (see Exp. 1, ¶ 2). A similar method is employed with a set of electrical resistances (Exp. 86).

A great many physical instruments show only which of two quantities is the greater, without indicating how great the difference is between them. The best results are obtained with such instruments by the methods of trial described above. When, however, it is possible to calculate approximately the magnitude of an unknown quantity from the results of one or more trials, this method may be greatly shortened. Thus, by observing how much the temperature of a mixture is lowered by cooling one of the ingredients a certain number of degrees, we may calculate roughly how many degrees this ingredient must be warmed or cooled to bring about any desired temperature in the mixture (see ¶ 99, I.) A series of trials may be arranged in this way so that each is much closer than the one before it. This is called the "method of trial and error," or the "method of successive approximations" (Pickering, Physical Manipulation, vol. i., page 10).

§ 36. **Methods of Graduation and Calibration.** — (1) PRODUCTION OF A SET OF STANDARDS. The purposes of physical measurement frequently require the production of a set of standards, each of which must be an accurate multiple of a given unit. Let us first suppose that a suitable standard unit can be obtained. The first step is to make an accurate copy of this unit. This requires the aid of some instrument capable of detecting the slightest difference between two quantities (§ 42). With such an instrument, the copy is made as nearly as possible like the original by the method of trial and error (§ 35). Let us call the original A, and the copy B. The two are then combined, and by the aid of the same instrument two standards, C and D, are prepared, each equal to the sum of the standards A and B, — that is, $2\,A$, nearly. There are then two ways of producing a standard E equal to $5\,A$. We may combine C, D, and A; or C, D, and B. The former is preferred because, in employing the original standard A, instead of a copy of it, there is one less chance of error; see (4). By combining A, C, D, and E, two standards, F and G, may be produced, each equal to $10\,A$, nearly. There are, then, two ways of making a standard, H, equal to $20\,A$. One way is to combine F and G, the other is to combine one of these — F, for instance — with A,

C, D, and E. The latter is preferred because it makes use of the sum of the standards (A, C, D, and E) instead of a copy of this sum; see (4). In a similar manner, we may prepare standards of the magnitudes 50 A, 100 A, &c.

Let us now suppose that a suitable standard unit cannot be obtained, and that the only available standard is some multiple of this unit, as for instance 1000 A. We then assume a provisional unit of any magnitude, x, and construct a series of provisional standards, of the magnitudes 2 x, 5 x, 10 x, &c., until we reach a value as great as the given standard. Then by the method of trial (§ 35) we find how many provisional units are equal to this standard. The values in the provisional series are now known; and by making and copying the proper combinations of this series, we may construct a series of standards which are more or less accurate multiples of the standard unit which we desire to represent.

It would be out of place to consider here the mechanical operations by which graduated scales and circles are produced. Standards must in general be subjected to a series of tests, as will be explained in (2) and (3).

(2) Testing a set of standards. The construction of a set of standards may be considered as a first step toward the accuracy of results; but no matter how carefully such a set may be prepared, it is almost always possible to detect a difference between any two combinations of nominally the same value. It is generally easier to measure and allow

for such differences than it is to avoid them. A set of standards may accordingly be tested by a series of comparisons involving essentially the same combinations as those employed in processes of construction; see (1). Instead, however, of comparing H with $A + C + D + E + F$, we should in practice compare it with $F + G$, since the latter combination $(F + G)$, being more frequently employed, — see (4), — needs to be known with greater precision. We prefer, in fact, tests involving the use of the smallest possible number of standards.

In addition to a series of comparisons by which we may determine the relative values of different standards in a set (see Exp. 7), either the sum of the set or one or more of the larger standards which it contains should be compared with some standard of known value.

(3). Calibration. Variations in the bore or "calibre" of a tube may evidently give rise to errors in the estimation of its contents by means of a scale attached to the tube. Any process by which such errors may be eliminated is properly called "calibration" (see ¶¶ 68 and 71, Exps. 25 and 26). This term has, however, been extended to the correction of a scale of any sort.

To obtain accurate results with an ordinary scale of length, it is obviously necessary that all the intervals of a given nominal value should be equal, or at least that they should not differ from one another by a perceptible amount. A simple way to test the accuracy of a scale is to lay beside it another scale graduated in exactly the same manner. Let a, b, c, &c., represent the spaces on one scale, and a', b', c', &c., those on the other scale, and let us suppose that the division lines between these spaces are opposite one another. Then $a = a'$, $b = b'$, $c = c'$, &c. The first scale is then to be moved along so that a may come opposite to b'. If the division lines again come opposite, $a = b'$, $b = c'$, &c. Since in the first case $b' = b$, and in the second case $b' = a$, it follows that $a = b$, and in the same way all the intervals, a, b, c, &c., must be equal.

To test, accordingly, the uniformity of the millimetre divisions on a metre rod, we place two such rods side by side, then we move one of them along 1 *mm.* The equality of the centimetre spaces may be similarly established by moving one of the rods 1 *cm.*, and the decimetres may be tested by moving the rod 10 *cm.* It must not be imagined, because there is no perceptible irregularity in the millimetre divisions, that there can be none in the centimetre or in the decimetre divisions. If for instance, the first 100 *mm.* spaces on each rod were longer than the next 100 *mm.* spaces by $\frac{1}{100}$ *mm.* in each case, we should hardly notice the difference between them; but the first decimetre would be longer than the second by a whole millimetre. For a similar reason it is important to compare the two halves of a scale,—see (4),—the two quarters into which each half may be divided, &c. (see Exp. 24).

The relations between the magnitudes compared in testing a graduated scale or circle are, to a certain extent, the same as in the case of a set of standards; see (2).

When there is no other way of testing the relative values of different scale indications, we do so by measuring with the scale different quantities bearing known ratios to one another (Exp. 96); the scale may then be used for relative indications. Every scale which is to be depended upon for absolute results must be compared in one case at least with a standard of known absolute value.

(4) DIRECT AND INDIRECT PROCESSES. The correction of a scale or of a set of standards usually depends, as we have seen, upon a series of comparisons, each of which must introduce a certain chance for error in the result. Standards should evidently be compared *directly* with the originals which they are intended to represent, whenever it is possible to do so, rather than with copies of these originals. Again, the two halves of a scale should be compared *directly* with one another, not indirectly, by means of the spaces into which they are subdivided; see (3). Short and direct methods of comparison are always preferable, other things being equal, to long and indirect processes.

It will be seen from (1) that in certain cases the sum of several weights is more reliable than a single weight of the same nominal value. In general, however, each weight in a set is subject to a certain error, especially when the set has been copied from another set, or when the weights are worn or corroded. In such cases the chances for error in weighing increase in proportion to the number of weights which we employ. For this reason, as well as for convenience in manipulation, we make it a general rule to use as few weights as possible. Further illustrations of the principles which underlie this rule will be found in § 38.

§ 37. **Methods of Subdivision.**[1] — The subdivision of a scale or of a set of standards may be carried theoretically to almost any extent by ordinary methods of graduation (§ 36); but there is always a practical limit to the process. The smallest quantity actually indicated by a given instrument is called the "least count" of that instrument. Errors due to "least count" may easily arise. Their influence on a result may be lessened by methods of multiplication or repetition (§ 39) or by methods of "least error" in general (§ 38). It is, nevertheless, desirable that the "least count" of an instrument should be reduced to the smallest practicable amount.

Even with the best analytical balances, weights smaller than 1 milligram are seldom employed. The fractions of a milligram are usually estimated by means of a "rider" or small weight sliding along a graduated scale on the beam of a balance. It has been found similarly impracticable to make use of standards of electrical resistance less than one tenth of an ohm. Fractions of the smallest available standards are estimated in general by methods of interpolation (§ 41).

[1] References in this edition to the Method of Multiplication or Repetition should read § 39, not § 37.

In the measurement of length, there are certain methods of subdivision by which the least count of a scale may be greatly diminished without a proportionate increase in the number of divisions. Thus a centimetre scale 1 metre long, requires for its production 100 lines besides the zero; but if the first centimetre be divided into 100 parts, we may with 200 lines measure any length less than a metre to a tenth of a millimetre.

When this method of subdivision is employed, the application of corrections for errors in graduation (§ 36) is comparatively simple, since a given measurement can be made in only one way. We lose, however, the advantage which is sometimes gained by making measurements in different parts of the scale, and averaging the results (see § 46). For this reason there would be an obvious advantage in using a short movable scale very finely divided, in connection with a scale of centimetres. This principle has been applied in the construction of various sliding-scales or gauges. It is found, however, impracticable to read any scale with the naked eye unless the divisions are at least $\frac{1}{5}$ of a millimetre apart. The use of sliding scales was therefore very limited until Vernier showed how, by a slight modification in these scales, comparatively accurate results could be obtained. The divisions of a Vernier scale are made nearly but not exactly equal to one or more main-scale divisions.

A common form of Vernier gauge consists of a fixed scale in millimetres and a sliding piece with ten or eleven marks, each nine tenths of a millimetre from the next (see Fig. 2). The first of these, numbered 0, points out the reading of the instrument, in millimetres, upon the main scale, just as if there were no "vernier." It comes opposite a millimetre mark only when the reading is a whole number of millimetres. In this case the next mark on the vernier (No. 1), being $\frac{9}{10}$ *mm.* further on, falls $\frac{1}{10}$ *mm.* short of the nearest main-scale division; No. 2 falls $\frac{2}{10}$ *mm.* short, and so on. Hence if the sliding scale be moved along $\frac{1}{10}$ *mm.*, the mark No. 1 will come opposite a mark on the main scale (not the one nearest the zero of the vernier), and if the vernier is moved $\frac{2}{10}$ *mm.* along, mark No. 2 will be exactly opposite still

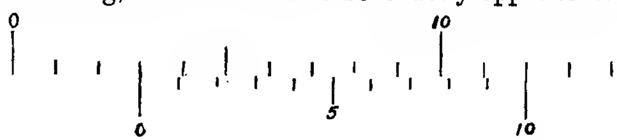


Fig. 2.

another mark on the main scale. In the same way Nos. 3, 4, 5, &c., will come opposite various marks in the main scale, when the vernier is respectively $\frac{3}{10}$, $\frac{4}{10}$, $\frac{5}{10}$, &c., *mm.* beyond the original position. Obviously we have only to find the number of the vernier line which is opposite a line on the main scale (no matter which) to determine the number of tenths of a millimetre between the zero of the vernier and the line just below it on the main scale.

The same principle holds in the case of any vernier. By a series of steps, easily counted, the spaces on the vernier gain or lose one space with respect to the main scale. The reading of the main scale is thus practically divided into as many parts as there are steps in the gain or loss of one space.

It often happens that in comparing the vernier and the main scale, no two lines are found to be exactly opposite, so as to form a single continuous line; instead, two lines are found, which, though nearly continuous, show, when closely examined, more or less dislocation. We then estimate by the eye the relative amount of dislocation in each case, and reduce the result as accurately as possible to decimals. Thus if in a vernier the third and fourth lines are equally dislocated, the reading is .35; if the third line is only

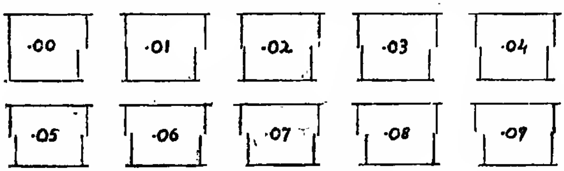


FIG. 3.

one fourth as much dislocated as the fourth, then the reading is .32. By reference to the diagrams in Fig. 3, it will generally be possible to express the reading of the gauge to hundredths of a millimetre, and with almost as much accuracy as if the vernier contained a hundred lines.

The use of a vernier for the subdivision of a scale is closely related to the method of coincidences (§ 40), and may be considered also as one of the various methods of interpolation (§ 41) by which fractions of the smallest available standards are customarily estimated.

§ 38. **Methods of Least Error.** — It is desirable in physical measurement that observations should be accurate; it is equally desirable that the conditions under which they are made should be favorable for the exact determination of results. There are certain general principles by which experiments are, when possible, arranged so that a given error in the observations may cause the least possible error in the result. Any method in which these principles are applied may be called a method of least error.

The advantages of direct methods of comparison have been already pointed out (§ 36). We prefer, in general, determinations which depend upon the fewest data, assume the fewest laws, and make use of the fewest and best-known physical constants. The present section is devoted especially to the relations which should exist between physical instruments and the quantities which they are used to measure.

The delicacy of most instruments is somewhat diminished by an increase in the magnitude of the quantities measured, but *not in proportion* to this increase. The best results are accordingly obtained with quantities nearly as great as the capacity of the instrument will admit. We employ, for instance, large quantities of a substance in determinations of specific gravity by means of a balance. On the other hand, it would be impracticable to measure accurately the weight of copper deposited (Exp. 81) on an electrode weighing several thousand times as much as the deposit in question; for a balance capable of weighing the electrode would not be sensitive enough for the deposit. While, therefore, it is desirable to increase the deposit of copper, the weight of the electrode should obviously be diminished. We avoid, in general, determinations of the difference between two nearly equal quantities depending upon observations of the quantities themselves. Such differences should be measured *directly* if possible (§§ 41, 42).

Some instruments are particularly adapted to measuring quantities of a given magnitude. A tangent galvanometer, for instance, gives the best results with electrical currents which deflect it 45°. Let us suppose that when three turns of wire are used, the needle points to 26°; with six turns, to 45°; with 12 turns, to 63°. An error of observation equal to + 1° would give 27° instead of 26°, 46° instead of 45°, and 64° instead of 63°. Now the results depend upon the *tangents* of the observed angles (see Exp. 78). The tangents of 26° and 27° differ (see Table 5) by about 4.4 %, and the tangents of 63° and 64° differ in the same proportion; but the tangents of 45° and 46° agree within 3.6 %. We should obviously employ 6 turns of wire in preference to 3 or 12.

In making selections or modifications of the instruments which we employ, we must consider, in general, the nature of the formulæ by which the results are to be reduced. It will be found, for instance, that a 1 % error in a quantity causes an error of about 2 % in estimating the square of that quantity but only about ½ of 1 % in the estimation of its square root (see § 57). We prefer, accordingly, determinations depending on roots rather than on powers of the quantities directly observed. The relative value of different determinations must be judged, not by the accuracy of the observations, but by that of the results.

The principles of "least error" may require, under certain circumstances, the use of the method of multiplication or repetition (see § 39), the method of coincidences (see § 40), or the method of reversal or interchange (see § 44).

§ 39. **Methods of Multiplication and Repetition.**[1] — It would be impossible to weigh a single drop of water very accurately on a coarse balance; but if we knew under what circumstances the drop was formed it might be possible to produce a thousand drops of almost exactly the same size, and by finding their combined weight to arrive at that of a single drop.

The error in measuring 1000 drops may not be perceptibly greater than in the case of a single drop, and since in the process of reduction this error is divided by 1000, we may obtain at least a comparatively accurate result. The use of any means for increasing the magnitude of a quantity in a given proportion for the purpose of finding a more accurate measure of that quantity constitutes in general a "method of multiplication." The value of such methods evidently depends on the accuracy with which a quantity may be reproduced as compared with the accuracy of a direct measurement.

[1] References in this edition to the Method of Graduation or Calibration should read § 37, not § 39

We may find, for instance, the weight of mercury required to fill a capillary tube by emptying the contents of the tube several times in succession into a vessel, in which the mercury is collected and weighed. The same method could not, however, be employed with water, on account of the considerable portion which sometimes adheres to the tube.

The method of multiplication is often used in the determination of times of vibration; for it may be proved mathematically (see § 111) that successive vibrations executed under certain conditions do not differ by a perceptible amount. The rate of a pendulum should accordingly be determined by a long series of observations. Such a series may be extended, by a system of mechanical counting, for days or even for months. There must evidently be no break in the series. The method of multiplication is applicable only to *consecutive intervals* in the measurement of time.

The method of multiplication is sometimes used for the estimation or detection of a series of small impulses given to a pendulum or to a vibrating needle at the middle point of a swing, so that the effects may be added together. A large allowance must sometimes be made for the effects of friction, or other causes tending to destroy the motion. For the "method of multiplication and recoil" see Kohlrausch, Physical Measurement, Art. 76.

The method of multiplication is applied in the construction and use of an ordinary galvanometer or "multiplier," the object of which is to increase the effect of an electrical current in a known or measurable proportion. Methods of multiplication are also applied in the measurement of length.

There are various mechanical devices by which a body may be moved in a straight line through successive distances, each equal (or nearly equal) to its own length. We have an example in the ordinary method of measuring distances with a rod or chain. This is, however, more or less inaccurate on account of the uncertainty of the marks which show where the ends of the measure are placed. One method by which greater precision may be obtained is to place a block end to end in front of a measuring rod, then to remove the rod, to place a second block behind the first, just touching it, then to remove the first block and to put the rod in front of the second block. This process is then repeated over and over until the length of the rod has been multiplied, or, as we say technically, "repeated," a sufficient number of times. By this means very long distances may be quite accurately measured even with a short millimetre scale. This and similar methods are properly called "methods of repetition."

Methods of repetition are frequently used in the measurement of angles. Let us suppose that a given angle, cut out of thin metal, reaches from the zero of a circle, graduated in degrees, to a point between 40° and 41°; and that by some method of repetition similar to that just described, the angle is found to reach from the last point (between 40° and 41°) to one between 80° and 81°, &c. We should obtain in this way a series of observations like the following: — 0°, 40° +, 80° +, 120° +, 160° +, 200° +, 240° +, 280° +, 320° +, 360° +, 401° — 441° —, &c. We see from any two successive observations that the angle must lie between 40° and 41°, but we have no means of estimating the fraction of a degree over 40. If however, we consider the first and last observations, we see that the angle must be less then $\frac{1}{11}$ of 441°, which gives $40\frac{1}{11}$ as the superior limit of the angle. In other words, the angle becomes known within $\frac{1}{11}$ of a degree. By considering two observations which differ by 360° (or any multiple of 360°) we escape from a great variety of errors by which the results obtained with graduated circles are apt to be affected. A method by which we may utilize, not simply the first and last, but nearly all of a series of consecutive observations will be considered in § 61.

§ 40. **Method of Coincidences.** — We have seen (§ 37) that some lines on a vernier come almost exactly opposite the lines nearest them on the main scale, while others do not. In the same way, when any two scales are compared together, cases of more or less approximate "coincidence" usually occur. Every fifth inch on an English scale coincides, for instance, as nearly as the eye can judge, with every 127th division on a millimetre scale. We should evidently prefer to calculate the length of the inch in millimetres from a case of perfect coincidence than from one where a given number of inches was found to be greater or less than a given number of millimetres by a fraction which could only be estimated by the eye.

The method of coincidences may be used with advantage to avoid errors due to "least count" (§ 37) in the comparison of any two sets of standards of the same sort, no matter what kind of physical quantity they represent. 11 Troy ounces happen, for instance, to balance 342 grams within a few milligrams. With two ordinary sets of weights, the smallest of which are 1 ounce and 1 gram respectively, it is possible accordingly, to find the value of the Troy ounce in grams within a small fraction of a milligram.

The most important application of the method of coincidences is, however, in the comparison of intervals of time. Let us suppose that two pendula differ slightly in their rates of oscillation, so that one gains slowly upon the other, and that they start together at a given point of time. After a certain number of oscillations have been executed by one of the pendula, the two will be swinging in opposite ways, and again after a given number of oscillations, they will be swinging the same way. The relative rate of oscillation may be accurately determined by counting the number of oscillations in question. If, for instance, the faster pendulum makes n vibrations between two successive coincidences, the slower pendulum must make $n - 1$; hence the relative rate is $n \div n - 1$. Let us suppose that through an error in observation $n + 1$ oscillations were counted instead of n; the relative rate would then be estimated as $n + 1 \div n$. The error committed would therefore be,

$$\frac{n+1}{n} - \frac{n}{n-1} = \frac{n^2-1}{n^2-n} - \frac{n^2}{n^2-n} = \frac{-1}{n^2-n}.$$

If n is moderately large such an error would be inappreciable.

§ 41. **Methods of Interpolation.** — We have seen that errors due to the "least count" of an instrument may be almost indefinitely reduced by the methods of multiplication, repetition, and coincidences (§§ 39, 40). Such methods cannot, however, always be applied. The value of an observed quantity, q, is usually found to lie between two limits, one A, the other $A + a$, where a represents the "least count" or smallest change which can be produced in a set of standards. That is, we have —

$$A + a > q > A.$$

If more precise results are required, we seek some instrument or indicator by which we may estimate, relatively at least, the differences between the quantity q and the two nearest values of the standards, A and $A + a$, with which we are able to compare it.

The sensitiveness of any instrument used as an indicator may be defined as the number of scale divisions by which its reading changes when the smallest possible change (a) is made in the standards. We will first suppose the sensitiveness to be known. Let the quantity q be compared with the combination of standards (A) just below it in magnitude, and let the indicator show a motion of x scale divisions. Then since s divisions correspond to the quantity a, we may infer that x divisions must correspond to x s^{ths} of a, hence the true magnitude of q is —

$$q = A + \frac{xa}{s}.$$

In the same way, if the indicator shows a motion of y scale divisions when the quantity q is compared with the combination of standards $(A + a)$ just above it, we have —

$$q = A + a - \frac{ya}{s} = A + \frac{(s - y)\,a}{s}.$$

By comparing this equation with the last, we see that x must be equal to $s - y$, or —

$$x + y = s.$$

The last equation enables us to calculate the sensitiveness of any indicator from two deflections, obtained as stated above. The value of s may vary according to circumstances. The special value here determined is the sensitiveness of the indicator to a change of the magnitude a in the quantity q. The process of estimating a quantity (q) from the relative differences (x and y) separating it from two magnitudes (A and $A + a$) between which it lies is called "interpolation" ("putting in between").

We have instances of the method of interpolation when, in the use of a Nicholson's Hydrometer (Exps. 2, 3, 4), the distances of a certain mark above or below the surface of the water are used to estimate fractions of a centigram, or when in the use of a vernier (§ 37), the relative dislocations of two lines are used to estimate hundredths of a millimetre. The vernier itself may be considered as one means of interpolation. The use of a "rider" (¶ 259) enables us to determine weights exactly by interpolation even if the weight of the rider be unknown. The indications of the pointer of a balance afford another means of interpolation in weighing (see ¶ 20). The deflections of a galvanometer are similarly used (see Exp. 93) to estimate small differences between two opposing electromotive forces which we seek to bring into equilibrium.

§ 42. **Null Methods.** — Most physical quantities cannot, like scales of length, be directly compared with one another, but are measurable only through the effects which they produce upon some instrument. Electrical currents, for instance, are usually determined by their action upon the needle of a galvanometer. When two effects lie in the same direction, they are generally compared by the method of substitution (§ 43). It is, however, frequently desirable to *oppose* two effects, especially when they are nearly equal, in order that the difference between them may be directly measured (see § 38). In weighing with a balance, the effects of two nearly equal weights upon the instrument are thus opposed. Any method by which two effects may be made to neutralize or *annul* each other may be called a *null method*.

In electrical measurements, the term "null method" is usually applied to cases where two equal electromotive forces are opposed to one another so as to produce no current through a delicate galvanometer. Null methods are characterized by the fact that the conditions of perfect adjustment between the different parts of an apparatus is shown by the *absence of any indication* on the part of some delicate instrument.

Null methods do not require the use of instruments which indicate the magnitude of the difference between two nearly equal quantities, although it is often convenient to employ such instruments for purposes of interpolation (see § 41). It is only necessary that an instrument should show whether two quantities are equal or unequal. Being used solely to detect differences, such instruments are sometimes called "detectors." They take the place of sight, touch, or hearing (§ 23) with quantities which do not affect these senses.

There are two principal precautions to be observed in the use of null methods. One is to make sure that the instrument employed responds to the slightest variation in either of the two quantities which are compared; the other is to test the zero of the instrument (§ 32). Errors may occur, for instance, from a break or from a cross-connection in the circuit of a galvanometer; for in this case there will be no perceptible deflection, no matter how great may be the difference between the electromotive forces which are compared together. Again, if the needle of a galvanometer does not naturally point to zero, it may require a current to make it do so (see Exps. 89, 90). We should infer wrongly in such a case that the current had been reduced to zero.

Null methods usually depend upon the use of very sensitive instruments; but the conclusions which we draw from them, being founded upon purely negative indications, must be examined with great care. Null methods are considered highly desirable on account of their precision, but they need in general some kind of confirmation.

§ 43. **Method of Substitution.** — The "method of substitution" is the fundamental method for testing any result the accuracy of which is questioned. It is so called because a known quantity is *substituted* for an unknown. Thus if the resistance of a wire has been found by means of any electrical combination sensitive to variations in resistance (Exps. 86, 87) to be equivalent to 10 ohms, we have only to substitute for it a resistance *known* to be 10 ohms to find whether there is or is not any error in our work.

The scale of a densimeter (Exp. 15) may be tested by substituting a liquid of known, for one of unknown density, or the indications of a volt-meter (Exp. 96) by substituting known for unknown electromotive forces. The method of substitution is often used where no other is possible, as in Experiments 2, 3, and 4. It depends upon the principle that two quantities must be equal if they can be substituted one for the other without affecting a combination sensitive to variations in the magnitude of the quantities in question. Evidently the known and unknown quantities thus compared should be as nearly equal as possible.

In the method of substitution, as in null methods (§ 42), we must make sure that the instrument which we employ is free to move, since otherwise very unequal quantities might apparently produce the same effect upon it. The "zero-error" of an instrument (§ 32), and instrumental errors in general (§ 31), are usually eliminated by the method of substitution. Borda's method of weighing is to counterpoise accurately an unknown weight in one pan of a balance with material of any sort in the opposite pan, then to substitute known weights for the unknown until an exact balance is again established. In a similar manner, when, in electrical measurements, null methods (§ 42) are employed, it is well to test the accuracy of the results by substituting known for unknown quantities. The use of the method of substitution in combination with null methods is the most general way of obtaining both accuracy and precision in physical measurement.

§ 44. **Methods of Interchange and Reversal.** — In the ordinary method of double weighing (see Exp. 8) an unknown weight is first placed in the left-hand pan of a balance, and a known weight in the right-hand pan. Let us suppose that the former is greater than the latter by a small amount, which is sufficient to send the pointer of the balance x divisions to the right of its natural resting-point. The unknown weight is next placed in the right-hand pan, and the known weight in the left-hand pan. The pointer will evidently move about x scale divisions to the left of its natural resting-point. The total movement produced by interchanging the weights will therefore be about 2 x scale-divisions. If, however, the unknown weight were exactly counterpoised, the substitution of the known weight for it would cause a motion of the pointer through only x scale divisions. It is easier, accordingly, to detect a difference between two weights by the method of interchange than by the method of substitution (§ 43).

The method of interchange is generally used in connection with null methods of comparison (§ 42) when *reversible instruments* are employed. Whatever may be the difference between the two nearly equal quantities thus compared, its effect upon a reversible instrument is doubled by interchanging these quantities. For this reason the method of interchange, when applicable, is always preferred to the method of substitution.

A similar method is employed in case of reversible instruments in general. Thus an electrical current which deflects a galvanometer needle $x°$ to the east of north, should if reversed deflect it $x°$ to the west of north. The needle is thus moved, by a reversal of the current, through $2\,x°$. Since an angle of $2\,x°$ can be measured as accurately as an angle of $x°$, the method of reversal has to a certain extent the advantage of a method of multiplication (§ 39). In the methods of interchange and reversal "zero-errors" are eliminated (§ 32), for the increase of one reading due to an error in the zero will be nearly offset by a decrease in the reversed reading. Methods of reversal are always, when practicable, employed.

§ 45. **Check Methods.** The methods of substitution and of reversal are instances of check methods. In physical measurement, as in arithmetic, an indefinite number of such methods may be devised. The use of check methods is not, however, limited to such as yield accurate measurements. We often find an advantage in checking results which we believe to be precise, with others obtained by different methods, which we consider comparatively unreliable. It is in this way, principally, that gross mistakes are discovered, such as are otherwise likely to be repeated over and over. But the use of check methods is also important in the detection of smaller errors. Even if a method is uncertain, there is probably some limit to its inaccuracy, and if the results fail to agree with those of a different method by an amount greater than this limit, we are led immediately to suspect an unknown source of error in one of these methods. The densimeter, for instance (Exp. 15), though not nearly so exact as the specific gravity bottle (Exp. 14) should be accurate at least within 1%: hence if the results differ by more than 1% we at once repeat the determination with the specific gravity bottle. On the other hand an agreement of the two results within 1 % indicates the absence of gross mistakes in either determination.

Whenever the results of check methods, however rough, agree with previous results as closely as may be expected, there is always a certain degree of mutual confirmation. It should be remembered, however, that a check method is such only in so far as it makes use of different data, different constants, different instruments, and different laws or principles from those already employed. Accuracy in physical measurement is generally obtained only when every possible variation has been made in the conditions of an experiment, the results compared, and the differences between them explained.

§ 46. **Method of Averages.** — When finally all possible care has been taken to avoid sources of constant error, and to increase the accuracy of determinations, there remains one general method of escaping from what are known as accidental errors (§ 24), or those which tend sometimes to increase, and at other times to diminish, the result. This method is simply to take a great number of measurements, and to find the average. It is not likely, for instance, that in ten observations all should by accident be greater, or all less, than in the long run; in fact, the chances are more than one thousand to one against it. It is much more likely that three or four should be affected one way, and the rest the other way. In fact, we must expect that the errors due to chance shall to a certain extent offset one another. The consequence is that the average of several observations is more reliable than any one alone. For a discussion of the advantages gained by taking the average of several observations, see § 51.

§ 47. **Allowance for Errors.** — We have considered, so far, the principal methods by which errors may be eliminated from physical measurement. There are, however, certain errors which cannot thus be avoided. The effect of some of these may be submitted to calculation. The buoyancy of air, for instance, is computed and allowed for in all accurate weighings (§ 67). There is another class of errors which cannot be calculated in this way from data already in our possession. The causes from which such errors arise may require separate investigation. Thus the heat lost in transferring a hot body from one place to another can be estimated only by comparing results of different experiments (see Part I. ¶¶ 93, 94).

No single observer can expect to discover all the sources of error which are likely to arise in measurements. Our knowledge of the corrections which are to be applied in the determination of a given physical quantity is one of slow historical growth. It is necessary to refer continually to examples which have stood the test of long criticism. At the same time, each observer must be on the alert against new sources of error. The slightest alteration in the conditions of an experiment may entirely change the nature of the corrections to be applied.

Errors of greater or less magnitude are sure to creep into our work notwithstanding every possible effort to avoid them. The student is advised not to pay too close attention to fine corrections, lest in so doing he may overlook others of much greater importance. It is a well-known fact that the accuracy of results is apt to be grossly overestimated (see Introduction). Sufficient allowance for errors is seldom if ever made.

The application of corrections to the results of physical measurement must be considered separately in connection with each experiment or class of experiments. The discussion of errors and corrections belongs perhaps to the "Reduction of Results" (Chap. IV.), rather than to "General Methods" of measurement. The student must not, however, forget that a just allowance for errors constitutes one of the most important parts of an accurate physical measurement.

§ 48. **Standard of Accuracy.** — The distinction between accuracy and precision has been pointed out in the Introduction. One generally knows by experience, roughly at least, what degree of accuracy is attainable with a given instrument. Thus a weighing with ordinary prescription scales will doubtless be accurate to centigrams, but not to milligrams; temperatures taken with a common laboratory thermometer are reliable to degrees, but not generally to tenths of degrees; lengths may be true to hundredths, but not perhaps to thousandths of a centimetre. From such data we may generally estimate roughly the degree of accuracy attainable in the final result. All parts of a measurement should be made with a corresponding degree of accuracy.

Let us suppose, for instance, that it is desired to determine the density of alcohol at a given temperature (*e. g.* 20°) within a few hundredths of 1 % by means of a specific gravity bottle (see Exp. 14) of about 100 *cu. cm.* capacity. To do this, the weight of water and the weight of alcohol required to fill the bottle must be determined within a few centigrams; the temperature of the water must be known within about 1° (see Table 25), and that of the alcohol within a few tenths of 1° (see Table 27). The real difficulty in this experiment consists accordingly in the accurate determination of the *temperature of the alcohol*, — a point to which the student's attention needs generally to be directed. An accurate reading of the barometer would be wholly out of place in such a determination, since an error of several centimetres (see Table 22) would scarcely affect the last significant figure (§ 55) in the result.

§ 49. **Distribution of Time.** — Time is often misspent in the exact determination of quantities which have comparatively little influence in the result. Thus the correction for atmospheric pressure seldom affects the decigrams in a weighing, and ordinary variations make only a few milligrams' difference in the result. It is therefore unnecessary, in many experiments, to read a mercurial barometer closer than to millimetres, much less to correct it for variations of temperature, for capillarity, or for the tension of mercurial vapor. A double weighing, with a rough allowance for the buoyancy of air, takes about the same time as a single weighing with the exact correction, and is, with rough balances, decidedly to be preferred.

When a measurement depends on several determinations of about the same degree of precision, we generally devote an equal amount of time to each; but if we can see that the result will be affected by the errors in one case more than in another, the number of observations is increased *in proportion*. Thus in the determination of the volume of a cylinder from its length and diameter we take twice as many observations of the latter as of the former, because the diameter occurs twice as a factor, while the length occurs only once in the calculation of the result. A fuller discussion of this principle will be found in Part IV.

CHAPTER IV.

REDUCTION OF RESULTS.

§ 50. **Probable Error.** — When several observations of a given quantity have been made, their "probable error" may be found roughly by the following rule: throw out alternately the highest and lowest values until only a majority remains; take half the range of that majority as the probable error of a single observation.

Thus from the ten following observations of the boiling-point of alcohol —

78°.79	78°.33	78° 02	78°.93	78°.46
78°.67	78°.00	78°.81	78°.43	78°.56

we have, throwing out 78°.93, 78°.00, 78°.81 and 78°.02, a majority of six, ranging from 78°.33 to 78°.79, that is, through 0°.46. The probable error of a single observation is therefore about 0°.23.

In saying that the probable error is 0°.23, we do not mean that this error is more probable than any other, 0°.20 for instance. We mean simply that in the long run more than half the errors will probably be less than 0°.23 (see Table 7), and hence, as some errors are positive and others negative, that a majority of the observations will be scattered through a range not exceeding 0°.46. This is evidently the case if the observations above are a fair sample of those which would be obtained in an extended series.

§ 51. **Probable Error of an Average.** — To find the probable error of the average of several observations, we divide that of a single observation by the square root of the number of observations.

Thus if the probable error of a single observation of temperature is, as in the last section, 0°.23, that of the mean of ten observations is $0°.23 \div \sqrt{10}$, or less than 0°.08.

The relation between the probable error of an average and that of a single observation is established by the theory of the combination of errors as explained in Part IV.

§ 52. **Probable Error of a Result.** — The probable error of a result can be calculated if we know that of each datum upon which it depends, as will be explained in Part IV. It is often, however, less laborious to work out several independent results, the probable error of which can be found by inspection, as shown at the beginning of this chapter. Thus instead of calculating the density of a block (in Experiment 1) from its *average* weight, length, breadth, and thickness, we may use each measurement of length, breadth, and thickness for a separate calculation, and average the results. In all such cases the probable error should be determined.

§ 53. **Representation of Probable Error.** — The average of the ten observations of the boiling-point of alcohol mentioned in § 50 is 78°.50; the probable error of this average as found in § 51 is 0°.08. We say, accordingly, that alcohol boils (probably) at 78°.50 ± 0°.08.

In the same way the probable error of any result is often written after it with the "plus - or - minus" sign.

§ 54. **Notation.** — It is convenient for many reasons to express results in units of such magnitude that the probable error may lie below the decimal point. When no such units exist, we introduce as a factor 10 raised to the necessary power. Thus the mechanical equivalent of the unit of heat is not written 41,660,000 ergs, but 41.66 megergs, or 4.166×10^7 ergs.

In this notation we escape any possible confusion between ciphers which are the result of actual measurement and those which we are obliged to use from the necessity of the case.

Ciphers are used in physical measurement at the end of a decimal as freely as any other figure. Thus the average of ten observations in the last section was written 78°.50. The cipher informs us that the average was between 78°.495 and 78°.505. Without the cipher we should infer simply that the average was between 78°.45 and 78°.55. The existence of a cipher in the last decimal place has therefore as much significance as that of any other figure. The question how many figures it is advisable to retain is discussed in the next section.

§ 55. **Significant Figures.** — In arithmetic any number of figures may be significant. In physical measurement those figures only are significant to the left of which the probable error does not extend.

Thus, in the observations at the beginning of this chapter, the degrees and tenths are significant, but the hundredths are not, because the probable error is 0°.23. In the average of the ten observations, the hundredths, also, are significant, since the probable error is 0°.08. One figure is generally enough to describe the probable error. The place which this figure occupies is the same as that of the last significant figure.

It is customary to retain only significant figures either in an observation or in a result. Some authorities use two or more places affected by probable error. When the probable error is stated, there is no objection to this practice. Otherwise it is equivalent to a false pretension to accuracy.[1]

§ 56. **Use of Significant Figures.** — Labor is saved in physical reductions by using only significant figures. The rejection of subsequent figures is not found in practice to impair the accuracy of the result. In deciding how many places to retain, the following approximate rules may be of assistance: —

1st. In addition or subtraction, retain the same number of *decimal places* throughout, — as many as are significant in the least accurate of all the terms.

2d. In multiplication or division, retain the same number of *figures* throughout, — as many as are sig-

[1] The student is cautioned in particular against cases where the result of some mathematical process is to generate an indefinite number of figures. It is true that a metre is about $3\frac{1}{3}$ feet; but it would be misleading to state that it is about 3.33333, etc., feet.

nificant in the least accurate of the factors, — not counting, of course, initial ciphers.

2d. In logarithmic work, use as many decimal places as there are significant figures in the least accurate of the arguments.

Thus in weighings with a balance accurate only to a fraction of a centigram, we carry out corrections only as far as the milligrams. Again, in calorimetry, where results are often proportional to differences of temperature less than 10° and accurate only to tenths, these results seldom contain more than three significant figures, and corrections not affecting the third figure may be disregarded.

§ 57. **Rules for Approximation.** — A great deal of time is often saved by applying rules which give approximate but not rigorously accurate results. Thus to add 1% or 2% to any quantity corresponds nearly to adding twice that per cent to the square of that quantity, three times that per cent to its cube, half that per cent to its square root, or to subtracting the original per cent from its reciprocal. The truth of these assertions will be seen by reference to Table 2.

It is obviously the same thing to add a certain per cent to a quantity as to add it to a product in which that quantity occurs as a factor; and nearly the same thing, if the per cent is small, as to subtract it from a quotient obtained with the quantity as a divisor.

One of the most valuable rules for approximation is that used in finding the product of several quantities, each nearly equal to unity. Instead of multiplying, we *add them together*. The resulting decimal is approximately the same. Since the product cannot be far from unity, the figure in the unit's place is easily supplied.

Thus if the ratio of the arms of a balance is 0.99996, the correction for the use of brass weights in air 0.99984, for the buoyancy of air on water 1.00122, and the space occupied by 1 gram of water is 1.00175, the volume of water is found by multiplying its apparent weight by the factors $0.99996 \times 0.99984 \times 1.00122 \times 1.00175$. The product found by the ordinary laborious process is 1.0027715+, or, to five places of decimals, 1.00277. The same decimal is found by adding the four numbers together.

The arithmetic mean (or half-sum) of two quantities differing by less than 2% may usually be substituted for their geometric mean (or square root of their product) which is harder to calculate.

It will be noticed in Table 3, *b*, *c*, *d*, and *e*, that the sine, tangent, arc, and chord of small angles are approximately equal. It is frequently useful to substitute one for the other. It is also seen that the cosine of a small angle is nearly equal to unity, so that the difference may often be disregarded.

The above rules for approximation may be applied without injury to all results which are not expected to contain more than four significant figures, provided that the corrections do not exceed 2% nor the angles 2°.

§ 58. **Use of Tables.** — The reductions in physical measurement are often facilitated by the use of tables. There are two kinds of these: one in which the quantity sought is given in terms of a single argument; the other where it is given in terms of two arguments. The first kind is readily understood by any one who has used logarithms. In one column, generally at the left of the page, we find the argument; in the next column, the corresponding values of the quantity sought. Generally, however, there are ten such columns on the same page. The argument is not printed at the left of each column, but, to save space, the last figure of it is at the head of the column and the rest at its left in the first column on the page. The numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 at the head of different columns usually indicate a table of the first kind.

When the argument lies between two values in the table, we cannot directly find the quantity which we seek. We have to make use of interpolation, the rules for which need hardly be explained.

Interpolation depends upon the principle that slight differences in any quantity are nearly proportional to the corresponding differences in its argument, and upon the application of the rules of simple proportion to the differences in question.

The second kind of table is similar to the first, only that at the head of the different columns is contained a second and independent argument upon which the quantities in the body of the table also depend.

Thus the density of air at different pressures and temperatures is contained in Table 19. We follow the line corresponding to a given pressure until we reach the column corresponding to the given temperature, and there find the density in question.

Interpolation in such a table is more difficult than in one of the first kind, because the variation due to both arguments must be taken into account, as explained in ¶ 153. Interpolation is, however, unnecessary when the quantities are, as in Table 20, close enough together, or where only a rough value is required.

§ 59. **Graphical Method.** — Co-ordinate paper (that is, paper ruled in small squares) is useful in many experiments, both for representing results so that any gross error is visible to the eye, and for purposes of interpolation. At the left of the paper there is usu-

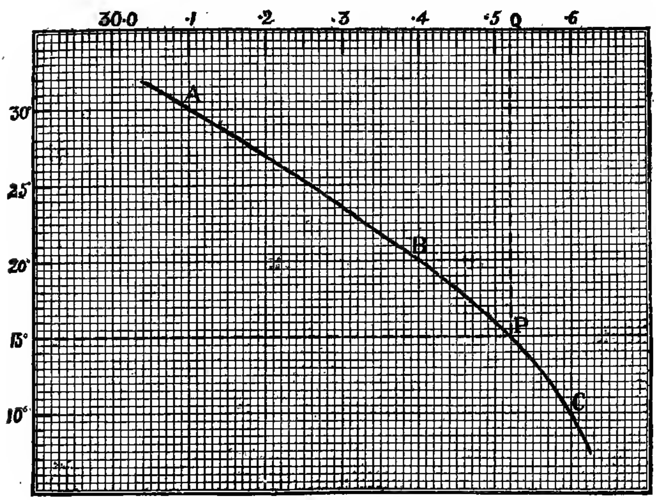


FIG. 4.

ally constructed a vertical scale, like the scale of degrees in the diagram. At the top there is a horizontal scale, like that in the diagram representing the weights floated by a Nicholson's hydrometer. The correspondence of two values is represented by a point opposite the two values in question. Thus in Fig. 4, *A* represents that at 30° the hydrometer floats 30.1 grams; *B*, that at 20° it floats 30.4 grams; *C*, that at 10° it floats 30.6 grams. The dotted line *ABC* drawn with a bent ruler thus supplies an indefinite number of approximate values. To find the weight floated at 15°, we find a point *P* opposite 15°, and then a point *Q* opposite *P*. The answer is 30.52 grams. In the same way the relation between any two quantities can be represented by points, and intermediate values found.

§ 60. **Use of Rough Methods.**—It is always prudent to revise any reduction involving much numerical work, applying the various tests which arithmetics contain. It is, however, easier to reason clearly about small quantities than about large ones, since the former only can be carried in the head. Mistakes in reasoning can often be discovered by rough mental processes when no error can be detected in the figuring.

Thus, if the buoyancy of air relieves water of a little more than a thousandth part of its weight, 50 grams will lose a little over 5 centigrams. If we find that we have introduced a correction of 6 decigrams or 6 milligrams, we at once detect the mistake.

The use even of rough tables, when they can be found, is a very convenient check upon numerical work. When a multiplication runs into the millions, logarithms will be useful,—not always, however, five places. Gross errors are most easily detected by logarithms carried out only to a single place of decimals, the whole attention being placed upon the characteristic. It is thought advisable in physics to use negative characteristics in preference to subtracting from 10. The student may be reminded that a most serious and at the same time a most common mistake in calculation is the misplacement of the decimal point.

§ 61. **Reduction of Consecutive Observations.** — In § 38 we obtained the following series of angles: 0°, 40°+, 80°+, 120°+, 160°+, 200°+, 240°+, 280°+, 320°+, 360°+, 401°—, and 441°—; the first and last give us a difference of 441° —, indicating less than $40\frac{1}{11}°$ for the angle; the second and next to the last give less than $40\frac{1}{9}°$, but the 3d and 3d from the last as well as the 4th and 4th from the last give each 40°. The average of these four results is $40\frac{5}{99}°$, or 40°.05 nearly.

Again, the 1st and 9th, the 2d and 10th, the 3d and 11th, and the 4th and 12th give respectively 40°+, 40°, $40\frac{1}{8}°$ —, and $40\frac{1}{8}°$—; the average of these four values is $40\frac{1}{16}°$, or 40°.06 nearly. Either of these methods of reduction is accurate enough for the measurements in question. In each case the 5th, 6th, 7th, and 8th observations were omitted. By using them we could have obtained two more pairs of observations; but the shortness of the interval between them takes off from their value. The probable error of the result would actually be increased by treating them as we have the others. It is generally advisable to omit in this way the middle third of a series of consecutive observations.

There is a third way of reducing consecutive intervals against which the student must be cautioned. The differences between the 1st and 2d, the 2d and 3d, etc., are in 10 cases 40°, in one 41°. There is a common fallacy to the effect that the average of these, $40\frac{1}{11}°$, makes use of all the observations. It is easy, however, to see that in taking the average we must first add the intervals together, and that we shall obtain as a result the interval between the 1st and 12th observations, since the whole is equal to the sum of all its parts. We subsequently divide by 11, but the result depends solely upon the 1st and 12th, and not in any way upon the intermediate observations, the value of which is therefore completely lost.

This method of averaging consecutive intervals should be accounted a serious error, not simply because it is unnecessarily laborious, but because of the self-deception which it involves.

CHAPTER V.

HYDROSTATICS.

§ 62. **Pascal's Principle.** — From experiments in weighing liquids we might infer that their weight exerted simply a downward action. By immersing a pressure-gauge[1] in any liquid we find, however, that at a given depth the liquid exerts an equal force upon it in all directions, whether horizontal, vertical, or oblique, whether up or down. The same instrument shows that when a fluid is at rest the pressure is the same at all points on the same level. If this were not so, a perfect fluid would evidently be unable to remain at rest. Conversely, all points in a stationary liquid which are subject to a given pressure are found on a given level.[2]

§ 63. **Hydrostatic Pressure.** — If we have a column of liquid in a tube with vertical sides which it cannot cling to, the whole weight of the column must rest upon the bottom of the tube. Let the tube be 1 *sq. cm.* in section; then the weight of the whole column

[1] For the construction of such a gauge see Descriptive list of Experiments in Elementary Physics, 1889, Exercise 5. This experiment is due to Professor Hall.

[2] When (see Fig. 60, page 127) the air-pressure is greater on one part of a liquid surface (*c*) than on another (*b*), the liquid stands at unequal heights in two parts of the apparatus, but if the air-pressure is the same it stands at the same level in both places (Fig. 61). That part of a liquid in a U-tube which lies below a given level transmits or communicates pressure along this level without increasing or diminishing it.

rests upon a surface 1 *sq. cm.* in area, and the pressure in dynes per *sq. cm.* is numerically equal to this weight reduced to dynes. The weight of the column is evidently the product of its volume in *cu. cm.*, the density (or weight of 1 *cu. cm.* in grams), and the intensity of gravity (or weight of 1 gram in dynes); and as the tube has a unit cross section, the volume is numerically equal to its height. The hydrostatic pressure (that is, the pressure of the liquid per unit of area) at the bottom of a tube is therefore the product of the depth and density of the fluid and the intensity of the earth's gravitation. It is clear that the size of the tube makes no difference, for in a tube of twice the cross-section we should have twice the weight distributed over twice the area, and the pressure per *sq. cm.* would be the same. Since pressure is the same in all directions, we may therefore state as a general principle that pressure increases with the depth.

§ 64. **Principle of Archimedes.** — Suppose we suspend a solid in a fluid. The pressure on the solid will of course be greater the more we lower it into the fluid, but the pressure on the bottom of the solid will always be greater than on the top; hence the fluid will buoy up the solid more or less. One can calculate the amount of this buoyancy by the principles which have already been stated if the shape of the solid is not too complex, but there is a much simpler way of arriving at the result. Imagine the solid out of the fluid, and its place filled by a separate portion of that fluid, having the same shape and bounding surfaces as the solid. The pressures on this new portion of the fluid must be the same as on the actual solid, because the surfaces and their depths are the same; but the forces produced result simply in holding the fluid in place, hence their resultant is equal and opposite to the weight of a portion of the fluid equal to the solid in bulk. This principle is known by the name of its discoverer, Archimedes, (287 to 212 B. C.), and may be thus stated: a solid immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced. The difference between the weight of a body and the buoyant force of a fluid in which it is submerged may be called the *effective weight* of the body in that fluid.

§ 65. **Buoyancy of Air.** — According to the principle of Archimedes just explained, a body loses weight in air just as it would in any other fluid. Seven grams of brass displace, for instance, about five-sixths of a cubic centimetre of air; that is, about one milligram, or one 7000th of their nominal value. Bodies weighed against them also lose in weight according to the amount of air displaced. Ordinary weighing consists, therefore, in a comparison of effective weights. The number of grams which balance a body in air is called its apparent weight in air. If, however, the body is in water (the weights being as before in air), we find what is called the apparent weight in water. The effective weights in air or in water can always be found roughly from the corresponding apparent weights by subtracting, for reasons above explained, one part in 7000 from the nominal values of the brass weights. The exact correction is given in § 67. Only apparent weights are obtained by Nicholson's hydrometer, by the hydrostatic balance, or by the specific-gravity bottle.

§ 66. **Apparent Specific Gravities.** — It is obvious that in weighing a body first in air, then in water, as in Experiments 2, 3, and 4, or 8 and 9, we find first the apparent difference between the weight of the body and that of an equal bulk of air, and second, the apparent difference between the weight of the body and that of an equal bulk of water. Subtracting the latter from the former we have the apparent difference of weight between the water and air displaced, or what is the same thing,[1] the apparent weight in air of an equal bulk of water. The ratio between the apparent weight of a body (in air) and that of an equal bulk of water (in air) is called the apparent specific gravity of the body. Without corrections for the buoyancy of air, we can obviously find only apparent specific gravities.

§ 67. **Correction of Apparent Weights.** — Given the apparent weight of a body in air and in water, we usually proceed as follows: First calculate by subtraction the weight of an equal bulk of water, as explained in § 66. Multiply this by the space apparently occupied by 1 gram (see Table 22) to find the volume in question. This is obviously equal to the number of *cu. cm.* of air displaced by the substance. Multiply it, therefore, by the weight of

[1] This holds strictly for *effective* weights from the principle of Archimedes; hence also for apparent weights, to which the former are proportional. See § 65.

1 *cu. cm.* of air (see Tables 19 and 20) to find the weight of air displaced. Next multiply the weight in grams of the body in air by the weight of air displaced by 1 gram of brass (Table 20, A) to find the weight of air displaced by the brass weights. Subtract the latter from the apparent weight of the body in air to find its effective weight in air (§§ 64, 65). Add to this the weight of air displaced by the body to find its true weight *in vacuo.*

When the density of a substance is approximately known, either by reference to Tables 8–11, or from an actual determination of its apparent specific gravity, we may at once reduce its apparent weight to *vacuo* by applying the appropriate coefficient from Table 21.

The apparent weight of a liquid, obtained either by methods of displacement or by the specific gravity bottle, must be reduced to *vacuo*, like any other apparent weight, starting with either (1) the volume, or (2) the density of the liquid, or (3) with the weight of an equal bulk of water. The apparent weight of a *body in a liquid* needs, however, to be corrected only, as has been explained above, for the buoyancy of air on the brass weights by which the body is counterpoised.

§ 68. **Correction of apparent Specific Gravities.** — To find the density of a body, we first find, as explained in § 67, the *volume* of the body from the apparent weight of water displaced, and second the weight of the body *in vacuo.* The weight *in vacuo* is then simply divided by the volume to find the true density of the substance at the given temperature and pressure.

In case we have given, as in Experiment 13, not the apparent weight of water displaced by a solid, but that of some other fluid of known density, we may divide the corrected weight of the fluid, *in vacuo*, obtained as above, by the density of the fluid, to find the space occupied; or we may divide its apparent weight by its apparent specific gravity, if we know it, to find the apparent weight of an equivalent bulk of water, and work out the result as before.

We notice that, in reducing apparent specific gravity to density, we apply to the numerator of a fraction a factor from one table, and to the denominator a factor from another table. The same result, essentially,[1] may be obtained (see § 57), by a single process. Subtract the factor in Table 22 from that in Table 21, multiply the apparent specific gravity by the algebraic difference, and apply the correction thus found. The difference between density and specific gravity is usually less than one per cent.

§ 69. **Density and Specific Gravity distinguished.** — Specific gravity is defined as relative density. Hence density bears to specific gravity (referred to water) the same ratio that the density of water bears to unity. (See Table 25.) By the specific gravity of a substance at a given temperature, we understand, in the absence of any statement to the contrary, the proportion between its weight and that of an equal bulk

[1] Results thus reduced show a slight error, usually confined to the sixth place of decimals.

of water at the same temperature. It is understood also, unless otherwise stated, that both bodies are under atmospheric pressure (76 *cm.*). Specific gravities of gases, however, are often stated with respect to hydrogen or air at the same temperature and pressure. Specific gravities are also referred to water at its temperature of maximum density. Having accepted the value 1.00001 for the maximum density of water, we see that such specific gravities are less than densities by an amount (10 parts in a million) which is small compared with the probable error of observation.

§ 70. **Calculation of Difference of Density.** — Since density, D, is the quotient of mass, M, by volume, V, or

$$D = \frac{M}{V},$$

two bodies having the same volume, V, densities D_1, D_2, and masses M_1, M_2, have a difference of density equal to the difference in their masses divided by the volume, that is,

$$D_2 - D_1 = \frac{M_2}{V} - \frac{M_1}{V} = \frac{M_2 - M_1}{V}.$$

Hence we may find the difference in density between two liquids or two gases (as in Experiment 18) from the difference in weight of a flask of known capacity filled first with one, then with the other. It is obvious that in weighing a flask filled first with air, then with a liquid (as in Experiments 11 and 14), we might determine in this way the difference of density between the liquid and air, and that by adding to this result the density of air, D_1 (from Tables 19 and 20), we should find the density, D_2, of the liquid in question; that is,

$$D_2 = D_1 + \frac{M_2 - M_1}{V}.$$

When the substance weighed is (as in Experiment 18) lighter than air, the difference of density may be considered negative, and must be subtracted numerically from the density of air as indicated by the formula identical with the above,

$$D_2 = D_1 - \frac{M_1 - M_2}{V}.$$

§ 71. **Accuracy of Meteorological Instruments.** The density of the atmosphere is found to affect all delicate weighings. For many purposes it is sufficiently accurate to assume a mean density of 1.2 *mgr.* to the cubic centimetre;[1] but for the most accurate determinations we need to correct it for temperature, pressure, and humidity. The corrections are so slight that a rough estimate is sufficient for this course of measurements, and hence we may accept provisionally the indications of such weather instruments as may be found in the laboratory. We shall learn, later on, the means of detecting errors in these indications, and shall expect to prove that these errors have not perceptibly affected our results.

In place of the ordinary weather instruments, we may employ a sensitive baroscope, or *barodeik*, consisting of a hollow cylinder which has been counterpoised *in vacuo* against a weight occupying say 1000

[1] The probable error under this assumption may be estimated as between 1 part in 10,000 and 1 part in 100,000.

cu. cm. less space than itself. The apparent difference of weight between the hollow cylinder and its counterpoise indicates at once the actual density of the atmosphere.

§ 72. **Accuracy of Gram-Weights.** We must choose between accepting such copies of the gram as are attainable, and determining independently the weight of a cubic centimetre of water. Experience shows that weights can be copied (and that they generally are copied) with a very great degree of precision, while it is comparatively difficult to copy standards of length, and still more difficult to reproduce them.[1] There is also more or less uncertainty as to the temperature at which a cubic centimetre of water may be assumed to weigh one gram (see § 6 and Table 25), and it is by no means easy to find the weight of a cubic centimetre of water with any degree of precision. It is, moreover, important to express our results in *conventional* units. For these reasons we prefer to accept a set of gram-weights, provided, however, that we are not able to detect any gross error in them by such means as are in our power.

§ 73. **The Density of Water.** — On account of the inaccuracy of our standards of length we are unable to determine the volume of a body very accurately from its length, breadth, and thickness; and hence we cannot find its density absolutely, as in Experiment 1, with any degree of precision. The same inaccuracy affects the volume of water which such a body dis-

[1] The error in the original determinations was nearly a tenth of one per cent. (See § 5.)

places, and hence also the density of water, which is found by comparing the weight and volume displaced. We prefer, therefore, to accept the results of a great number of determinations (see Table 25) rather than any rough measurements of our own, and we make use of this table of density for testing or correcting our standards of length, and not of our standards of length for the determination of a new table of densities. It is thought that measurements of length corrected in this way will be nearer the conventional standard than those depending directly on such rough copies as are found in the market. The approximate agreement of our actual standards of length and mass is the first of a series of tests to which these standards must be subjected, and through which, finally, any gross error in either is sure of detection.

CHAPTER VI.

HEAT.

§ 74. **Temperature.** — Temperature is believed to depend upon the vibration of the molecules of which a body is composed, and hence be akin to what we call heat. Temperature is not, however, heat, but the state of saturation with heat which determines, under certain conditions, whether heat will be imparted or absorbed. Bodies which can communicate heat to others are said to have a higher temperature. Two bodies in contact are said to have the same temperature when no heat flows from one to the other. It is found that two bodies at the same temperature as a third are themselves in thermal equilibrium. Heat corresponds in a certain sense to quantity, temperature to intensity of vibration (see § 84). The temperature of a gas is seen from its nature to be intimately connected with pressure; for pressure is explained as the effect of the perpetual bombardment of the molecules against the sides of a vessel which contains them.

§ 75. **Absolute Zero.** — We must distinguish the absolute zero of temperature from that which we have provisionally adopted. At the absolute zero, the particles of a body are supposed to be at rest. Gases therefore exert no pressure at this temperature, and occupy no space, save that which their molecules take up when closely packed together.[1] The absolute zero must be the same for all bodies, since when their heat is wholly taken away they cannot communicate any from one to another, and hence have, by definition, the same temperature. There is reason to believe that the absolute zero of temperature is, on our provisional scale, about 273° centigrade below the freezing-point of water.

§ 76. **Absolute Temperatures.** — We have seen that the temperature and pressure of gases are intimately connected. The absolute scale of temperature is founded upon this fact. By definition, *absolute temperature is proportional to the pressure of a perfect gas confined to a constant volume.* All permanent gases are found to be essentially perfect in this sense.

To compare absolute temperatures, we may seal up a mercurial barometer in a tube, or an aneroid barometer in a preserving jar. The corrected indication of the pressure of the air enclosed will be proportional to the absolute temperature.

We are still at liberty to adopt any length of degree which we please, and for convenience we will choose that of the centigrade scale. Let us suppose that the barometer rises ten inches when we heat the air from the freezing to the boiling point of water. Then a tenth of an inch will represent a degree. The abso-

[1] The molecules are thought to occupy at least one half as much space as the liquid formed by the condensation of a gas.

lute temperature of freezing or boiling can now be found from the corresponding pressure of the barometer in tenths of an inch. We discover in this way that water freezes at 273°, and boils at 373° on this absolute scale.

Whatsoever means we adopt for estimating the pressure of a confined gas, the same result is obtained, since the pressure at boiling is to that at freezing as 373 is to 273.

It is found that all temperatures on the mercurial thermometer may be converted approximately to the absolute scale by adding 273°.

§ 77. **Velocity of Molecules.** — From the definition of force (§ 12) depending on mass, time, and change of velocity, it is clear that the pressure of a gas must depend both upon the number and upon the velocity of the molecules which strike a given surface in a given time. If we double the velocity of the molecules without changing the distance they must travel before hitting the sides of the vessel, the blows will be twice as frequent and twice as strong; hence the pressure will be quadrupled, — also, by definition, the absolute temperature, as the volume remains the same. So, in general, temperature may be shown to vary as the square of the molecular velocity.

We do not know the mass of a single molecule, except within wide limits; but we can find the weight of a cubic centimetre of a gas, and thus independently of the number of molecules in the given space, we can calculate the average velocity which will account for a given pressure. Molecular velocity is not therefore a matter simply of conjecture.[1]

§ 78. **Pressure and Density of Gases.** — The density of a gas is evidently proportional, other things being equal, to the number of molecules in a given space. In the case of exceedingly rarefied gases, the molecules are so far apart as not practically to interfere with one another; hence each will hit the sides of the vessel as often as if the others were not present.[2] It follows from the principles explained in the last section that in such a case pressure and density are proportional when the average velocity, or temperature, remains the same. Hence at a constant temperature, *the pressure of a perfect gas varies with the density.* Experiment confirms this assumption in the case of exceedingly rarefied gases.

As a gas becomes more and more condensed, there is less and less space between the molecules free for vibration, and cohesion may come into play, particularly in the case of a vapor near its point of condensation. In such cases the law connecting density and pressure cannot be applied. Even the most permanent gases are more or less compressible than theory would indicate (see Table 12), though in most experiments the variation is barely perceptible.

§ 79. **Law of Boyle and Mariotte.** — As the volume of a gas increases, the density obviously diminishes,

[1] The average velocity of a hydrogen molecule at 0° is found to be not far from a mile per second; that of oxygen is one fourth as great. For a further discussion of this subject, see Maxwell's Theory of Heat, chapter 22.

[2] See Daniell's Principles of Physics, page 224.

and the pressure, as we have seen, diminishes in proportion. Hence *the volume of a perfect gas at a given temperature varies inversely as its pressure.*

§ 80. **Law of Charles.** — As the volume of a gas increases, the pressure diminishes; but as the absolute temperature increases, the pressure increases. It follows that if both the volume and the absolute temperature increase in the same proportion, the pressure will remain the same. Hence *the volume of a perfect gas at a constant pressure is proportional to its absolute temperature.*

By this principle absolute temperature can be estimated from the volume of a gas at a constant pressure as in Experiment 26, as well as from the pressure of a gas at a constant volume, as in Experiment 27 (see § 76).

§ 81. **Reduction of Density to Standard Temperature and Pressure.** — If D is the density of a gas, P its pressure, and T its absolute temperature, then the pressure, P_1, at the standard temperature, T_0, will be given by the proportion, $P_1 : P :: T_0 : T$, or $P_1 = PT_0 \div T$; the density, D_0, at the standard pressure, P_0, is given by the proportion, $D_0 : D :: P_0 : P_1$; whence $D_0 = D\ P_0 \div P_1 = D\ P_0 \div (P\ T_0 \div T) = D\ P_0\ T \div P\ T_0$.

If the pressure, p, is expressed in centimetres of mercury, and the temperature, t, is on the ordinary centigrade scale, we have

$$D_0 = D \times \frac{76}{p} \times \frac{273 + t}{273}.$$

§ 82. **Expansion of Solids and Liquids.** — In the case of solids and liquids, the effects of temperature in causing expansion are slight in comparison with those in the case of gases. It is probable that the cohesive forces which bind their particles together leave very little available space for their vibration, and it is quite possible that this available space obeys the same laws in general as in the case of gases. We have, however, several cases where bodies contract with heat, the most notable of which is water below 4°. Such cases may be explained as the result of the gradual re-arrangement of the particles consequent on a rise of temperature, — that is, to the same cause which makes water occupy about ten per cent less space than the same weight of ice.

§ 83. **Linear and Cubical Co-efficients of Expansion.** — A co-efficient of expansion is a number which always occurs as a factor or *co-efficient* in calculating expansion produced by heat. The increase of the volume of one cubic centimetre caused by a rise of 1° in temperature is called the cubical co-efficient of expansion of a substance. The increase of the length of 1 *cm.* is called the linear co-efficient of expansion. Unless otherwise stated, the co-efficient of expansion of gases and liquids is assumed to be cubical; that of solids, linear, affecting length, breadth, and thickness alike, and hence only one-third as great as the corresponding cubical co-efficient.

§ 84. **Relation between Heat and Temperature.** — The relation which temperature bears to heat is analogous to that which hydrostatic pressure bears to water. Heat flows from high temperature to low temperature, water from high level to low level. When we pour water into a vessel, the level rises; so heat increases the temperature of a body. It takes more water to fill a large jar to a given depth than a small one, more heat to warm a heavy body to a given temperature than a light one. Heat, like water, is indestructible, though it can be transformed into many shapes. We usually estimate quantities of heat relatively to a certain unit, which has been defined (§ 16), or, in the absolute system, by the quantity of work to which it is equivalent.

§ 85. **Thermal Capacity.** — The thermal capacity of a substance may be defined as the total amount of heat necessary to raise its temperature one degree. It corresponds to the cross-section of a vessel. A common measuring-glass, flaring a little at the top, requires more and more water to raise the level by a given amount. So most substances require more heat to raise their temperature one degree as the temperature increases. The variation is, however, frequently imperceptible.

§ 86. **Specific Heat.** — If we put pebbles into a vessel it will take less water to fill it than before; still less if the spaces between the pebbles are filled with sand.

Specific heat corresponds to the material which a vessel contains before water is added. It is something irrespective of the weight or bulk of a body which gives it a greater or less capacity for heat. From experiments in mechanics we infer that the fineness of subdivision of the particles of a body is what fits them to be set in vibration, that is, to absorb heat. Specific heats accordingly increase as what we call the "molecular" weight diminishes. In the case of elementary substances this can almost be called a law.[1]

§ 87. **Latent Heat.** — If a small vessel is put inside a large one, and water poured into the space between, the level rises up to the edge of the small vessel, then is constant until the small vessel is filled, after which it rises again. So when ice is heated it rises in temperature until it begins to melt, then the temperature is constant until the ice is all converted into water, then it rises again.

A certain quantity of heat disappears in melting the ice, without raising the temperature, just as a certain quantity of water disappears in filling the inner vessel. The quantity which is thus absorbed in melting a gram of a substance is called its latent heat of liquefaction. In the same way heat disappears when a liquid is changed into a vapor. The amount of heat necessary to convert a gram of a liquid into a vapor is called its latent heat of vaporization.

Thus it takes about 80 units of heat (or 3,300 megergs) to change a gram of ice at 0° into a gram of water at 0°. The water is not any warmer than the ice, because water and ice may remain indefinitely in contact and yet perfectly distinct. In the same way

[1] The products of the atomic weights and the corresponding specific heats (see Table 8, *a*) will be found in most cases to be nearly equal to the number 6.

it takes about 536 units[1] of heat (or 22,000 megergs) to change a gram of water at 100° into a gram of steam at 100° when the atmospheric pressure has to be overcome.

§ 88. **Explanation of Latent Heat.** — When the particles of a body are separated in such a way as to overcome certain forces called cohesive, because they tend to hold particles together, it is clear that work must be done. If a particle of ether escaping from a drop of that fluid is held back by the attraction of that drop, it will evidently lose a part of its velocity; and as only the swiftest particles can escape at all, the slowest must remain, and the drop will grow cooler and cooler. The work done in evaporation is at the expense of temperature. When finally the liquid has been all converted into vapor, heat must be communicated to the latter to restore to it the *same* temperature that it had in the liquid state. The boiling of a liquid depends upon the *continuous* communication of heat necessary to maintain a *constant temperature*. This heat is said to be latent, because it does not affect the thermometer. It can, however, be recovered; for the heat absorbed in vaporization is given back in the act of condensation. The process is in fact reversed. A particle of vapor is accelerated by the attraction of the liquid mass into which it falls, and gains in velocity what before it lost.

§ 89. **Law of Cooling.** — There are three ways in which heat is likely to escape from a calorimeter:

[1] Of this, about 40 units are consumed in overcoming the pressure of the atmosphere.

first by conduction, or passing from one particle to another; second by convection, or being carried bodily by currents of air; and third by radiation, or directly passing from one place to another as the sun's heat does in waves or rays. When all these causes have been guarded against, there is apt to be a very slight loss of heat, which has to be allowed for. In all three ways in which heat can escape the amount is found to be proportional, nearly, to the difference of temperature between the contents of the calorimeter and the surrounding air. Hence we have Newton's law of cooling: *Loss of heat per unit of time is proportional to difference of temperature.*

If, for instance, the temperature within the calorimeter is 40° and that outside of it 20° and the rate of cooling 1° in 5 minutes, we should infer that if the calorimeter were at 30° the temperature would fall only about 1° in 10 minutes. We are thus able to estimate the temperature at a point of time when observation would be impracticable. (See Experiment 31.)

§ 90. **Principle of Calorimetry.** — When substances at different temperatures are mechanically mixed in a calorimeter so that no chemical or physical reaction takes place, with the exception of a small quantity of heat which escapes as has just been explained, the total amount remains constant. What is lost by one body is therefore taken up by another.

If m_1 is the mass of one body, s_1 its specific heat, t_1 its temperature before mixture, and t its temperature after mixture, then the number of units it has absorbed is $m_1 s_1 \times (t-t_1)$. If it has lost heat instead of gaining it, the expression will be negative. Denoting by subscripts 1, 2, 3, &c. in the same way the properties of the several substances contained in the calorimeter, we have

$$m_1 s_1 (t-t_1) + m_2 s_2 (t-t_2) + m_3 s_3 (t-t_3) + etc. = 0.$$

The temperature of the mixture, t, is the same for all. The products $m_1 s_1$, $m_2 s_2$, $m_3 s_3$, *etc.*, are evidently the thermal capacities of the bodies in question. For if s is the heat required to raise 1 gram 1°, $m s$ will be that required to raise m grams 1°.

To calculate the thermal capacity of a calorimeter, we multiply the weight of the *inner vessel* in grams by the specific heat (from Table 8, *a*) of the material, usually brass, of which it is composed. The thermal capacity of a stirrer attached to the bulb of a thermometer is calculated in the same way. The thermal capacity of a thermometer is about one-half of the number of cubic centimetres immersed, whether of mercury or of glass, — more exactly, $\frac{4}{9}$ in the case of mercury. The various methods of calculating specific heat by the above principles will be explained in Experiments 32, 33 and 34.

§ 91. **Heat Developed in a Calorimeter.** — When a substance contained in a calorimeter undergoes a change of state, whether physical or chemical, heat is usually developed or absorbed. The fact is recognized by the departure of the temperature of the mixture from that which it would be expected to have if the mixture were purely mechanical. The heat developed or absorbed when a gram of a solid is dissolved is called the (latent) heat of solution; when it unites chemically with another substance, it is the heat of combination; or if it *burns* in the process, the heat of combustion. The calculation of these heats is explained in Experiments 35–38.

CHAPTER VII.

SOUND AND LIGHT.

§ 92. **Wave Motion.** — When a row of marbles is set in a crack of the floor, and one at the end of the row is hit, it strikes the one next to it and comes to rest after giving up nearly all its motion, the second marble gives up its motion to the third, and so on, until finally the last marble is set in motion. In the same way a string can transmit a pulse. The string however, has generally a lateral motion and each portion pulls the next one side instead of pushing it forward. A wave of sound in air is transmitted like a pulse through a row of marbles, a wave of light like a pulse through a string. In both cases, however, the pulse, if not obstructed, is carried from the origin not simply in one direction but in all. The different paths by which light spreads out are illustrated by a system of strings radiating in all directions from a given point. These strings represent also what are called rays of light. To explain the distribution of sound we may imagine a space filled with solid bodies having springs of some sort between them so as to keep them apart and yet allow any one to transmit a blow to its neighbor, as in the case of the marbles.

§ 93. **The Air and the Ether.** — A pulse of sound in air is in reality transmitted by the impact of the molecules of air, which are perfectly elastic, whereas marbles are not. The velocity of sound in air is a little over 33 thousand *cm. per sec.* While sound is intercepted by what we call a vacuum (there being no molecules to transmit it), light passes more easily through a vacuum than through air. What carries light we do not know. We call it *the ether*. The ether, like air, is perfectly elastic; but it has no weight, and no perceptible resistance to motion through it; it seems to pass between the particles of the densest solids "as freely as the wind passes through a grove of trees."[1] And yet it transmits, as we have seen, transverse vibrations, after the manner of a string.

In some respects the ether reminds us of magnetism, which, though perfectly immaterial, can hold a piece of iron firmly through a piece of glass. Electricity, however, affords the only true analogy to light. It is well known that telephone messages are carried from one wire to another, either through a vacuum or through almost any medium which we can interpose. The fact is certainly significant that electrical vibrations may pass in this way with the velocity of light (30 thousand million *cm. per sec.*), and the belief is gaining ground that light is carried by what is called electromagnetic induction from one particle to another.

§ 94. **Law of Inverse Squares.** — Since both sound and light spread out equally in every direction, a pulse

[1] Lloyd's Undulatory Theory of Light, § 21.

naturally takes the form of a hollow shell, perfectly spherical, and growing larger as the wave passes farther from the source. The area of such a shell is proportional to the square of its radius; hence the intensity of sound or light per square centimetre varies inversely as the square of the distance, — the same amount of energy being distributed over a greater amount of surface. The transmission of sound and light without any perceptible loss affords another illustration of the principle of the conservation of energy.

§ 95. **Relation of Wave-Front and Rays.** — The surface of a shell such as is formed by a pulse spreading out in all directions, or any portion of such a surface, is called a wave-front. It is clear that a wave-front is perpendicular at every point to the ray of light passing through that point, as the radius of a sphere is perpendicular to its surface. When a portion of a wave passes through an orifice, the rest being interrupted, most of it still continues to advance very much as if the whole wave were present. It is found, indeed, that waves tend to move in straight lines, and in all cases in a direction at right angles to their front. It follows that any cause which can change the direction of the wave-front will also cause a bending of the rays. In the absence of any such cause, the general direction will remain constant.

This tendency of waves to move in straight lines is much more marked when a great number of pulses are sent one behind the other, as is always, practically, the case. The wave-fronts then find it impossible to bend much without interfering with one another.

A series of wave-fronts issuing from an orifice constitutes in the case of light what is called a beam. The middle part of a beam is perfectly straight; the bending is confined to an almost imperceptible portion at the edges. Sound shows also a tendency to move in straight lines; but, owing to the great distance between the pulses, not nearly to the same extent.

§ 96. **Frequency of Vibration.** — When a toothed wheel, by striking on a card, gives a regular series of pulses to the air, a musical note is often produced. The pitch of the note depends on the number of pulses per second. There are three classes of notes, one in which the pulses are too infrequent to produce a continuous effect upon the ear, the second audible (say from 30 to 30,000 pulses per second), and the third too rapid to be heard. In the same way there are three classes of vibration in light; one too slow to affect our organs of sight, a second visible (from 400 to 800 millions of millions per second), and a third more rapid still and in consequence invisible.

When sound is intercepted, it is usually changed into heat. All kinds of light when absorbed by an opaque body are generally transformed into heat. In all such cases the heat is equivalent, erg for erg, to the energy spent in producing the vibrations in question. All kinds of light act on a photographic plate, but principally those of the third class alluded to, often called actinic. In sunlight the principal source of energy is from invisible vibrations of the first class, often called calorific for this reason.[1]

[1] See Tyndall's Fragments of Science, pages 182–184.

§ 97. **Reflection.** — All waves are reflected from a surface as an elastic ball is from the floor. That part of the motion which is perpendicular to the surface is reversed, and that parallel to it preserved; hence the path of the ball makes the same angle with the surface before and after reflection. One can see, without a special examination of the motion of separate particles, that a reversal of one component accounts for a similar change of direction in a wave.

§ 98. **Wave-length.** When sound is reflected back and forth between two walls, an echo is heard at intervals corresponding to the time it takes sound to traverse the distance back and forth between the walls. When the walls are only a few feet apart, the echo may become so frequent as to produce a musical note. Thus a tube closed at both ends exhibits this phenomenon. The distance which sound travels between two successive pulses is called in general a wave-length, and is clearly equal in this case to twice the length of the tube. When a particular color is produced in the same way by the reflection of light back and forth between two pieces of glass very close together, its wave-length is twice the thickness of the space between the glasses.

§ 99 **Resonance.** — The vibration of a tube closed at both ends may be described as a periodic rush of air from one half to the other and back again. When such a tube is cut in two in the middle, each half has the power of vibrating essentially as before. The atmosphere receives the rush of air out of the tube and supplies air to fill the vacuum thus caused, taking in fact to each half the same place as the other half of the tube. Since the whole tube was equal to half a wave in length, the halves will be nearly quarter-wave-lengths; but as the vibration extends a little beyond the open ends,[1] a tube closed at one end only is not quite a quarter of the length of the wave to which it responds.

When a tuning-fork emitting the corresponding note is held near the mouth of the tube, the sound is greatly increased. The downward pulses from the fork are reflected from the bottom of the tube so as to reach it in the middle of its upward motion, which is therefore reinforced in its effect upon the air. The slightest variation in the length of the tube causes the phenomenon to disappear; but if the tube is made just one half a wave-length longer, or any number of half-wave-lengths, the reflected pulses, traversing the distance twice, are retarded a whole wave-length or several whole wave-lengths, meet the fork as before, and resonance reappears.

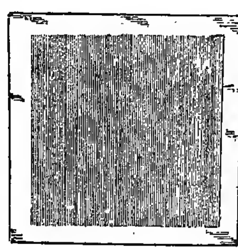
A tube open at one end therefore responds to a given note when its depth is equal to $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$, etc., wave-lengths or thereabouts. The first quarter-wave-length is approximate; the other lengths are greater than the first by exactly $\frac{1}{2}$, 1, $1\frac{1}{2}$, etc., wave-lengths respectively.

§ 100. **Interference.** — When two series of pulses arrive at the same place at the same time the effect

[1] It has been estimated that the vibration virtually extends beyond the open end of a tube to a distance equal to a fourth or a fifth part of its diameter.

is greatly increased; but if they arrive at different times, each tends to fill up the gaps in the other, and thus often to diminish the effect. Hence if a musical sound enters a room by two windows, a person standing between the windows on the opposite side might receive the pulses from each at the same time, while one by his side, being nearer one window than the other, would receive the pulses at different times.

Again, a person still further to one side would receive pulse No. 1 from the further window at the same time as pulse No. 2 from the nearer window, and the sound would be reinforced. Evidently the difference of his distances from the two windows must be the same as that between two pulses, or in other words, a wave-length. There will be reinforcement again when one window is 2, 3, 4, etc., wave-lengths further off than the other; but whenever there is a fraction of a wave-length involved there will be more or less interference. The same holds for a series of windows, or when sound arrives by any two channels whatsoever. We can always find the wave-length of a given note if we know the smallest difference in the length of different channels producing reinforcement or interference.



DIFFRACTION-GRATING.

§ 101. **Diffraction-Grating.** — Precisely the same method is applied to light. An ordinary diffraction-grating (see illustration) consists of a series of lines with slits between them, through which light passes.

We find the difference in length of the paths followed by the light arriving at a given point by two successive slits, and this is the wave-length of the light which is reinforced at that point by the grating.

There is an obvious advantage in employing a grating with a large number of lines, let us say a thousand. If each line is exactly one wave-length further off than the next, a thousand pulses will arrive simultaneously at the eye; but if there is the least error in adjustment, let us say a thousandth of a wave-length, the pulses will all arrive at different times, and thus produce complete interference.

It is to be observed that waves of light and sound tend to move in straight lines only when the breadth of the waves is considerably greater than the distance between them; hence the phenomena of bending or diffraction in passing through narrow orifices. Sound-waves, being on the average a million times farther apart than waves of light, bend much more readily, and require a screen proportionally broad to produce a distinct "sound-shadow." The longest light-waves are, however, comparable with the shortest waves of sound. All waves bend round a small obstacle very much like the waves of the sea.

§ 102. **Refraction.** — If a line of soldiers should march obliquely into a swamp, those who met it first would be most retarded, and their front would change its direction. In the same way a wave changes its direction in entering a medium in which it moves more slowly. Let AB (Fig. 5) be the wave-front *in vacuo* advancing in the direction AC at right angles to $A B$; and let $C D$ be the wave-front advancing in the direction $B D$ at right angles to $C D$, after passing through the surface $B C$ of a refracting medium. Since the time in passing from A to C is the same as from B to D, $A C$ is to $B D$ as the velocity *in vacuo* is to the velocity in the refracting

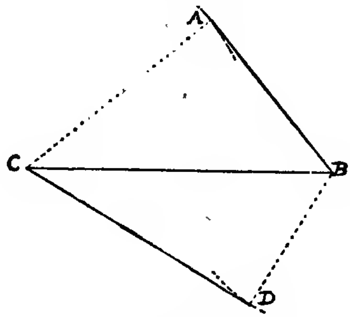


FIG. 5.

medium; but $\dfrac{A C}{B C}$ is the sine of $A B C$, which may be called the angle of incidence (i), and $\dfrac{B D}{B C}$ is the sine of $B C D$, the angle of refraction (r); hence

$$\frac{\sin i}{\sin r} = \frac{A C}{B C} \div \frac{B D}{B C} = \frac{A C}{B D}.$$

The ratio of $A C$ to $B D$, or the velocity *in vacuo* to the velocity in a given medium, is called the index of refraction of that medium, μ, and hence is calculated by the formula

$$\mu = \frac{\sin i}{\sin r}.$$

The index of refraction of glass, for instance, is given as 1.5, nearly. This means that light travels half as fast again *in vacuo* as in glass.

§ 103. **Law of Lenses.** — When waves of light diverging from a point B (Fig. 6) pass through a lens $A\,I$, and converge to a point H, the central portions are clearly retarded by a constant amount $D\,F$ in-

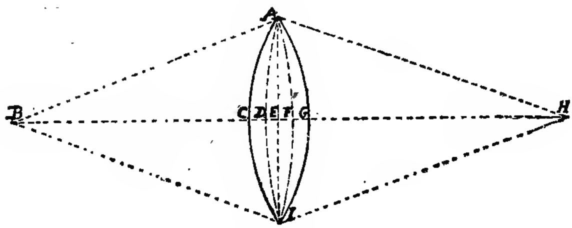


FIG. 6.

cluded between two spherical surfaces $A\,F\,I$ and $A\,D\,I$ with B and H respectively as centres. $D\,F$ may be divided by a plane $A\,E\,I$ into two portions, $D\,E$ and $E\,F$, which, by geometry, are inversely as the distances $B\,E$ and $H\,E$ (nearly), called conjugate focal lengths. As $D\,F$ must be constant, $D\,E + E\,F$ must be constant, — hence also the sum of the reciprocals of the conjugate focal lengths.

When rays emanate from a distant point, like a star, so as to be nearly parallel, they are focussed at the shortest possible distance by a given lens. This distance is called the principal focal length. As its conjugate is very large, the reciprocal of this conjugate may be neglected. Hence the law of lenses: *The reciprocal of the principal focal length* (F_0) *is equal to the sum of the reciprocals of any two conjugate focal lengths* (F_1 *and* F_2), *or*

$$\frac{1}{F_0} = \frac{1}{F_1} + \frac{1}{F_2}$$

The calculation of the index of refraction of a lens will be explained in Part IV.

§ 104. **Images.** — If waves of light emanate, not from a single point, as B in Fig. 6, but from several such points, as B, B', B'' (Fig. 7), they will be focussed at several points, as H, H', H'', so situated as to be in the straight lines $B E H$, $B' E H'$, $B'' E H''$, as the middle of a lens, having two parallel surfaces, does not bend the rays.

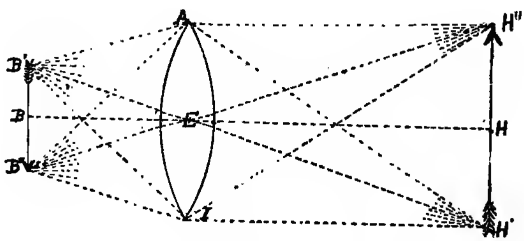


FIG. 7.

Since every point B is represented, we find at H a perfect image of an object at B, but completely inverted; and the separation between any two points is clearly proportional to the relative distance of the image and object from the lens.

We distinguish between real and virtual images. H, H', H'' is a real image of B, B', B'', because the rays of light from B, B', B'' actually meet at $H H', H''$, respectively, and again diverge from these points as from a real object. A photograph requires a real image for its production. On the other hand, an image in a looking-glass is virtual, because rays do not really meet in it or diverge from it.

A virtual image may be located as in experiment 43. When for instance an object is too near a convex lens to have a real image on the opposite side, we may still find a virtual image behind the object. That is, rays diverging from the object may, after passing through the lens, seem to diverge from a more distant point on the same side of the lens as the object.[1] Concave mirrors furnish similar examples of real and virtual images. Convex mirrors and concave lenses do not tend to bring rays to a focus, and give therefore only virtual images.

[1] By a construction similar to Fig. 6 it may be shown that in such cases the reciprocal of the principal focal length is equal to the *difference* of the reciprocals of two conjugate focal lengths.

CHAPTER VIII.

FORCE AND WORK.

§ 105. **Components and Resultants.** — When a body moves from A to B (Fig. 8), then from B to C, it passes of course from A to C; the two motions $A B$ and $B C$ may also be thought of as relative motions taking place at the same time. Let the points A, B, and C all start at A; let B move with respect to A, through the distance and in the direction $A B$, and at the same time let C move *with respect to* B through the distance and in the direction $B C$; then clearly C has moved with respect to A through the distance and in the direction $A C$.

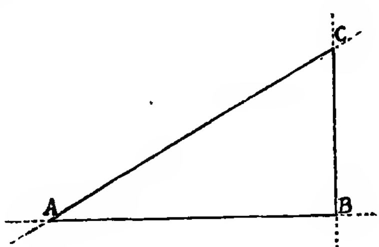


FIG. 8.

We express this fact by calling the motion $A C$ the *resultant* of the two motions $A B$ and $B C$, and by calling $A B$ and $B C$ *components* of $A C$, because when compounded together they produce $A\,C$. We shall have occasion to consider only components which are at right-angles.

If $A\,B$ and $B\,C$ are motions which take place in the unit of time, they represent velocities; hence clearly the resultant of two velocities $A\,B$ and $B\,C$ is $A\,C$.

Again $A\,B$ and $B\,C$ may represent component velocities which a body acquires in the unit of time; in other words, component accelerations (§ 11); evidently the resultant of two accelerations $A\,B$ and $B\,C$ must be an acceleration $A\,C$.

Finally, we may multiply the accelerations $A\,B$, $B\,C$, and $A\,C$ by the mass of the body which they affect, without disturbing their relative values; but the products of mass and acceleration are forces (§ 12); hence two component forces, $A\,B$ and $B\,C$, must give a resultant force $A\,C$.

In fact it is evident that all quantities involving distance and direction, whether motions, velocities, accelerations, or forces, must be compounded by the same rules as lines in geometry.

Now since $A\,B$ and $B\,C$ are geometrically equivalent to $A\,C$, $B\,C$ must be the geometrical difference between $A\,B$ and $A\,C$. Hence a change of velocity from $A\,B$ to $A\,C$ means the acquisition of a new velocity, $B\,C$. We are thus able to represent the change of velocity consequent on a change of direction as well as from a change in magnitude.

Again, a motion $A\,C$ carries a body as far away from the line $A\,B$ as the motion $B\,C$, and a motion $A\,C$ carries it as much nearer to $B\,C$ as a motion $A\,B$.

Hence if the components, $A B$ and $B C$, are at right-angles, $A B$ and $B C$ measure respectively the effects of a motion $A C$, in the general directions $A B$ and $B C$.[1]

§ 106. **Absolute Measurement of Force.** — If a body is free to move in every way, the force acting upon it is always said to have the same direction as the velocity which the body acquires, as explained in the last section. It is also said to have a magnitude such that the product of the force f and the time t it acts is equal to the product of the mass m acted upon and the velocity v acquired. This *definition* of force is expressed also by the formula

$$ft = mv.$$

Experience shows that force defined as above corresponds to that which we ordinarily measure with a spring-balance.

The student should bear in mind that the fundamental law of motion contained in the formula applies only to bodies perfectly free to move, like masses in astronomy. It is a common fallacy to suppose that force is necessary to *maintain* motion. Our formula

[1] The relation between the components and resultants of forces may be illustrated by the strains which they produce. Let A be the head of a nail bent by one force from A to B, and by another force from B to C. As a result, it is bent from A to C. Now by Hooke's Law, as explained in § 114 below, forces are proportional (with certain limitations) to the strains produced; hence two forces $A B$ and $B C$ must have a resultant $A C$ when estimated in this way.

Again, a nail bent from A to C is bent in the general direction $A B$ by the same amount as if bent from A to B; and in the general direction $B C$ the same as if bent from B to C. Hence $A B$ and $B C$ are the components of $A C$ in their respective directions.

expresses the fact that, in the absence of friction or other interference, motion maintains itself; for if $f = 0$, $v = 0$, — that is, in the absence of force there is no change of velocity either in magnitude or in direction. This is essentially Newton's first law of motion. The force which one body exerts upon another is found to be equal and opposite to that with which the second body reacts upon the first. It is necessary, therefore, to measure only one of these forces.

§ 107. **Average Velocity.** — If we take any series of consecutive numbers beginning at 0, we shall find the average value to be half the last value. Thus the average of 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 is 5. So if we begin with a body at rest, and increase its velocity uniformly up to a given point, the average velocity will be half the final velocity.

The average velocity is also found if we divide the distance traversed by the time; or the distance a body moves is the product of the average velocity and the time.

§ 108. **Laws of Falling Bodies.** — The force in dynes which gravity exerts upon a body is the product of the mass m in grams and the intensity of gravity g, in dynes per gram. Substituting mg for f in the general formula of § 106, we have

$$mgt = mv, \quad \text{or}$$
$$gt = v.$$

The velocity acquired by a falling body is therefore proportional to the intensity of gravity and to the time it acts.

The final velocity is, by the last section, equal to $\frac{1}{2}v$; and the distance d traversed, being the product of the average velocity and the time, is

$$d = \tfrac{1}{2}\, vt.$$

Substituting the value of v above we have

$$d = \tfrac{1}{2}\, gt \times t = \tfrac{1}{2}\, gt^2.$$

In other words the distance a body falls is proportional to the intensity of gravity and to the square of the time.

Again, we find the value of t,

$$t = \frac{v}{g};$$

and substituting this in the last formula, we have

$$d = \tfrac{1}{2}\, g \times \frac{v^2}{g^2} = \tfrac{1}{2}\, \frac{v^2}{g}.$$

The square of the velocity which a body acquires is therefore proportional to the distance fallen.

The same formulæ express the relation between the velocity lost by a body projected vertically upwards, the time it takes it to reach its highest point, and the distance it rises in so doing.

§ 109. **Ballistic Pendulum.** — When a body A suspended by a vertical cord AC (Fig. 9) is given a horizontal velocity v along the arc AB, it continues until it reaches a point B at a vertical height AD above A the same as if it had been projected vertically upwards. The reason of this will be seen later on, when we have considered problems in the conservation of energy. We have from the last section

$$AD = \tfrac{1}{2}\, \frac{v^2}{g}.$$

Drawing the diameter $A E$, and the chords $A B$ and $B E$, we have in the similar triangles $A B E$ and $A D B$,

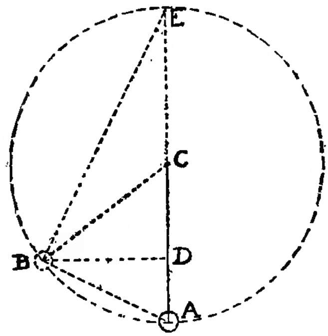


FIG. 9.

$\overline{A D} : \overline{A B} :: \overline{A B} : \overline{A E}$, or $A D = \overline{A B}^2 \div \overline{A E}$. Hence, substituting,

$$\overline{A B}^2 \div \overline{A E} = v^2 \div 2\,g\,;$$

and as $\overline{A E} = 2\,\overline{A C}$,

$$v^2 = \overline{A B}^2 \times g \div A\,C,$$

$$v = A\,B\sqrt{\frac{g}{A\,C}}.$$

The velocity of a pendulum at its middle point is therefore proportional to the distance $A B$ of the point where it turns, measured in a straight line; that is the velocity is proportional to the chord of the arc $A B$. This is the principle used in comparing velocities by the ballistic pendulum.

We shall see that a suspended magnet differs from a pendulum chiefly in the nature of the force which causes it to return to its normal position. When a needle, previously at rest, is given a sudden angular velocity, the arc through which it swings is called the *throw* of the needle. The velocity is therefore proportional to the chord of the throw.

§ 110. **Laws of Vibration.** — The square of the velocity of a pendulum at the middle point of its swing resulting from a given displacement is seen from the last section to vary as the intensity of gravity, and inversely as the length of the pendulum. We may infer that the length of a pendulum is proportional to the square of the time occupied by a single swing; and the force acting upon it is proportional to the square of its rapidity of oscillation.

The same principle applies to a magnetic needle, and is frequently used in comparing the strength of the forces which are exerted upon it. See Experiments 75 and 82.

§ 111. **Isochronism.** — It is well known that a pendulum vibrating in a very small arc keeps almost exactly the same time as in a comparatively large one. This shows that the average velocity of the pendulum (§ 107) must be proportional to the arc. The explanation is simply this, that the force urging the pendulum towards its middle point becomes greater as the arc increases. This force is proportional to *A F* (Fig. 10), perpendicular to *B C*, drawn as in § 109 and hence approximately equal to the distance *A B* which the pendulum must travel. We have already seen that the velocity acquired in reaching the middle point is proportional to the chord *A B* and hence approximately to the arc.

From the fact, however, that the lines $A F$ and $A B$ are not quite equal to the arc $A B$, we infer that a common pendulum is not perfectly isochronous. The effect of different arcs on the rate of vibration will be

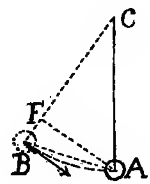


FIG. 10.

found in Table 3, column g. In all experiments with a pendulum or with a vibrating needle, we must limit the arc of oscillation according to the degree of accuracy required.

§ 112. **Point of Application of a System of Forces.** — It may be observed that the weight of a body acts as if a single force were applied to a certain point called the centre of gravity, and that it must be sustained by a single force, or its equivalent, applied in the same vertical line with the centre of gravity, equal and opposite to the weight of the body in question, in order that the body may remain at rest. In the case of a magnet the forces which it exerts act for most purposes as if they came from two points, represented in Fig. 13, § 126. We say therefore that the point of application of the forces exerted by gravity is at the centre of gravity, while the centres of magnetic forces are at two points called poles.

§ 113. **Couples.** — A pair of forces equal in magnitude but opposite in direction are said to constitute a *couple*. The perpendicular distance between the lines in which they act is called the arm of the couple; the product of the magnitude of either force and the arm of the couple is called the *magnitude* of the couple.

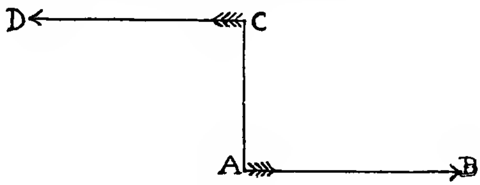


FIG. 11.

Thus AB and CD (Fig. 11) constitute a couple with an arm AC, and magnitude $AB \cdot AC$. The effect of a couple in a given plane ($ABCD$) does not depend upon the location or direction of the arm AC with respect to the (rigid) body acted upon, and it is indifferent at what points in the lines AB and CD the corresponding forces are applied. A left-handed couple ($AB \cdot AC$) can be balanced only by an equal and opposite right-handed couple ($A'B' \cdot A'C'$) such that

$$AB : A'B' :: A'C' : AC.$$

§ 114. **Hooke's Law.** — The effect of a force applied at the end of a rod is either to stretch or to bend it; the effect of a couple is to twist a rod. These effects are found to be proportional to the magnitude of the forces or couples in question. Hooke's law "ut tensio sic vis" may be translated, *strains are proportional to stresses*. (See § 22.) The ratio of a stress to a strain constitutes what is called a *modulus of elasticity*.

§ 115. **Laws of Flexure.** — The force required to bend a beam is evidently proportional to its breadth, but the thickness must be taken three times into account, first, because a greater strain or distortion necessarily accompanies a given amount of bending; second, because (as in the case of breadth) there is more material to be bent, and third, because the force has less purchase upon the material.

The force required is in fact proportional to the cube of the thickness. It can be shown in a similar way to be inversely as the cube of the length, for less force will be required, first, because it has a greater purchase; second, because the longer the beam is, the less sharply need it be bent to deflect it through a given angle; and third, because it takes a smaller angle to produce a given deflection.

§ 116. **Laws of Torsion.** — The couple required to twist a rod of a given shape increases with its breadth or thickness, first, because the average strain or distortion is greater — at the edges, for instance; second, because the purchase of the forces is less; third, because the material acted upon is proportional to the breadth; and fourth, because the material is also proportional to the thickness. In the case of a square or round rod the couple is therefore[1] proportional to the fourth power of the diameter. It is also inversely as the length, because the strain is less in proportion to the length of the rod for a given amount of twisting.

[1] It may be remarked that if there are N *independent* reasons why one quantity should increase in proportion to another quantity, the former always varies, other things being equal, as the N^{th} power of the latter.

§ 117. **Measurement of Work.** — Work is measured by multiplying together the distance through which a point has moved and the force which has been overcome. Thus the work transmitted through a belt can be found if we know the difference of tension between the two portions moving respectively to and from the driving-wheel, and the total distance traversed. If the belt is prevented from moving, as in Experiment 69, we can find the work done by the wheel in rubbing against the belt. We multiply together in this case the difference of tension in the belt and the distance traversed by the rim of the wheel. The work in question is transformed by friction into heat, but it could easily be utilized by allowing the belt to turn machinery. The measurement of work transmitted through a belt while in motion is more or less complicated.

§ 118. **Work of Water under Pressure.** — The work represented by a flow of water under pressure is easily calculated. Suppose the orifice to be 1 *sq. cm.* in section; then the force behind the stream is numerically equal to the pressure (see § 63). Let the stream advance 1 *cm.*; then the work done, being the force times the distance, or in this case the pressure times the distance, is also numerically equal to the pressure. The volume of water which escapes from the orifice is clearly 1 *cu. cm.* Hence the work done on 1 *cu. cm.* is numerically equal to the pressure. The same is also true, no matter what the size of the orifice may be; for with a given pressure per *sq. cm.* the force must vary with the cross section of the stream, and hence also the work represented by an advance of 1 *cm.*; but the volume in *cu. cm.* delivered also increases in the same proportion, and therefore the work per *cu. cm.* remains the same.

Since pressure in dynes per square centimetre is numerically the same as work in ergs per cubic centimetre, we have the following rule: To find the work in ergs represented by a flow of water under pressure, multiply together the flow in cubic centimetres and the pressure in dynes per square centimetre.

§ 119. **Work done by Oblique Forces.** — When the direction of the force and the motion is not the same, we consider only the effect or component of the force in the direction of the motion (see § 105); or we may, on the other hand, take the component of the motion in the direction of the force, and multiply by the whole force in question; because in taking the component of either the force or the motion we reduce it in a given proportion determined by the angle between the two directions in question (see § 105). Evidently it makes no difference which of the two terms in a product is thus reduced.

§ 120. **Conservation of Work.** — It follows from the principle set down in the last section that moving from A to B (Fig. 12), then from B to C, against a force acting in any fixed direction, FF', requires the same amount of work as in moving directly from A to C. For if we drop perpendiculars AA', BB', CC', upon the line FF' representing the direction of the force, the components of the motions are $A'B'$, $B'C'$, and $A'C'$ respectively, and since these are in the same straight line, $A'B' + B'C' = A'C'$. That is, the sum of the component motions is the same by a direct or by an indirect path, and hence also the work required,

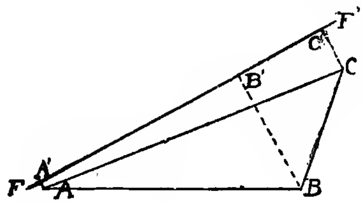


FIG. 12.

or the product of these components by the whole force in question. The fact that no work is gained or lost by choosing different paths is an illustration of the more general principle of the conservation of energy.

§ 121. **Energy of a Moving Body.** — A question which often arises is, how much work is stored up in a moving body, as for instance in a gram of matter with a velocity one *cm. per sec.* Suppose a dyne to act on a gram at rest, we know that it would give it, by definition (§ 12), in one second a velocity of one *cm. per sec.* We know (by § 107) that the average velocity for this second will be half a centimetre per second, or that the gram will have moved $\frac{1}{2}$ *cm.* The work done upon it is therefore $\frac{1}{2}$ *dyne-cm.* $= \frac{1}{2}$ *erg.*

To give a gram twice the velocity in the same time would require twice the force and double the average velocity; the distance would also be doubled. This would mean four times the work. In the same way three times the velocity would mean nine times the work, or in general the work done upon a moving body is proportional to the square of its velocity. It is obviously also proportional to the mass; and as 1 gram with a velocity of 1 *cm. per sec.* has been found to contain $\frac{1}{2}$ erg, we have the following rule: Multiply the mass in grams by the square of the velocity in centimetres *per sec.* and divide by 2 to find the work in ergs which a moving body contains.

It is easily found by calculation that a moving body in coming to rest can do the same amount of work as was required to set it in motion. A gram, for instance, with a velocity of 1 *cm. per sec.* will be brought to rest by a force of 1 dyne in 1 second. The average velocity is therefore $\frac{1}{2}$ *cm. per sec.*; the distance traversed $\frac{1}{2}$ *cm*; the work done against 1 dyne through a distance of $\frac{1}{2}$ *cm.* is $\frac{1}{2}$ erg,—the same that was required to start it in motion.

§ 122. **Conservation of Energy in Mechanics.**—Work stored in a body is often called energy. Energy is again defined as the power of doing work. We distinguish between the energy of motion of a body (kinetic energy) and its energy of position (potential energy), due to the level, for instance, to which it has been raised. All kinds of energy are measured in ergs.

We have seen that it takes the same amount of work to raise a body from one level to another, no matter by what path it may be raised (§ 120). When it returns to the original level the work is given back. The energy spent in setting a body in motion is also restored when the body comes to rest (§ 121).

Energy of position may be changed into energy of motion and the reverse, as is particularly evident in the case of falling bodies or bodies projected into the air; but in mechanics *no energy is ever lost.* This statement is an illustration of a more general principle known as the "Conservation of Energy" (§ 149).

CHAPTER IX.

ELECTRICITY AND MAGNETISM.

§ 123. **Nature of Electricity and Magnetism.** — We do not know what electricity and magnetism are; that is, we are ignorant of their fundamental relations to matter and motion. Electricity circulating around the particles of steel is believed by many to be the sole cause of its magnetism. This hypothesis accounts for all the observed effects. It has been suggested by leading scientific men that the rapidity with which light is transmitted may be due to electrical action (see § 93), and it is suspected that chemical affinity is closely related to electricity. (See §§ 142–144.) We speak of electricity as if it were a fluid; but there are three reasons why neither electricity nor magnetism can be regarded as a fluid in the ordinary sense: first, they have no inertia (or resistance to being set in motion); second, they have no weight (or attraction for ordinary matter under the law of universal gravitation); and third, they repel, instead of attracting their own kind.

In the first two respects electricity and magnetism resemble heat more than a fluid. It has been suggested that they may be forms of energy; but there are more objections to this view than to the other, and comparatively little help is to be derived from it. Even if electricity were proved to be a kind of motion, we should still think of it as a fluid, as we do of heat when it is said to *flow* from one point to another (§ 74).

§ 124. **Positive and Negative Electricity.** — As compressed air can be distinguished from rarefied air, so positive may be distinguished from negative electricity. When mixed together they neutralize one another; and in this neutral condition, electricity, like the atmosphere, seems to be everywhere present. Positive electricity can be separated from negative by various means; but we produce in all cases equal quantities of both. For instance, glass rubbed with a piece of silk receives a positive charge; an equal charge of negative electricity is found in the silk. Some writers maintain that there are really two distinct kinds of electricity which unite, somewhat as an acid does with a base to form a neutral compound; and mathematicians are apt to take this view, finding it convenient to treat electricity as incompressible. Positive electricity may, however, be thought of as under greater pressure than negative, whether it yields to that pressure or not. We imagine that it is this pressure which causes electricity to flow from one place to another. We consider only the flow of positive electricity; though it is maintained by some that half the effect is due to the flow of an equal quantity of negative electricity in the opposite direction.

§ 125. **Electrical Attractions and Repulsions.** — Two bodies charged with positive electricity repel each other, or two charged with negative electricity repel each other; but a body charged with positive electricity attracts one with a negative charge. The force exerted is proportional to the charge, or quantity of electricity in each body. It is, in fact, equal to the product of the two charges, divided by the square of the distance between them. There is also a mutual repulsion between different portions of the same charge, which tend therefore to fly as far apart as possible. Hence electricity collects in the surfaces of bodies which conduct it, and (except while flowing through them) is never found at any appreciable depth.

§ 126. **Nature of a Magnet.** — In a similar way positive and negative charges of magnetism may be sepa-

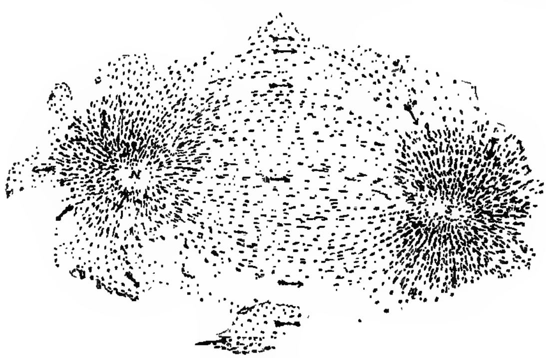


FIG. 13.

rated, but only in a few substances like steel. With magnetism, as with electricity, a positive charge implies an equal negative charge; but in the case of magnetism both charges are always found in *the same* body. Such a body constitutes a magnet, and is said to have two poles, corresponding to the centres of positive and negative magnetism. The position of the poles *N* and *S* (Fig. 13) is shown by sprinkling iron-filings on a piece of glass over the magnet. The iron-filings arrange themselves in lines as in the diagram, radiating from the two poles *N* and *S*. One of these poles, *N*, is called north because, when the magnet is freely suspended, it tends to point approximately in that direction;[1] the other is called the south pole. The direction in which a magnet is said to point is always determined by its north pole.

§ 127. **Lines of Force.** — The iron-filings arrange themselves along what are called "lines of force." A small compass-needle placed close to the glass always points parallel to the lines of iron-filings, and gives the direction of the lines of force, as indicated by arrows in the diagram. The lines accordingly are said to come *from* the north pole, and go *to* the south pole. It is found that where the lines are closest, the magnetism is strongest. A strong horseshoe magnet can hold a solid mass of iron-filings between its poles.

§ 128. **Field of Force.** — The space around or between the poles of a magnet, wherever its action is felt, is called the *field* of force, or simply the *field* of that magnet. By the *intensity* of this field we mean the force exerted by the magnet on a unit quantity of magnetism (§ 17) placed at any point of the field.

[1] At Cambridge, Massachusetts, a magnet points very nearly north by west.

The intensity varies in different parts of the field. At a given point the intensity of the field due to a single magnetic pole is equal to the strength of the pole divided by the square of its distance from the point in question. Both poles of a magnet must, however, be taken into account in calculating the intensity of a field. The resultant (§ 105) of the forces upon a unit of positive or north[1] magnetism determines, by its direction and magnitude, both the direction of the lines of force, and the intensity of the field.

The earth, for example, is a great, though weak magnet. The intensity of its field at Cambridge, Massachusetts, is about 1 dyne per unit of magnetism; or more exactly, $\frac{3}{4}$ dyne. The lines of force are, however, more nearly vertical than horizontal, and only their horizontal component, or about one quarter of the whole effect, is felt by a compass. The angle between the lines of force and a horizontal plane (70°–80°) is called the magnetic dip.

The field of a dynamo machine may be several thousand times stronger than that of the earth.

§ 129. **Magnetic Attractions and Repulsions.** — Two north poles, or two south poles, repel each other; a north and a south attract; the force exerted is proportional to what we call the *strength* of each pole — in the case of two poles, it is equal to the product of their

[1] By "north magnetism" we mean the kind of magnetism contained in that end of a magnet which points north. This is evidently the opposite kind to that which we find in the north polar regions of the earth, since only dissimilars attract. The "magnetic north pole" of the earth is therefore technically a negative or south pole.

strengths divided by the square of the distance between them. Comparing this statement with that in § 128, we see that the force acting on a magnetic pole is equal to the product of its strength, and that of the *field of force* in which it is placed. The strengths of the north and south poles of a given magnet are always alike.

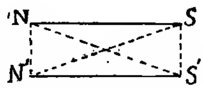


FIG. 14.

When two magnets with poles, N, S, N', S', of nearly equal strengths, $\pm s$, and $\pm s'$, are placed parallel and opposite to one another, as in Fig. 14, if the distance between them is d, there is a perpendicular repulsion between N and N' equal to $ss' \div d^2$; and one between S and S', of the same amount. There is furthermore an oblique attraction between N and S', also between N' and S; but if the distances NS' and $N'S$ are great in comparison with NN', or d, the oblique forces may be disregarded.[1] The resultant is therefore approximately equal to $2ss' \div d^2$.

By supposing one of the magnets reversed, we find in the same way a resultant attraction nearly equal to $2ss' \div d^2$. Counting attractive forces as negative, the

[1] The effective components of the oblique forces bear to the perpendicular forces a ratio equal to $(NN' \div NS')^3$. If NS' is 5 times as great as NN', the error committed by disregarding the oblique forces will be less than 1 per cent. The chief source of error in the application of the principles contained in this section lies in the fact that magnetic forces are only approximately centred in poles.

algebraic difference,[1] Δ, between the repulsion and the attraction will be

$$\Delta = 4\,\frac{s\,s'}{d^2}, \text{ nearly.}$$

We measure Δ by an ordinary balance in experiment 72, with a small distance, d, between two nearly equal magnets, and thus determine roughly the mean strength of the poles in question.

§ 130. **Action of Currents on Magnets.** — When an electric current flows through a wire, it affects all magnetic bodies in its vicinity. It creates, in fact, a magnetic field. When only a short portion of the wire is considered, the intensity of the field due to this portion is proportional to its length and to the strength of the current passing through it; the intensity also varies inversely as the square of the distance from the wire. The lines of force are perpendicular to the wire at every point. They are in fact circles with the wire at their centre, as shown by the arrangement of iron-filings about a vertical current, in Fig. 15. Hence, a magnet tends to point at right angles to an electric current, and to the line joining the two. To remember which way the magnet points, place the thumb across the forefinger of the right hand; if the

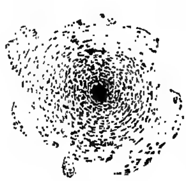


FIG. 15.

[1] Charges of magnetism which each magnet "induces" upon the other increase the mutual attraction of the magnets, but decrease their mutual repulsion by a nearly equal amount. The algebraic difference remains essentially the same.

finger represents the direction of the current, the thumb shows how the north pole of a magnet points.

§ 131. **Action of Magnets on Currents.** — Conversely, an electric current is acted upon by magnetic bodies in its neighborhood. It is, in short, affected by a magnetic field. The effect is equal, under the most favorable circumstances, to the product of the length of wire, the strength of the current, and the intensity of the field. In general, however, we consider only that portion or component of a current which is perpendicular to the lines of force. The direction in which a field acts upon a current is at right angles to the lines of force and to the current. To remember which way the field acts on the current, let the thumb represent a north pole as before, and the forefinger a current; then the thumb will point in the direction in which the pole is urged; hence as action and reaction are equal and opposite, the current must be urged towards the base of the thumb.

The lines of force due to the current are, as we have seen, parallel to the thumb; but those due to the pole are perpendicular both to the thumb and to the forefinger. They issue in fact from the north pole (see § 127) and follow, accordingly, the *line of pressure* between the thumb and forefinger. It is these lines alone which affect the current. Neither the pole nor the current is influenced by the field of force which it itself creates.

§ 132. **Magnetic Current Measure.** — From our definition of the unit of current (see § 18) and the laws stated in the last section, it is clear that the field of force due to a current C flowing through a length of wire L at a distance D is equal to $CL \div D^2$, and that the action of a field of force F on the same current, if they are at right angles, is CLF. These expressions enable us to measure a current through its magnetic action, as will be explained further in §§ 133–135.

§ 133. **Constant of a Coil.** — The constant of a coil of wire is defined as the field at its centre due to a unit of current passing through the wire. If the radius of a circular coil is r, the number of turns of wire n, the length of wire is $2\pi r \times n$, every portion of which acts in the same direction on a magnet at the centre (see § 130); hence the constant is

$$K = \frac{L}{D^2} = \frac{2\pi rn}{r^2} = \frac{2\pi n}{r}.$$

§ 134. **Magnetic Area.** — A rectangular coil, $abcd$, of wire in the plane of this paper, would be acted upon differently in different parts by a field of force in the same plane. Suppose that the current C circulates with the hands of a watch; and that a field acts from left to right. Then (by § 131) the sides ab and cd (Fig. 16) will not be affected; $\overline{ad}$ will be depressed with a force $C \times \overline{ad} \times F$, and $\overline{bc}$ will be raised with the same force; the two forces then constitute a couple, with an arm ab and magnitude $CF \times \overline{ab} \times \overline{ad}$. The couple acting on a rectangle, $abcd$, is therefore equal to the product of the current and field of force multi-

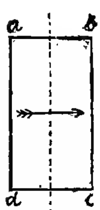
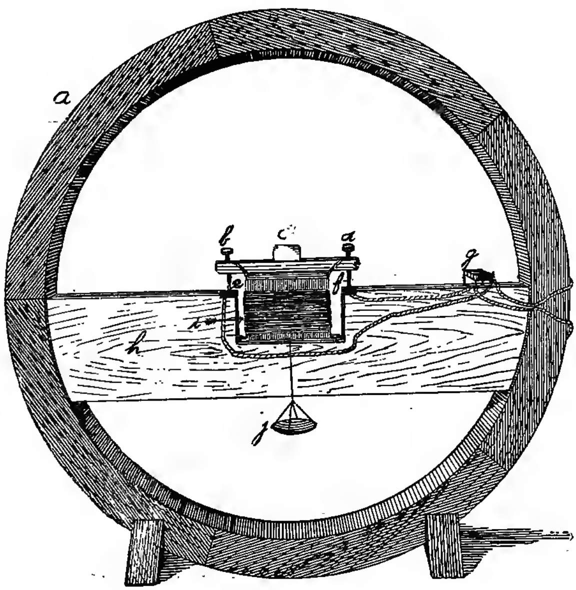


FIG. 16.

plied by the area of that rectangle. The same clearly holds for any number of rectangles or for their sum. A rectangular coil of wire consists essentially of a series of rectangles, *a b c d*, each carrying the current, *C*. The total area, *A*, enclosed by these rectangles is called the *magnetic area* of the coil, and determines the couple, $C F A$, acting upon the coil in a magnetic field, *F*, in its own plane.

§ 135. **Electro-Dynamometer.** — A common form of electro-dynamometer consists (see illustration) in a



ELECTRO-DYNAMOMETER.

coil of wire *a*, with a smaller coil *i*, at right angles to it near its centre. The larger coil is usually circular;

the smaller may be rectangular. If K is the constant of the large coil, a current C, circulating through this coil, will cause a field of force ($F = C K$) to act on the small coil; if the magnetic area of this is A, and the same current, C, passes through the small coil, the couple acting on the latter will be $C F A = C^2 K A$.

When the constant K and magnetic area A are known it is only necessary to measure the couple in order to determine the current. A current is thus primarily measured by the force with which it acts on itself. We shall not need to consider currents through long conductors, except where, as in § 133 or in § 134, every portion is similarly situated with respect to the forces in question.

CHAPTER X.

ELECTROMOTIVE FORCE AND RESISTANCE.

§ 136. **Heating by Electricity.** — When a current of electricity passes through a wire, heat is developed in proportion to the square of the current and also to what we call the *electrical resistance* of the conductor. This is known as Joules's Law. When the power, or the rate at which heat is generated, reduced to watts (see § 15) is P, when the current in ampères (§ 19) is C, and when the resistance in ohms (§ 20) is R, we have $P = C^2 R$.

The resistance R of a conductor is thus easily found if we know the amount of heat developed in it by a given current in a given time. (See ¶ 172.)

§ 137. **Electrical Power.** — The work spent in one second in maintaining a current is obviously the same thing as the power, P; and the quantity of electricity flowing in one second is by definition equal to the current C; the ratio of the power to the current is therefore the same thing as the work spent per unit of electrical quantity, and is defined as electromotive force, E. Electromotive force corresponds therefore to hydrostatic pressure (see § 118), or rather, to a difference of hydrostatic pressure.

We have, therefore,

$$E = P \div C \text{ or } P = CE;$$

that is, electrical power (in watts) is equal to the product of the current (in ampères) by the electromotive force (in volts).

§ 138. **Ohm's Law.** — Since in the last section we found $E = P \div C$, and in the section before, $P = C^2 R$; we have, substituting, $E = C^2 R \div C = C R$. In other words, the electromotive force (in volts) is equal to the product of the current (in ampères) and the resistance (in ohms). It follows that the current (in ampères) is equal to the electromotive force (in volts) divided by the resistance (in ohms), or

$$C = \frac{E}{R}.$$

This is known as Ohm's Law.

A similar law discovered by Poiseuille holds for the flow of liquids through capillary tubes. If R is the resistance of such a tube as defined in § 20, E the hydrostatic pressure in *dynes per sq. cm.*, and C the current in *cu. cm. per sec.*, we have

$$C = \frac{E}{R}.$$

§ 139. **Electrical Potential.** — Electrical potential is analogous to pressure, or head of water. As water flows through a horizontal tube from places of high pressure to places of low pressure, so electricity flows from points of high potential to points of low potential. The electromotive force of a battery is the same thing as the difference in potential which it is capable of producing. Hence we may apply Ohm's Law as follows: the current (in ampères) through any conductor (containing no source of electricity) is equal to the difference in potential of its two extremes (in volts) divided by the resistance (in ohms) between them, no matter how the difference of potential is kept up; and the difference of potential at the two extremes of such a conductor (in volts) is the product of the current (in ampères) and the resistance (in ohms). Denoting by c the current, by r the resistance, and by e, the difference of potential in any portion of the conductor, we have

$$e = c\,r.$$

Clearly, when a given current of electricity, c, travels along a wire it loses in potential by an amount, e, proportional at any point to the resistance, r, which has been overcome.

§ 140. **Resistance in Series and in Multiple Arc.** — When a current passes first through one conductor then through another, as we say in *series*, the total resistance is clearly the sum of the separate resistances; but if the current has a choice of two paths, like a congregation dispersing through two doors, it is less retarded than if confined to one alone.

Let $A\,B\,C$ and $A\,D\,C$ (Fig. 17) be two such channels as we say, in *multiple arc;*

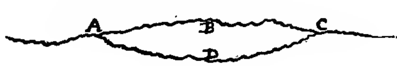


FIG. 17.

if the resistance of $A\,B\,C$ is R_1, and that of $A\,D\,C$, R_2, and the difference of potential between A and C is E, then the current C_1 through $A\,B\,C$ is $C_1 = E \div R_1$; that through $A\,D\,C$ is $C_2 = E \div R_2$; the total current is $C = C_1 + C_2 = E \div R_1 + E \div R_2$. But if the combined resistance is R, we have $C = E \div R$. Equating the two values of C, and cancelling E, we have

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}; \text{ or}$$

the reciprocal of the combined resistance of two (or more) conductors in multiple arc is equal to the sum of the reciprocals of the separate resistances.

We notice also that the current through each channel is inversely as its resistance, or $C_1 : C_2 :: R_2 : R_1$, from which $C_1 : C :: R_2 : R_1 + R_2$, etc.

§ 141. **Wheatstone's Bridge.**—We have seen (§ 139) that loss of potential is proportional by a given path to the resistance overcome. Since in Fig. 17, § 140, in passing by either path from A to C, the total loss of potential must be the same, the loss in reaching B will be the same as in reaching D if the resistances of $A\,B$ and $A\,D$ bear the same proportion to the total resistances of $A\,B\,C$ and $A\,D\,C$ respectively. In this case no current will flow through a wire joining B and D (Fig. 18), since these points will have the same po-

FIG. 18.

tential. A cross connection $B\,D$, between two parallel circuits $A\,C$, is called a Wheatstone's Bridge; and the absence of any current through it shows that the four resistances AB, BC, AD, and DC are in proportion; that is,

$$AB : BC :: AD : DC.$$

§ 142. **Electrolysis.** — When a current of electricity passes into and out of a fluid by means of two conductors, often called electrodes, the liquid is almost always decomposed, and its constituents liberated. The metallic elements are generally carried with the current, the acid constituents against it until they reach the electrodes. There they are either deposited, as in electroplating, or set free in the gaseous form, as in the electrolysis of water, or made to combine with the material of one of the electrodes, as the acid does with the zinc of an ordinary battery.

§ 143. **Electro-chemical Equivalents.** — As concerns the quantity of a given substance acted upon in electrolysis, neither the surface of the electrode nor the chemical nature of the reaction seems to have any effect. A given quantity of electricity always affects a given quantity of a given substance. Thus one ampère in one second causes about one 3000th of a gram of zinc to be dissolved from a zinc plate forming one of the electrodes, or deposits about three times as much mercury. The quantity of mercury is found to be the same, whether the nitrate or chloride is used; and a similar uniformity is found, in the case of other elementary substances, in regard to the quantity set free from their various salts. The weight of a substance acted upon by the unit quantity of electricity is called its electro-chemical equivalent. (See Tables 8 *b*, 11 and 12.)

§ 144. **Law of Electro-chemical Equivalents.** — Observation shows that the electro-chemical equivalents of different substances are to each other as their chemical combining proportions. Thus 2 parts of hydrogen combine with 16 parts of oxygen to form water, or with 71 parts of chlorine to form muriatic acid; again, 71 parts of chlorine or 16 parts of oxygen unite with 63 parts of copper or 65 parts of zinc; one ampère in about 192 seconds sets free 2 *mgr.* of hydrogen, 16 *mgr.* of oxygen, 71 *mgr.* of chlorine, dissolves 65 *mgr.* of zinc, and precipitates 63 *mgr.* of copper. There is evidently an intimate connection between electricity and the bonds which bind atoms chemically together; though no one as yet has offered a satisfactory explanation of the law of electro-chemical equivalents.

§ 145. **Calculation of Electromotive Force.** — Since we know the quantity of zinc dissolved by one ampère in a second ($\frac{1}{3000}$ *g*), the amount of heat which a gram of zinc gives out in combining with nitric acid (about 1500 units), and the value of one unit of heat per second in watts (4.2 nearly), we can evidently find the power spent on one ampère by multiplying these three together, and this should be (§ 137) the electromotive force developed by the action. Hence a battery in which the only reaction is the dissolving of zinc in nitric acid should have an electromotive force of about $\frac{1}{3000} \times 1500 \times 4.2$, or 2.1 volts.

The electromotive force E generated by any chemical action is accordingly 4.2 times the product of the electro-chemical equivalent and heat of combination in question. In the Daniell cell we must offset against the electromotive force due to the solution of zinc, that due to the precipitation of copper, which is about one-half of the former, because the copper which is separated from the acid with which it is combined has very nearly half as much affinity for it as the zinc which takes its place. The electromotive force of a Daniell cell is therefore about 1 volt.

Experiment shows that electromotive forces can be calculated with more or less exactness in this way, as nearly all of the chemical energy is spent on the electric current. The actual electromotive force can never exceed its theoretical value.

§ 146. **Arrangement of Batteries.** — When we join several batteries together in multiple arc (Fig. 19), the

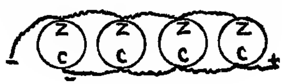


FIG. 19.

zinc poles having all the same potential, and the copper (or carbon) poles all the same potential, we gain nothing in electromotive force, any more than we should gain in pressure by connecting two reservoirs on the same level. The current is, however, often increased, owing to the diminished resistance (see § 140).

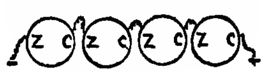


FIG. 20.

When, however, we join batteries in series (Fig. 20), so that the current passes in all cases from zinc to copper, a given amount of work is done on the same current by each cell, as explained in the last section, and hence the electromotive force is increased in proportion to the number of cells. Unfortunately, the resistance is also increased in the same proportion, (§ 140).

In seeking to increase a current, it is as important to diminish resistance as to increase electromotive force (see Ohm's Law, § 138); and a practical rule often of service in the arrangement of a battery is to reduce the resistance of a battery by arrangement in multiple arc or to increase its electromotive force by arrangement in series until the internal resistance is equal as nearly as may be to the resistance of the outside circuit which is to be overcome.[1] In this way the greatest possible current will be obtained from a given number of cells through a given outside resistance. Thus for a very long telegraph line we prefer an arrangement of batteries in series; for a very short circuit an arrangement in multiple arc.

§ 147. **Induction of Electricity.** — When a wire of length L, carrying a current C, at right angles to the lines of force of a magnetic field F, is moved at right angles both to these lines and to itself with a velocity V, against the forces acting on it, evidently power is required of the magnitude $C\,L\,F\,V$ ergs per second; for the force overcome is $C\,L\,F$ (§ 132) and the distance traversed in one second is V. The power required *per unit of current* to keep up the motion is

[1] A similar rule applies to the arrangement of several electrical instruments, but from lack of space it cannot be dwelt upon here.

therefore $C L F V \div C$, or $L F V$. Experiment shows that this power is not spent, as one might expect, in heating the wire, but, through some agency which we do not understand, it acts upon the current in the wire. It produces, in fact, an electromotive force, E, which we have seen (§ 137) is equal to the power per unit of current.[1] That is $E = L F V$. The current is given accordingly by Ohm's Law (§ 138), if the resistance of the circuit is known. We are thus able, given the phenomenon, to anticipate the law governing what is called the *induction of electricity*.

We make use of induced currents, in Experiment 76, to compare the intensity of two fields of force; and in Experiment 77, to compare the intensity of the same field in two directions. In each case the motion of the wires is limited to a certain distance. If the distance is traversed rapidly we get a strong current for a short time; if slowly, a small current for a long time; the sudden throw of a galvanometer-needle (see § 109) is therefore dependent simply upon the strength of the magnetic field.

§ 148. **Thermo-Electricity.** — In regard to the electric current generated by heating or cooling a junction of two dissimilar metals, we observe that the electromotive force is approximately proportional to the temperature of the junction, within narrow limits. As

[1] The electromotive force in this formula is expressed in ergs per second per unit of current. Reducing the power to watts and the current to ampères, we find that the electromotive force in volts is equal to the product of the length of wire in centimetres, its velocity in centimetres per second, and the strength of the field in dynes per unit of magnetism divided by 100,000,000.

one junction in an electrical circuit implies another, it is the difference of temperature of these two junctions which we take into account.

When the range of temperature is considerable, the thermo-electric force is rarely proportional to the difference of temperature of the two junctions. Thus the current which flows ordinarily from copper to iron through a hot junction, increases up to 275°, then diminishes, and is reversed at a still higher temperature.

§ 149. **Conservation of Energy.** — The principle of the conservation of energy explained at the end of chapter VIII., applies to all transformations of energy, and forms the basis, as we have seen, of most important calculations. Whatever light, electricity, and magnetism may be, they return to us eventually in some form the energy spent in creating them. Energy, like matter, may be transformed or scattered, but cannot be destroyed.

ADDENDA.

AMBIGUOUS TERMS.

§ 150. **Gravity**. — Ordinary matter has two characteristic properties: inertia (§ 151), and gravity. The continual changes which take place in the velocities of heavenly bodies, or in the directions of their motions, are attributed to gravity. To account for these changes, it is necessary to suppose an attraction between different bodies which, other things being equal, varies inversely as the square of the distance between them. This is known as Newton's Law of Universal Gravitation. It is not confined to heavenly bodies alone, but holds for any two bodies of matter, however small; though the operation of the law may be concealed by other phenomena. That property in matter which makes it attract other matter is properly called its *gravity*. We say, for instance, that "gravity" draws all bodies toward the centre of the earth. In such expressions as the "acceleration of gravity," the earth's gravity is usually referred to. A body cannot strictly be said to fall under the influence of *its own* gravity. Gravitation is a *mutual* attraction, existing only between two different bodies of matter. We must distinguish between forces of gravitation, which depend upon the distances between bodies, and their gravity proper, which is invariable so long as no change is made in the *quantity of matter* which they contain. An estimate of the quantity of matter, founded upon this invariable property is usually designated by the word *mass*, notwithstanding the fact that "mass" is strictly defined without any reference to gravity whatsoever (see § 152). It is also designated by the word "weight," though this has properly an entirely different signification (see § 153).

Either the word "mass" or the word "weight" *may* mean, accordingly, an *estimate of the quantity of matter which a body contains, founded upon gravitation.* Thus the number of grams (§ 6) by which a body can be balanced determines its "weight in grams." The word "weight" should always be qualified in this way when it refers to a quantity of matter; and when thus qualified it is preferable to the word "mass" as applied to measurements depending upon gravity.

§ 151. **Inertia.** — Bodies do not move instantly from one place to another under the action of forces. More or less time is always required to set a body in motion, to turn it one side, or to bring it to rest. These facts are explained as the result of a universal property of matter called *inertia.* There is, however, no agreement amongst scientific men as to the exact meaning of this term. Inertia is described by some writers (in accordance with the original meaning of the Latin word) as the "inability" of matter to move itself. According to Ganot,[1] "Inertia is a purely negative, though universal, property of matter." Other writers associate with inertia a certain power or necessity. An old term, *vis inertiæ* (force of inertia), illustrates this view. Inertia has been defined as "that property of matter which makes the application of a force necessary for any change in the magnitude or direction of a body's motion."[2] "The fundamental principle of physics," says Deschanel,[3] "is the inertia of matter."

We must distinguish between the so-called *forces of inertia* — that is, forces of greater or less magnitude required under different conditions to produce changes in the motion of a body — and the inertia proper of a given body, which, like its gravity (§ 150), depends only upon the quantity of matter which it contains. An estimate of a quantity of matter, founded upon this invariable property, is designated by the word *mass* in its strict scientific signification (see § 152).

§ 152. **Mass.** — The word *mass* is thought to have the same origin as the German Maas, and to denote, literally, a *measure* of the quantity of matter which a body contains. The mass of a body is strictly defined as the number of standard units of quantity (§ 6) to which a body is equivalent in respect to inertia (§ 151). This is what is always meant by the "dynamical mass" of a body. There are various dynamical devices by which masses may be compared

[1] Ganot's Physics, § 19. [2] Hall's Elementary Ideas, page 5.
[3] Deschanel's Natural Philosophy, 1878, § 6.

(Exps. 59–60); but none leading to very accurate results. It is, however, inferred from results obtained with pendula constructed of different materials (Exp. 58), that there is no perceptible difference between the mass and the weight of a body when both are estimated in grams. The best comparisons of mass are made, accordingly, by means of an ordinary balance. In practice the word "mass" means the number of grams to which a body is equivalent in respect to weight. It is in other words (practically) the same thing as "weight in grams" (§ 150).

§ 153. **Weight.** — Weight is, as we have seen (§ 150), sometimes used to denote the quantity of matter which a body contains. The proper use of the term is, however, in the sense of a force. The weight of a body is strictly defined as the force with which it is attracted by the earth's gravity. In this sense weights should be accordingly expressed in dynes (§ 12). To avoid confusion between the different meanings of the word "weight," it is well to qualify it even when used in its strictest sense. To speak, for instance, of the "weight in dynes" of a body leaves no doubt that it is the idea of force which we wish to convey.

It may be observed that the "weight in dynes" of a body varies with the intensity of the force of gravity exerted upon it; but that the "weight in grams," being practically the same thing as the mass of the body, remains always the same.

§ 154. **Density.** — The density of a body is strictly defined as the ratio of its mass to its volume (§ 9).

Since, however, we usually estimate masses by balancing them with gram weights, and since volumes are measured in cubic centimetres (§ 9), density means in practice the quotient obtained when the weight in grams of a body is divided by its volume in cubic centimetres. The weight is supposed in all cases to be corrected for the buoyancy of air, or in other words, *reduced to vacuo* (§ 67); the volume is supposed to be measured at 0° or reduced to 0°, unless the temperature of the experiment is stated.

If V is the volume of a body in *cu. cm.*, M its mass (or practically its weight) in grams, and D its density, we have accordingly —

$$D = \frac{M}{V}, \tag{1}$$

whence

$$M = DV, \tag{2}$$

and

$$V = \frac{M}{D}. \tag{3}$$

It follows that the density of a substance is numerically equal to the number of grams contained in 1 *cu. cm.* Thus 1 *cu. cm.* of lead weighs (see Table 8) from 11.3 to 11.4 grams; and 1 *cu. cm.* of dry air usually weighs (see Table 19) from .0011 to .0013 grams.

§ 155. **Specific Volume.** — The specific volume of a body is defined as the ratio of its volume to its mass. It is found in practice by dividing its volume in cubic centimetres by its weight in grams. The specific volume (S) of a substance is accordingly the reciprocal of its density; that is (see § 154),

$$S = \frac{1}{D}, \tag{1}$$

whence

$$S = \frac{V}{M}, \tag{2}$$

$$V = MS, \tag{3}$$

and

$$M = \frac{S}{V}. \tag{4}$$

We must distinguish apparent specific volumes from true specific volumes. The true specific volume of a substance is the space occupied by a quantity of that substance weighing 1 gram *in vacuo*. The *apparent* specific volume is the space occupied by a quantity weighing apparently 1 gram in air. Apparent specific volumes are accordingly affected by the density of air. The apparent specific volumes of water under different conditions are contained in Table 22, and are useful in calculations of volumes in hydrostatics. If w is the apparent weight of water, and s its apparent specific volume, the true volume v is given by the equation (see 3),

$$v = ws. \tag{5}$$

§ 156. **Correction and Error.** — Mistakes sometimes arise from confusion between the terms "correction" and "error." If o is the observed magnitude of a quantity, q, the error of observation is $o - q$. A correction is defined as a quantity which added algebraically to an observed magnitude (o) will give the true magnitude (q). It is equal, accordingly, to $q - o$. If the observed value is greater than the true value, it follows that the error is positive, the correction negative; but if the observed value is less than the true value, the error is negative and the correction positive. In every case the correction and the error are equal and opposite.

If e is the "*probable error*" of observation (see § 50), we have by definition,

$$o + e > q > o - e, \textit{ probably},$$

or in the conventional system of representation (§ 53),

$$q = o \pm e.$$

The student must not be led by this expression to imagine that the "probable error" of a result is to be added to it or subtracted from it. He should bear in mind that the so-called "probable error" is not literally a probable error (see § 50), but simply a limit within which the error is probably confined. Even if we knew the magnitude of the error, it would still be impossible to correct for it, since the sign is unknown. No matter how great the probable error of our observations may be, results strictly calculated from these observations are generally *less improbable* than those obtained by making allowances for errors which we do not know to exist.

NOTES

ON THE

ARRANGEMENT OF MATHEMATICAL AND PHYSICAL TABLES.

METHODS OF CONDENSATION.

THE object of constructing mathematical or physical tables is to condense into a small space a large number of results obtained either by calculation or by observation. There are various well-known methods by which condensation may be effected. Thus, instead of writing

The square of the number 1 is 1.
The square of the number 2 is 4. I.
The square of the number 3 is 9.
etc. etc. etc.

we may express these results more concisely as follows: —

Numbers.	Squares.	Numbers.	Squares.	Numbers.	Squares.	
1	1	3	9	5	25	II.
2	4	4	16	6	36	

or in a still more condensed form: —

Numbers.	1	2	3	4	5	6	7	8	9	III.
Squares.	1	4	9	16	25	36	49	64	81	

The fact that a certain column or line of figures contains numbers, another the squares of these numbers, is indicated by the words "numbers" or "squares" at the beginning of the column or line. It is not, however, explicitly stated which number each square corresponds to; this is left to be inferred from the proximity of the printed figures by which the squares and the numbers are represented. Thus in either of the tables II. or III. above, the fact that 25 is the square of 5 is indicated by printing the 5 much nearer to the 25 than to any of the other squares contained in the table.

Sometimes a heavy or a double line is used, as between the 9 and the 5 of the second table (II.), to indicate a wide separation. In this case an arrangement of figures similar to that in Table II., is to be interpreted in accordance with the fact that 25 (not 9) is the square of 5, even if the 9 is closer than the 25 to the figure 5.

It is occasionally desirable to print side by side on the same page the results of performing different operations upon a given number. "Reciprocals," "square roots," "squares," and "cubes" might thus be represented: —

Numbers.	Reciprocals.	Numbers.	Square Roots.
1	1	1	1
2	0.5	2	1.41
&c.	&c.	&c.	&c.

IV.

Numbers.	Squares.	Numbers.	Cubes.
1	1	1	1
2	4	2	8
&c.	&c.	&c.	&c.

It is obviously unnecessary in such cases to repeat the same numbers in each alternate column; and by omitting to do so, as in V., considerable space is gained.

Numbers.	Reciprocals.	Square Roots.	Squares.	Cubes.	
1	1	1	1	1	
2	0 5	1 41	4	8	V.
&c.	&c.	&c.	&c.	&c.	

ARGUMENT, VARIABLE, AND FUNCTION DEFINED.

Starting in such a table (see Table 2, page 798), in the left-hand column, with any number between 1 and 100, we find in a line with it its reciprocal, square root, square, or cube. The number which one starts with is called the *argument*. Different values of the "argument" are almost always placed in the left-hand column of a table, and are printed in heavy type, so as to be distinguished from the rest of the table. The "arguments" represent certain values of a quantity which may or may not vary between wide limits. This quantity is called in any case the "variable." It will be seen by reference to Table 2 (page 798) that when a number increases, its reciprocal diminishes; but that its square and its cube increase faster than the number itself. The reciprocal, square, cube, &c., of a variable are called *functions* of that variable (*fungo*, to perform). Logarithms, sines, cosines, &c., are also called "functions," and in general, whenever two variable quantities are connected together, either by mathematical or by physical laws, so that if the first is given the second may be found, the second is called a "function" of the first. The name of a table relates to the function which it represents. If several functions are given in the same table (see V.) the name of each is usually printed at the head of each column or at the beginning of each line containing the function in question.

ORDINARY MATHEMATICAL TABLES.

When the argument and the function require each 3 or 4 figures to represent it, the same page cannot conveniently contain more than 200 or 300 values of each. If, however, the argument increases regularly (as is generally the case), it is not necessary that it should be printed opposite each value of the function. It is, in fact, sufficient that the argument should be given for every 10th value of the function, since the intermediate values of the argument can be easily supplied. This principle is utilized in the ordinary arrangement of mathematical tables, and affords a considerable saving of space.

Different values of the argument, corresponding in such tables to every 10th value of the function, are placed in a column at the left of the page. Opposite them, in a second column, the corresponding values of the function are given.

Thus in the first two columns of Table 3, *G* (page 810), relating to the areas of circles, we find

10	78.5	
11	95.0	VI.
12	113.1	
etc.	etc.	

The letters *Diam.* are printed over the first column to show that it relates to the diameters of circles. The words "Areas of Circles" apply to the second as well as to the succeeding columns. We see, therefore, that a circle having a diameter equal to 10 units of length, must have an area equal to 78.5 units of area (as nearly as the result can be expressed by three figures). The use of the first two columns by themselves does not differ in any respect from cases which we have already examined.

It has, however, been stated that the first two columns give only every 10th value of the argument and function. The functions of "round numbers" are in fact confined to the second column, which is accordingly headed **0**. Intermediate values of the function are contained in the succeeding columns, headed by the numbers **1, 2, 3, 4, 5, 6, 7, 8, 9**. The values are arranged so as to follow in regular succession when read from left to right like a page of ordinary print. This succession should be continued in passing from a number in the column headed **9**, to the number in the next line in the column headed **0**. The table for the areas of circles becomes accordingly:—

Diam.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	
10	78.5	80.1	81.7	83.3	84.9	86.6	88.2	89.9	91.6	93.3	
11	95.0	96.8	98.5	100.3	102.1	103.9	105.7	107.5	109.4	111.2	VII.
12	113.1	115.0	116.9	118.8	120.8	122.7	124.7	126.7	128.7	130.7	
&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	

The chief peculiarity of a table constructed in this way is that, instead of printing the argument at the left of each value of the function, as in IV., part of the argument is to be found at the left of the line containing the function, while the remainder of the argument — usually a single figure — is placed at the head of the column containing the function. The areas in the first line of the main body of the table (VII.) correspond, accordingly, to the diameters 10.0, 10.1, 10.2, &c., those in the next line to 11.0, 11.1, 11.2, &c., &c.

The argument corresponding to any number in a given column and line may always be found by the following rule: *Add the figure at the head of the column to the figures at the left of the line to find the argument in question.* In making this addition, attention must of course be paid to *decimal points*, which in cases of doubt are given both at the head of each column and at the left of each line. If the decimal point is omitted in either of these two places, it may be taken for granted that the figure at the head of the column is to be written *after* the figures at the left of the line.[1] Thus in Tables 47 and 48, page 897, since the first column contains latitudes 0°, 10°, &c., while at the head of the columns we find 0°, 1°, 2°, &c., we infer that the first line refers to latitudes 0°. 1°, 2°, &c., while the second refers to 10°, 11°, 12°, &c. In table 3, *F* (page 809), however, in the absence of any decimal point in the left-hand column, we infer that the figures in that column, 10,

[1] For an arrangement of tables (having certain advantages) in which the *reverse* is taken for granted, see Pickering's Physical Manipulation, Vol. II.

11, &c., are simply to be prefixed to the figures 10, 20, &c., in the top line.

The first line of "circumferences" relates, accordingly, to circles with the following diameters: 1000, 1010, 1020, &c.; while the diameters corresponding to the second line of circumferences are 1100, 1110, 1120, &c.

EXTENSION OF TABLES.

Most tables contain arguments reaching from 1, 10, or 100 to a value 10 times as great,[1] so that it is possible to find the value of a function corresponding, if not to a given argument, at least to some decimal multiple or submultiple of that argument. From this the desired result may often be obtained by pointing off the proper number of decimal places. Thus to find the circumference of a circle 300 *cm.* in diameter, we observe that 300 *cm.* = 3000 *mm.*, and that the corresponding circumference is (see Table 3, *F*, page 808) 9425 *mm.*, or 942.5 *cm.* Again, in finding the area of this circle, we reduce the diameter (300 *cm.*) to decimetres; and starting with the result (30.0 decim.) as an argument, in Table 3, *G* (page 810), we find the area to be 706.9 *sq. decim.* or 70690 *sq. cm.* (since 1 *sq. decim.* = 100 *sq.*

[1] Tables of reciprocals, squares, cubes, logarithms, &c., often reach from 1 to 11 instead of from 1 to 10. The extension of such tables from 10 to 11, though strictly involving a repetition, is of great convenience in physical problems in which factors just above unity are of comparatively frequent occurrence.

cm.). In finding the volume of a sphere with the same diameter (300 *cm.*), we should reduce this diameter to metres; then with the result (3.00 metres) as an argument, we should find the volume of the sphere to be 14.14 *cubic metres*, according to Table 3, *H* (page 812), or 14,140,000 *cu. cm.* (since 1 cubic metre = 1,000,000 *cu. cm.*). For the extension of trigonometric or logarithmic tables beyond their natural limits, special rules must be observed (see explanation of the tables, page 761 *et seq.*).

OMISSION OF CIPHERS, ETC.

It may be remarked that it is not customary to repeat *initial* ciphers or decimal points throughout the whole of a table. These are given either at the head of each column, or at the beginning of each 5th line. In some books other omissions take place. It is well always to look through a new table carefully before deciding how it is to be read, and where the decimal point is to be placed. A negative sign placed *before* a number applies not only to the integral part of that number, but also to the decimal part which follows. A negative sign placed *over* a figure applies only to that figure. If the figure is an integer followed by a decimal, the integer is negative, the decimal positive. In logarithmic tables, decimal points are frequently omitted both in the argument and in the logarithm. In such cases they are always understood to exist *after* the first figure of the argument and *before* the first figure of the logarithm.

COMPLEMENTARY ARGUMENTS.

Some tables (for instance, Table 4, page 814) contain two arguments. One of these is printed in the ordinary manner, partly at the left and partly at the top of the page, and is to be used in connection with the function mentioned at the top of the page. The other argument is printed partly at the right and partly at the bottom of the page, and is to be used in connection with the function named at the bottom of the page. The object of this arrangement is to make a *double use* of the figures in the body of the table. An extra column of figures is usually added to avoid certain difficulties. No number is placed at the head of this column, and no attention is to be paid to it in dealing with the functions named at the top of the page. The argument corresponding to the function at the bottom of the page is found, in the case of a number in a given column and line, by adding the figure at the bottom of the column to the figures at the right of the line. The values of the argument at the right of a page increase upwards; those at the bottom of the page increase from right to left.

INDEPENDENT ARGUMENTS.

The two arguments employed in the class of tables mentioned above are not independent, but represent quantities each of which is usually the "complement" of the other. The use of *two independent arguments* introduces an entirely different kind of tables. The two arguments correspond in these tables to *two independent variables* upon which the value of the function depends. The first argument is arranged in a column, usually at the left of the table; the second is arranged in a line, usually across the top of the table. To find the value of a function corresponding to given values of both arguments, we follow the line containing the given value of the first argument until we reach the column containing the given value of the second argument. Table 1 (which is a form of multiplication table, see page 797) is an example of the use of two independent arguments. The first argument is a series of factors from 1 to 100, arranged in column at the left of either half of the table. The second argument is an independent series of factors, .1, .2, .3 .4, .5, .6, .7, .8, .9, in the head-line of either half of the table. The body of the table consists in results obtained by multiplying these two sets of factors together. The number occupying a place in a given column and line is the product of the number at the left of the line and the number at the head of the column.

In a table with two independent arguments, the nature of the function is usually given either in the title or at one side of the figures representing the function; the nature of the first argument is given at the head or at one side of the column containing it; while the nature of the second argument is given either at the beginning of the head-line of the table, or just above this head-line.

There is a second method of arranging tables with two arguments, namely: to calculate a separate table of the ordinary sort for each value of one of the arguments. Thus Table 16, *A*, consists of two parts, one calculated for a value of the acceleration of gravity equal to 980, the other for the value 981 *cm. per sec. per sec.* A still greater number of such tables would be necessary to cover all variations in gravity (from 978 to 983) on the earth's surface.

The only way in which it is practicable to represent the value of a function depending upon three independent variables is by means of a series of tables containing two independent arguments, each table being calculated for a special value of the third variable. A complete 2-place table containing three independent arguments, each varying from 1 to 10, would ordinarily occupy about the same space as a 4-place table with a single argument, varying from 1 to 1000, let us say 2 pages. A table with two independent arguments must occupy about 20 pages in order that 3 figures should be significant, and about 2000 pages to give significance to 4 figures. The addition of a third independent argument in the latter case would increase the table to about 2,000,000 pages. It is obvious that the use of tables containing more than 1 independent argument is practically reduced to cases where a rough knowledge of a function is sufficient (as in the calculation of corrections) or where one at least of the variables, like the acceleration of gravity on the earth's surface, or the ordinary condition of atmospheric temperature and pressure, is confined within narrow limits.

PHYSICAL TABLES.

We have seen that, in representing functions of two variables, one argument is usually printed at the left of the table, the other at the head of the table. A similar arrangement is adopted when it is desired to represent simultaneous variations in different physical quantities due to temperature, pressure, or any other single cause. The values of a given physical quantity are arranged either, as in Table 28, in a column opposite the values of the argument to which they correspond, or else, as in Table 31, in a line *underneath* the corresponding values of the argument. The second argument in such tables is *replaced by names*, referring to a series of physical quantities. These are usually different properties of a given substance, or a given property of different substances; but the arrangement may be applied to any set of quantities which are affected by changes in a given variable.

We have, furthermore, an arrangement peculiar to purely physical tables, in which one argument consists of a series of physical properties, while the other argument consists of a series of substances to which these properties belong. This arrangement is adopted in Tables 8, 9, 10, 11, 12, &c. The names of different substances are arranged in a column at the left of the table; the names of different physical properties are printed at the heads of a series of columns so as to form a line across the top of the table. The body of the table contains numerical values. The name of the property to which a given number relates is to be found at the head of the column containing that number; the name of the substance to which it applies is to be found at the left of the table in line with the number in question. The names of the properties and of the substances should be such that, when combined together, they form complete definitions of the physical quantities to which the table relates. The numerical values are in each column reduced, when practicable, to the C. G. S. system; when this is not practicable, a factor by which this reduction may be effected, is placed in the first line of the column. In any case the reduction consists simply in moving the decimal point.

DIFFERENCES.

The differences between adjacent numbers in a purely physical table (especially when, as in the cases which follow, an alphabetical order is observed) have in general no special significance. In mathematical tables, on the other hand, the use of such differences is exceedingly important.

The difference between two adjacent numbers in a table should theoretically, if represented at all, be printed half-way between them as in VIII.

1	1	2	1	3	1	4	1	5	
5		5		5		5		5	
6	1	7	1	8	1	9	1	10	VIII.
5		5		5		5		5	
11	1	12	1	13	1	14	1	15	

It is, however, customary if a given line or column of differences is constant, or nearly constant, to omit this line or column, and instead to print the average value of the differences thus omitted near where the end of the line or column of differences would naturally have come. Table VIII. would thus assume one of the following forms: —

						Dif.	
	1	2	3	4	5	1	
	6	7	8	9	10	1	IX.
	11	12	13	14	15	1	
Dif.	5	5	5	5	5		

										Dif.	
	1		2		3		4		5		
	6		7		8		9		10	5	
	11		12		13		14		15	5	X.
Dif.		1		1		1		1			

	1		2		3		4		5	
	6		7		8		9		10	
	11		12		13		14		15	XI.
Dif.	5		5		5		5		5	
Dif.		1		1		1		1		

					Dif.	Dif.	
1	2	3	4	5	1		
6	7	8	9	10	1	5 5	XII.
11	12	13	14	15	1		

Differences printed, as in IX., *on* a given line or *in* a given column relate accordingly to pairs of adjacent numbers in that line or column. On the other hand, differences printed, as in X., *between* two lines or *between* two columns relate to pairs of adjacent numbers *one in each line* or *one in each column*. Either set of differences, if not needed, may of course be omitted. Table 3, *D* (page 806), corresponds, for instance, to form IX. without the lower line, or to form XII. without the right-hand column of differences.

Instead of printing a series of numbers in the column of differences when they are exactly alike, it is customary to print only one of them, situated as nearly as possible in the middle of the space which the whole series would occupy. This method of representing differences is adopted in Tables 3 *A*, 3 *C*, 3 *G*, 4, 4 *A*, 5, 5 *A*, &c. The difference between any two consecutive values of the function is, in these tables, approximately equal to the *nearest* number in the column of differences. The use of this column of differences will be found to effect a considerable saving of time[1] in processes of interpolation. To effect a still greater saving of time in these processes, a small table of "proportional parts" has been printed in the table of logarithms (Table 6), beneath each difference. The use of proportional parts for interpolation will be explained below (see explanation of Table 1).

[1] It may be remarked that owing to necessary irregularities in the differences which most tables of functions contain, the most accurate results require that these differences should be calculated by actual subtraction in each case.

USE AND EXPLANATION OF MATHEMATICAL AND PHYSICAL TABLES.

TABLE 1 consists of products obtained by multiplying any of the whole numbers (from 1 to 100) in the left-hand column of either half of the table by the decimals .1, .2, .3, .4, .5, .6, .7, .8, .9 at the head of the table. The decimal part of the product is rejected in every case, the units being increased by 1 if the fraction is .5 or more. The table is useful in dividing differences into *parts proportional* to the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, whence the name of the table. It may be used in connection with any of the tables which follow. Let us suppose that it is required to find the sine of 12°.34 in Table 4, page 814. We find the sine of 12°.3 (in the line with 12° and in the column with .3) to be .2130, while the sine of 12°.4 is .2147. The first number (.2130) is too small; the second (.2147) is too great. The difference between them is .0017, or 17 units in the last place, as indicated by the nearest number in the column of differences. If 0°.1 makes a difference of 17 units, 0°.04 should make a difference of .04 ÷ 0.1 × 17, that is, 6.8 or (nearly) 7 units in the last place. The *same result* may be found by seeking in Table 1 a number opposite the difference (17) and under the figure (4) for which the interpolation takes place. The result (7 units in the last place) is to be *added to* the sine of 12°.3, because the sines increase when the angles increase — in other words, because the differences are positive. The sine of 12°.34 is accordingly 0.2130 + .0007 = 0.2137.

Again, to find the reciprocal of 6.789, by Table 3 A, page 802, we observe that the reciprocal of 6.78 is .14749, while that of 6.79 is .14728. The difference between these reciprocals is —.00021, because the reciprocals decrease as the numbers increase. Opposite 21 and under .9 in Table 1 we find 19; hence the answer is .14749 — 00019 = .14730. If we had used the nearest number (22) in the column of differences of Table 3 A., instead of the actual difference (21), we should have found similarly .14729 instead of .14730. The true reciprocal happens to lie between these two values.

Table 1 can be used also in *inverse* processes. Let us suppose that it is required to find the cube root of 800, by Table 3 D, page 806. We notice that the cube of 9.28 is 799.2, just below 800, while the cube of 9.29 is 801.8, just above 800; the difference being 26 units in the last place. The difference between 799.2 and 800.0 is 8 units in the last place. In line with the number 26 in the left-hand column of Table 1, and *over* the number 8,[1] we find .3. We see

[1] When the exact number cannot be found amongst the proportional parts we choose the one nearest to it.

therefore that the cube of 9.283 would be 800.0; hence, conversely, 9.283 is the cube root of 800.

The use of proportional parts is especially recommended when accuracy in the last figure is important. The tables which follow have, however, been constructed with such fulness that interpolation will generally be unnecessary, or readily carried on in the head.

TABLE 2 contains several functions often needed, and is intended for rough and rapid work. More exact values of the functions will be found in Tables 3 A — 3 H, which follow.

Column a contains the "reciprocals" of the numbers in the first column from 1 to 100. The reciprocal of a number is defined as the quotient obtained when unity is divided by the number in question. Example: the reciprocal of 30 is .0333.

Column b contains the square roots of numbers from 1 to 100. The square root of a number is defined as a number which multiplied by itself would give a product equal to the original number. Example: the square root of 49 is 7.00.

Column c contains the squares of numbers from 1 to 100; that is, the products obtained when each number is multiplied by itself. Example: the square of 40 is 1600.

Column d contains the cubes of numbers from 1 to 100. The cube of a number is defined as the result of multiplying that number by the square of that number; or as the result of multiplying that number three times into unity. Example: the cube of 5 is 125.

Column e contains three-place logarithms (see under Table 6) from 0.1 to 10.0. Example: the logarithm of 2 is 0.301, correct to 3 places of decimals.

Column f contains the circumferences of circles having diameters from .1 to 10.0. The circumference is given in the same units as the diameter. Example: given the diameter 2.0 *cm.*, the circumference is 6.28 *cm.*

Column g contains the areas of circles having diameters from .1 to 10.0. The area is given in units corresponding to the unit of length employed in measuring the diameter. Example: given the diameter 2.0 *cm.*, the area of the circle is 3.14 *sq. cm.*

Column h contains the volumes of spheres having diameters from .1 to 10.0. The volume is given in units corresponding to the unit of length employed in measuring the diameter. Example: given the diameter 2.0 *cm.*, the volume of the sphere is 4.19 *cu. cm.*

TABLE 3 contains principally 3-place trigonometric functions, and is, like Table 2, intended for rough and rapid work.

Column a contains angles from 0° to 90°; covering in all a right-angle.

Column b contains the tangents of angles. The tangent of an (acute) angle is defined, with reference to a right-angled triangle, as the ratio of the side opposite it to the (shorter) adjacent side. Example: the tangent of 15° is 0.268.

Column c contains "arcs;" that is, in a circle of radius unity, the length of the arcs intercepted by angles with their vertices at the centre of the circle. "Arcs" are also called the "circular measures" of angles. Example: 15° is equal to 0.262 in circular measure; or the arc of 15° is 0.262.

Column d contains the "chords" of angles. The chord of an angle is defined, with reference to an isosceles triangle, as the ratio of the side opposite the vertical angle to either of the two equal sides. Example: the chord of 15° is 0.261.

Column e contains natural sines. The sine of an angle is defined, with respect to a right-angled triangle, as the ratio of the side opposite that angle to the longest side, or hypothenuse. Example: the sine of 15° is 0.259.

Column f contains natural cosines. The cosine of an angle is defined as the sine of the complement of that angle (see *i*). Example: the cosine of 15° is 0.966.

Column g contains rates of vibration corresponding to different arcs from 0° to 45°, through which for instance a pendulum is vibrating. The arcs are measured from one side of the vertical to the other. The rate of vibration in a very small arc is taken as 1. Example I.: if a pendulum vibrates once a second in a very small arc, it will vibrate .99893 times a second in an arc of 15° (i. e. $7\frac{1}{2}$° on each side of the vertical). Example II.: given the time of oscillation of a magnet equal to 10 seconds in an arc of 45°; required its time of oscillation in a very small arc. Answer, $10 \times .99037 = 9.9037$ sec. Column *g* contains also *coversines* from 45° to 90°.

The coversine of an angle is defined as unity less the sine of the angle. It is the same thing as the versine of the complement of the angle. Versines and coversines measure various errors introduced into physical measurement when two lines which ought to be parallel or perpendicular are inclined at a given angle. The inclination of the two lines is to be found in column *a* or in column *i* as the case may be. Example I.: the shaft of a cathetometer (¶ 262) makes an angle of 89° with the horizon; required the error introduced in the measurement of vertical distances. Answer, .00015 parts in 1, or $\frac{15}{1000}$ of 1 %. Example II.: a magnet which should be horizontal dips 10°; required the error in estimating its magnetism. Answer, .0152, or 1 $\frac{52}{100}$ %.

Column h contains secants, or the reciprocals of cosines. Example: the secant of 15° is 1.035.

Column i contains the complements of the angles contained in column *a*; that is, the results of subtracting these angles from 90°. Example: the complement of 15° is 75°.

It may be remarked that the cotangent of an angle is the tangent of its complement; the cochord of an angle is the chord of its complement; the cosecant of an angle is the secant of its complement. These may all be found, accordingly, by Table 3. Examples: —

The cotangent of 15° = tangent of 75° = 3.732
The cochord of 15° = chord of 75° = 1.218
The cosecant of 15° = secant of 75° = 3.864

To find any function of the *complement* of an angle, we have only to look up that angle in *column i*, instead of in *column a*.

TABLE 3 A is essentially a 4-place table of reciprocals from 1.00 to 11.09, carried out, however, to 5 places between 6.00 and 9.99. Examples: the reciprocal of 2.73 is .3663; the reciprocal of 273 is .003663.

TABLE 3 C is a 4-place table of squares from 1.00 to 9.99, carried out to 5 places between 10.0 and 11.09. Examples: the square of 3.14 is 9.860; the square of 31.4 is 986.0. The square root of 1.25 is is 1.12 nearly, or more exactly, 1.118 (see under Table 1).

TABLE 3 D is a 4-place table of cubes from 1.00 to 9.99, carried out to 5 places from 10.0 to 11.09. Examples: the cube of 5.55 is 171.0; the cube of .555 is .1710. The cube root of 800 is 9.283 (see under Table 1).

TABLE 3 F contains the circumferences of circles with diameters (*diam.*) varying from 1000 to 10090 by 10 units at a time. The results are carried out to units. The differences in this table are either 31 or 32, from beginning to end. The mean difference is 31.416. Proportional parts corresponding to this mean difference are printed at the bottom of the table. The circumference is given in units of the same magnitude as the diameter. Example I.: the circumference of a circle 3600 *cm.* in diameter is 11310 *cm.* Example II.: given a circumference 10,000 metres, the diameter is 3180 metres, nearly; or more exactly, 3183 metres (see under Table 1).

TABLE 3 G is a 4-place table containing the areas of circles corresponding to diameters (*diam.*) from 10.0 to 100.9. The area is given in units corresponding to the unit of length in which the diameter is measured. Example I.: diameter = 15.0 *cm.*, area = 176.7 *sq. cm.* Example II.: diameter = 55.5 *mm.*, area = 2419 *sq. mm.* = 24.19 *sq. cm.* Example III.: area = 4000 *sq. cm.*, diameter = 71.4 *cm.*, nearly; more exactly, 71.36 *cm.* (see under Table 1).

TABLE 3 H contains the volumes of spheres corresponding to diameters from 1.00 to 10.09. The volume is given in units corresponding to the unit of length in which the diameter is measured. Example I.: diameter = 11.1 *mm.* = 1.11 *cm.*: volume = .539 *cu. cm.* = 539 *cu. mm.* Example II.: volume = 35.00 *cu. cm.*, diameter = 4.06 *cm.*, nearly; or more exactly, 4.058 *cm.* (see under Table 1).

TABLE 4 is a 4-place table giving the natural sines of angles from 0°.0 to 89°.9, when interpreted in the ordinary manner by means of the argument at the left and at the top of the page. Natural cosines may also be found by means of this table, by using the argument at the right and at the bottom of the page. Example I.: the sine of 30°.0 is 0.5000. Example II.: the cosine of 30°.0 is .8660.

TABLE 4 A is a 4-place table giving the logarithmic sines (that is the logarithms of the sines) of angles from 0°.0 to 89°.9, when read in the ordinary way. Logarithmic cosines may be found through the argument at the right and bottom of the page. Example I.: the logarithm of the sine of 30° is $\bar{1}.6990$.

Example II.: the logarithm of the cosine of 30° is $\bar{1}.9375$.

TABLE 5 contains the natural tangents of angles from 0°.0 to 89°.9. Natural cotangents from 45°.0 to 89°.9 may also be found by using the argument at the right and bottom of the first half of the table. Below this limit, they are not given; but they may be found by calculating the complement of the angle and looking up its tangent. Example I.: the tangent of 30° is 0.5774. Example II.: the cotangent of 22°.5 = tangent of 77°.5 = 4.511.

TABLE 5 A is a 4-place table giving the logarithmic tangents (that is, the logarithms of the tangents) of angles when read in the ordinary way. Logarithmic cotangents may also be found by using the argument at the right and at the bottom of the page. Example I.: the logarithm of the tangent of 30°.0 is $\bar{1}.7614$. Example II.: the logarithm of the cotangent of 30° is 0.2386.

TABLE 6 is a 5-place table of the logarithms of numbers from 1,000 to 11,009. A decimal point is understood after the first figure of each number and before the first figure of each logarithm. Example: the logarithm of 2.000 is .30103.

When the decimal point of a number does not follow the first figure, the corresponding logarithm consists of two parts. The first part is a whole number called the "characteristic" of the logarithm; the second or decimal part is called the "mantissa."

The "characteristic" of a logarithm is not to be found in Table 6, but is to be supplied by inspection.

Its numerical value is equal to the number of figures between the decimal point of the argument and the space following the first figure of the argument.

Thus the logarithm of the number 1.11 has the characteristic 0; while the characteristics of 11.1 and 111 are 1 and 2 respectively. The sign of the characteristic is positive if the decimal point is at the right of the first figure of the argument; if it is at the left, the sign is negative. Thus the characteristic of the logarithm of .1111 is — 1., the characteristic of the logarithm of .01111 is — 2., &c. The negative sign is in practice written *over* the characteristic, as it affects this characteristic alone.

It is a peculiarity of logarithms that the "mantissa" is not affected by the location of the decimal point in the original number. The logarithm of 1.111 (namely, 0.04571) is, for instance, the same as the logarithm of 1,111. (namely, 3.04571), as far as the mantissa is concerned. The mantissa or decimal part of the logarithm of any number may be found, accordingly, by Table 6, by considering only the figures of which the number is composed.

Initial and final ciphers may be thrown off *ad libitum* in this process; but ciphers in the middle of a number form an essential part of it. Thus in finding the decimal part of the logarithm of .000,100,100, we need to consider only the figures 1001, since these are preceded and followed only by ciphers; but the ciphers between the first and last figures cannot be neglected. The following logarithms from Table 6 may also serve as examples: —

The logarithm	of	3.1416	is	0.49715
" "	"	980	"	2.99123
" "	"	41,700,000	"	7.62014
" "	"	.00367	"	$\bar{3}$.56467

Conversely, in finding the number corresponding to a a given logarithm, we first obtain the figures of which the number is composed by considering simply the mantissa, or decimal part of the logarithm, and to these figures we add as many initial or final ciphers as may be needed; then starting with the space at the right of the first figure (disregarding initial ciphers) we count off to the right if the characteristic of the logarithm is positive (or to the left if negative) a number of spaces equal to the characteristic in question, in order to locate the decimal point. In any case the number of figures between the decimal point and the space following the first figure of the answer must be equal to the characteristic of the logarithm.

Example I.: given the logarithm 0.14860, the figures of the corresponding number are 1408; the characteristic of the logarithm being 0, the answer is 1.408. Example II.: given the logarithm 3.14860, the mantissa being .14860 as before, we find the same figures, 1408. Since the characteristic (3) is positive, the decimal point is at the right of the first figure, and since 3 figures must come between the decimal point and the space following the first figure, the answer is 1,408.

The following rules embody the most important applications of logarithms, — namely, to problems of multiplication and division.

Rule 1. To multiply two or more numbers together, find the logarithm of each and add the logarithms together. The number corresponding to their sum is the required product. Example: to multiply 2×4.

The logarithm of 2 is	0.30103
" " " 4 is	0.60206
The sum of these logarithms is	0.90309,

which is the logarithm of 8, the answer. Numbers involving more than 3 significant figures may be multiplied together by the aid of logarithms with greater ease than by arithmetical processes.

Rule 2. To divide one number by another, find the logarithm of the first, subtract the logarithm of the second; the remainder is the logarithm of the answer. Example: to divide 4 by 8,

The logarithm of 4 is	0.60206
" " " 8 "	0.90309
The difference is	$\bar{1}.69997$,

which is the logarithm of 0.5, the answer.

Rule 3. To find the value of a fraction with several factors, find the logarithm of each factor in the numerator, and add the logarithms together. Then find the logarithm of each term in the denominator, and add these logarithms together. Subtract the latter sum from the former sum. The remainder is the logarithm of the answer. Example: to find the value of the fraction

$$\frac{.2345 \times 45.67 \times 6{,}789}{1.234 \times 34.56 \times 567.8}, \text{ we find —}$$

(1) log. .2345	$= \bar{1}.37014$	(5) log. 1.234	$= 0.09132$
(2) " 45.67	$= 1.65963$	(6) " 34.56	$= 1.53857$
(3) " 6789	$= 3.83181$	(7) " 567.8	$= 2.75420$
(4) sum	$= 4.86158$	(8) sum	$= 4.38409$
(9) subtract	4.38409		
(10) remainder	$= 0.47749 = \log.\ 3.002 +$, ans.		

Rule 4. To raise a number to any power, find its logarithm, multiply by the power, and the product is the logarithm of the answer. Example: to find the 4th power of 2. The logarithm of 2 is 0.30103; which multiplied by 4 gives 1.20412. This is the logarithm of 16, the answer.

Rule 5. To extract any root of a number, find the logarithm of the number and divide by the root in question; the quotient is the logarithm of the answer. Example: to find the 12th root of 2. The logarithm of 2 is 0.30103; this divided by 12 gives .02509, which is the logarithm of 1.0595, the answer. (This is the value of the interval called 1 semitone on the tempered scale.)

TABLE 7 contains the probability of an error's exceeding limits bearing to the "probable error" (§ 50) the ratios represented in the left-hand column. The probability is expressed as so many chances in 1. Example I.: the probable error of a weighing is 1 centigram; what are the chances of an error greater than 1 centigram? Answer, by definition, an *even chance* or 0.50000. Example II.: under the same circumstances, what are the chances of an error's exceeding 2 centigrams? Answer, 0.17734, i. e. 17,734 chances in 100,000, or about 1 chance in 6. Example III.: under the same circumstances, what are the chances of an error's exceeding 5 centigrams? Answer, 0.00075, or less than 1 in 1000.[1]

TABLES 8, 9, 10, 11, and 12 contain (1) the names, (2) the chemical symbols, and (3) the atomic weights of various substances, and deal with the following physical properties: (4) the specific gravity (§ 69) of gases and vapors referred to hydrogen at the same temperature and pressure; (5) the density (§ 15) of substances at 0° under the ordinary atmospheric pressure; (6) the "viscosity" of liquids at about 20°, or the force in dynes required to maintain a relative velocity of 1 *cm. per sec.* between two surfaces 1 *cm.* square and 1 *cm.* apart; (7) the "surface tension" of liquids (¶ 169) at about 20°, or the force in dynes with which *each surface* of a liquid film 1 *cm.* broad tends to contract; (8) the "breaking strength" of solids, or the force in kilo-megadynes[2] required to break a wire 1 *sq. cm.* in cross section; (9) the "crushing strength" of solids, or the force in kilo-megadynes required to crush a block 1 *sq. cm.*

[1] The chances relate only to "accidental errors" (§ 24). The chances of "mistakes" are not included.

[2] 1 kilo-megadyne = 1.02 "tonne weight," or 1 English ton weight, nearly.

in cross section; (10) the "shearing strength" of solids, or the force in kilo-megadynes required to cut a wire 1 *sq. cm.* in cross section; (11) the "hardness" of solids according to Mohs' arbitrary scale (page 587); (12) the "simple rigidity" of solids, or the force in kilo-megadynes required to make two surfaces 1 *cm.* square and 1 *cm.* apart move parallel to one another through a thousandth of a centimetre (.001 *cm.*), (13) "Young's modulus," or the force in kilo-megadynes required to pull two such surfaces apart through one thousandth of a centimetre (.001 *cm.*); (14) the "resilience of volume" or the pressure in kilo-megadynes required to compress a centimetre cube by one cubic millimetre; (15) the average cubical "coefficient of expansion" of substances[1] (§ 83) between 0° and 100° under a constant pressure of 76 *cm.* of mercury; (16) the "melting-point" of solids, or the "freezing-point" of liquids on the Centigrade scale; (17) the "boiling-point" of liquids, or the "temperature of condensation" of vapors at the atmospheric pressure; (18) the "critical temperature" of liquids and vapors, — that is, the temperature at which the properties of the liquid and its vapor become indistinguishable; (19) the "critical pressure" of liquids and vapors, that is the pressure of the vapor of a liquid at the critical temperature in megadynes *per sq. cm.*; (20) the "pressure

[1] When a change of state takes place between 0° and 100°, the averages in question refer only to that part of the interval (0° to 100°) in which the substance exists in the state named at the head of the table.

of vapors" at 20°, in megadynes *per sq. cm.*; (21) the average specific heat of substances[1] (§ 86) between 0° and 100°, under the "constant pressure" of 76 *cm.* of mercury; (22) the average specific heat of substances[1] between 0° and 100°, when prevented from expanding; that is, confined to a "constant volume;" (23) the "latent heat of melting" of solids, or the "latent heat of liquefaction" of liquids; that is, the number of units of heat required to convert 1 gram of a solid, at its melting-point, into a liquid at the same temperature under a pressure of 1 atmosphere; (24) the "latent heat of vaporization" of liquids, or the "latent heat of condensation" of vapors; that is, the number of units of heat required to convert 1 gram of a liquid at the boiling-point into vapor at the same temperature under the atmospheric pressure; (25) the "heat conductivity" of substances, or the number of units of heat conducted in one second between two opposing faces of a centimetre cube differing 1° in temperature; (26) the "electrical conductivity" of substances, or the current in ampères flowing between two opposing faces of a centimetre cube differing 1 microvolt (.000,001 volt) in electrical potential (§ 139); (27) the "thermo-electric heights" of conductors, or the electromotive force in microvolts developed by a thermo-electric junction of which one element is lead, corresponding to a difference of temperature of 1°; (28) "electro-chemical equivalents," or the weight in milligrams of various

[1] See footnote, page 775.

elementary substances affected by a current of 10 ampères in 1 second; (29) the specific inductive capacity of substances determined by currents alternating several hundred times per second (¶ 256); (30) the minimum "extraordinary index of refraction" of optical materials; (31) the "ordinary index of refraction" of uniaxial crystals, or the "medium" index of refraction of biaxial crystals; (32) the maximum "extraordinary index of refraction" of different substances — these three indices referring to the sodium (D) line; (33) the ordinary (or medium) "index of dispersion," or the difference between the ordinary (or medium) indices of refraction for the lines *A* and *H* of the solar spectrum; and finally (34) the solubility of solids in water, expressed in per-cents by weight, and the solubility of gases, also in per-cents by weight, under a pressure of 1 atmosphere.

The first line of each table contains factors by which the values given in the column below them may be reduced to the C. G. S. system. Thus the coefficient of resilience of aluminum (Table 8) is 0.5 (?) $\times 10^{12} = 500{,}000{,}000{,}000$(?), and the thermo-electric height of copper is about $4 \times 100 = 400$ absolute units.

TABLE 8 contains the properties of elementary substances.

TABLE 9 contains the properties of solids remarkable especially for their strength or for other properties rendering them suitable for building materials or for the construction of machines.

TABLE 9 A contains the properties of certain chemical salts and other substances in ordinary use.

TABLE 10 contains the properties of solids remarkable for their optical or other allied properties.

TABLE 11 contains properties of liquids.

TABLE 12 contains properties of gases and vapors.

TABLES 13 A, B, and C, give the (maximum) pressure in megadynes *per sq. cm.* of the vapor arising from various liquids at different temperatures.

TABLE 13 A contains substances which are for the most part gaseous at ordinary temperatures.

TABLE 13 B contains more or less volatile liquids.

TABLE 13 C gives the pressure of the vapor of mercury, sulphur, and water, including the vapor of water arising from sulphuric acid of different strengths.

TABLE 13 D contains the "density of steam," or the maximum density of aqueous vapor at different temperatures.

TABLE 14 gives the boiling-points of water corresponding to different barometric pressures from 68.0 to 77.9 centimetres of mercury reduced to latitude 45° (see Landolt and Börnstein, Table 20). Example: when the barometer stands at 75.0 *cm.*, water boils at 99°.63.

TABLE 14 A gives dew-points (calculated from Regnault's data) corresponding to different degrees of temperature and "relative humidity." The "dew-point" means that temperature at which moisture would (barely) be precipitated out of the air (as when dew is formed); the "relative humidity" is the proportion which the moisture contained in the air at a given temperature bears to the maximum possible amount which it can hold at that temperature. Example I.: the air of a room at 20° is half saturated with moisture (i. e. the relative humidity = 50 %); required the dew-point. Answer, 9° Centigrade by Table 14 A. Example II.: sea air saturated at 9° with moisture is warmed to 20°; required the relative humidity. Answer, 50 %.

TABLE 15 shows at a given temperature (T) the maximum pressure (P) of aqueous vapor in centimetres of mercury, the maximum density (D) of aqueous vapor, and the factor (F) by which the difference between the readings of a wet and a dry bulb thermometer must be multiplied in order to find the difference between the dew-point and the temperature (T) of the air. The data have been taken from Kohlrausch, Table 13, Landolt and Börnstein, Tables 18 *a* and 23, and from Everett's "Units and Physical Constants," Art. 124. The first three columns are an amplification of results contained in Table 13. The last column is useful in hygrometry. Example: if the dry-bulb thermometer reads 20°, and the wet-bulb thermometer reads 15°, so that the difference between them is 5°, we have (since F = 1.8), 5° × 1.8 = 9°, which subtracted from 20° gives 11° for the dew-point.

TABLE 16 A gives the specific heat of moist air at about 50°, corresponding to different dew-points under a constant pressure of 76 *cm.* of mercury. The specific heat of dry air at 0° (.2383) is the mean between the results obtained by Regnault and E. Wiedemann. The other specific heats have been calculated by interpolation between the specific heats of air and of steam (.4805).

TABLE 15 B gives the velocity of sound in atmospheric air calculated for different degrees of temperature and relative humidity, allowing for the effect of moisture on the density of air and on the ratio of the two specific heats of air under constant pressure and under constant volume. The barometric pressure (which has hardly a perceptible influence on the result) was assumed to be 76 *cm.* of mercury.

TABLE 15 C contains coefficients of interdiffusion of gases. The values (due to Maxwell) are taken from Everett's "Units and Physical Constants" (Art. 131). If two reservoirs filled with different gases are connected by a tube 1 *cm.* long, the numbers in Table 15 C show the mean velocity in *cm. per sec.* with which a stream of gas flows through the tube from each reservoir into the other.

TABLE 16 is intended for the reduction of barometric readings, when given in inches, to centimetres. The last line of the table contains "proportional parts" (see under Table 1).

TABLES 16 A and B are intended for the reduction of barometric readings in *cm.* of mercury at 0° to megadynes *per sq. cm.* A is calculated for a value of the acceleration of gravity (g) equal to 980 *cm. per sec. per sec.*; B for the value $g = 981$. The two tables differ by about 10 units in the last place. For values of g between 980 and 981, or just outside of these limits, results may be easily obtained by interpolation. Example: $g = 980.4$; required the value of 1 atmosphere (76 *cm.*) in megadynes *per sq. cm.* Answer, $1.0126 + 11 \times .4 = 1.0130$ megadynes *per sq. cm.*

Table 17 gives the elevation in metres above the sea-level corresponding to different barometric pressures at 10° Centigrade. It has been calculated for dry air in latitude 45° by the formula

$$h = 190790 \,(\log. 76 - \log. p)\,(1 + .000{,}0001\, h).$$

It is used in estimating heights by the barometer. Example I.: the mean barometric pressure is 70.0 *cm.* at the top of a hill rising out of the sea, the sides of the hill having a mean temperature of about 10°; required the height of the hill. Answer, about 681 metres. Example II.: the barometric pressures at a given instant are 75.1 *cm.* at the foot of a hill, and 74.2 *cm.* at the top of the hill,—the mean temperature being about 10°; required the height of the hill. Answer, $199 - 99 = 100$ metres.

TABLES 17 A and 17 B give corrections in per cent to be added to or subtracted from the results of Table 17, according to the mean temperature and dew-point between the observing stations. Thus for a mean temperature 23° and the dew-point + 8° add $4.6 + 0.4 = 5.0\ \%$ to all results. This would make the height of the hill in Example II., 105 (instead of 100) metres.

TABLE 18 *a* gives the correction in centimetres to be subtracted (on account of expansion) from the reading of a mercurial barometer provided with a brass scale reaching from its zero in the surface of mercury in the reservoir to the free surface of mercury in the tube. In calculating this table, the coefficient of expansion of mercury was assumed to be .000180 + .000,000,036 t; the value .000019 was taken for the coefficient of expansion of brass. Example I.: the mercurial column is 76 *cm.* long, measured by a brass scale, its temperature is 20°, we subtract 0.245 *cm.*, and find 75.755 *cm.* for the value at 0°. Example II.: same as I. except that a *glass* scale is used; corrected value the same less .016 *cm.*, that is, 75.739 *cm.*

TABLE 18 *b* gives the mean correction to be *added to* the apparent height of the mercurial column on account of "capillarity," that is, the tendency of capillary or in general *small* tubes to depress a mercurial column (see Everett, 46 A, and Pickering, Table 12). The correction depends, however, not only upon the internal diameter of the barometer tube at the point where the mercury stands, but also upon the height of the "meniscus," which is different according to the direction in which the mercurial column has been moving. Corrections corresponding to different heights of the meniscus are taken from Kohlrausch, Table 15, 6th ed. The results in this table differ widely from those quoted in the 2d edition.

TABLE 18 *c* contains corrections for the pressure of mercurial vapor. They have been obtained by averaging the results of Regnault, Hagen, and Hertz, quoted in Landolt and Börnstein, Table 27. The results in question differ in some cases even in regard to the position of the decimal point.

On account of the great discrepancy between the results obtained by different observers, barometric readings, even when corrected by Tables 18 *a*, 18 *b*, and 18 *c*, are significant only as far as hundredths of a centimetre.

TABLES 18 *d*, 18 *e*, 18 *f*, and 18 *g*, contain factors for the reduction of either the density or the volume of a gas to 0° or to 76 *cm*. Example I.: the density of coal-gas being .0005 at 20° and 75.0 *cm*., required its density at 0° and 76 *cm*. Answer, $.0005 \times 1.0734 \times 1.0133 = .00054+$. **Example** II.: the volume of a gas at 20° and 75 *cm*. is 100 *cu. cm.*; required its volume at 0° and 76 *cm*. Answer, $100 \times 0.9316 \times 0.9868 = 91.9$ *cu. cm.* If the gas were collected over water at 20° we should subtract 1.74 *cm*. (see Table 15) from the apparent pressure (75 *cm*.) and find 73.26 *cm*. for the pressure of the gas. This would give a factor .9640 instead of .9868, and a result 89.8 *cu. cm*. in the example above.

TABLE 19 contains the density (or weight of 1 *cu. cm.*) of air corresponding to different temperatures and pressures, and has been taken from Kohlrausch, 2d ed., Table 6. It was calculated from Regnault's observations for latitude 45°.

TABLE 20 contains corrections for the results in Table 19 to be applied on account of moisture. Example: required the density of air at 20° and 76 *cm*. pressure when the dew-point is + 4° Centigrade. Answer, $.001204 - .000004 = .001200$.

TABLE 20 A contains the weight of air displaced by 1 gram of brass of the density 8.4, and is useful in calculating effective weights (§ 64). Example: a body is balanced by 100 grams of brass in air of the density .001200; required the effective weight of the body. Answer, 100 grams *minus* 100 × 0.000143 grams, or 99.9857 grams.

TABLE 21 contains factors for reducing apparent weighings with brass weights to vacuo. The factors correspond to different densities of the substance weighed, as well as of the air in which the weighing takes place. Example: a piece of glass of the density 2.5 is balanced by 100 grams of brass, in air of the density .00120; required its true weight *in vacuo.* Answer, 100 × 1.00034 = 100.034 grams.

TABLE 22 contains "apparent specific volumes" of water; that is, the space in cubic centimetres occupied by a quantity of water weighing apparently 1 gram when counterpoised in air with brass weights of the density 8.4. The apparent specific volumes correspond to different temperatures and different conditions of atmospheric density, and are useful especially in calculations of volume or capacity in hydrostatics. Example: a flask holds apparently 1000 grams (1 litre, nearly) of water at 20°, when weighed in air of the density .00120; required the capacity of the flask. Answer, 1000 × 1.00279 = 1002.79 *cu. cm.*

TABLE 23 contains true "specific volumes" of water; that is, the space in cubic centimetres occupied at various temperatures by a quantity of water weighing actually 1 gram *in vacuo.* These values are reciprocals of those in Table 24, and are to be used for the calculation of volumes corresponding to *true* weights *in vacuo*. Example: a piece of steel displaces 100 grams of boiling water; required its volume. Answer, $100 \times 1.04311 = 104.311$ *cu. cm.*

TABLE 23 A gives the true specific volume of mercury at different temperatures, and is used like Table 23. In calculating this table Regnault's value (13.596) for the density of mercury at 0° was used, and a coefficient of expansion $.000180 + .000,000,036\ t$.

TABLE 23 B gives apparent specific volumes of mercury when balanced by brass weights of the density 8.4 in air of the density .0012. It is used, like Table 22, to calculate volumes and capacities. Example: the apparent weight of mercury required to fill a tube at 20° is 100 grams; required the capacity of the tube. Answer, $100 \times 0.073812 = 7.3812$ *cu. cm.*

TABLE 24 contains the density of mercury at different temperatures. The values are reciprocals of those contained in Table 23 A.

TABLE 25 contains the density of water at different temperatures. A mean value, 1.00001, was taken for the maximum density of water (Kupffer's value is 1.000013). The relative densities lie between the estimates of Rossetti and Volkmann, founded upon observations by Despretz, Hagen, Jolly, Kopp, Matthiessen, Pierre, and Rossetti.

TABLE 26 contains the density of commercial glycerine, calculated from observations made in the Jefferson Physical Laboratory.

TABLE 27 contains the density of dilute alcohol corresponding to different temperatures and different strengths. The values are a mean between results obtained by numerous observers.

TABLE 28 gives the density, at 15°, of acids and saline solutions corresponding to various strengths, and is useful in making tests with a densimeter. See Storer's "Dictionary of Solubilities." Example: the density of some sulphuric acid is 1.807 at (about) 15°; required its strength. Answer (about) 88 %.

TABLE 29 gives the boiling-points of solutions of various strengths estimated by interpolation from data contained in Storer's "Dictionary of Solubilities." It furnishes an independent (and in processes of concentration by boiling a very convenient) method of estimating the strength of such solutions. Thus a solution of hydrate of sodium boiling at 120° is known to have a strength of about 40 %.

TABLE 30 gives the specific heats of solutions of different strengths at about 20°. It is useful in certain processes in calorimetry (see ¶¶ 99–100). The numbers were obtained by interpolation from results contained in Landolt and Börnstein, Tables 71 and 72. Those nearest the observed values are printed in heavier type.

TABLE 31 A gives the electrical conductivity of solutions at about 18°. It shows the current in ampères which an electromotive force of one volt would cause to flow through a metre-cube of the solutions in question, or through a voltameter with plates 1 decimetre square and 1 *cm.* apart, filled with these solutions, neglecting the effects of polarization. The results must be multiplied by 10^{-11} (.000,000,000,01) to reduce them to the C. G. S. system. The relative values of different results are probably accurate within 5 or 10 per cent, but their absolute values are much less reliable.

TABLE 31 B gives Refractive and Dispersive indices corresponding to the sodium (D) line for solutions of different strengths, and was obtained by interpolation from results quoted by Landolt and Börnstein.

TABLE 31 C is intended to facilitate the preparation of solutions of any desired strength, and for the calculation of per cent contents from the ratio of two constituents. Example: how many parts of salt must be added to 100 of water to make a 20 % solution? Let A = salt; B = water, — the answer is 25 parts. Example II.: a solution contains 100 parts of sulphuric acid to 150 of water; required its strength. Let A = water, B = sulphuric acid; the answer is: 60 % water, 40 % sulphuric acid.

TABLE 31 D gives coefficients of diffusion of saline solutions in water at about 20°. The values were calculated from Graham's data quoted in Cooke's "Chemical Physics." Example: how much common salt would escape by diffusion into pure water from a 20 % solution in 600,000 seconds through a layer 1.2 *cm.* thick and 8 *sq. cm.* in cross section? Answer, 20 % of $600{,}000 \times 8 \times .000{,}0046 \div 1.2 = 3.68$ grams.

TABLES 31 E and F give the rotation in degrees of the plane of polarization of different kinds of light corresponding to the Fraunhofer lines A to H. *E* refers to dilute solutions having such a depth that a beam of light passing through an orifice 1 *cm.* square meets just one gram of the dissolved substance.[1] *F* refers to the effect of plates 1 *cm.* thick.

TABLE 31 G relates to the effect of a magnetic field in rotating the plane of polarization of light parallel to the lines of force.

TABLE 31 H relates to (1) Magnetic Susceptibility, (2) Saturation, and (3) Permanent Magnetism,—that is, the magnetic moment of a unit cube of different materials (1) in a unit magnetic field, (2) in an infinite magnetic field, and (3) in space after the magnetizing influence has been removed. The results are taken from Everett and Ganot.

TABLE 31 I contains some of Weisbach's results for the coefficient of friction of water moving with different velocities through tubes not far from 1 *cm.* in diameter. The results have been reduced to the the C. G. S. system.

TABLE 31 J gives coefficients of friction of solids on solids, taken from De Labarpe's "Notes et Formules de l'Ingénieur."

TABLE 31 K contains coefficients of reflection, absorption, and transmission of radiant heat, from Ganot's Physics.

TABLE 31 L contains estimates (by the author) of the heat radiated at different temperatures by 1 *sq.*

[1] The rotation is proportional, within more or less narrow limits, to the strength of the solution; but may vary widely outside of these limits. Cases of reversal even occur. See Landolt and Börnstein.

cm. of blackened or perfectly radiating surface surrounded by perfectly absorbing walls, or space at 0°. The table was calculated by the formula —

$$q = \log.^{-1} (.0013 \times (t^\circ + 273^\circ) - 1.8249) - .034,$$

which was found to reconcile various well-known facts. Example: how much heat is required to maintain 1 *sq. cm.* of platinum at its melting-point (1900°) for 1 sec.? Answer, 10 (?) units.[1]

TABLES 32 A and 32 B give heats of combustion[2] in oxygen and in chlorine respectively, from data quoted by Everett, by Landolt and Börnstein, and by other authorities. The chemical reactions are not in all cases such as actually take place; but the table gives the heat which it is supposed *would be* developed if the reactions did take place. The last column gives the electromotive forces developed by or necessary to undo some of the reactions. Example: 2 grams of hydrogen uniting with 16.0 grams of oxygen give out 69,000 units of heat, or 34,500 units per gram of hydrogen. This is equivalent to 1440 megergs per *mgr.* of hydrogen consumed. To decompose water, an electromotive force of 1.49 volts is required.

TABLE 33 gives "heats of combination" involving more complicated chemical reactions than those which take place in simple combustion.

[1] This corresponds to 8 + volt-ampères per candle-power.

[2] The heat of combustion of many substances can be inferred only from indirect processes. See experiment 38.

TABLE 34 gives contact differences of electrical potential in volts. The data are taken from Everett's "Units and Physical Constants," Art. 206. Example: a piece of zinc is brought into contact with a piece of copper; required the difference of electrical potential. Answer, the zinc is positively electrified with respect to the copper; the difference of potential is 0.750 volts.

TABLE 35 gives the electromotive force in volts of voltaic cells of various sorts.

TABLE 36 gives the relation between electromotive forces and the length in *mm.* of the spark which they produce in ordinary atmospheric air, calculated from Everett, Art. 192. Example: an induction machine produces sparks 2.5 *mm.* long; required the difference of potential between its poles at the instant. Answer, 9000 volts. Only the first two figures are significant in this answer.

TABLES 37 *a* and *b* give specific resistances of conductors and insulators at 0°. The last column gives the per cent of increase of all these resistances due to a rise of temperature of 1° Centigrade.

TABLE 38 gives the specific resistances of electrolytes corresponding to various strengths. The resistances are in ohms, and apply to a centimetre cube of the liquid. The probable error of the results is about 10 %. *Relative* values are probably not so inaccurate. Example: required the resistance of a cubical Daniell cell, with a plate of copper 10 × 10 *cm.*, separated by a layer of 20 % (crystallized) sulphate of copper 5 *cm.* deep, and by a layer of 20 % (crystallized) sulplate of zinc, also 5 *cm.* deep, from a plate of zinc 10 $\times$ 10 *cm.* Answer: the resistance of the copper solution is $20 \times 5 \div (10 \times 10) = 1$ ohm; that of the zinc solution is the same; hence the resistance of the battery is 2 ohms.

TABLE 39 gives a comparison between the Fahrenheit and Centigrade thermometers. Example: 98°.6, F = 37°.0, C.

TABLE 40 (Pickering, Table 14) gives a comparison of hydrometer scales. Example: 40 Beaumé for liquids lighter than water corresponds to the density 0.830.

TABLE 41 gives lengths of waves of light in air, intermediate between the numerous results quoted by Landolt and Börnstein. The probable error is about 1 unit in the last figure. Example: the Fraunhofer lines D_1 and D_2, together designated Na (or D), are due to sodium (symbol, Na) and occur in the yellow of the spectrum. They correspond to number 50 on Bunsen's scale, to numbers 1003 and 1007 on (Bunsen and) Kirchoff's scale, and have the wave-lengths 0.00005896 *cm.* and 0.00005890 *cm.* respectively.

TABLE 42 A refers to the imperial wire gauge adopted by the Board of Trade (Stewart & Gee, I. B.).

TABLE 42 B gives the Birmingham wire gauge (B. W. G.). The results are intermediate between those quoted in English, French, and German books. The probable error is about 1 unit in the last figure.

TABLE 43 gives the number of vibrations corresponding to a series of musical notes on the tempered or isotonic scale, one half of a semitone apart. The designation of some of these notes is given in the left-hand or in the right-hand column. The former is to be used for "physical pitch," in which all powers of the number 2 represent the note C; the right-hand column may be used for notes given by American instruments tuned to "concert pitch." The numbers *between* those corresponding to a given note in the first and last columns may be taken to represent the same note according to the old Stuttgart standard of pitch (A = 440, C = 264). Example: the "middle C" of an American piano (in the little octave), makes about 135.6 vibrations per second, and corresponds to C♯ physical pitch.

TABLE 44 A gives reductions of minutes and seconds to thousandths of a degree. The number of minutes is first sought; the tenths of a degree will be found next to it. Then *in the same section of the table* (there are 6 sections) the nearest number of seconds is found, and next to it the hundredths and thousandths of a degree. Example: $23°27'13'' = 23°. + 0°.4 + 0°.054$, nearly, or $23°.454$.

TABLE 44 B gives the correction to be added to dates in different years to compare them with the year 1891. Thus, Jan. 1, 10h. 0m. 0s., A. M., 1899; corresponds to Jan. 1, 10h. 0m. 0s. + 1h. 29m. 28s. A.M. = Jan. 1, 11h. 29m. 28s. A.M. 1891. The declination of the sun and the equation of time will, for instance, be the same on these two dates.

TABLE 44 C gives the gain of sidereal over mean solar time.

TABLE 44 D gives the sidereal time at Greenwich mean noon for the 10th, 20th, and last day of every month of the year 1891.

TABLE 44 E gives the semidiameter of the sun at different times in the year.

TABLE 44 F gives the declination of the sun at Greenwich mean noon for the year 1891. The sign of the declination is to be found at the head of the several columns (+ north, — south).

TABLE 44 G gives the "equation of time" at Greenwich mean noon for the year 1891. The signs + and — at the head of the columns and elsewhere show whether the sun is "fast" or "slow" respectively; + indicates that the sun passes the meridian before noon; — after noon.

TABLE 44 H gives certain astronomical data relating to the solar system.

TABLE 45 gives the Right Ascension and Declination of some of the most important stars.

TABLE 46 gives latitudes, longitudes, and elevations of certain important places.

TABLES 47 and 48 give respectively the acceleration of gravity and the length of the seconds pendulum corresponding to different latitudes. Example: since the latitude of the Jefferson Physical Laboratory of Harvard College is 42°22½′ or 42°.38, the acceleration of gravity is 980.37 *cm. per sec. per sec.*

TABLE 49 A and 49 B relate to the reduction of measures to and from the C. G. S. system.

TABLE 50 contains mathematical and physical constants in frequent use.

SOURCES OF AUTHORITY.

TABLES 1–3 H were prepared, in so far as possible, from existing tables, by rejecting decimal places when necessary. More than 3,000 values (including all doubtful cases) were confirmed or determined by an independent calculation. The results were printed with the ordinary precautions to avoid typographical errors. Tables 4–5 A were obtained by transposing Pickering's tables 6–9.

The logarithms of numbers from 1,000 to 10,000, in Table 6, were printed directly from a copy of the tables arranged by Mr. Oliver Whipple Huntington, of Harvard College. The proofs were compared with Bowditch's 5-place tables (Government Printing Office, Washington, 1882). The logarithms of numbers from 10,000 to 11,000 were obtained by rejecting figures in Chamber's 7–8-place tables. A special investigation was made in cases where the rejected figures were 50 or 500. Stereotype-proofs of all the logarithms were compared with the tables of Gauss. The table of probabilities as far as 5.0 is due to Chauvenet. The remainder of the table was the result of special calculation.

The physical tables (Nos. 8 to 50) were compiled for the most part by the aid of results contained in Landolt and Börnstein's "Physico-Chemical Tables,"[1] to which the reader is referred for a full exposition of the evidence upon which the selection of values has been made. The author wishes to thank Professors Landolt and Börnstein for looking over his manuscript, for several useful suggestions, and for their kind permission to utilize their results.

The author has quoted numerous data from Everett's "Units and Physical Constants" (Macmillan, 1886). He has also made use of information given by Professor Everett in choosing the unusually low value (4.17×10^7) for the mechanical equivalent of heat.

Among other books from which results have been taken are the following: Cooke's Chemical Philosophy, Deschanel's Natural Philosophy, Ganot's Physics, Hoffmann's Tabellen für Chemiker, Kohlrausch's Leitfaden der Praktischen Physik, das Nautisches Jahrbuch, 1891, Pickering's Physical Manipulation, Stewart and Gee's Practical Physics, Storer's Dictionary of Solubilities, Trowbridge's New Physics, and Weisbach's Mechanics.

These and other sources of authority have been acknowledged in connection with the explanation of the tables above; but it was found impossible, in the limited space which could be devoted to the tables, to give authority for the separate data. It was,

[1] Physikalisch-Chemische Tabellen von Dr. H. Landolt und Dr. Richard Börnstein, Professoren. Verlag von Julius Springer, Montbijou Platz 3, Berlin.

moreover, considered inexpedient to present to students, who would naturally be unaccustomed to weighing evidence, the conflicting statements from which the probable values of many of the physical constants have to be estimated by scientific men.

Care has been taken, in all such cases, to give results *intermediate between* those obtained by different observers. To do this, a considerable number of figures was sometimes required; but the use of figures, *not really significant*, has been in so far as possible avoided. The last figure quoted in the results is in general the only one in regard to which a difference of opinion was found to exist.

It is regretted that, owing to the necessity of entrusting the composition to foreign printers, obvious imperfections of type will be found, especially in the mathematical tables. In the expectation of reprinting these tables at no distant date, corrections and suggestions will be most gladly received.

№	.1	.2	.3	.4	.5	.6	.7	.8	.9	№	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0	0	0	0	0	0	0	0	0	50	5	10	15	20	25	30	35	40	45
1	0	0	0	0	1	1	1	1	1	51	5	10	15	20	26	31	36	41	46
2	0	0	1	1	1	1	1	2	2	52	5	10	16	21	26	31	36	42	47
3	0	1	1	1	2	2	2	2	3	53	5	11	16	21	27	32	37	42	48
4	0	1	1	2	2	2	3	3	4	54	5	11	16	22	27	32	38	43	49
5	1	1	2	2	3	3	4	4	5	55	6	11	17	22	28	33	39	44	50
6	1	1	2	2	3	4	4	5	5	56	6	11	17	22	28	34	39	45	50
7	1	1	2	3	4	4	5	6	6	57	6	11	17	23	29	34	40	46	51
8	1	2	2	3	4	5	6	6	7	58	6	12	17	23	29	35	41	46	52
9	1	2	3	4	5	5	6	7	8	59	6	12	18	24	30	35	41	47	53
10	1	2	3	4	5	6	7	8	9	60	6	12	18	24	30	36	42	48	54
11	1	2	3	4	6	7	8	9	10	61	6	12	18	24	31	37	43	49	55
12	1	2	4	5	6	7	8	10	11	62	6	12	19	25	31	37	43	50	56
13	1	3	4	5	7	8	9	10	12	63	6	13	19	25	32	38	44	50	57
14	1	3	4	6	7	8	10	11	13	64	6	13	19	26	32	38	45	51	58
15	2	3	5	6	8	9	11	12	14	65	7	13	20	26	33	39	46	52	59
16	2	3	5	6	8	10	11	13	14	66	7	13	20	26	33	40	46	53	59
17	2	3	5	7	9	10	12	14	15	67	7	13	20	27	34	40	47	54	60
18	2	4	5	7	9	11	13	14	16	68	7	14	20	27	34	41	48	54	61
19	2	4	6	8	10	11	13	15	17	69	7	14	21	28	35	41	48	55	62
20	2	4	6	8	10	12	14	16	18	70	7	14	21	28	35	42	49	56	63
21	2	4	6	8	11	13	15	17	19	71	7	14	21	28	36	43	50	57	64
22	2	4	7	9	11	13	15	18	20	72	7	14	22	29	36	43	50	58	65
23	2	5	7	9	12	14	16	18	21	73	7	15	22	29	37	44	51	58	66
24	2	5	7	10	12	14	17	19	22	74	7	15	22	30	37	44	52	59	67
25	3	5	8	10	13	15	18	20	23	75	8	15	23	30	38	45	53	60	68
26	3	5	8	10	13	16	18	21	23	76	8	15	23	30	38	46	53	61	68
27	3	5	8	11	14	16	19	22	24	77	8	15	23	31	39	46	54	62	69
28	3	6	8	11	14	17	20	22	25	78	8	16	23	31	39	47	55	62	70
29	3	6	9	12	15	17	20	23	26	79	8	16	24	32	40	47	55	63	71
30	3	6	9	12	15	18	21	24	27	80	8	16	24	32	40	48	56	64	72
31	3	6	9	12	16	19	22	25	28	81	8	16	24	32	41	49	57	65	73
32	3	6	10	13	16	19	22	26	29	82	8	16	25	33	41	49	57	66	74
33	3	7	10	13	17	20	23	26	30	83	8	17	25	33	42	50	58	66	75
34	3	7	10	14	17	20	24	27	31	84	8	17	25	34	42	50	59	67	76
35	4	7	11	14	18	21	25	28	32	85	9	17	26	34	43	51	60	68	77
36	4	7	11	14	18	22	25	29	32	86	9	17	26	34	43	52	60	69	77
37	4	7	11	15	19	22	26	30	33	87	9	17	26	35	44	52	61	70	78
38	4	8	11	15	19	23	27	30	34	88	9	18	26	35	44	53	62	70	79
39	4	8	12	16	20	23	27	31	35	89	9	18	27	36	45	53	62	71	80
40	4	8	12	16	20	24	28	32	36	90	9	18	27	36	45	54	63	72	81
41	4	8	12	16	21	25	29	33	37	91	9	18	27	36	46	55	64	73	82
42	4	8	13	17	21	25	29	34	38	92	9	18	28	37	46	55	64	74	83
43	4	9	13	17	22	26	30	34	39	93	9	19	28	37	47	56	65	74	84
44	4	9	13	18	22	26	31	35	40	94	9	19	28	38	47	56	66	75	85
45	5	9	14	18	23	27	32	36	41	95	10	19	29	38	48	57	67	76	86
46	5	9	14	18	23	28	32	37	41	96	10	19	29	38	48	58	67	77	86
47	5	9	14	19	24	28	33	38	42	97	10	19	29	39	49	58	68	78	87
48	5	10	14	19	24	29	34	38	43	98	10	20	29	39	49	59	69	78	88
49	5	10	15	20	25	29	34	39	44	99	10	20	30	40	50	59	69	79	89
50	5	10	15	20	25	30	35	40	45	100	10	20	30	40	50	60	70	80	90

No.	a. Reciprocal	b. Square Root	c. Square	d. Cube	No.	a. Reciprocal	b. Square Root	c. Square	d. Cube
0	∞	0.00	0	0	50	.0200	7.07	2500	125000
1	1.000	1.00	1	1	51	196	7.14	2601	132651
2	0.500	1.41	4	8	52	192	7.21	2704	140608
3	333	1.73	9	27	53	189	7.28	2809	148877
4	250	2.00	16	64	54	185	7.35	2916	157464
5	0.200	2.24	25	125	55	.0182	7.42	3025	166375
6	167	2.45	36	216	56	179	7.48	3136	175616
7	143	2.65	49	343	57	175	7.55	3249	185193
8	125	2.83	64	512	58	172	7.62	3364	195112
9	111	3 00	81	729	59	169	7.68	3481	205379
10	0.100	3.16	100	1000	60	.0167	7.75	3600	216000
11	.0909	3.32	121	1331	61	164	7.81	3721	226981
12	833	3.46	144	1728	62	161	7.87	3844	238328
13	769	3.61	169	2197	63	159	7.94	3969	250047
14	714	3.74	196	2744	64	156	8.00	4096	262144
15	.0667	3.87	225	3375	65	.0154	8.06	4225	274625
16	625	4.00	256	4096	66	152	8.12	4356	287496
17	588	4.12	289	4913	67	149	8.19	4489	300763
18	556	4.24	324	5832	68	147	8.25	4624	314432
19	526	4.36	361	6859	69	145	8.31	4761	328509
20	.0500	4.47	400	8000	70	.0143	8.37	4900	343000
21	476	4.58	441	9261	71	141	8.43	5041	357911
22	455	4.69	484	10648	72	139	8.49	5184	373248
23	435	4.80	529	12167	73	137	8.54	5329	389017
24	417	4.90	576	13824	74	135	8.60	5476	405224
25	.0400	5.00	625	15625	75	0133	8.66	5625	421875
26	385	5.10	676	17576	76	132	8.72	5776	438976
27	370	5.20	729	19683	77	130	8.77	5929	456533
28	357	5.29	784	21952	78	128	8 83	6084	474552
29	345	5.39	841	24389	79	127	8.89	6241	493039
30	.0333	5.48	900	27000	80	.0125	8.94	6400	512000
31	323	5.57	961	29791	81	123	9.00	6561	531441
32	313	5.66	1024	32768	82	122	9.06	6724	551368
33	303	5.74	1089	35937	83	120	9.11	6889	571787
34	294	5.83	1156	39304	84	119	9.17	7056	592704
35	.0286	5.92	1225	42875	85	.0118	9.22	7225	614125
36	278	6.00	1296	46656	86	116	9.27	7396	636056
37	270	6.08	1369	50653	87	115	9.33	7569	658503
38	263	6.16	1444	54872	88	114	9.38	7744	681472
39	256	6.24	1521	59319	89	112	9.43	7921	704969
40	.0250	6.32	1600	64000	90	.0111	9.49	8100	729000
41	244	6 40	1681	68921	91	110	9.54	8281	753571
42	238	6.48	1764	74088	92	109	9.59	8464	778688
43	233	6.56	1849	79507	93	108	9.64	8649	804357
44	227	6.63	1936	85184	94	106	9.70	8836	830584
45	.0222	6.71	2025	91125	95	.0105	9.75	9025	857375
46	217	6.78	2116	97336	96	104	9.80	9216	884736
47	213	6.86	2209	103823	97	103	9.85	9409	912673
48	208	6.93	2304	110592	98	102	9.90	9604	941192
49	204	7.00	2401	117649	99	101	9.95	9801	970299
50	.0200	7.07	2500	125000	100	.0100	10.00	10000	1000000

Diameter	e. Logarithm	f. Circumference	g. Area of Circle	h. Volume of Sphere	Diameter	e. Logarithm	f. Circumference	g. Area of Circle	h. Volume of Sphere
.0	$-\infty$	0.00	0.00	.000	5.0	0.699	15.71	19.6	65
.1	$\bar{1}$.000	31	01	.001	5.1	708	16.02	20.4	69
.2	301	63	03	.004	5.2	716	16.34	21.2	74
.3	477	94	07	.014	5.3	724	16.65	22.1	78
.4	602	1.26	13	.034	5.4	732	16.96	22.9	82
.5	$\bar{1}$.699	1.57	0.20	.065	5.5	0.740	17.28	23.8	87
.6	778	1.88	28	.113	5.6	748	17.59	24.6	92
.7	845	2.20	38	.180	5.7	756	17.91	25.5	97
.8	903	2.51	50	.268	5.8	763	18.22	26.4	102
.9	954	2.83	64	.382	5.9	771	18.54	27.3	108
1.0	0.000	3.14	0.79	0.52	6.0	0.778	18.85	28.3	113
1.1	041	3.46	0.95	70	6.1	785	19.16	29.2	119
1.2	079	3.77	1.13	90	6.2	792	19.48	30.2	125
1.3	114	4.08	1.33	1.15	6.3	799	19.79	31.2	131
1.4	146	4.40	1.54	1.44	6.4	806	20.11	32.2	137
1.5	0.176	4.71	1.77	1.77	6.5	0.813	20.42	33.2	144
1.6	204	5.03	2.01	2.14	6.6	820	20.73	34.2	151
1.7	230	5.34	2.27	2.57	6.7	826	21.05	35.3	157
1.8	255	5.65	2.54	3.05	6.8	833	21.36	36.3	165
1.9	279	5.97	2.84	3.59	6.9	839	21.68	37.4	172
2.0	0.301	6.28	3.14	4.19	7.0	0.845	21.99	38.5	180
2.1	322	6.60	3.46	4.85	7.1	851	22.31	39.6	187
2.2	342	6.91	3.80	5.58	7.2	857	22.62	40.7	195
2.3	362	7.23	4.15	6.37	7.3	863	22.93	41.9	204
2.4	380	7.54	4.52	7.24	7.4	869	23.25	43.0	212
2.5	0.398	7.85	4.91	8.2	7.5	0.875	23.56	44.2	221
2.6	415	8.17	5.31	9.2	7.6	881	23.88	45.4	230
2.7	431	8.48	5.73	10.3	7.7	886	24.19	46.6	239
2.8	447	8.80	6.16	11.5	7.8	892	24.50	47.8	248
2.9	462	9.11	6.61	12.8	7.9	898	24.82	49.0	258
3.0	0.477	9.42	7.07	14.1	8.0	0.903	25.13	50.3	268
3.1	491	9.74	7.55	15.6	8.1	908	25.45	51.5	278
3.2	505	10.05	8.04	17.2	8.2	914	25.76	52.8	289
3.3	519	10.37	8.55	18.8	8.3	919	26.08	54.1	299
3.4	531	10.68	9.08	20.6	8.4	924	26.39	55.4	310
3.5	0.544	11.00	9.6	22.4	8.5	0.929	26.70	56.7	322
3.6	556	11.31	10.2	24.4	8.6	934	27.02	58.1	333
3.7	568	11.62	10.8	26.5	8.7	940	27.33	59.4	345
3.8	580	11.94	11.3	28.7	8.8	944	27.65	60.8	357
3.9	591	12.25	11.9	31.1	8.9	949	27.96	62.2	369
4.0	0.602	12.57	12.6	33.5	9.0	0.954	28.27	63.6	382
4.1	613	12.88	13.2	36.1	9.1	959	28.59	65.0	395
4.2	623	13.19	13.9	38.8	9.2	964	28.90	66.5	408
4.3	633	13.51	14.5	41.6	9.3	968	29.22	67.9	421
4.4	643	13.82	15.2	44.6	9.4	973	29.53	69.4	435
4.5	0.653	14.14	15.9	47.7	9.5	0.978	29.85	70.9	449
4.6	663	14.45	16.6	51.0	9.6	982	30.16	72.4	463
4.7	672	14.77	17.3	54.4	9.7	987	30.47	73.9	478
4.8	681	15.08	18.1	57.9	9.8	991	30.79	75.4	493
4.9	690	15.39	18.9	61.6	9.9	996	31.10	77.0	508
5.0	0.699	15.71	19.6	65.4	10.0	1.000	31.42	78.5	524

a. Angle.	b. Tangent.	c. Arc.	d. Chord.	e. Sine.	f. Cosine.	g. Rate of Vibration.	h. Secant.	i. Complement.
0°	0.000	0.000	0.000	0.000	1.000	1.00000	1.000	90°
1	017	017	017	017	1.000	1.00000	1.000	89
2	035	035	035	035	0.999	0.99998	1.001	88
3	052	052	052	052	999	99996	1.001	87
4	070	070	070	070	998	99992	1.002	86
5	0.087	0.087	0.087	0.087	0.996	0.99988	1.004	85
6	105	105	105	105	995	99983	1.006	84
7	123	122	122	122	993	99977	1.008	83
8	141	140	140	139	990	99970	1.010	82
9	158	157	157	156	988	99961	1.012	81
10	0.176	0.175	0.174	0.174	0.985	0.99952	1.015	80
11	194	192	192	191	982	99942	1.019	79
12	213	209	209	208	978	99931	1.022	78
13	231	227	226	225	974	99920	1.026	77
14	249	244	244	242	970	99907	1.031	76
15	0.268	0.262	0.261	0.259	0.966	0.99893	1.035	75
16	287	279	278	276	961	99878	1.040	74
17	306	297	296	292	956	99862	1.046	73
18	325	314	313	309	951	99846	1.051	72
19	344	332	330	326	946	99828	1.058	71
20	0.364	0.349	0.347	0.342	0.940	0.99810	1.064	70
21	384	367	364	358	934	99790	1.071	69
22	404	384	382	375	927	99770	1·079	68
23	424	401	399	391	921	99749	1.086	67
24	445	419	416	407	914	99726	1.095	66
25	0.466	0.436	0.433	0.423	0.906	0.99703	1.103	65
26	488	454	450	438	899	99678	1.113	64
27	510	471	467	454	891	99653	1.122	63
28	532	489	484	469	883	99627	1.133	62
29	554	506	501	485	875	99600	1.143	61
30	0.577	0.524	0.518	0.500	0.866	0.99572	1.155	60
31	601	541	534	515	857	99543	1.167	59
32	625	559	551	530	848	99513	1.179	58
33	649	576	568	545	839	99482	1.192	57
34	675	593	585	559	829	99450	1.206	56
35	0.700	0.611	0.601	0.574	0.819	0.99417	1.221	55
36	727	628	618	588	809	99384	1.236	54
37	754	646	635	602	799	99349	1.252	53
38	781	663	651	616	788	99314	1.269	52
39	810	681	668	629	777	99277	1.287	51
40	0.839	0.698	0.684	0.643	0.766	0.99239	1.305	50
41	869	716	700	656	755	99200	1.325	49
42	900	733	717	669	743	99161	1.346	48
43	933	750	733	682	731	99121	1.367	47
44	966	768	749	695	719	99079	1.390	46
45°	1.000	0.785	0.765	0.707	0.707	0.99037	1.414	45°

a. Angle.	b. Tangent.	c. Arc.	d. Chord.	e. Sine.	f. Cosine.	g. Coversine	h. Secant.	i. Complement.
45°	1.000	0.785	0.765	0.707	0.707	0.293	1.414	45°
46	1.036	0.803	781	719	695	281	1.440	44
47	1.072	0.820	797	731	682	269	1.466	43
48	1.111	0.838	813	743	669	257	1.494	42
49	1.150	0.855	829	755	656	245	1.524	41
50	1.192	0.873	0.845	0.766	0.643	0.234	1.556	40
51	1.235	0.890	861	777	629	223	1.589	39
52	1.280	0.908	877	788	616	212	1.624	38
53	1.327	0.925	892	799	602	201	1.662	37
54	1.376	0.942	908	809	588	191	1.701	36
55	1.428	0.960	0.923	0.819	0.574	0.181	1.743	35
56	1.483	0.977	939	829	559	171	1.788	34
57	1.540	0.995	954	839	545	161	1.836	33
58	1.600	1.012	970	848	530	152	1.887	32
59	1 664	1.030	985	857	515	143	1 942	31
60	1.732	1.047	1.000	0.866	0.500	0.134	2.000	30
61	1.804	1.065	1.015	875	485	125	2.063	29
62	1.881	1.082	1.030	883	469	117	2.130	28
63	1.963	1.100	1.045	891	454	109	2.203	27
64	2.050	1.117	1.060	899	438	101	2.281	26
65	2.145	1.134	1.075	0 906	0.423	0.094	2.366	25
66	2.246	1.152	1.089	914	407	086	2.459	24
67	2.356	1.169	1.104	921	391	079	2.559	23
68	2.475	1.187	1.118	927	375	073	2.669	22
69	2.605	1.204	1.133	934	358	066	2.790	21
70	2.747	1.222	1.147	0.940	0.342	0.060	2.924	20
71	2.904	1.239	1.161	946	326	054	3.072	19
72	3.078	1.257	1.176	951	309	049	3.236	18
73	3.271	1.274	1.190	956	292	044	3.420	17
74	3.487	1.292	1.204	961	276	039	3.628	16
75	3.732	1.309	1 218	0.966	0.259	0.034	3.864	15
76	4.011	1.326	1.231	970	242	030	4.134	14
77	4.331	1.344	1.245	974	225	026	4.445	13
73	4.705	1.361	1.259	978	208	022	4.810	12
79	5.145	1.379	1.272	982	191	018	5.241	11
80	5.671	1.396	1.286	0 985	0.174	0.0152	5.759	10
81	6.314	1.414	1 299	988	156	0123	6.392	9
82	7.115	1.431	1.312	990	139	0097	7.185	8
83	8.144	1.449	1.325	993	122	0075	8.206	7
84	9.514	1.466	1.338	995	105	0055	9.567	6
85	11.43	1.484	1.351	0.996	0.087	0.00381	11.47	5
86	14 30	1.501	1 364	998	070	00244	14.34	4
87	19.08	1.518	1.377	999	052	00137	19.11	3
88	28.64	1.536	1.389	999	035	00061	28.65	2
89	57.29	1.553	1 402	1.000	017	00015	57.30	1
90°	∞	1.571	1 414	1.000	0.000	0.00000	∞	0°

№	0	1	2	3	4	5	6	7	8	9	Dif.
1.0	1.0000	9901	9804	9709	9615	9524	9434	9346	9259	9174	92
1.1	0.9091	9009	8929	8850	8772	8696	8621	8547	8475	8403	76
1.2	8333	8264	8197	8130	8065	8000	7937	7874	7813	7752	65
1.3	7692	7634	7576	7519	7463	7407	7353	7299	7246	7194	55
1.4	7143	7092	7042	6993	6944	6897	6849	6803	6757	6711	48
1.5	0.6667	6623	6579	6536	6494	6452	6410	6369	6329	6289	42
1.6	6250	6211	6173	6135	6098	6061	6024	5988	5952	5917	37
1.7	5882	5848	5814	5780	5747	5714	5682	5650	5618	5587	33
1.8	5556	5525	5495	5464	5435	5405	5376	5348	5319	5291	29
1.9	5263	5236	5208	5181	5155	5128	5102	5076	5051	5025	26
2.0	0.5000	4975	4950	4926	4902	4878	4854	4831	4808	4785	24
2.1	4762	4739	4717	4695	4673	4651	4630	4608	4587	4566	22
2.2	4545	4525	4505	4484	4464	4444	4425	4405	4386	4367	20
2.3	4348	4329	4310	4292	4274	4255	4237	4219	4202	4184	18
2.4	4167	4149	4132	4115	4098	4082	4065	4049	4032	4016	17
2.5	0.4000	3984	3968	3953	3937	3922	3906	3891	3876	3861	15
2.6	3846	3831	3817	3802	3788	3774	3759	3745	3731	3717	14
2.7	3704	3690	3676	3663	3650	3636	3623	3610	3597	3584	13
2.8	3571	3559	3546	3534	3521	3509	3496	3484	3472	3460	12
2.9	3448	3436	3425	3413	3401	3390	3378	3367	3356	3344	12
3.0	0.3333	3322	3311	3300	3289	3279	3268	3257	3247	3236	11
3.1	3226	3215	3205	3195	3185	3175	3165	3155	3145	3135	10
3.2	3125	3115	3106	3096	3086	3077	3067	3058	3049	3040	9
3.3	3030	3021	3012	3003	2994	2985	2976	2967	2959	2950	9
3.4	2941	2933	2924	2915	2907	2899	2890	2882	2874	2865	
3.5	0.2857	2849	2841	2833	2825	2817	2809	2801	2793	2786	8
3.6	2778	2770	2762	2755	2747	2740	2732	2725	2717	2710	
3.7	2703	2695	2688	2681	2674	2667	2660	2653	2646	2639	7
3.8	2632	2625	2618	2611	2604	2597	2591	2584	2577	2571	
3.9	2564	2558	2551	2545	2538	2532	2525	2519	2513	2506	
4.0	0.2500	2494	2488	2481	2475	2469	2463	2457	2451	2445	6
4.1	2439	2433	2427	2421	2415	2410	2404	2398	2392	2387	
4.2	2381	2375	2370	2364	2358	2353	2347	2342	2336	2331	
4.3	2326	2320	2315	2309	2304	2299	2294	2288	2283	2278	
4.4	2273	2268	2262	2257	2252	2247	2242	2237	2232	2227	5
4.5	0.2222	2217	2212	2208	2203	2198	2193	2188	2183	2179	
4.6	2174	2169	2165	2160	2155	2151	2146	2141	2137	2132	
4.7	2128	2123	2119	2114	2110	2105	2101	2096	2092	2088	
4.8	2083	2079	2075	2070	2066	2062	2058	2053	2049	2045	
4.9	2041	2037	2033	2028	2024	2020	2016	2012	2008	2004	
5.0	0.2000	1996	1992	1988	1984	1980	1976	1972	1969	1965	4
5.1	1961	1957	1953	1949	1946	1942	1938	1934	1931	1927	
5.2	1923	1919	1916	1912	1908	1905	1901	1898	1894	1890	
5.3	1887	1883	1880	1876	1873	1869	1866	1862	1859	1855	
5.4	1852	1848	1845	1842	1838	1835	1832	1828	1825	1821	
5.5	0.1818	1815	1812	1808	1805	1802	1799	1795	1792	1789	
5.6	1786	1783	1779	1776	1773	1770	1767	1764	1761	1757	
5.7	1754	1751	1748	1745	1742	1739	1736	1733	1730	1727	3
5.8	1724	1721	1718	1715	1712	1709	1706	1704	1701	1698	
5.9	1695	1692	1689	1686	1684	1681	1678	1675	1672	1669	
6.0	0.1667	1664	1661	1658	1656	1653	1650	1647	1645	1642	

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6.0	0.16667	16639	16611	16584	16556	16529	16502	16474	16447	16420	27
6.1	16393	16367	16340	16313	16287	16260	16234	16207	16181	16155	26
6.2	16129	16103	16077	16051	16026	16000	15974	15949	15924	15898	26
6.3	15873	15848	15823	15798	15773	15748	15723	15699	15674	15649	25
6.4	15625	15601	15576	15552	15528	15504	15480	15456	15432	15408	24
6.5	0.15385	15361	15337	15314	15291	15267	15244	15221	15198	15175	23
6.6	15152	15129	15106	15083	15060	15038	15015	14992	14970	14948	23
6.7	14925	14903	14881	14859	14837	14815	14793	14771	14749	14728	22
6.8	14706	14684	14663	14641	14620	14599	14577	14556	14535	14514	21
6.9	14493	14472	14451	14430	14409	14388	14368	14347	14327	14306	21
7.0	0.14286	14265	14245	14225	14205	14184	14164	14144	14124	14104	20
7.1	14085	14065	14045	14025	14006	13986	13966	13947	13928	13908	
7.2	13889	13870	13850	13831	13812	13793	13774	13755	13736	13717	19
7.3	13699	13680	13661	13643	13624	13605	13587	13569	13550	13532	
7.4	13514	13495	13477	13459	13441	13423	13405	13387	13369	13351	18
7.5	0.13333	13316	13298	13280	13263	13245	13228	13210	13193	13175	
7.6	13158	13141	13123	13106	13089	13072	13055	13038	13021	13004	17
7.7	12987	12970	12953	12937	12920	12903	12887	12870	12853	12837	
7.8	12821	12804	12788	12771	12755	12739	12723	12706	12690	12674	16
7.9	12658	12642	12626	12610	12594	12579	12563	12547	12531	12516	
8.0	0.12500	12484	12469	12453	12438	12422	12407	12392	12376	12361	
8.1	12346	12330	12315	12300	12285	12270	12255	12240	12225	12210	15
8.2	12195	12180	12165	12151	12136	12121	12107	12092	12077	12063	
8.3	12048	12034	12019	12005	11990	11976	11962	11947	11933	11919	
8.4	11905	11891	11876	11862	11848	11834	11820	11806	11792	11779	14
8.5	0.11765	11751	11737	11723	11710	11696	11682	11669	11655	11641	
8.6	11628	11614	11601	11587	11574	11561	11547	11534	11521	11507	
8.7	11494	11481	11468	11455	11442	11429	11416	11403	11390	11377	13
8.8	11364	11351	11338	11325	11312	11299	11287	11274	11261	11249	
8.9	11236	11223	11211	11198	11186	11173	11161	11148	11136	11123	
9.0	0.11111	11099	11086	11074	11062	11050	11038	11025	11013	11001	
9.1	10989	10977	10965	10953	10941	10929	10917	10905	10893	10881	12
9.2	10870	10858	10846	10834	10823	10811	10799	10787	10776	10764	
9.3	10753	10741	10730	10718	10707	10695	10684	10672	10661	10650	
9.4	10638	10627	10616	10604	10593	10582	10571	10560	10549	10537	
9.5	0.10526	10515	10504	10493	10482	10471	10460	10449	10438	10428	11
9.6	10417	10406	10395	10384	10373	10363	10352	10341	10331	10320	
9.7	10309	10299	10288	10277	10267	10256	10246	10235	10225	10215	
9.8	10204	10194	10183	10173	10163	10152	10142	10132	10121	10111	
9.9	10101	10091	10081	10070	10060	10050	10040	10030	10020	10010	
10.0	0.10000	.09990	9980	9970	9960	9950	9940	9930	9921	9911	10
10.1	.09901	9891	9881	9872	9862	9852	9843	9833	9823	9814	
10.2	9804	9794	9785	9775	9766	9756	9747	9737	9728	9718	
10.3	9709	9699	9690	9681	9671	9662	9653	9643	9634	9625	
10.4	9615	9606	9597	9588	9579	9569	9560	9551	9542	9533	
10.5	0.09524	9515	9506	9497	9488	9479	9470	9461	9452	9443	9
10.6	9434	9425	9416	9407	9398	9390	9381	9372	9363	9355	
10.7	9346	9337	9328	9320	9311	9302	9294	9285	9276	9268	
10.8	9259	9251	9242	9234	9225	9217	9208	9200	9191	9183	
10.9	9174	9166	9158	9149	9141	9132	9124	9116	9107	9099	
11.0	0.09091	9083	9074	9066	9058	9050	9042	9033	9025	9017	8

№	0	1	2	3	4	5	6	7	8	9	DD.
1.0	1.000	1.020	1.040	1.061	1.082	1.103	1.124	1.145	1.166	1.188	21
1.1	1.210	1.232	1.254	1.277	1.300	1.323	1.346	1.369	1.392	1.416	23
1.2	1.440	1.464	1.488	1.513	1.538	1.563	1.588	1.613	1.638	1.664	25
1.3	1.690	1.716	1.742	1.769	1.796	1.823	1.850	1.877	1.904	1.932	27
1.4	1.960	1.988	2.016	2.045	2.074	2.103	2.132	2.161	2.190	2.220	29
1.5	2.250	2.280	2.310	2.341	2.372	2.403	2.434	2.465	2.496	2.528	31
1.6	2.560	2.592	2.624	2.657	2.690	2.723	2.756	2.789	2.822	2.856	33
1.7	2.890	2.924	2.958	2.993	3.028	3.063	3.098	3.133	3.168	3.204	35
1.8	3.240	3.276	3.312	3.349	3.386	3.423	3.460	3.497	3.534	3.572	37
1.9	3.610	3.648	3.686	3.725	3.764	3.803	3.842	3.881	3.920	3.960	39
2.0	4.000	4.040	4.080	4.121	4.162	4.203	4.244	4.285	4.326	4.368	41
2.1	4.410	4.452	4.494	4.537	4.580	4.623	4.666	4.709	4.752	4.796	43
2.2	4.840	4.884	4.928	4.973	5.018	5.063	5.108	5.153	5.198	5.244	45
2.3	5.290	5.336	5.382	5.429	5.476	5.523	5.570	5.617	5.664	5.712	47
2.4	5.760	5.808	5.856	5.905	5.954	6.003	6.052	6.101	6.150	6.200	49
2.5	6.250	6.300	6.350	6.401	6.452	6.503	6.554	6.605	6.656	6.708	51
2.6	6.760	6.812	6.864	6.917	6.970	7.023	7.076	7.129	7.182	7.236	53
2.7	7.290	7.344	7.398	7.453	7.508	7.563	7.618	7.673	7.728	7.784	55
2.8	7.840	7.896	7.952	8.009	8.066	8.123	8.180	8.237	8.294	8.352	57
2.9	8.410	8.468	8.526	8.585	8.644	8.703	8.762	8.821	8.880	8.940	59
3.0	9.000	9.060	9.120	9.181	9.242	9.303	9.364	9.425	9.486	9.548	61
3.1	9.610	9.672	9.734	9.797	9.860	9.923	9.986	10.05	10.11	10.18	—
3.2	10.24	10.30	10.37	10.43	10.50	10.56	10.63	10.69	10.76	10.82	
3.3	10.89	10.96	11.02	11.09	11.16	11.22	11.29	11.36	11.42	11.49	
3.4	11.56	11.63	11.70	11.76	11.83	11.90	11.97	12.04	12.11	12.18	
3.5	12.25	12.32	12.39	12.46	12.53	12.60	12.67	12.74	12.82	12.89	7
3.6	12.96	13.03	13.10	13.18	13.25	13.32	13.40	13.47	13.54	13.62	
3.7	13.69	13.76	13.84	13.91	13.99	14.06	14.14	14.21	14.29	14.36	
3.8	14.44	14.52	14.59	14.67	14.75	14.82	14.90	14.98	15.05	15.13	
3.9	15.21	15.29	15.37	15.44	15.52	15.60	15.68	15.76	15.84	15.92	
4.0	16.00	16.08	16.16	16.24	16.32	16.40	16.48	16.56	16.65	16.73	8
4.1	16.81	16.89	16.97	17.06	17.14	17.22	17.31	17.39	17.47	17.56	
4.2	17.64	17.72	17.81	17.89	17.98	18.06	18.15	18.23	18.32	18.40	
4.3	18.49	18.58	18.66	18.75	18.84	18.92	19.01	19.10	19.18	19.27	
4.4	19.36	19.45	19.54	19.62	19.71	19.80	19.89	19.98	20.07	20.16	
4.5	20.25	20.34	20.43	20.52	20.61	20.70	20.79	20.88	20.98	21.07	9
4.6	21.16	21.25	21.34	21.44	21.53	21.62	21.72	21.81	21.90	22.00	
4.7	22.09	22.18	22.28	22.37	22.47	22.56	22.66	22.75	22.85	22.94	
4.8	23.04	23.14	23.23	23.33	23.43	23.52	23.62	23.72	23.81	23.91	
4.9	24.01	24.11	24.21	24.30	24.40	24.50	24.60	24.70	24.80	24.90	
5.0	25.00	25.10	25.20	25.30	25.40	25.50	25.60	25.70	25.81	25.91	10
5.1	26.01	26.11	26.21	26.32	26.42	26.52	26.63	26.73	26.83	26.94	
5.2	27.04	27.14	27.25	27.35	27.46	27.56	27.67	27.77	27.88	27.98	
5.3	28.09	28.20	28.30	28.41	28.52	28.62	28.73	28.84	28.94	29.05	
5.4	29.16	29.27	29.38	29.48	29.59	29.70	29.81	29.92	30.03	30.14	
5.5	30.25	30.36	30.47	30.58	30.69	30.80	30.91	31.02	31.14	31.25	11
5.6	31.36	31.47	31.58	31.70	31.81	31.92	32.04	32.15	32.26	32.38	
5.7	32.49	32.60	32.72	32.83	32.95	33.06	33.18	33.29	33.41	33.52	
5.8	33.64	33.76	33.87	33.99	34.11	34.22	34.34	34.46	34.57	34.69	
5.9	34.81	34.93	35.05	35.16	35.28	35.40	35.52	35.64	35.76	35.88	
6.0	36.00	36.12	36.24	36.36	36.48	36.60	36.72	36.84	36.97	37.09	

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6.0	36.00	36.12	36.24	36.36	36.48	36.60	36.72	36.84	36.97	37.09	1
6.1	37.21	37.33	37.45	37.58	37.70	37.82	37.95	38.07	38.19	38.32	
6.2	38.44	38.56	38.69	38.81	38.94	39.06	39.19	39.31	39.44	39.56	
6.3	39.69	39.82	39.94	40.07	40.20	40.32	40.45	40.58	40.70	40.83	
6.4	40.96	41.09	41.22	41.34	41.47	41.60	41.73	41.86	41.99	42.12	
6.5	42.25	42.38	42.51	42.64	42.77	42.90	43.03	43.16	43.30	43.43	13
6.6	43.56	43.69	43.82	43.96	44.09	44.22	44.36	44.49	44.62	44.76	
6.7	44.89	45.02	45.16	45.29	45.43	45.56	45.70	45.83	45.97	46.10	
6.8	46.24	46.38	46.51	46.65	46.79	46.92	47.06	47.20	47.33	47.47	
6.9	47.61	47.75	47.89	48.02	48.16	48.30	48.44	48.58	48.72	48.86	
7.0	49.00	49.14	49.28	49.42	49.56	49.70	49.84	49.98	50.13	50.27	14
7.1	50.41	50.55	50.69	50.84	50.98	51.12	51.27	51.41	51.55	51.70	
7.2	51.84	51.98	52.13	52.27	52.42	52.56	52.71	52.85	53.00	53.14	
7.3	53.29	53.44	53.58	53.73	53.88	54.02	54.17	54.32	54.46	54.61	
7.4	54.76	54.91	55.06	55.20	55.35	55.50	55.65	55.80	55.95	56.10	
7.5	56.25	56.40	56.55	56.70	56.85	57.00	57.15	57.30	57.46	57.61	15
7.6	57.76	57.91	58.06	58.22	58.37	58.52	58.68	58.83	58.98	59.14	
7.7	59.29	59.44	59.60	59.75	59.91	60.06	60.22	60.37	60.53	60.68	
7.8	60.84	61.00	61.15	61.31	61.47	61.62	61.78	61.94	62.09	62.25	
7.9	62.41	62.57	62.73	62.88	63.04	63.20	63.36	63.52	63.68	63.84	
8.0	64.00	64.16	64.32	64.48	64.64	64.80	64.96	65.12	65.29	65.45	16
8.1	65.61	65.77	65.93	66.10	66.26	66.42	66.59	66.75	66.91	67.08	
8.2	67.24	67.40	67.57	67.73	67.90	68.06	68.23	68.39	68.56	68.72	
8.3	68.89	69.06	69.22	69.39	69.56	69.72	69.89	70.06	70.22	70.39	
8.4	70.56	70.73	70.90	71.06	71.23	71.40	71.57	71.74	71.91	72.08	
8.5	72.25	72.42	72.59	72.76	72.93	73.10	73.27	73.44	73.62	73.79	17
8.6	73.96	74.13	74.30	74.48	74.65	74.82	75.00	75.17	75.34	75.52	
8.7	75.69	75.86	76.04	76.21	76.39	76.56	76.74	76.91	77.09	77.26	
8.8	77.44	77.62	77.79	77.97	78.15	78.32	78.50	78.68	78.85	79.03	
8.9	79.21	79.39	79.57	79.74	79.92	80.10	80.28	80.46	80.64	80.82	
9.0	81.00	81.18	81.36	81.54	81.72	81.90	82.08	82.26	82.45	82.63	18
9.1	82.81	82.99	83.17	83.36	83.54	83.72	83.91	84.09	84.27	84.46	
9.2	84.64	84.82	85.01	85.19	85.38	85.56	85.75	85.93	86.12	86.30	
9.3	86.49	86.68	86.86	87.05	87.24	87.42	87.61	87.80	87.98	88.17	
9.4	88.36	88.55	88.74	88.92	89.11	89.30	89.49	89.68	89.87	90.06	
9.5	90.25	90.44	90.63	90.82	91.01	91.20	91.39	91.58	91.78	91.97	19
9.6	92.16	92.35	92.54	92.74	92.93	93.12	93.32	93.51	93.70	93.90	
9.7	94.09	94.28	94.48	94.67	94.87	95.06	95.26	95.45	95.65	95.84	
9.8	96.04	96.24	96.43	96.63	96.83	97.02	97.22	97.42	97.61	97.81	
9.9	98.01	98.21	98.41	98.60	98.80	99.00	99.20	99.40	99.60	99.80	
10.0	100.00	100.20	100.40	100.60	100.80	101.00	101.20	101.40	101.61	101.81	20
10.1	102.01	102.21	102.41	102.62	102.82	103.02	103.23	103.43	103.63	103.84	
10.2	104.04	104.24	104.45	104.65	104.86	105.06	105.27	105.47	105.68	105.88	
10.3	106.09	106.30	106.50	106.71	106.92	107.12	107.33	107.54	107.74	107.95	
10.4	108.16	108.37	108.58	108.78	108.99	109.20	109.41	109.62	109.83	110.04	
10.5	110.25	110.46	110.67	110.88	111.09	111.30	111.51	111.72	111.94	112.15	21
10.6	112.36	112.57	112.78	113.00	113.21	113.42	113.64	113.85	114.06	114.28	
10.7	114.49	114.70	114.92	115.13	115.35	115.56	115.78	115.99	116.21	116.42	
10.8	116.64	116.86	117.07	117.29	117.51	117.72	117.94	118.16	118.37	118.59	
10.9	118.81	119.03	119.25	119.46	119.68	119.90	120.12	120.34	120.56	120.78	
11.0	121.00	121.22	121.44	121.66	121.88	122.10	122.32	122.54	122.77	122.99	22

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1.0	1.000	1.030	1.061	1.093	1.125	1.158	1.191	1.225	1.260	1.295	33
1.1	1.331	1.368	1.405	1.443	1.482	1.521	1.561	1.602	1.643	1.685	39
1.2	1.728	1.772	1.816	1.861	1.907	1.953	2.000	2.048	2.097	2.147	47
1.3	2.197	2.248	2.300	2.353	2.406	2.460	2.515	2.571	2.628	2.686	55
1.4	2.744	2.803	2.863	2.924	2.986	3.049	3.112	3.177	3.242	3.308	63
1.5	3.375	3.443	3.512	3.582	3.652	3.724	3.796	3.870	3.944	4.020	72
1.6	4.096	4.173	4.252	4.331	4.411	4.492	4.574	4.657	4.742	4.827	82
1.7	4.913	5.000	5.088	5.178	5.268	5.359	5.452	5.545	5.640	5.735	92
1.8	5.832	5.930	6.029	6.128	6.230	6.332	6.435	6.539	6.645	6.751	101
1.9	6.859	6.968	7.078	7.189	7.301	7.415	7.530	7.645	7.762	7.881	114
2.0	8.000	8.121	8.242	8.365	8.490	8.615	8.742	8.870	8.999	9.129	126
2.1	9.261	9.394	9.528	9.664	9.800	9.938	10.08	10.22	10.36	10.50	—
2.2	10.65	10.79	10.94	11.09	11.24	11.39	11.54	11.70	11.85	12.01	15
2.3	12.17	12.33	12.49	12.65	12.81	12.98	13.14	13.31	13.48	13.65	16
2.4	13.82	14.00	14.17	14.35	14.53	14.71	14.89	15.07	15.25	15.44	18
2.5	15.63	15.81	16.00	16.19	16.39	16.58	16.78	16.97	17.17	17.37	19
2.6	17.58	17.78	17.98	18.19	18.40	18.61	18.82	19.03	19.25	19.47	21
2.7	19.68	19.90	20.12	20.35	20.57	20.80	21.02	21.25	21.48	21.72	22
2.8	21.95	22.19	22.43	22.67	22.91	23.15	23.39	23.64	23.89	24.14	24
2.9	24.39	24.64	24.90	25.15	25.41	25.67	25.93	26.20	26.46	26.73	26
3.0	27.00	27.27	27.54	27.82	28.09	28.37	28.65	28.93	29.22	29.50	28
3.1	29.79	30.08	30.37	30.66	30.96	31.26	31.55	31.86	32.16	32.46	30
3.2	32.77	33.08	33.39	33.70	34.01	34.33	34.65	34.97	35.29	35.61	32
3.3	35.94	36.26	36.59	36.93	37.26	37.60	37.93	38.27	38.61	38.96	34
3.4	39.30	39.65	40.00	40.35	40.71	41.06	41.42	41.78	42.14	42.51	36
3.5	42.88	43.24	43.61	43.99	44.36	44.74	45.12	45.50	45.88	46.27	38
3.6	46.66	47.05	47.44	47.83	48.23	48.63	49.03	49.43	49.84	50.24	40
3.7	50.65	51.06	51.48	51.90	52.31	52.73	53.16	53.58	54.01	54.44	42
3.8	54.87	55.31	55.74	56.18	56.62	57.07	57.51	57.96	58.41	58.86	44
3.9	59.32	59.78	60.24	60.70	61.16	61.63	62.10	62.57	63.04	63.52	47
4.0	64.00	64.48	64.96	65.45	65.94	66.43	66.92	67.42	67.92	68.42	49
4.1	68.92	69.43	69.93	70.44	70.96	71.47	71.99	72.51	73.03	73.56	52
4.2	74.09	74.62	75.15	75.69	76.23	76.77	77.31	77.85	78.40	78.95	54
4.3	79.51	80.06	80.62	81.18	81.75	82.31	82.88	83.45	84.03	84.60	57
4.4	85.18	85.77	86.35	86.94	87.53	88.12	88.72	89.31	89.92	90.52	60
4.5	91.13	91.73	92.35	92.96	93.58	94.20	94.82	95.44	96.07	96.70	62
4.6	97.34	97.97	98.61	99.25	99.90	100.6	101.2	101.8	102.5	103.2	—
4.7	103.8	104.5	105.2	105.8	106.5	107.2	107.9	108.5	109.2	109.9	7
4.8	110.6	111.3	112.0	112.7	113.4	114.1	114.8	115.5	116.2	116.9	7
4.9	117.6	118.4	119.1	119.8	120.6	121.3	122.0	122.8	123.5	124.3	7
5.0	125.0	125.8	126.5	127.3	128.0	128.8	129.6	130.3	131.1	131.9	8
5.1	132.7	133.4	134.2	135.0	135.8	136.6	137.4	138.2	139.0	139.8	8
5.2	140.6	141.4	142.2	143.1	143.9	144.7	145.5	146.4	147.2	148.0	8
5.3	148.9	149.7	150.6	151.4	152.3	153.1	154.0	154.9	155.7	156.6	9
5.4	157.5	158.3	159.2	160.1	161.0	161.9	162.8	163.7	164.6	165.5	9
5.5	166.4	167.3	168.2	169.1	170.0	171.0	171.9	172.8	173.7	174.7	9
5.6	175.6	176.6	177.5	178.5	179.4	180.4	181.3	182.3	183.3	184.2	10
5.7	185.2	186.2	187.1	188.1	189.1	190.1	191.1	192.1	193.1	194.1	10
5.8	195.1	196.1	197.1	198.2	199.2	200.2	201.2	202.3	203.3	204.3	10
5.9	205.4	206.4	207.5	208.5	209.6	210.6	211.7	212.8	213.8	214.9	11
6.0	216.0	217.1	218.2	219.3	220.3	221.4	222.5	223.6	224.8	225.9	11

	0	1	2	3	4	5	6	7	8	9	Dif.
6.0	216.0	217.1	218.2	219.3	220.3	221.4	222.5	223.6	224.8	225.9	11
6.1	227 0	228.1	229.2	230.3	231.5	232.6	233.7	234.9	236.0	237.2	11
6.2	238.3	239.5	240.6	241.8	243.0	244.1	245.3	246.5	247.7	248.9	12
6.3	250.0	251.2	252.4	253.6	254.8	256.0	257.3	258.5	259.7	260.9	12
6.4	262.1	263.4	264.6	265.8	267.1	268.3	269.6	270.8	272.1	273.4	12
6.5	274.6	275.9	277.2	278.4	279.7	281.0	282.3	283.6	284.9	286.2	13
6.6	287.5	288.8	290 1	291.4	292.8	294.1	295.4	296.7	298.1	299.4	13
6.7	300.8	302.1	303.5	304.8	306.2	307.5	308.9	310.3	311.7	313.0	14
6.8	314.4	315.8	317.2	318.6	320.0	321.4	322.8	324.2	325.7	327.1	14
6.9	328.5	329.9	331.4	332.8	334.3	335.7	337.2	338.6	340.1	341.5	15
7.0	343.0	344.5	345.9	347.4	348.9	350.4	351.9	353.4	354.9	356.4	15
7.1	357.9	359.4	360.9	362.5	364.0	365.5	367.1	368.6	370.1	371.7	15
7.2	373.2	374.8	376.4	377.9	379.5	381.1	382.7	384.2	385.8	387.4	16
7.3	389.0	390.6	392.2	393.8	395.4	397.1	398.7	400.3	401.9	403.6	16
7.4	405.2	406.9	408.5	410.2	411.8	413.5	415.2	416.8	418.5	420.2	16
7.5	421.9	423.6	425.3	427.0	428.7	430.4	432.1	433.8	435.5	437.2	17
7.6	439.0	440.7	442.5	444.2	445.9	447.7	449.5	451.2	453.0	454.8	17
7.7	456 5	458.3	460.1	461.9	463.7	465.5	467.3	469.1	470.9	472.7	18
7.8	474.6	476.4	478.2	480.0	481.9	483.7	485.6	487.4	489.3	491.2	18
7.9	493.0	494.9	496.8	498.7	500.6	502.5	504.4	506.3	508.2	510.1	19
8.0	512.0	513.9	515.8	517.8	519.7	521.7	523.6	525.6	527.5	529.5	19
8.1	531.4	533.4	535.4	537.4	539.4	541.3	543.3	545.3	547.3	549.4	20
8.2	551.4	553.4	555.4	557.4	559.5	561.5	563.6	565.6	567.7	569.7	20
8.3	571.8	573.9	575.9	578.0	580.1	582.2	584.3	586.4	588.5	590.6	21
8.4	592.7	594.8	596.9	599.1	601.2	603.4	605.5	607.6	609.8	612.0	21
8.5	614.1	616.3	618.5	620.7	622.8	625.0	627.2	629.4	631.6	633.8	22
8.6	636.1	638.3	640.5	642.7	645 0	647.2	649.5	651.7	654.0	656.2	22
8.7	658.5	660.8	663.1	665.3	667.6	669.9	672.2	674.5	676.8	679.2	23
8.8	681.5	683.8	686.1	688.5	690.8	693.2	695.5	697.9	700.2	702.6	23
8.9	705.0	707.3	709.7	712.1	714.5	716.9	719.3	721.7	724.2	726.6	24
9.0	729.0	731.4	733.9	736.3	738.8	741.2	743.7	746.1	748.6	751.1	25
9.1	753.6	756.1	758.6	761.0	763.6	766.1	768.6	771.1	773.6	776.2	25
9.2	778.7	781.2	783.8	786.3	788.9	791.5	794.0	796.6	799.2	801.8	26
9.3	804.4	807.0	809.6	812.2	814.8	817.4	820.0	822.7	825.3	827.9	26
9.4	830.6	833.2	835.9	838.6	841.2	843.9	846.6	849.3	852.0	854.7	27
9.5	857.4	860.1	862.8	865.5	868.3	871.0	873.7	876.5	879.2	882.0	27
9.6	884.7	887.5	890.3	893.1	895.8	898.6	901.4	904.2	907.0	909.9	28
9.7	912.7	915.5	918.3	921.2	924.0	926.9	929.7	932.6	935.4	938.3	28
9.8	941.2	944.1	947.0	949.9	952.8	955.7	958.6	961.5	964.4	967.4	29
9.9	970.3	973.2	976.2	979.1	982.1	985.1	988.0	991.0	994.0	997.0	30
10.0	1000.0	1003.0	1006.0	1009.0	1012.0	1015.1	1018.1	1021.1	1024.2	1027.2	30
10.1	1030.3	1033.4	1036.4	1039.5	1042.6	1045.7	1048.8	1051.9	1055.0	1058.1	31
10.2	1061.2	1064.3	1067.5	1070.6	1073.7	107.69	1080.0	1083.2	1086.4	1089.5	31
10.3	1092.7	1095.9	1099.1	1102.3	1105.5	1108.7	1111.9	1115.2	1118.4	1121.6	32
10.4	1124.9	1128.1	1131.4	1134.6	1137.9	1141.2	1144.4	1147.7	1151.0	1154.3	33
10.5	1157.6	1160.9	1164.3	1167.6	1170.9	1174.2	1177.6	1180.9	1184.3	1187.6	33
10.6	1191.0	1194.4	1197.8	1201.2	1204.6	1207.9	1211.4	1214.8	1218.2	1221.6	34
10.7	1225.0	1228.5	1231.9	1235.4	1238.8	1242.3	1245.8	1249.2	1252.7	1256.2	35
10.8	1259.7	1263.2	1266.7	1270.2	1273.8	1277.3	1280.8	1284.4	1287.9	1291.5	35
10.9	1295.0	1298.6	1302.2	1305.8	1309.3	1312.9	1316.5	1320.1	1323.8	1327.4	36
11.0	1331.0	1334.6	1338.3	1341.9	1345.6	1349.2	1352.9	1356.6	1360.3	1363.9	37

Diam.	00.	10.	20.	30.	40.	50.	60.	70.	80.	90.
10	3142	3173	3204	3236	3267	3299	3330	3362	3393	3424
11	3456	3487	3519	3550	3581	3613	3644	3676	3707	3738
12	3770	3801	3833	3864	3896	3927	3958	3990	4021	4053
13	4084	4115	4147	4178	4210	4241	4273	4304	4335	4367
14	4398	4430	4461	4492	4524	4555	4587	4618	4650	4681
15	4712	4744	4775	4807	4838	4869	4901	4932	4964	4995
16	5027	5058	5089	5121	5152	5184	5215	5246	5278	5309
17	5341	5372	5404	5435	5466	5498	5529	5561	5592	5623
18	5655	5686	5718	5749	5781	5812	5843	5875	5906	5938
19	5969	6000	6032	6063	6095	6126	6158	6189	6220	6252
20	6283	6315	6346	6377	6409	6440	6472	6503	6535	6566
21	6597	6629	6660	6692	6723	6754	6786	6817	6849	6880
22	6912	6943	6974	7006	7037	7069	7100	7131	7163	7194
23	7226	7257	7288	7320	7351	7383	7414	7446	7477	7508
24	7540	7571	7603	7634	7665	7697	7728	7760	7791	7823
25	7854	7885	7917	7948	7980	8011	8042	8074	8105	8137
26	8168	8200	8231	8262	8294	8325	8357	8388	8419	8451
27	8482	8514	8545	8577	8608	8639	8671	8702	8734	8765
28	8796	8828	8859	8891	8922	8954	8985	9016	9048	9079
29	9111	9142	9173	9205	9236	9268	9299	9331	9362	9393
30	9425	9456	9488	9519	9550	9582	9613	9645	9676	9708
31	9739	9770	9802	9833	9865	9896	9927	9959	9990	10022
32	10053	10085	10116	10147	10179	10210	10242	10273	10304	10336
33	10367	10399	10430	19462	10493	10524	10556	10587	10619	10650
34	10681	10713	10744	10776	10807	10838	10870	10901	10933	10964
35	10996	11027	11058	11090	11121	11153	11184	11215	11247	11278
36	11310	11341	11373	11404	11435	11467	11498	11530	11561	11592
37	11624	11655	11687	11718	11750	11781	11812	11844	11875	11907
38	11938	11969	12001	12032	12064	12095	12127	12158	12189	12221
39	12252	12284	12315	12346	12378	12409	12441	12472	12504	12535
40	12566	12598	12629	12661	12692	12723	12755	12786	12818	12849
41	12881	12912	12943	12975	13006	13038	13069	13100	13132	13163
42	13195	13226	13258	13289	13320	13352	13383	13415	13446	13477
43	13509	13540	13572	13603	13635	13666	13697	13729	13760	13792
44	13823	13854	13886	13917	13949	13980	14012	14043	14074	14106
45	14137	14169	14200	14231	14263	14294	14326	14357	14388	14420
46	14451	14483	14514	14546	14577	14608	14640	14671	14703	14734
47	14765	14797	14828	14860	14891	14923	14954	14985	15017	15048
48	15080	15111	15142	15174	15205	15237	15268	15300	15331	15362
49	15394	15425	15457	15488	15519	15551	15582	15614	15645	15677
50	15708	15739	15771	15802	15834	15865	15896	15928	15959	15991
51	16022	16054	16085	16116	16148	16179	16211	16242	16273	16305
52	16336	16368	16399	16431	16462	16493	16525	16556	16588	16619
53	16650	16682	16713	16745	16776	16808	16839	16870	16902	16933
54	16965	16996	17027	17059	17090	17122	17153	17185	17216	17247
55	17279	17310	17342	17373	17404	17436	17467	17499	17530	17562
Dif.	(Mean)	(1) 3	(2) 6	(3) 9	(4) 13	(5) 16	(6) 19	(7) 22	(8) 25	(9) 28

Diam.	00.	10.	20.	30.	40.	50.	60.	70.	80.	90.
55	17279	17310	17342	17373	17404	17436	17467	17499	17530	17562
56	17593	17624	17656	17687	17719	17750	17781	17813	17844	17876
57	17907	17938	17970	18001	18033	18064	18096	18127	18158	18190
58	18221	18253	18284	18315	18347	18378	18410	18441	18473	18504
59	18535	18567	18598	18630	18661	18692	18724	18755	18787	18818
60	18850	18881	18912	18944	18975	19007	19038	19069	19101	19132
61	19164	19195	19227	19258	19289	19321	19352	19384	19415	19446
62	19478	19509	19541	19572	19604	19635	19666	19698	19729	19761
63	19792	19823	19855	19886	19918	19949	19981	20012	20043	20075
64	20106	20138	20169	20200	20232	20263	20295	20326	20358	20389
65	20420	20452	20483	20515	20546	20577	20609	20640	20672	20703
66	20735	20766	20797	20829	20860	20892	20923	20954	20986	21017
67	21049	21080	21112	21143	21174	21206	21237	21269	21300	21331
68	21363	21394	21426	21457	21488	21520	21551	21583	21614	21646
69	21677	21708	21740	21771	21803	21834	21865	21897	21928	21960
70	21991	22023	22054	22085	22117	22148	22180	22211	22242	22274
71	22305	22337	22368	22400	22431	22462	22494	22525	22557	22588
72	22619	22651	22682	22714	22745	22777	22808	22839	22871	22902
73	22934	22965	22996	23028	23059	23091	23122	23154	23185	23216
74	23248	23279	23311	23342	23373	23405	23436	23468	23499	23531
75	23562	23593	23625	23656	23688	23719	23750	23782	23813	23845
76	23876	23908	23939	23970	24002	24033	24065	24096	24127	24159
77	24190	24222	24253	24285	24316	24347	24379	24410	24442	24473
78	24504	24536	24567	24599	24630	24662	24693	24724	24756	24787
79	24819	24850	24881	24913	24944	24976	25007	25038	25070	25101
80	25133	25164	25196	25227	25258	25290	25321	25353	25384	25415
81	25447	25478	25510	25541	25573	25604	25635	25667	25698	25730
82	25761	25792	25824	25855	25887	25918	25950	25981	26012	26044
83	26075	26107	26138	26169	26201	26232	26264	26295	26327	26358
84	26389	26421	26452	26484	26515	26546	26578	26609	26641	26672
85	26704	26735	26766	26798	26829	26861	26892	26923	26955	26986
86	27018	27049	27081	27112	27143	27175	27206	27238	27269	27300
87	27332	27363	27395	27426	27458	27489	27520	27552	27583	27615
88	27646	27677	27709	27740	27772	27803	27835	27866	27897	27929
89	27960	27992	28023	28054	28086	28117	28149	28180	28212	28243
90	28274	28306	28337	28369	28400	28431	28463	28494	28526	28557
91	28588	28620	28651	28683	28714	28746	28777	28808	28840	28871
92	28903	28934	28965	28997	29028	29060	29091	29123	29154	29185
93	29217	29248	29280	29311	29342	29374	29405	29437	29468	29500
94	29531	29562	29594	29625	29657	29688	29719	29751	29782	29814
95	29845	29877	29908	29939	29971	30002	30034	30065	30096	30128
96	30159	30191	30222	30254	30285	30316	30348	30379	30411	30442
97	30473	30505	30536	30568	30599	30631	30662	30693	30725	30756
98	30788	30819	30850	30882	30913	30945	30976	31008	31039	31070
99	31102	31133	31165	31196	31227	31259	31290	31322	31353	31385
100	31416	31447	31479	31510	31542	31573	31604	31636	31667	31699
Dif.	(Mean)	(1) 3	(2) 6	(3) 9	(4) 13	(5) 16	(6) 19	(7) 22	(8) 25	(9) 28

Diam.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Dif.
10	78.5	80.1	81.7	83.3	84.9	86.6	88.2	89.9	91.6	93.3	17
11	95.0	96.8	98.5	100.3	102.1	103.9	105.7	107.5	109.4	111.2	18
12	113.1	115.0	116.9	118.8	120.8	122.7	124.7	126.7	128.7	130.7	20
13	132.7	134.8	136.8	138.9	141.0	143.1	145.3	147.4	149.6	151.7	21
14	153.9	156.1	158.4	160.6	162.9	165.1	167.4	169.7	172.0	174.4	23
15	176.7	179.1	181.5	183.9	186.3	188.7	191.1	193.6	196.1	198.6	24
16	201.1	203.6	206.1	208.7	211.2	213.8	216.4	219.0	221.7	224.3	26
17	227.0	229.7	232.4	235.1	237.8	240.5	243.3	246.1	248.8	251.6	28
18	254.5	257.3	260.2	263.0	265.9	268.8	271.7	274.6	277.6	280.6	29
19	283.5	286.5	289.5	292.6	295.6	298.6	301.7	304.8	307.9	311.0	31
20	314.2	317.3	320.5	323.7	326.9	330.1	333.3	336.5	339.8	343.1	32
21	346.4	349.7	353.0	356.3	359.7	363.1	366.4	369.8	373.3	376.7	34
22	380.1	383.6	387.1	390.6	394.1	397.6	401.1	404.7	408.3	411.9	35
23	415.5	419.1	422.7	426.4	430.1	433.7	437.4	441.2	444.9	448.6	37
24	452.4	456.2	460.0	463.8	467.6	471.4	475.3	479.2	483.1	487.0	38
25	490.9	494.8	498.8	502.7	506.7	510.7	514.7	518.7	522.8	526.9	40
26	530.9	535.0	539.1	543.3	547.4	551.5	555.7	559.9	564.1	568.3	42
27	572.6	576.8	581.1	585.3	589.6	594.0	598.3	602.6	607.0	611.4	43
28	615.8	620.2	624.6	629.0	633.5	637.9	642.4	646.9	651.4	656.0	45
29	660.5	665.1	669.7	674.3	678.9	683.5	688.1	692.8	697.5	702.2	46
30	706.9	711.6	716.3	721.1	725.8	730.6	735.4	740.2	745.1	749.9	48
31	754.8	759.6	764.5	769.4	774.4	779.3	784.3	789.2	794.2	799.2	50
32	804.2	809.3	814.3	819.4	824.5	829.6	834.7	839.8	845.0	850.1	51
33	855.3	860.5	865.7	870.9	876.2	881.4	886.7	892.0	897.3	902.6	53
34	907.9	913.3	918.6	924.0	929.4	934.8	940.2	945.7	951.1	956.6	54
35	962	968	973	979	984	990	995	1001	1007	1012	
36	1018	1024	1029	1035	1041	1046	1052	1058	1064	1069	
37	1075	1081	1087	1093	1099	1104	1110	1116	1122	1128	
38	1134	1140	1146	1152	1158	1164	1170	1176	1182	1188	6
39	1195	1201	1207	1213	1219	1225	1232	1238	1244	1250	
40	1257	1263	1269	1276	1282	1288	1295	1301	1307	1314	
41	1320	1327	1333	1340	1346	1353	1359	1366	1372	1379	
42	1385	1392	1399	1405	1412	1419	1425	1432	1439	1445	
43	1452	1459	1466	1473	1479	1486	1493	1500	1507	1514	
44	1521	1527	1534	1541	1548	1555	1562	1569	1576	1583	
45	1590	1598	1605	1612	1619	1626	1633	1640	1647	1655	7
46	1662	1669	1676	1684	1691	1698	1706	1713	1720	1728	
47	1735	1742	1750	1757	1765	1772	1780	1787	1795	1802	
48	1810	1817	1825	1832	1840	1847	1855	1863	1870	1878	
49	1886	1893	1901	1909	1917	1924	1932	1940	1948	1956	
50	1963	1971	1979	1987	1995	2003	2011	2019	2027	2035	
51	2043	2051	2059	2067	2075	2083	2091	2099	2107	2116	8
52	2124	2132	2140	2148	2157	2165	2173	2181	2190	2198	
53	2206	2215	2223	2231	2240	2248	2256	2265	2273	2282	
54	2290	2299	2307	2316	2324	2333	2341	2350	2359	2367	
55	2376	2384	2393	2402	2411	2419	2428	2437	2445	2454	

Diam.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Dif.
55	2376	2384	2393	2402	2411	2419	2428	2437	2445	2454	
56	2463	2472	2481	2489	2498	2507	2516	2525	2534	2543	
57	2552	2561	2570	2579	2588	2597	2606	2615	2624	2633	9
58	2642	2651	2660	2669	2679	2688	2697	2706	2715	2725	
59	2734	2743	2753	2762	2771	2781	2790	2799	2809	2818	
60	2827	2837	2846	2856	2865	2875	2884	2894	2903	2913	
61	2922	2932	2942	2951	2961	2971	2980	2990	3000	3009	
62	3019	3029	3039	3048	3058	3068	3078	3088	3097	3107	
63	3117	3127	3137	3147	3157	3167	3177	3187	3197	3207	
64	3217	3227	3237	3247	3257	3267	3278	3288	3298	3308	10
65	3318	3329	3339	3349	3359	3370	3380	3390	3400	3411	
66	3421	3432	3442	3452	3463	3473	3484	3494	3505	3515	
67	3526	3536	3547	3557	3568	3578	3589	3600	3610	3621	
68	3632	3642	3653	3664	3675	3685	3696	3707	3718	3728	
69	3739	3750	3761	3772	3783	3794	3805	3816	3826	3837	
70	3848	3859	3871	3882	3893	3904	3915	3926	3937	3948	11
71	3959	3970	3982	3993	4004	4015	4026	4038	4049	4060	
72	4072	4083	4094	4106	4117	4128	4140	4151	4162	4174	
73	4185	4197	4208	4220	4231	4243	4254	4266	4278	4289	
74	4301	4312	4324	4336	4347	4359	4371	4383	4394	4406	
75	4418	4430	4441	4453	4465	4477	4489	4501	4513	4524	
76	4536	4548	4560	4572	4584	4596	4608	4620	4632	4645	12
77	4657	4669	4681	4693	4705	4717	4729	4742	4754	4766	
78	4778	4791	4803	4815	4827	4840	4852	4865	4877	4889	
79	4902	4914	4927	4939	4951	4964	4976	4989	5001	5014	
80	5027	5039	5052	5064	5077	5090	5102	5115	5128	5140	
81	5153	5166	5178	5191	5204	5217	5230	5242	5255	5268	
82	5281	5294	5307	5320	5333	5346	5359	5372	5385	5398	13
83	5411	5424	5437	5450	5463	5476	5489	5502	5515	5529	
84	5542	5555	5568	5581	5595	5608	5621	5635	5648	5661	
85	5675	5638	5701	5715	5728	5741	5755	5768	5782	5795	
86	5809	5822	5836	5849	5863	5877	5890	5904	5917	5931	
87	5945	5958	5972	5986	5999	6013	6027	6041	6055	6068	
88	6082	6096	6110	6124	6138	6151	6165	6179	6193	6207	
89	6221	6235	6249	6263	6277	6291	6305	6319	6333	6348	14
90	6362	6376	6390	6404	6418	6433	6447	6461	6475	6490	
91	6504	6518	6533	6547	6561	6576	6590	6604	6619	6633	
92	6648	6662	6677	6691	6706	6720	6735	6749	6764	6778	
93	6793	6808	6822	6837	6851	6866	6881	6896	6910	6925	
94	6940	6955	6969	6984	6999	7014	7029	7044	7058	7073	
95	7088	7103	7118	7133	7148	7163	7178	7193	7208	7223	15
96	7238	7253	7268	7284	7299	7314	7329	7344	7359	7375	
97	7390	7405	7420	7436	7451	7466	7482	7497	7512	7528	
98	7543	7558	7574	7589	7605	7620	7636	7651	7667	7682	
99	7698	7713	7729	7744	7760	7776	7791	7807	7823	7838	
100	7854	7870	7885	7901	7917	7933	7949	7964	7980	7996	16

Diam.	0	1	2	3	4	5	6	7	8	9	Dif.
1.0	.524	.539	.556	.572	.589	.606	.624	.641	.660	.678	17
1.1	.697	.716	.736	.755	.776	.796	.817	.839	.860	.882	21
1.2	.905	.928	.951	.974	.998	1.023	1.047	1.073	1.098	1.124	25
1.3	1.150	1.177	1.204	1.232	1.260	1.288	1.317	1.346	1.376	1.406	29
1.4	1.437	1.468	1.499	1.531	1.563	1.596	1.630	1.663	1.697	1.732	33
1.5	1.767	1.803	1.839	1.875	1.912	1.950	1.988	2.026	2.065	2.105	38
1.6	2.145	2.185	2.226	2.268	2.310	2.352	2.395	2.439	2.483	2.527	43
1.7	2.572	2.618	2.664	2.711	2.758	2.806	2.855	2.903	2.953	3.003	48
1.8	3.054	3.105	3.157	3.209	3.262	3.315	3.369	3.424	3.479	3.535	54
1.9	3.591	3.648	3.706	3.764	3.823	3.882	3.942	4.003	4.064	4.126	60
2.0	4.189	4.252	4.316	4.380	4.445	4.511	4.577	4.644	4.712	4.780	66
2.1	4.849	4.919	4.989	5.060	5.131	5.204	5.277	5.350	5.425	5.500	73
2.2	5.575	5.652	5.729	5.806	5.885	5.964	6.044	6.125	6.206	6.288	80
2.3	6.371	6.454	6.538	6.623	6.709	6.795	6.882	6.970	7.059	7.148	87
2.4	7.238	7.329	7.421	7.513	7.606	7.700	7.795	7.890	7.986	8.083	94
2.5	8.18	8.28	8.38	8.48	8.58	8.68	8.78	8.89	8.99	9.10	10
2.6	9.20	9.31	9.42	9.53	9.63	9.74	9.85	9.97	10.08	10.19	11
2.7	10.31	10.42	10.54	10.65	10.77	10.89	11.01	11.13	11.25	11.37	12
2.8	11.49	11.62	11.74	11.87	11.99	12.12	12.25	12.38	12.51	12.64	13
2.9	12.77	12.90	13.04	13.17	13.31	13.44	13.58	13.72	13.86	14.00	14
3.0	14.14	14.28	14.42	14.57	14.71	14.86	15.00	15.15	15.30	15.45	15
3.1	15.60	15.75	15.90	16.06	16.21	16.37	16.52	16.68	16.84	17.00	16
3.2	17.16	17.32	17.48	17.64	17.81	17.97	18.14	18.31	18.48	18.65	17
3.3	18.82	18.99	19.16	19.33	19.51	19.68	19.86	20.04	20.22	20.40	18
3.4	20.58	20.76	20.94	21.13	21.31	21.50	21.69	21.88	22.07	22.26	19
3.5	22.45	22.64	22.84	23.03	23.23	23.43	23.62	23.82	24.02	24.23	20
3.6	24.43	24.63	24.84	25.04	25.25	25.46	25.67	25.88	26.09	26.31	21
3.7	26.52	26.74	26.95	27.17	27.39	27.61	27.83	28.06	28.28	28.50	22
3.8	28.73	28.96	29.19	29.42	29.65	29.88	30.11	30.35	30.58	30.82	23
3.9	31.06	31.30	31.54	31.78	32.02	32.27	32.52	32.76	33.01	33.26	24
4.0	33.51	33.76	34.02	34.27	34.53	34.78	35.04	35.30	35.56	35.82	26
4.1	36.09	36.35	36.62	36.88	37.15	37.42	37.69	37.97	38.24	38.52	27
4.2	38.79	39.07	39.35	39.63	39.91	40.19	40.48	40.76	41.05	41.34	29
4.3	41.63	41.92	42.21	42.51	42.80	43.10	43.40	43.70	44.00	44.30	30
4.4	44.60	44.91	45.21	45.52	45.83	46.14	46.45	46.77	47.08	47.40	31
4.5	47.71	48.03	48.35	48.67	49.00	49.32	49.65	49.97	50.30	50.63	33
4.6	50.97	51.30	51.63	51.97	52.31	52.65	52.99	53.33	53.67	54.02	34
4.7	54.36	54.71	55.06	55.41	55.76	56.12	56.47	56.83	57.18	57.54	36
4.8	57.91	58.27	58.63	59.00	59.37	59.73	60.10	60.48	60.85	61.22	37
4.9	61.60	61.98	62.36	62.74	63.12	63.51	63.89	64.28	64.67	65.06	38
5.0	65.45	65.84	66.24	66.64	67.03	67.43	67.83	68.24	68.64	69.05	40
5.1	69.46	69.87	70.28	70.69	71.10	71.52	71.94	72.36	72.78	73.20	42
5.2	73.62	74.05	74.47	74.90	75.33	75.77	76.20	76.64	77.07	77.51	43
5.3	77.95	78.39	78.84	79.28	79.73	80.18	80.63	81.08	81.54	81.99	45
5.4	82.45	82.91	83.37	83.83	84.29	84.76	85.23	85.70	86.17	86.64	47
5.5	87.11	87.59	88.07	88.55	89.03	89.51	90.00	90.48	90.97	91.46	48

Diam.	0	1	2	3	4	5	6	7	8	9	
5.5	87.1	87.6	88.1	88.5	89.0	89.5	90.0	90.5	91.0	91.5	Dif.
5.6	92.0	92.4	92.9	93.4	93.9	94.4	94.9	95.4	95.9	96.5	5
5.7	97.0	97.5	98.0	98.5	99.0	99.5	100.1	100.6	101.1	101.6	
5.8	102.2	102.7	103.2	103.8	104.3	104.8	105.4	105.9	106.4	107.0	
5.9	107.5	108.1	108.6	109.2	109.7	110.3	110.9	111.4	112.0	112.5	
6.0	113.1	113.7	114.2	114.8	115.4	115.9	116.5	117.1	117.7	118.3	
6.1	118.8	119.4	120.0	120.6	121.2	121.8	122.4	123.0	123.6	124.2	6
6.2	124.8	125.4	126.0	126.6	127.2	127.8	128.4	129.1	129.7	130.3	
6.3	130.9	131.5	132.2	132.8	133.4	134.1	134.7	135.3	136.0	136.6	
6.4	137.3	137.9	138.5	139.2	139.8	140.5	141.2	141.8	142.5	143.1	
6.5	143.8	144.5	145.1	145.8	146.5	147.1	147.8	148.5	149.2	149.8	
6.6	150.5	151.2	151.9	152.6	153.3	154.0	154.7	155.4	156.1	156.8	7
6.7	157.5	158.2	158.9	159.6	160.3	161.0	161.7	162.5	163.2	163.9	
6.8	164.6	165.4	166.1	166.8	167.6	168.3	169.0	169.8	170.5	171.3	
6.9	172.0	172.8	173.5	174.3	175.0	175.8	176.5	177.3	178.1	178.8	
7.0	179.6	180.4	181.1	181.9	182.7	183.5	184.3	185.0	185.8	186.6	
7.1	187.4	188.2	189.0	189.8	190.6	191.4	192.2	193.0	193.8	194.6	8
7.2	195.4	196.2	197.1	197.9	198.7	199.5	200.4	201.2	202.0	202.9	
7.3	203.7	204.5	205.4	206.2	207.1	207.9	208.8	209.6	210.5	211.3	
7.4	212.2	213.0	213.9	214.8	215.6	216.5	217.4	218.3	219.1	220.0	
7.5	220.9	221.8	222.7	223.6	224.4	225.3	226.2	227.1	228.0	228.9	9
7.6	229.8	230.8	231.7	232.6	233.5	234.4	235.3	236.3	237.2	238.1	
7.7	239.0	240.0	240.9	241.8	242.8	243.7	244.7	245.6	246.6	247.5	
7.8	248.5	249.4	250.4	251.4	252.3	253.3	254.3	255.2	256.2	257.2	
7.9	258.2	259.1	260.1	261.1	262.1	263.1	264.1	265.1	266.1	267.1	10
8.0	268.1	269.1	270.1	271.1	272.1	273.1	274.2	275.2	276.2	277.2	
8.1	278.3	279.3	280.3	281.4	282.4	283.4	284.5	285.5	286.6	287.6	
8.2	288.7	289.8	290.8	291.9	292.9	294.0	295.1	296.2	297.2	298.3	
8.3	299.4	300.5	301.6	302.6	303.7	304.8	305.9	307.0	308.1	309.2	11
8.4	310.3	311.4	312.6	313.7	314.8	315.9	317.0	318.2	319.3	320.4	
8.5	321.6	322.7	323.8	325.0	326.1	327.3	328.4	329.6	330.7	331.9	
8.6	333.0	334.2	335.4	336.5	337.7	338.9	340.1	341.2	342.4	343.6	
8.7	344.8	346.0	347.2	348.4	349.6	350.8	352.0	353.2	354.4	355.6	12
8.8	356.8	358.0	359.3	360.5	361.7	362.9	364.2	365.4	366.6	367.9	
8.9	369.1	370.4	371.6	372.9	374.1	375.4	376.6	377.9	379.2	380.4	
9.0	381.7	383.0	384.3	385.5	386.8	388.1	389.4	390.7	392.0	393.3	13
9.1	394.6	395.9	397.2	398.5	399.8	401.1	402.4	403.7	405.1	406.4	
9.2	407.7	409.1	410.4	411.7	413.1	414.4	415.7	417.1	418.4	419.8	
9.3	421.2	422.5	423.9	425.2	426.6	428.0	429.4	430.7	432.1	433.5	
9.4	434.9	436.3	437.7	439.1	440.5	441.9	443.3	444.7	446.1	447.5	14
9.5	448.9	450.3	451.8	453.2	454.6	456.0	457.5	458.9	460.4	461.8	
9.6	463.2	464.7	466.1	467.6	469.1	470.5	472.0	473.5	474.9	476.4	
9.7	477.9	479.4	480.8	482.3	483.8	485.3	486.8	488.3	489.8	491.3	15
9.8	492.8	494.3	495.8	497.3	498.9	500.4	501.9	503.4	505.0	506.5	
9.9	508.0	509.6	511.1	512.7	514.2	515.8	517.3	518.9	520.5	522.0	
10.0	523.6	525.2	526.7	528.3	529.9	531.5	533.1	534.7	536.3	537.9	16

Angle	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Complement		Dif.
0°	0 0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	0175	89°	
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	0349	88	
2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	0523	87	
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	0698	86	
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	0872	85	
5	0.0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	1045	84	
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	1219	83	
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	1392	82	
8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	1564	81	
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	1736	80	
10	0.1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	1908	79	
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	2079	78	
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	2250	77	17
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	2419	76	
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	2588	75	
15	0.2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	2756	74	
16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	2924	73	
17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3090	72	
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3256	71	
19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3420	70	
20	0.3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3584	69	
21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3746	68	
22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3907	67	
23	3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	4067	66	16
24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	4226	65	
25	0.4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	4384	64	
26	4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	4540	63	
27	4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	4695	62	
28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	4848	61	
29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	5000	60	
30	0.5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	5150	59	15
31	5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	5299	58	
32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	5446	57	
33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	5592	56	
34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	5736	55	
35	0.5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	5878	54	
36	5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	6018	53	14
37	6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	6157	52	
38	6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	6293	51	
39	6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	6428	50	
40	0.6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	6561	49	
41	6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	6691	48	13
42	6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	6820	47	
43	6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	6947	46	
44°	6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	7071	45°	
Complement		.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	Angle	

Natural Cosines.

Angle	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Complement	Dif.
45°	0.7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	7193 44°	
46	7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	7314 43	12
47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	7431 42	
48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	7547 41	
49	7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	7660 40	
50	0.7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	7771 39	
51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	7880 38	11
52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	7986 37	
53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	8090 36	
54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	8192 35	
55	0.8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	8290 34	10
56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	8387 33	
57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	8480 32	
58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	8572 31	
59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	8660 30	9
60	0.8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	8746 29	
61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	8829 28	
62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	8910 27	8
63	8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	8988 26	
64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	9063 25	
65	0.9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	9135 24	
66	9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	9205 23	7
67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	9272 22	
68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	9336 21	
69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	9397 20	6
70	0.9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	9455 19	
71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	9511 18	
72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	9563 17	
73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	9613 16	5
74	9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	9659 15	
75	0.9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	9703 14	
76	9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	9744 13	4
77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	9781 12	
78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	9816 11	
79	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	9848 10	
80	0.9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	9877 9	3
81	9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	9903 8	
82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	9925 7	
83	9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	9945 6	2
84	9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	9962 5	
85	0.9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	9976 4	
86	9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	9986 3	1
87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	9994 2	
88	9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	9998 1	
89°	9998	9999	9999	9999	9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000 0°	0
Complement	.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	Angle	

Natural Cosines.

Logarithmic Sines.

Angle	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Complement	Dif.
0°	$-\infty$	$\bar{3}.2419$	$\bar{3}.5429$	$\bar{3}.7190$	$\bar{3}.8439$	$\bar{3}.9408$	$\bar{2}.0200$	$\bar{3}.0870$	$\bar{2}.1450$	$\bar{2}.1961$	$\bar{2}.2419$	89°	—
1	$\bar{2}.2419$	$\bar{2}.2832$	$\bar{2}.3210$	$\bar{2}.3558$	$\bar{2}.3880$	$\bar{2}.4179$	$\bar{2}.4459$	$\bar{2}.4723$	$\bar{2}.4971$	$\bar{2}.5206$	$\bar{2}.5428$	88	—
2	5428	5640	5842	6035	8220	6397	6567	6731	6889	7041	7188	87	—
3	7188	7330	7468	7602	7731	7857	7979	8098	8213	8326	8436	86	—
4	8436	8543	8647	8749	8849	8946	9042	9135	9226	9315	9403	85	—
5	$\bar{2}.9403$	$\bar{2}.9489$	$\bar{2}.9573$	$\bar{2}.9655$	$\bar{2}.9736$	$\bar{2}.9816$	$\bar{2}.9894$	$\bar{2}.9970$	$\bar{1}.0046$	$\bar{1}.0120$	$\bar{1}.0192$	84	79
6	$\bar{1}.0192$	$\bar{1}.0264$	$\bar{1}.0334$	$\bar{1}.0403$	$\bar{1}.0472$	$\bar{1}.0539$	$\bar{1}.0605$	$\bar{1}.0670$	$\bar{1}.0734$	$\bar{1}.0797$	$\bar{1}.0859$	83	67
7	0859	0920	0981	1040	1099	1157	1214	1271	1326	1381	1436	82	58
8	1436	1489	1542	1594	1646	1697	1747	1797	1347	1895	1943	81	51
9	1943	1991	2038	2085	2131	2176	2221	2266	2310	2353	2397	80	46
10	$\bar{1}.2397$	$\bar{1}.2439$	$\bar{1}.2482$	$\bar{1}.2524$	$\bar{1}.2565$	$\bar{1}.2606$	$\bar{1}.2647$	$\bar{1}.2687$	$\bar{1}.2727$	$\bar{1}.2767$	$\bar{1}.2806$	79	21
11	2806	2845	2883	2921	2959	2997	3034	3070	3107	3143	3179	78	87
12	3179	3214	3249	3284	3319	6353	3387	3421	3455	3488	3521	77	34
13	3521	3554	3586	6618	3650	6682	3713	3745	3775	6806	3837	76	31
14	3837	3867	3897	3927	3957	6986	4015	4044	4073	4102	4130	75	49
15	$\bar{1}.4130$	$\bar{1}.4158$	$\bar{1}.4186$	$\bar{1}.4214$	$\bar{1}.4242$	$\bar{1}.4269$	$\bar{1}.4296$	$\bar{1}.4323$	$\bar{1}.4350$	$\bar{1}.4377$	$\bar{1}.4403$	74	27
16	4403	4430	4456	4482	4508	4533	4559	4584	4609	4634	4659	73	26
17	4659	4684	4709	4733	4757	4781	4805	4829	4853	4876	4900	72	24
18	4900	4923	4946	4969	4992	5015	5037	5060	5082	5104	5126	71	23
19	5126	5148	5170	6192	5213	5235	5256	5278	5299	5320	6341	70	22
20	$\bar{1}.5341$	$\bar{1}.5361$	$\bar{1}.5382$	$\bar{1}.5402$	$\bar{1}.5423$	$\bar{1}.5443$	$\bar{1}.5463$	$\bar{1}.5484$	$\bar{1}.5504$	$\bar{1}.5523$	$\bar{1}.5543$	69	20
21	5543	5563	5583	5602	5621	5641	5660	5679	5698	5717	5736	68	19
22	5736	5754	5773	5792	5810	6828	5847	5865	5883	5901	5919	67	18
23	5919	5937	5954	5972	5990	6007	6024	6042	6059	6076	6093	66	18
24	6093	6110	6127	6144	6161	6177	8194	6210	6227	6243	6259	65	17
25	$\bar{1}.6259$	$\bar{1}.6276$	$\bar{1}.6292$	$\bar{1}.6308$	$\bar{1}.6324$	$\bar{1}.6340$	$\bar{1}.6356$	$\bar{1}.6371$	$\bar{1}.6387$	$\bar{1}.6403$	$\bar{1}.6418$	64	16
26	6418	6434	6449	6465	6480	6495	6510	6526	6541	6556	6570	63	16
27	6570	6585	6600	6615	6629	6644	6659	6673	6687	6702	8716	62	15
28	6716	6730	6744	6759	6773	6787	6801	6814	6828	6843	6856	61	14
29	6858	8869	6833	6898	6910	6923	6937	6950	6963	6977	6990	60	
30	$\bar{1}.6990$	$\bar{1}.7003$	$\bar{1}.7016$	$\bar{1}.7029$	$\bar{1}.7042$	$\bar{1}.7055$	$\bar{1}.7068$	$\bar{1}.7080$	$\bar{1}.7093$	$\bar{1}.7106$	$\bar{1}.7118$	59	18
31	7118	7131	7144	7158	7168	7181	7193	7205	7318	7230	7242	58	
32	7242	7254	7266	7278	7290	7602	7314	7326	7338	7349	7361	57	12
33	7361	7373	7384	7396	7407	7419	7430	7442	7453	7464	7478	56	
34	7478	7487	7498	7509	7520	7561	7542	7553	7564	7575	7586	55	11
35	$\bar{1}.7586$	$\bar{1}.7597$	$\bar{1}.7607$	$\bar{1}.7618$	$\bar{1}.7629$	$\bar{1}.7640$	$\bar{1}.7650$	$\bar{1}.7661$	$\bar{1}.7671$	$\bar{1}.7682$	$\bar{1}.7692$	54	
36	7692	7703	7713	7723	7734	7744	7754	7764	7774	7785	7795	53	
37	7795	7805	7815	7825	7835	7844	7854	7864	7874	7884	7893	52	10
38	7893	7903	7913	7922	7932	7941	7951	7960	7970	7979	7939	51	
39	7989	7998	8007	8017	8026	8035	8044	8053	8063	8072	8081	50	
40	$\bar{1}.8081$	$\bar{1}.8090$	$\bar{1}.8099$	$\bar{1}.8108$	$\bar{1}.8117$	$\bar{1}.8125$	$\bar{1}.8134$	$\bar{1}.8143$	$\bar{1}.8152$	$\bar{1}.8161$	$\bar{1}.8169$	49	9
41	8169	8178	8187	8195	8204	8213	8221	8230	8238	8247	8255	48	
42	8255	8264	8272	8280	8289	8297	8605	8313	8322	8330	8338	47	
43	8338	8346	8354	8862	8370	8378	8386	8394	8402	8410	8418	46	
44°	8418	8426	8433	8441	8449	8457	8464	8472	8480	8487	8495	45°	8
Complement	.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	Angle		

Logarithmic Cosines.

Logarithmic Sines.

Angle	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Complement	Diff.
45°	$\bar{1}$.8495	$\bar{1}$.8502	$\bar{1}$.8510	$\bar{1}$.8517	$\bar{1}$.8525	$\bar{1}$.8532	$\bar{1}$.8540	$\bar{1}$.8547	$\bar{1}$.8555	$\bar{1}$.8562	$\bar{1}$.8569	44°	
46	8569	8577	8584	8591	8598	8606	8613	8620	8627	8634	8641	43	
47	8641	8648	8655	8662	8669	8676	8683	8690	8697	8704	8711	42	7
48	8711	8718	8724	8731	8738	8745	8751	8758	8765	8771	8778	41	
49	8778	8784	8791	8797	8804	8810	8817	8823	8830	8836	8843	40	
50	$\bar{1}$.8843	$\bar{1}$.8849	$\bar{1}$.8855	$\bar{1}$.8862	$\bar{1}$.8868	$\bar{1}$.8874	$\bar{1}$.8880	$\bar{1}$.8887	$\bar{1}$.8893	$\bar{1}$.8899	$\bar{1}$.8905	39	
51	8905	8911	8917	8923	8929	8935	8941	8947	8953	8959	8965	38	6
52	8965	8971	8977	8983	8989	8995	9000	9006	9012	9018	9023	37	
53	9023	9029	9035	9041	9046	9052	9057	9063	9069	9074	9080	36	
54	9080	9085	9091	9096	9101	9107	9112	9118	9123	9128	9134	35	
55	$\bar{1}$.9134	$\bar{1}$.9139	$\bar{1}$.9144	$\bar{1}$.9149	$\bar{1}$.9155	$\bar{1}$.9160	$\bar{1}$.9165	$\bar{1}$.9170	$\bar{1}$.9175	$\bar{1}$.9181	$\bar{1}$.9186	34	
56	9186	9191	9196	9201	9206	9211	9216	9221	9226	9231	9236	33	5
57	9236	9241	9246	9251	9255	9260	9265	9270	9275	9279	9284	32	
58	9284	9289	9294	9298	9303	9308	9312	9317	9322	9326	9331	31	
59	9331	9335	9340	9344	9349	9353	9358	9362	9367	9371	9375	30	
60	$\bar{1}$.9375	$\bar{1}$.9380	$\bar{1}$.9384	$\bar{1}$.9388	$\bar{1}$.9393	$\bar{1}$.9397	$\bar{1}$.9401	$\bar{1}$.9406	$\bar{1}$.9410	$\bar{1}$.9414	$\bar{1}$.9418	29	
61	9418	9422	9427	9431	9435	9439	9443	9447	9451	9455	9459	28	
62	9459	9463	9467	9471	9475	9479	9483	9487	9491	9495	9499	27	4
63	9499	9503	9506	9510	9514	9518	9522	9525	9529	9533	9537	26	
64	9537	9540	9544	9548	9551	9555	9558	9562	9566	9569	9573	25	
65	$\bar{1}$.9573	$\bar{1}$.9576	$\bar{1}$.9580	$\bar{1}$.9583	$\bar{1}$.9587	$\bar{1}$.9590	$\bar{1}$.9594	$\bar{1}$.9597	$\bar{1}$.9601	$\bar{1}$.9604	$\bar{1}$.9607	24	
66	9607	9611	9614	9617	9621	9624	9627	9631	9634	9637	9640	23	
67	9640	9643	9647	9650	9653	9656	9659	9662	9665	9669	9672	22	
68	9672	9675	9678	9681	9684	9687	9690	9693	9696	9699	9702	21	3
69	9702	9704	9707	9710	9713	9716	9719	9722	9724	9727	9730	20	
70	$\bar{1}$.9730	$\bar{1}$.9733	$\bar{1}$.9735	$\bar{1}$.9738	$\bar{1}$.9741	$\bar{1}$.9743	$\bar{1}$.9746	$\bar{1}$.9749	$\bar{1}$.9751	$\bar{1}$.9754	$\bar{1}$.9757	19	
71	9757	9759	9762	9764	9767	9770	9772	9775	9777	9780	9782	18	
72	9782	9785	9787	9789	9792	9794	9797	9799	9801	9804	9806	17	
73	9806	9808	9811	9813	9815	9817	9820	9822	9824	9826	9828	16	
74	9828	9831	9833	9835	9837	9839	9841	9843	9845	9847	9849	15	
75	$\bar{1}$.9849	$\bar{1}$.9851	$\bar{1}$.9853	$\bar{1}$.9855	$\bar{1}$.9857	$\bar{1}$.9859	$\bar{1}$.9861	$\bar{1}$.9863	$\bar{1}$.9865	$\bar{1}$.9867	$\bar{1}$.9869	14	2
76	9869	9871	9873	9875	9876	9878	9880	9882	9884	9885	9887	13	
77	9887	9889	9891	9892	9894	9896	9897	9899	9901	9902	9904	12	
78	9904	9906	9907	9909	9910	9912	9913	9915	9916	9918	9919	11	
79	9919	9921	9922	9924	9925	9927	9928	9929	9931	9932	9934	10	
80	$\bar{1}$.9934	$\bar{1}$.9935	$\bar{1}$.9936	$\bar{1}$.9937	$\bar{1}$.9939	$\bar{1}$.9940	$\bar{1}$.9941	$\bar{1}$.9943	$\bar{1}$.9944	$\bar{1}$.9945	$\bar{1}$.9946	9	
81	9946	9947	9949	9950	9951	9952	9953	9954	9955	9956	9958	8	
82	9958	9959	9960	9961	9962	9963	9964	9965	9966	9967	9968	7	1
83	9968	9968	9969	9970	9971	9972	9973	9974	9975	9975	9676	6	
84	9976	9977	9978	9978	9979	9980	9981	9981	9982	9983	9983	5	
85	$\bar{1}$.9983	$\bar{1}$.9984	$\bar{1}$.9985	$\bar{1}$.9985	$\bar{1}$.9986	$\bar{1}$.9987	$\bar{1}$.9987	$\bar{1}$.9988	$\bar{1}$.9988	$\bar{1}$.9989	$\bar{1}$.9989	4	
86	9989	9990	9990	9991	9991	9992	9992	9993	9993	9994	9994	3	
87	9994	9994	9995	9995	9996	9996	9996	9996	9997	9997	9997	2	
88	9997	9998	9998	9998	9998	9999	9999	9999	9999	9999	9999	1	
89°	9999	9999	0000	0000	0000	0000	0000	0000	0000	0000	0000	0°	0
Complement	.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	Angle		

Logarithmic Cosines.

Angle	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Complement	Dif.
0°	0.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	0175	**89°**	
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	0349	**88**	
2	0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	0524	**87**	
3	0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	0699	**86**	
4	0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	0875	**85**	
5	0.0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	1051	**84**	
6	1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	1228	**83**	
7	1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	1405	**82**	
8	1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	1584	**81**	
9	1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	1763	**80**	
10	0.1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	1944	**79**	18
11	1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	2126	**78**	
12	2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	2309	**77**	
13	2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	2493	**76**	
14	2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	2679	**75**	
15	0.2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	2867	**74**	
16	2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3057	**73**	19
17	3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3249	**72**	
18	3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3443	**71**	
19	3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3640	**70**	
20	0.3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3839	**69**	
21	3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	4040	**68**	20
22	4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	4245	**67**	
23	4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	4452	**66**	
24	4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4663	**65**	21
25	0 4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4877	**64**	
26	4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	5095	**63**	
27	5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	5317	**62**	22
28	5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	5543	**61**	
29	5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	5774	**60**	23
30	0.5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	6009	**59**	
31	6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	6249	**58**	24
32	6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	6494	**57**	
33	6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	6745	**56**	25
34	6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	7002	**55**	
35	0.7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	7265	**54**	26
36	7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	7536	**53**	27
37	7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	7813	**52**	28
38	7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	8098	**51**	28
39	8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	8391	**50**	28
40	0.8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	8693	**49**	30
41	8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	9004	**48**	31
42	9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	9325	**47**	32
43	9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	9657	**46**	33
44°	9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	1.0000	**45°**	34
Complement	.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	Angle		

Natural Cotangents.

Angle.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Dif.
45°	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319	36
46	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686	37
47	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067	38
48	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463	40
49	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875	41
50	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305	43
51	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753	45
52	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222	47
53	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713	49
54	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229	52
55	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770	54
56	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340	57
57	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941	60
58	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577	64
59	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251	68
60	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966	72
61	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728	77
62	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542	82
63	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413	88
64	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348	94
65	2.145	2.154	2.164	2.174	2.184	2.194	2.204	2.215	2.225	2.236	10
66	2.246	2.257	2.267	2.278	2.289	2.300	2.311	2.322	2.333	2.344	11
67	2.356	2.367	2.379	2.391	2.402	2.414	2.426	2.438	2.450	2.463	12
68	2.475	2.488	2.500	2.513	2.526	2.539	2.552	2.565	2.578	2.592	13
69	2.605	2.619	2.633	2.646	2.660	2.675	2.689	2.703	2.718	2.733	14
70	2.747	2.762	2.778	2.793	2.808	2.824	2.840	2.856	2.872	2.888	16
71	2.904	2.921	2.937	2.954	2.971	2.989	3.006	3.024	3.042	3.060	17
72	3.078	3.096	3.115	3.133	3.152	3.172	3.191	3.211	3.230	3.250	19
73	3.271	3.291	3.312	3.333	3.354	3.376	3.398	3.420	3.442	3.465	22
74	3.487	3.511	3.534	3.558	3.582	3.606	3.630	3.655	3.681	3.706	25
75	3.732	3.758	3.785	3.812	3.839	3.867	3.895	3.923	3.952	3.981	28
76	4.011	4.041	4.071	4.102	4.134	4.165	4.198	4.230	4.264	4.297	32
77	4.331	4.366	4.402	4.437	4.474	4.511	4.548	4.586	4.625	4.665	37
78	4.705	4.745	4.787	4.829	4.872	4.915	4.959	5.005	5.050	5.097	44
79	5.145	5.193	5.242	5.292	5.343	5.396	5.449	5.503	5.558	5.614	52
80	5.67	5.73	5.79	5.85	5.91	5.98	6.04	6.11	6.17	6.24	7
81	6.31	6.39	6.46	6.54	6.61	6.69	6.77	6.85	6.94	7.03	8
82	7.12	7.21	7.30	7.40	7.49	7.60	7.70	7.81	7.92	8.03	10
83	8.14	8.26	8.39	8.51	8.64	8.78	8.92	9.06	9.21	9.36	14
84	9.51	9.68	9.84	10.0	10.2	10.4	10.6	10.8	11.0	11.2	
85	11.4	11.7	11.9	12.2	12.4	12.7	13.0	13.3	13.6	14.0	3
86	14.3	14.7	15.1	15.5	15.9	16.3	16.8	17.3	17.9	18.5	6
87	19.1	19.7	20.4	21.2	22.0	22.9	23.9	24.9	26.0	27.3	
88	28.6	30.1	31.8	33.7	35.8	38.2	40.9	44.1	47.7	52.1	
89°	57.	64.	72.	82.	95.	115.	143.	191.	286.	573.	
Angle.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	

Angle	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Complement	Dif.
0°	$-\infty$	$\bar{3}.2419$	$\bar{3}.5429$	$\bar{3}.7190$	$\bar{3}.8439$	$\bar{3}.9409$	$\bar{2}.0200$	$\bar{2}.0870$	$\bar{2}.1450$	$\bar{2}.1962$	$\bar{2}.2419$	89°	—
1	$\bar{2}.2419$	$\bar{2}.2833$	$\bar{2}.3211$	$\bar{2}.3559$	$\bar{2}.3881$	$\bar{2}.4181$	$\bar{2}.4461$	$\bar{2}.4725$	$\bar{2}.4973$	$\bar{2}.5208$	$\bar{2}.5431$	88	—
2	5431	5643	5845	6033	6223	6401	6571	6736	6894	7046	7194	87	—
3	7194	7337	7475	7609	7739	7865	7988	8107	8223	8336	8446	86	—
4	8446	8554	8659	8762	8862	8960	9056	9150	9241	9331	9420	85	—
5	$\bar{2}.9420$	$\bar{2}.9506$	$\bar{2}.9591$	$\bar{2}.9674$	$\bar{2}.9756$	$\bar{2}.9836$	$\bar{2}.9915$	$\bar{2}.9992$	$\bar{1}.0068$	$\bar{1}.0143$	$\bar{1}.0218$	84	60
6	$\bar{1}.0216$	$\bar{1}.0289$	$\bar{1}.0360$	$\bar{1}.0430$	$\bar{1}.0499$	$\bar{1}.0567$	$\bar{1}.0633$	$\bar{1}.0699$	$\bar{1}.0764$	$\bar{1}.0828$	$\bar{1}.0891$	83	68
7	0891	0954	1015	1076	1135	1194	1252	1310	1367	1423	1478	82	59
8	1478	1533	1587	1640	1693	1745	1797	1848	1898	1948	1997	81	52
9	1997	2046	2094	2142	2189	2236	2282	2328	2374	2419	2463	80	48
10	$\bar{1}.2643$	$\bar{1}.2507$	$\bar{1}.2551$	$\bar{1}.2594$	$\bar{1}.2637$	$\bar{1}.2680$	$\bar{1}.2722$	$\bar{1}.2764$	$\bar{1}.2805$	$\bar{1}.2846$	$\bar{1}.2887$	79	42
11	2887	2927	2967	3006	3046	3085	3123	3162	3200	3237	3275	78	39
12	3275	3312	3349	3385	3422	3458	3493	3529	3564	3599	3634	77	36
13	3634	3668	3702	3736	3770	3804	3837	3870	3903	3935	3968	76	33
14	3968	4000	4032	4064	4095	4127	4158	4189	4220	4250	4281	75	31
15	$\bar{1}.4281$	$\bar{1}.4311$	$\bar{1}.4341$	$\bar{1}.4371$	$\bar{1}.4400$	$\bar{1}.4430$	$\bar{1}.4459$	$\bar{1}.4488$	$\bar{1}.4517$	$\bar{1}.4546$	$\bar{1}.4575$	74	29
16	4575	4603	4632	4660	4688	4716	4744	4771	4799	4826	4853	73	28
17	4853	4880	4907	4934	4961	4987	5014	5040	5066	5092	5118	72	27
18	5118	5143	5169	5195	5220	5245	5270	5295	5320	5345	5370	71	25
19	5370	5394	5419	5443	5467	5491	5516	5539	5563	5587	5611	70	24
20	$\bar{1}.5611$	$\bar{1}.5634$	$\bar{1}.5658$	$\bar{1}.5681$	$\bar{1}.5704$	$\bar{1}.5727$	$\bar{1}.5750$	$\bar{1}.5773$	$\bar{1}.5796$	$\bar{1}.5819$	$\bar{1}.5842$	69	23
21	5842	5864	5887	5909	5932	5954	5976	5998	6020	6042	6064	68	22
22	6064	6086	6108	6129	6151	6172	6194	6215	6236	6257	6279	67	21
23	6279	6300	6321	6341	6362	6383	6404	6424	6445	6465	6486	66	21
24	6486	6506	6527	6547	6567	6587	6607	6627	6647	6667	6687	65	20
25	$\bar{1}.6687$	$\bar{1}.6706$	$\bar{1}.6726$	$\bar{1}.6746$	$\bar{1}.6765$	$\bar{1}.6785$	$\bar{1}.6804$	$\bar{1}.6824$	$\bar{1}.6843$	$\bar{1}.6863$	$\bar{1}.6882$	64	
26	6882	6901	6920	6939	6958	6977	6996	7015	7034	7053	7072	63	19
27	7072	7090	7109	7128	7146	7165	7183	7202	7220	7238	7257	62	
28	7257	7275	7293	7311	7330	7348	7366	7384	7402	7420	7438	61	18
29	7438	7455	7473	7491	7509	7526	7544	7562	7579	7597	7614	60	
30	$\bar{1}.7614$	$\bar{1}.7632$	$\bar{1}.7649$	$\bar{1}.7667$	$\bar{1}.7684$	$\bar{1}.7701$	$\bar{1}.7719$	$\bar{1}.7736$	$\bar{1}.7753$	$\bar{1}.7771$	$\bar{1}.7788$	59	
31	7788	7805	7822	7839	7856	7873	7890	7907	7924	7941	7958	58	17
32	7958	7975	7992	8008	8025	8042	8059	8075	8092	8109	8125	57	
33	8125	8142	8158	8175	8191	8208	8224	8241	8257	8274	8290	56	
34	8290	8306	8323	8339	8355	8371	8388	8404	8420	8436	8452	55	
35	$\bar{1}.8452$	$\bar{1}.8468$	$\bar{1}.8484$	$\bar{1}.8501$	$\bar{1}.8517$	$\bar{1}.8533$	$\bar{1}.8549$	$\bar{1}.8565$	$\bar{1}.8581$	$\bar{1}.8597$	$\bar{1}.8613$	54	16
36	8613	8629	8644	8660	8676	8692	8708	8724	8740	8755	8771	53	
37	8771	8787	8803	8818	8834	8850	8865	8881	8897	8912	8928	52	
38	8928	8944	8959	8975	8990	9006	9022	9037	9053	9068	9084	51	
39	9084	9099	9115	9130	9146	9161	9176	9192	9207	9223	9238	50	
40	$\bar{1}.9238$	$\bar{1}.9254$	$\bar{1}.9269$	$\bar{1}.9284$	$\bar{1}.9300$	$\bar{9}.9315$	$\bar{1}.9330$	$\bar{1}.9346$	$\bar{1}.9361$	$\bar{1}.9376$	$\bar{1}.9392$	49	
41	9392	9407	9422	9438	9453	9468	9483	9499	9514	9529	9544	48	
42	9544	9560	9575	9590	9605	9621	9636	9651	9666	9681	9697	47	
43	9697	9712	9727	9742	9757	9772	9788	9803	9818	9833	9848	46	
44°	9848	9864	9879	9894	9909	9924	9939	9955	9970	9985	0000	45°	15
Complement		.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	Angle	

Logarithmic Cotangents.

Angle	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Complement	Dif.
45°	0.0000	0.0015	0.0030	0.0045	0.0061	0.0076	0.0091	0.0106	0.0121	0.0136	0.0152	44° 15
46	0152	0167	0182	0197	0212	0228	0243	0258	0273	0288	0303	43
47	0303	0319	0334	0349	0364	0379	0395	0410	0425	0440	0456	42
48	0456	0471	0486	0501	0517	0532	0547	0562	0578	0593	0608	41
49	0608	0624	0639	0654	0670	0685	0700	0716	0731	0746	0762	40
50	0.0762	0.0777	0.0793	0.0808	0.0824	0.0839	0.0854	0.0870	0.0885	0.0901	0.0916	39
51	0916	0932	0947	0963	0978	0994	1010	1025	1041	1056	1072	38
52	1072	1088	1103	1119	1135	1150	1166	1182	1197	1213	1229	37
53	1229	1245	1260	1276	1292	1308	1324	1340	1356	1371	1387	36
54	1387	1403	1419	1435	1451	1467	1483	1499	1516	1532	1548	35 16
55	0.1548	0.1564	0.1580	0.1596	0.1612	0.1629	0.1645	0.1661	0.1677	0.1694	0.1710	34
56	1710	1726	1743	1759	1776	1792	1809	1825	1842	1858	1875	33
57	1875	1891	1908	1925	1941	1958	1975	1992	2008	2025	2042	32
58	2042	2059	2076	2093	2110	2127	2144	2161	2178	2195	2212	31 17
59	2212	2229	2247	2264	2281	2299	2316	2333	2351	2368	2386	30
60	0.2386	0.2403	0.2421	0.2438	0.2456	0.2474	0.2491	0.2509	0.2527	0.2545	0.2562	29
61	2562	2580	2598	2616	2634	2652	2670	2689	2707	2725	2743	28 18
62	2743	2762	2780	2798	2817	2835	2854	2872	2891	2910	2928	27
63	2928	2947	2966	2985	3004	3023	3042	3061	3080	3099	3118	26 19
64	3118	3137	3157	3176	3196	3215	3235	3254	3274	3294	3313	25
65	0.3313	0.3333	0.3353	0.3373	0.3393	0.3413	0.3433	0.3453	0.3473	0.3494	0.3514	24 20
66	3514	3535	3555	3576	3596	3617	3638	3659	3679	3700	3721	23 21
67	3721	3743	3764	3785	3806	3828	3849	3871	3892	3914	3936	22 22
68	3936	3958	3980	4002	4024	4046	4068	4091	4113	4136	4158	21 22
69	4158	4181	4204	4227	4250	4273	4296	4319	4342	4366	4389	20 23
70	0.4389	0.4413	0.4437	0.4461	0.4484	0.4509	0.4533	0.4557	0.4581	0.4606	0.4630	19 24
71	4630	4655	4680	4705	4730	4755	4780	4805	4831	4857	4882	18 25
72	4882	4908	4934	4960	4986	5013	5039	5066	5093	5120	5147	17 27
73	5147	5174	5201	5229	5256	5284	5312	5340	5368	5397	5425	16 28
74	5425	5454	5483	5512	5541	5570	5600	5629	5659	5689	5719	15 29
75	0.5719	0.5750	0.5780	0.5811	0.5842	0.5873	0.5905	0.5936	0.5968	0.6000	0.6032	14 31
76	6032	6065	6097	6130	6163	6196	6230	6264	6298	6332	6366	13 33
77	6366	6401	6436	6471	6507	6542	6578	6615	6651	6688	6725	12 36
78	6725	6763	6800	6838	6877	6915	6954	6994	7033	7073	7113	11 39
79	7113	7154	7195	7236	7278	7320	7363	7408	7449	7493	7537	10 49
80	0.7537	0.7581	0.7626	0.7672	0.7718	0.7764	0.7811	0.7858	0.7906	0.7954	0.8003	9 47
81	8003	8052	8102	8152	8203	8255	8307	8360	8413	8467	8522	8 52
82	8522	8577	8633	8690	8748	8806	8865	8924	8985	9046	9109	7 59
83	9109	9172	9238	9301	9367	9433	9501	9570	9640	9711	9784	6 68
84	9784	9857	9932	1.0008	1.0085	1.0164	1.0244	1.0326	1.0409	1.0494	1.0580	5 60
85	1.0580	1.0669	1.0759	1.0850	1.0944	1.1040	1.1138	1.1238	1.1341	1.1446	1.1554	4 —
86	1554	1664	1777	1893	2012	2135	2261	2391	2525	2663	2806	3 —
87	2806	2954	3108	3264	3429	3599	3777	3962	4155	4357	4569	2 —
88	4569	4792	5027	5275	5539	5819	6119	6441	6789	7167	7581	1 —
89°	7581	8038	8550	9130	9800	2.0591	2.1561	2.2810	2.4571	2.7581	∞	0° —
Complement	.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	Angle	

Logarithmic Cotangents.

Logs.	0	1	2	3	4	5	6	7	8	9	Dif.
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	5
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	6
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	7
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	

Logs.	0	1	2	3	4	5	6	7	8	9	Dif.
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	8
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	9
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	10
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	11
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	12
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	13
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	14
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	15
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	16
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	17
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	18
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	19
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	20
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	21
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	21
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	22
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	22
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	23

Nos.	0	1	2	3	4	5	6	7	8	9	Dif.
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	42
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	38
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	35
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	32
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	30
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	28
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	26
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	25
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	23
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	22
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	21
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	20
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	19
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	18
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	18
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	17
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	16
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	16
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	15
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	15
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	14
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	13
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	12
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	11
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	10
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	9
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	8

Nos.	0	1	2	3	4	5	6	7	8	9	Dif.
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	7
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	6
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	5
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	

No.	0	1	2	3	4	5	6	7	8	9	Dif.
100	00000	00043	00087	00130	00173	00217	00260	00303	00346	00389	43: 1 4
101	00432	00475	00518	00561	00604	00647	00689	00732	00775	00817	2 9, 3 13
102	00860	00903	00945	00988	01030	01072	01115	01157	01199	01242	4 17, 5 22
103	01284	01326	01368	01410	01452	01494	01536	01578	01620	01662	6 26, 7 30
104	01703	01745	01787	01828	01870	01912	01953	01995	02036	02078	8 34, 9 39
105	02119	02160	02202	02243	02284	02325	02366	02407	02449	02490	41: 1 4
106	02531	02572	02612	02653	02694	02735	02776	02816	02857	02898	2 8, 3 12
107	02938	02979	03019	03060	03100	03141	03181	03222	03262	03302	4 16, 5 21
108	03342	03383	03423	03463	03503	03543	03583	03623	03663	03703	6 25, 7 29
109	03743	03782	03822	03862	03902	03941	03981	04021	04060	04100	8 33, 9 37
110	04139	04179	04218	04258	04297	04336	04376	04415	04454	04493	39: 1 4
111	04532	04571	04610	04650	04689	04727	04766	04805	04844	04883	2 8, 3 12
112	04922	04961	04999	05038	05077	05115	05154	05192	05231	05269	4 16, 5 20
113	05308	05346	05385	05423	05461	05500	05538	05576	05614	05652	6 23, 7 27
114	05690	05729	05767	05805	05843	05881	05918	05956	05994	06032	8 31, 9 35
115	06070	06108	06145	06183	06221	06258	06296	06333	06371	06408	37: 1 4
116	06446	06483	06521	06558	06595	06633	06670	06707	06744	06781	2 7, 3 11
117	06819	06856	06893	06930	06967	07004	07041	07078	07115	07151	4 15, 5 19
118	07188	07225	07262	07298	07335	07372	07408	07445	07482	07518	6 22, 7 26
119	07555	07591	07628	07664	07700	07737	07773	07809	07846	07882	8 30, 9 33
120	07918	07954	07990	08027	08063	08099	08135	08171	08207	08243	35: 1 4
121	08279	08314	08350	08386	08422	08458	08493	08529	08565	08600	2 7, 3 11
122	08636	08672	08707	08743	08778	08814	08849	08884	08920	08955	4 14, 5 18
123	08991	09026	09061	09096	09132	09167	09202	09237	09272	09307	6 21, 7 25
124	09342	09377	09412	09447	09482	09517	09552	09587	09621	09656	8 28, 9 32
125	09691	09726	09760	09795	09830	09864	09899	09934	09968	10003	34: 1 3
126	10037	10072	10106	10140	10175	10209	10243	10278	10312	10346	2 7, 3 10
127	10380	10415	10449	10483	10517	10551	10585	10619	10653	10687	4 14, 5 17
128	10721	10755	10789	10823	10857	10890	10924	10958	10992	11025	6 20, 7 24
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151	17898	17926	17955	17984	18013	18041	18070	18099	18127	18156	
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153	18469	18498	18526	18554	18583	18611	18639	18667	18696	18724	
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219	34044	34064	34084	34104	34124	34143	34163	34183	34203	34223	9 18
220	34242	34262	34282	34301	34321	34341	34361	34380	34400	34420	19
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238	37658	37676	37694	37712	37731	37749	37767	37785	37803	37822	
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241	38202	38220	38238	38256	38274	38292	38310	38328	38346	38364	1 2
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265	42325	42341	42357	42374	42390	42406	42423	42439	42455	42472	
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289	46090	46105	46120	46135	46150	46165	46180	46195	46210	46225	
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349	54283	54295	54307	54320	54332	54345	54357	54370	54382	54394	
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567	75358	75366	75374	75381	75389	75397	75404	75412	75420	75427	
568	75435	75442	75450	75458	75465	75473	75481	75488	75496	75504	
569	75511	75519	75526	75534	75542	75549	75557	75565	75572	75580	
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572	75740	75747	75755	75762	75770	75778	75785	75793	75800	75808	
573	75815	75823	75831	75838	75846	75853	75861	75868	75876	75884	
574	75891	75899	75906	75914	75921	75929	75937	75944	75952	75959	
575	75967	75974	75982	75989	75997	76005	76012	76020	76027	76035	
576	76042	76050	76057	76065	76072	76080	76087	76095	76103	76110	
577	76118	76125	76133	76140	76148	76155	76163	76170	76178	76185	
578	76193	76200	76208	76215	76223	76230	76238	76245	76253	76260	
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580	76343	76350	76358	76365	76373	76380	76388	76395	76403	76410	
581	76418	76425	76433	76440	76448	76455	76462	76470	76477	76485	
582	76492	76500	76507	76515	76522	76530	76537	76545	76552	76559	
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586	76790	76797	76805	76812	76819	76827	76834	76842	76849	76856	
587	76864	76871	76879	76886	76893	76901	76908	76916	76923	76930	
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589	77012	77019	77026	77034	77041	77048	77056	77063	77070	77078	
590	77085	77093	77100	77107	77115	77122	77129	77137	77144	77151	7
591	77159	77166	77173	77181	77188	77195	77203	77210	77217	77225	1 1
592	77232	77240	77247	77254	77262	77269	77276	77283	77291	77298	2 1
593	77305	77313	77320	77327	77335	77342	77349	77357	77364	77371	3 2
594	77379	77386	77393	77401	77408	77415	77422	77430	77437	77444	4 3
595	77452	77459	77466	77474	77481	77488	77495	77503	77510	77517	5 4
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599	77743	77750	77757	77764	77772	77779	77786	77793	77801	77808	9 6
600	77815	77822	77830	77837	77844	77851	77859	77866	77873	77880	

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600	77815	77822	77830	77837	77844	77851	77859	77866	77873	77880	
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604	78104	78111	78118	78125	78132	78140	78147	78154	78161	78168	
605	78176	78183	78190	78197	78204	78211	78219	78226	78233	78240	
606	78247	78254	78262	78269	78276	78283	78290	78297	78305	78312	
607	78319	78326	78333	78340	78347	78355	78362	78369	78376	78383	
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611	78604	78611	78618	78625	78633	78640	78647	78654	78661	78668	
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616	78958	78965	78972	78979	78986	78993	79000	79007	79014	79021	
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618	79099	79106	79113	79120	79127	79134	79141	79148	79155	79162	
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620	79239	79246	79253	79260	79267	79274	79281	79288	79295	79302	7
621	79309	79316	79323	79330	79337	79344	79351	79358	79365	79372	1 1
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623	79449	79456	79463	79470	79477	79484	79491	79498	79505	79511	3 2
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625	79588	79595	79602	79609	79616	79623	79630	79637	79644	79650	5 4
626	79657	79964	79671	79678	79685	79692	79699	79706	79713	79720	6 4
627	79727	79734	79741	79748	79754	79761	79768	79775	79782	79789	7 5
628	79796	79803	79810	79817	79824	79831	79837	79844	79851	79858	8 6
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635	80277	80284	80291	80298	80305	80312	80318	80325	80332	80339	
636	80346	80353	80359	80366	80373	80380	80387	80393	80400	80407	
637	80414	80421	80428	80434	80441	80448	80455	80462	80468	80475	
638	80482	80489	80496	80502	80509	80516	80523	80530	80536	80543	
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640	80618	80625	80632	80638	80645	80652	80659	80665	80672	80679	
641	80686	80693	80699	80706	80713	80720	80726	80733	80740	80747	
642	80754	80760	80767	80774	80781	80787	80794	80801	80808	80814	
643	80821	80828	80835	80841	80848	80855	80862	80868	80875	80882	
644	80889	80895	80902	80909	80916	80922	80929	80936	80943	80949	
645	80956	80963	80969	80976	80983	80990	80996	81003	81010	81017	
646	81023	81030	81037	81043	81050	81057	81064	81070	81077	81084	
647	81090	81097	81104	81111	81117	81124	81131	81137	81144	81151	
648	81158	81164	81171	81178	81184	81191	81198	81204	81211	81218	
649	81224	81231	81238	81245	81251	81258	81265	81271	81278	81285	
650	81291	81298	81305	81311	81318	81325	81331	81338	81345	81351	

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650	81291	81298	81305	81311	81318	81325	81331	81338	81345	81351	7
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654	81558	81564	81571	81578	81584	81591	81598	81604	81611	81617	4 3
655	81624	81631	81637	81644	81651	81657	81664	81671	81677	81684	5 4
656	81690	81697	81704	81710	81717	81723	81730	81737	81743	81750	6 4
657	81757	81763	81770	81776	81783	81790	81796	81803	81809	81816	7 5
658	81823	81829	81836	81842	81849	81856	81862	81869	81875	81882	8 6
659	81889	81895	81902	81908	81915	81921	81928	81935	81941	81948	9 6
660	81954	81961	81968	81974	81981	81987	81994	82000	82007	82014	
661	82020	82027	82033	82040	82046	82053	82060	82066	82073	82079	
662	82086	82092	82099	82105	82112	82119	82125	82132	82138	82145	
663	82151	82158	82164	82171	82178	82184	82191	82197	82204	82210	
664	82217	82223	82230	82236	82243	82249	82256	82263	82269	82276	
665	82282	82289	82295	82302	82308	82315	82321	82328	82334	82341	
666	82347	82354	82360	82367	82373	82380	82387	82393	82400	82406	
667	82413	82419	82426	82432	82439	82445	82452	82458	82465	82471	
668	82478	82484	82491	82497	82504	82510	82517	82523	82530	82536	
669	82543	82549	82556	82562	82569	82575	82582	82588	82595	82601	
670	82607	82614	82620	82627	82633	82640	82646	82653	82659	82666	
671	82672	82679	82685	82692	82698	82705	82711	82718	82724	82730	
672	82737	82743	82750	82756	82763	82769	82776	82782	82789	82795	
673	82802	82808	82814	82821	82827	82834	82840	82847	82853	82860	
674	82866	82872	82879	82885	82892	82898	82905	82911	82918	82924	
675	82930	82937	82943	82950	82956	82963	82969	82975	82982	82988	
676	82995	83001	83008	83014	83020	83027	83033	83040	83046	83052	
677	83059	83065	83072	83078	83085	83091	83097	83104	83110	83117	
678	83123	83129	83136	83142	83149	83155	83161	83168	83174	83181	
679	83187	83193	83200	83206	83213	83219	83225	83232	83238	83245	
680	83251	83257	83264	83270	83276	83283	83289	83296	83302	83308	6
681	83315	83321	83327	83334	83340	83347	83353	83359	83366	83372	1 1
682	83378	83385	83391	83398	83404	83410	83417	83423	83429	83436	2 1
683	83442	83448	83455	83461	83467	83474	83480	83487	83493	83499	3 2
684	83506	83512	83518	83525	83531	83537	83544	83550	83556	83563	4 2
685	83569	83575	83582	83588	83594	83601	83607	83613	83620	83626	5 3
686	83632	83639	83645	83651	83658	83664	83670	83677	83683	83689	6 4
687	83696	83702	83708	83715	83721	83727	83734	83740	83746	83753	7 4
688	83759	83765	83771	83778	83784	83790	83797	83803	83809	83816	8 5
689	83822	83828	83835	83841	83847	83853	83860	83866	83872	83879	9 5
690	83885	83891	83897	83904	83910	83916	83923	83929	83935	83942	
691	83948	83954	83960	83967	83973	83979	83985	83992	83998	84004	
692	84011	84017	84023	84029	84036	84042	84048	84055	84061	84067	
693	84073	84080	84086	84092	84098	84105	84111	84117	84123	84130	
694	84136	84142	84148	84155	84161	84167	84173	84180	84186	84192	
695	84198	84205	84211	84217	84223	84230	84236	84242	84248	84255	
696	84261	84267	84273	84280	84286	84292	84298	84305	84311	84317	
697	84323	84330	84336	84342	84348	84354	84361	84367	84373	84379	
698	84386	84392	84398	84404	84410	84417	84423	84429	84435	84442	
699	84448	84454	84460	84466	84473	84479	84485	84491	84497	84504	
700	84510	84516	84522	84528	84535	84541	84547	84553	84559	84566	

No.	0	1	2	3	4	5	6	7	8	9	Diff.
700	84510	84516	84522	84528	84535	84541	84547	84553	84559	84566	
701	84572	84578	84584	84590	84597	84603	84609	84615	84621	84628	
702	84634	84640	84646	84652	84658	84665	84671	84677	84683	84689	
703	84696	84702	84708	84714	84720	84726	84733	84739	84745	84751	
704	84757	84763	84770	84776	84782	84788	84794	84800	84807	84813	
705	84819	84825	84831	84837	84844	84850	84856	84862	84868	84874	
706	84880	84887	84893	84899	84905	84911	84917	84924	84930	84936	
707	84942	84948	84954	84960	84967	84973	84979	84985	84991	84997	
708	85003	85009	85016	85022	85028	85034	85040	85046	85052	85058	
709	85065	85071	85077	85083	85089	85095	85101	85107	85114	85120	
710	85126	85132	85138	85144	85150	85156	85163	85169	85175	85181	
711	85187	85193	85199	85205	85211	85217	85224	85230	85236	85242	
712	85248	85254	85260	85266	85272	85278	85285	85291	85297	85303	
713	85309	85315	85321	85327	85333	85339	85345	85352	85358	85364	
714	85370	85376	85382	85388	85394	85400	85406	85412	85418	85425	
715	85431	85437	85443	85449	85455	85461	85467	85473	85479	85485	
717	85491	85497	85503	85509	85516	85522	85528	85534	85540	85546	
716	85552	85558	85564	85570	85576	85582	85588	85594	85600	85606	
718	85612	85618	85625	85631	85637	85643	85649	85655	85661	85667	
719	85673	85679	85685	85691	85697	85703	85709	85715	85721	85727	
720	85733	85739	85745	85751	85757	85763	85769	85775	85781	85788	6
721	85794	85800	85806	85812	85818	85824	85830	85836	85842	85848	1 1
722	85854	85860	85866	85872	85878	85884	85890	85896	85902	85908	2 1
723	85914	85920	85926	85932	85938	85944	85950	85956	85962	85968	3 2
724	85974	85980	85986	85992	85998	86004	86010	86016	86022	86028	4 2
725	86034	86040	86046	86052	86058	86064	86070	86076	86082	86088	5 3
726	86094	86100	86106	86112	86118	86124	86130	86136	86141	86147	6 4
727	86153	86159	86165	86171	86177	86183	86189	86195	86201	86207	7 4
728	86213	86219	86225	86231	86237	86243	86249	86255	86261	86267	8 5
729	86273	86279	86285	86291	86297	86303	86308	86314	86320	86326	3 5
730	86332	86338	86344	86350	86356	86362	86368	86374	86380	86386	
731	86392	86398	86404	86410	86415	86421	86427	86433	86439	86445	
732	86451	86457	86463	86469	86475	86481	86487	86493	86499	86504	
733	86510	86516	86522	86528	86534	86540	86546	86552	86558	86564	
734	86570	86576	86581	86587	86593	86599	86605	86611	86617	86623	
735	86629	86635	86641	86646	86652	86658	86664	86670	86676	86682	
736	86688	86694	86700	86705	86711	86717	86723	86729	86735	86741	
737	86747	86753	86759	86764	86770	86776	86782	86788	86794	86800	
738	86806	86812	86817	86823	86829	86835	86841	86847	86853	86859	
739	86864	86870	86876	86882	86888	86894	86900	86906	86911	86917	
740	86923	86929	86935	86941	86947	86953	86958	86964	86970	86976	
741	86982	86988	86994	86999	87005	87011	87017	87023	87029	87035	
742	87040	87046	87052	87058	87064	87070	87075	87081	87087	87093	
743	87099	87105	87111	87116	87122	87128	87134	87140	87146	87151	
744	87157	87163	87169	87175	87181	87186	87192	87198	87204	87210	
745	87216	87221	87227	87233	87239	87245	87251	87256	87262	87268	
746	87274	87280	87286	87291	87297	87303	87309	87315	87320	87326	
747	87332	87338	87344	87349	87355	87361	87367	87373	87379	87384	
748	87390	87396	87402	87408	87413	87419	87425	87431	87437	87442	
749	87448	87454	87460	87466	87471	87477	87483	87489	87495	87500	
750	87506	87512	87518	87523	87529	87535	87541	87547	87552	87558	

No.	0	1	2	3	4	5	6	7	8	9	Dif.
750	87506	87512	87518	87523	87529	87535	87541	87547	87552	87558	
751	87564	87570	87576	87581	87587	87593	87599	87604	87610	87616	
752	87622	87628	87633	87639	87645	87651	87656	87662	87668	87674	
753	87679	87685	87691	87697	87703	87708	87714	87720	87726	87731	
754	87737	87743	87749	87754	87760	87766	87772	87777	87783	87789	
755	87795	87800	87806	87812	87818	87823	87829	87835	87841	87846	
756	87852	87858	87864	87869	87875	87881	87887	87892	87898	87904	
757	87910	87915	87921	87927	87933	87938	87944	87950	87955	87961	
758	87967	87973	87978	87984	87990	87996	88001	88007	88013	88018	
759	88024	88030	88036	88041	88047	88053	88058	88064	88070	88076	
760	88981	88087	88093	88098	88104	88110	88116	88121	88127	88133	
761	88138	88144	88150	88156	88161	88167	88173	88178	88184	88190	
762	88195	88201	88207	88213	88218	88224	88230	88235	88241	88247	
763	88252	88258	88264	88270	88275	88281	88287	88292	88298	88304	
764	88309	88315	88321	88326	88332	88338	88343	88349	88355	88360	
765	88366	88372	88377	88383	88389	88395	88400	88406	88412	88417	
766	88423	88429	88434	88440	88446	88451	88457	88463	88468	88474	
767	88480	88485	88491	88497	88502	88508	88513	88519	88525	88530	
768	88536	88542	88547	88553	88559	88564	88570	88576	88581	88587	
769	88593	88598	88604	88610	88615	88621	88627	88632	88638	88643	
770	88649	88655	88660	88666	88672	88677	88683	88689	88694	88700	6
771	88705	88711	88717	88722	88728	88734	88739	88745	88750	88756	1 1
772	88762	88767	88773	88779	88784	88790	88795	88801	88807	88812	2 1
773	88818	88824	88829	88835	88840	88846	88852	88857	88863	88868	3 2
774	88874	88880	88885	88891	88897	88902	88908	88913	88919	88925	4 2
775	88930	88936	88941	88947	88953	88958	88964	88969	88975	88981	5 3
776	88986	88992	88997	89003	89009	89014	89020	89025	89031	89037	6 4
777	89042	89048	89053	89059	89064	89070	89076	89081	89087	89092	7 4
778	89098	89104	89109	89115	89120	89126	89131	89137	89143	89148	8 5
779	89154	89159	89165	89170	89176	89182	89187	89193	89198	89204	9 5
780	89209	89215	89221	89226	89232	89237	89243	89248	89254	89260	
781	89265	89271	89276	89282	89287	89293	89298	89304	89310	89315	
782	89321	89326	89332	89337	89343	89348	89354	89360	89365	89371	
783	89376	89382	89387	89393	89398	89404	89409	89415	89421	89426	
784	89432	89437	89443	89448	89454	89459	89465	89470	89476	89481	
785	89487	89492	89498	89504	89509	89515	89520	89526	89531	89537	
786	89542	89548	89553	89559	89564	89570	89575	89581	89586	89592	
787	89597	89603	89609	89614	89620	89625	89631	89636	89642	89647	
788	89653	89658	89664	89669	89675	89680	89686	89691	89697	89702	
789	89708	89713	89719	89724	89730	89735	89741	89746	89752	89757	
790	89763	89768	89774	89779	89785	89790	89796	89801	89807	89812	
791	89818	89823	89829	89834	89840	89845	89851	89856	89862	89867	
792	89873	89878	89883	89889	89894	89900	89905	89911	89916	89922	
793	89927	89933	89938	89944	89949	89955	89960	89966	89971	89977	
794	89982	89988	89993	89998	90004	90009	90015	90020	90026	90031	
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797	90146	90151	90157	90162	90168	90173	90179	90184	90189	90195	
798	90200	90206	90211	90217	90222	90227	90233	90238	90244	90249	
799	90255	90260	90266	90271	90276	90282	90287	90293	90298	90304	
800	90309	90314	90320	90325	90331	90336	90342	90347	90352	90358	

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800	90309	90314	90320	90325	90331	90336	90342	90347	90352	90358	
801	90363	90369	90374	90380	90385	90390	90396	90401	90407	90412	
802	90417	90423	90428	90434	90439	90445	90450	90455	90461	90466	
803	90472	90477	90482	90488	90493	90499	90504	90509	90515	90520	
804	90526	90531	90536	90542	90547	90553	90558	90563	90569	90574	
805	90580	90585	90590	90596	90601	90607	90612	90617	90623	90628	
806	90634	90639	90644	90650	90655	90660	90666	90671	90677	90682	
807	90687	90693	90698	90703	90709	90714	90720	90725	90730	90736	
808	90741	90747	90752	90757	90763	90768	90773	90779	90784	90789	
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810	90849	90854	90859	90865	90870	90875	90881	90886	90891	90897	
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813	91009	91014	91020	91025	91030	91036	91041	91046	91052	91057	
814	91062	91068	91073	91078	91084	91089	91094	91100	91105	91110	
815	91116	91121	91126	91132	91137	91142	91148	91153	91158	91164	
816	91169	91174	91180	91185	91190	91196	91201	91206	91212	91217	
817	91222	91228	91233	91238	91243	91249	91254	91259	91265	91270	
818	91275	91281	91286	91291	91297	91302	91307	91312	91318	91323	
819	91328	91334	91339	91344	91350	91355	91360	91365	91371	91376	
820	91381	91387	91392	91397	91403	91408	91413	91418	91424	91429	5
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825	91645	91651	91656	91661	91666	91672	91677	91682	91687	91693	5 3
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832	92012	92018	92023	92028	92033	92038	92044	92049	92054	92059	
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835	92169	92174	92179	92184	92189	92195	92200	92205	92210	92215	
836	92221	92226	92231	92236	92241	92247	92252	92257	92262	92267	
837	92273	92278	92283	92288	92293	92298	92304	92309	92314	92319	
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842	92531	92536	92542	92547	92552	92557	92562	92567	92572	92578	
843	92583	92588	92593	92598	92603	92609	92614	92619	92624	92629	
844	92634	92639	92645	92650	92655	92660	92665	92670	92675	92681	
845	92686	92691	92696	92701	92706	92711	92716	92722	92727	92732	
846	92737	92742	92747	92752	92758	92763	92768	92773	92778	92783	
847	92788	92793	92799	92804	92809	92814	92819	92824	92829	92834	
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849	92891	92896	92901	92906	92911	92916	92921	92927	92932	92937	
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853	93095	93100	93105	93110	93115	93120	93125	93131	93136	93141	
854	93146	93151	93156	93161	93166	93171	93176	93181	93186	93192	
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858	93349	93354	93359	93364	93369	93374	93379	93384	93389	93394	
859	93399	93404	93409	93414	93420	93425	93430	93435	93440	93445	
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863	93301	93606	93611	93616	93621	93626	93631	93636	93641	93646	
864	93651	93656	93661	93666	93671	93676	93682	93687	93692	93697	
865	93702	93707	93712	93717	93722	93727	93732	93737	93742	93747	
866	93752	93757	93762	93767	93772	93777	93782	93787	93792	93797	
867	93802	93807	93812	93817	93822	93827	93832	93837	93842	93847	
868	93852	93857	93862	93867	93872	93877	93882	93887	93892	93897	
869	93902	93907	93912	93917	93922	93927	93932	93937	93942	93947	
870	93952	93957	93962	93967	93972	93977	93982	93987	93992	93997	5
871	94002	94007	94012	94017	94022	94027	94032	94037	94042	94047	1 1
872	94052	94057	94062	94067	94072	94077	94082	94086	94091	94096	2 1
873	94101	94106	94111	94116	94121	94126	94131	94136	94141	94146	3 2
874	94151	94156	94161	94166	94171	94176	94181	94186	94191	94196	4 2
875	94201	94206	94211	94216	94221	94226	94231	94236	94240	94245	5 3
876	94250	94255	94260	94265	94270	94275	94280	94285	94290	94295	6 3
877	94300	94305	94310	94315	94320	94325	94330	94335	94340	94345	7 4
878	94349	94354	94359	94364	94369	94374	94379	94384	94389	94394	8 4
879	94399	94404	94409	94414	94419	94424	94429	94433	94438	94443	9 5
880	94448	94453	94458	94463	94468	94473	94478	94483	94488	94493	
881	94498	94503	94507	94512	94517	94522	94527	94532	94537	94542	
882	94547	94552	94557	94562	94567	94571	94576	94581	94586	94591	
883	94596	94601	94606	94611	94616	94621	94626	94630	94635	94640	
884	94645	94650	94655	94660	94665	94670	94675	94680	94685	94689	
885	94694	94699	94704	94709	94714	94719	94724	94729	94734	94738	
886	94743	94748	94753	94758	94763	94768	94773	94778	94783	94787	
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889	94890	94895	94900	94905	94910	94915	94919	94924	94929	94934	
890	94939	94944	94949	94954	94959	94963	94968	94973	94978	94983	
891	94988	94993	94998	95002	95007	95012	95017	95022	95027	95032	
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893	95085	95090	95095	95100	95105	95109	95114	95119	95124	95129	
894	95134	95139	95143	95148	95153	95158	95163	95168	95173	95177	
895	95182	95187	95192	95197	95202	95207	95211	95216	95221	95226	
896	95231	95236	95240	95245	95250	95255	95260	95265	95270	95274	
897	95279	95284	95289	95294	95299	95303	95308	95313	95318	95323	
898	95328	95332	95337	95342	95347	95352	95357	95361	95366	95371	
899	95376	95381	95386	95390	95395	95400	95405	95410	95415	95419	
900	95424	95429	95434	95439	95444	95448	95453	95458	95463	95468	

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900	95424	95429	95434	95439	95444	95448	95453	95458	95463	95468	
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902	95521	95525	95530	95535	95540	95545	95550	95554	95559	95564	
903	95569	95574	95578	95583	95588	95593	95598	95602	95607	95612	
904	95617	95622	95626	95631	95636	95641	95646	95650	95655	95660	
905	95665	95670	95674	95679	95684	95689	95694	95698	95703	95708	
906	95713	95718	95722	95727	95732	95737	95742	95746	95751	95756	
907	95761	95766	95770	95775	95780	95785	95789	95794	95799	95804	
908	95809	95813	95818	95823	95828	95832	95837	95842	95847	95852	
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910	95904	95909	95914	95918	95923	95928	95933	95938	95942	95947	
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912	95999	96004	96009	96014	96019	96023	96028	96033	96038	96042	
913	96047	96052	96057	96061	96066	96071	96076	96080	96085	96090	
914	96095	96099	96104	96109	96114	96118	96123	96128	96133	96137	
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916	96190	96194	96199	96204	96209	96213	96218	96223	96227	96232	
917	96237	96242	96246	96251	96256	96261	96265	96270	96275	96280	
918	96284	96289	96294	96298	96303	96308	96313	96317	96322	96327	
919	96332	96336	96341	96346	96350	96355	96360	96365	96369	96374	
920	96379	96384	96388	96393	96398	96402	96407	96412	96417	96421	5
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922	96473	96478	96483	96487	96492	96497	96501	96506	96511	96515	2 1
923	96520	96525	96530	96534	96539	96544	96548	96553	96558	96562	3 2
924	96567	96572	96577	96581	96586	96591	96595	96600	96605	96609	4 2
925	96614	96619	96624	96628	96633	96638	96642	96647	96652	96656	5 3
926	96661	96666	96670	96675	96680	96685	96689	96694	96699	96703	6 3
927	96708	96713	96717	96722	96727	96731	96736	96741	96745	96750	7 4
928	96755	96759	96764	96769	96774	96778	96783	96788	96792	96797	8 4
929	96802	96806	96811	96816	96820	96825	96830	96834	96839	96844	9 5
930	96848	96853	96858	96862	96867	96872	96876	96881	96886	96890	
931	96895	96900	96904	96909	96914	96918	96923	96928	96932	96937	
932	96942	96946	96951	96956	96960	96965	96970	96974	96979	96984	
933	96988	96993	96997	97002	97007	97011	97016	97021	97025	97030	
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937	97174	97179	97183	97188	97192	97197	97202	97206	97211	97216	
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939	97267	97271	97276	97280	97285	97290	97294	97299	97304	97308	
940	97313	97317	97322	97327	97331	97336	97340	97345	97350	97354	
941	97359	97364	97368	97373	97377	97382	97387	97391	97396	97400	
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946	97589	97594	97598	97603	97607	97612	97617	97621	97626	97630	
947	97635	97640	97644	97649	97653	97658	97663	97667	97672	97676	
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949	97727	97731	97736	97740	97745	97749	97754	97759	97763	97768	
950	97772	97777	97782	97786	97791	97795	97800	97804	97809	97813	

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953	97909	97914	97918	97923	97928	97932	97937	97941	97946	97950	3 2
954	97955	97959	97964	97968	97973	97978	97982	97987	97991	97996	4 2
955	98000	98005	98009	98014	98019	98023	98028	98032	98037	98041	5 3
956	98046	98050	98055	98059	98064	98068	98073	98078	98082	98087	6 3
957	98091	98096	98100	98105	98109	98114	98118	98123	98127	98132	7 4
958	98137	98141	98146	98150	98155	98159	98164	98168	98173	98177	8 4
959	98182	98186	98191	98195	98200	98204	98209	98214	98218	98223	9 5
960	98227	98232	98236	98241	98245	98250	98254	98259	98263	98268	
961	98272	98277	98281	98286	98290	98295	98299	98304	98308	98313	
962	98318	98322	98327	98331	98336	98340	98345	98349	98354	98358	
963	98363	98367	98372	98376	98381	98385	98390	98394	98399	98403	
964	98408	98412	98417	98421	98426	98430	98435	98439	98444	98448	
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966	98498	98502	98507	98511	98516	98520	98525	98529	98534	98538	
967	98543	98547	98552	98556	98561	98565	98570	98574	98579	98583	
968	98588	98592	98597	98601	98605	98610	98614	98619	98623	98628	
969	98632	98637	98641	98646	98650	98655	98659	98664	98668	98673	
970	98677	98682	98686	98691	98695	98700	98704	98709	98713	98717	
971	98722	98726	98731	98735	98740	98744	98749	98753	98758	98762	
972	98767	98771	98776	98780	98784	98789	98793	98798	98802	98807	
973	98811	98816	98820	98825	98829	98834	98838	98843	98847	98851	
974	98856	98860	98865	98869	98874	98878	98883	98887	98892	98896	
975	98900	98905	98909	98914	98918	98923	98927	98932	98936	98941	
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981	99167	99171	99176	99180	99185	99189	99193	99198	99202	99207	
982	99211	99216	99220	99224	99229	99233	99238	99242	99247	99251	
983	99255	99260	99264	99269	99273	99277	99282	99286	99291	99295	
984	99300	99304	99308	99313	99317	99322	99326	99330	99335	99339	
985	99344	99348	99352	99357	99361	99366	99370	99374	99379	99383	
986	99388	99392	99396	99401	99405	99410	99414	99419	99423	99427	
987	99432	99436	99441	99445	99449	99454	99458	99463	99467	99471	
988	99476	99480	99484	99489	99493	99498	99502	99506	99511	99515	
989	99520	99524	99528	99533	99537	99542	99546	99550	99555	99559	
990	99564	99568	99572	99577	99581	99585	99590	99594	99599	99603	4
991	99607	99612	99616	99621	99625	99629	99634	99638	99642	99647	1 0
992	99651	99656	99660	99664	99669	99673	99677	99682	99686	99691	2 1
993	99695	99699	99704	99708	99712	99717	99721	99726	99730	99734	3 1
994	99739	99743	99747	99752	99756	99760	99765	99769	99774	99778	4 2
995	99782	99787	99791	99795	99800	99804	99808	99813	99817	99822	5 2
996	99826	99830	99835	99839	99843	99848	99852	99856	99861	99865	6 2
997	99870	99874	99878	99883	99887	99891	99896	99900	99904	99909	7 3
998	99913	99917	99922	99926	99930	99935	99939	99944	99948	99952	8 3
999	99957	99961	99965	99970	99974	99978	99983	99987	99991	99996	9 4
1000	00000	00004	00009	00013	00017	00022	00026	00030	00035	00039	

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1000	00000	00004	00009	00013	00017	00022	00026	00030	00035	00039	
1001	00043	00048	00052	00056	00061	00065	00069	00074	00078	00082	
1002	00087	00091	00095	00100	00104	00108	00113	00117	00121	00126	
1003	00130	00134	00139	00143	00147	00152	00156	00160	00165	00169	
1004	00173	00178	00182	00186	00191	00195	00199	00204	00208	00212	
1005	00217	00221	00225	00230	00234	00238	00243	00247	00251	00255	
1006	00260	00264	00268	00273	00277	00281	00286	00290	00294	00299	
1007	00303	00307	00312	00316	00320	00325	00329	00333	00337	00342	
1008	00346	00350	00355	00359	00363	00368	00372	00376	00381	00385	
1009	00389	00393	00398	00402	00406	00411	00415	00419	00424	00428	
1010	00432	00436	00441	00445	00449	00454	00458	00462	00467	00471	
1011	00475	00479	00484	00488	00492	00497	00501	00505	00509	00514	
1012	00518	00522	00527	00531	00535	00540	00544	00548	00552	00557	
1013	00561	00565	00570	00574	00578	00582	00587	00591	00595	00600	
1014	00604	00608	00612	00617	00621	00625	00629	00634	00638	00642	
1015	00647	00651	00655	00659	00664	00668	00672	00677	00681	00685	
1016	00689	00694	00698	00702	00706	00711	00715	00719	00724	00728	
1017	00732	00736	00741	00745	00749	00753	00758	00762	00766	00771	
1018	00775	00779	00783	00788	00792	00796	00800	00805	00809	00813	
1019	00817	00822	00826	00830	00834	00839	00843	00847	00852	00856	
1020	00860	00864	00869	00873	00877	00881	00886	00890	00894	00898	4
1021	00903	00907	00911	00915	00920	00924	00928	00932	00937	00941	1 0
1022	00945	00949	00954	00958	00962	00966	00971	00975	00979	00983	2 1
1023	00988	00992	00996	01000	01005	01009	01013	01017	01022	01026	3 1
1024	01030	01034	01038	01043	01047	01051	01055	01060	01064	01068	4 2
1025	01072	01077	01081	01085	01089	01094	01098	01102	01106	01111	5 2
1026	01115	01119	01123	01127	01132	01136	01140	01144	01149	01153	6 2
1027	01157	01161	01166	01170	01174	01178	01182	01187	01191	01195	7 3
1028	01199	01204	01208	01212	01216	01220	01225	01229	01233	01237	8 3
1029	01242	01246	01250	01254	01258	01263	01267	01271	01275	01280	9 4
1030	01284	01288	01292	01296	01301	01305	01309	01313	01317	01322	
1031	01326	01330	01334	01339	01343	01347	01351	01355	01360	01364	
1032	01368	01372	01376	01381	01385	01389	01393	01397	01402	01406	
1033	01410	01414	01418	01423	01427	01431	01435	01439	01444	01448	
1034	01452	01456	01460	01465	01469	01473	01477	01481	01486	01490	
1035	01494	01498	01502	01507	01511	01515	01519	01523	01528	01532	
1036	01536	01540	01544	01549	01553	01557	01561	01565	01569	01574	
1037	01578	01582	01586	01590	01595	01599	01603	01607	01611	01616	
1038	01620	01624	01628	01632	01636	01641	01645	01649	01653	01657	
1039	01662	01666	01670	01674	01678	01682	01687	01691	01695	01699	
1040	01703	01708	01712	01716	01720	01724	01728	01733	01737	01741	
1041	01745	01749	01753	01758	01762	01766	01770	01774	01778	01783	
1042	01787	01791	01795	01799	01803	01808	01812	01816	01820	01824	
1043	01828	01833	01837	01841	01845	01849	01853	01858	01862	01866	
1044	01870	01874	01878	01883	01887	01891	01895	01899	01903	01907	
1045	01912	01916	01920	01924	01928	01932	01937	01941	01945	01949	
1046	01953	01957	01961	01966	01970	01974	01978	01982	01986	01991	
1047	01995	01999	02003	02007	02011	02015	02020	02024	02028	02032	
1048	02036	02040	02044	02049	02053	02057	02061	02065	02069	02073	
1049	02078	02082	02086	02090	02094	02098	02102	02107	02111	02115	
1050	02119	02123	02127	02131	02135	02140	02144	02148	02152	02156	

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1050	02119	02123	02127	02131	02135	02140	02144	02148	02152	02156	
1051	02160	02164	02169	02173	02177	02181	02185	02189	02193	02197	
1052	02202	02206	02210	02214	02218	02222	02226	02230	02235	02239	
1053	02243	02247	02251	02255	02259	02263	02268	02272	02276	02280	
1054	02284	02288	02292	02296	02301	02305	02309	02313	02317	02321	
1055	02325	02329	02333	02338	02342	02346	02350	02354	02358	02362	
1056	02366	02371	02375	02379	02383	02387	02391	02395	02399	02403	
1057	02407	02412	02416	02420	02424	02428	02432	02436	02440	02444	
1058	02449	02453	02457	02461	02465	02469	02473	02477	02481	02485	
1059	02490	02494	02498	02502	02506	02510	02514	02518	02522	02526	
1060	02531	02535	02539	02543	02547	02551	02555	02559	02563	02567	
1061	02572	02576	02580	02584	02588	02592	02596	02600	02604	02608	
1062	02612	02617	02621	02625	02629	02633	02637	02641	02645	02649	
1063	02653	02657	02661	02666	02670	02674	02678	02682	02686	02690	
1064	02694	02698	02702	02706	02710	02715	02719	02723	02727	02731	
1065	02735	02739	02743	02747	02751	02755	02759	02763	02768	02772	
1066	02776	02780	02784	02788	02792	02796	02800	02804	02808	02812	
1067	02816	02821	02825	02829	02833	02837	02841	02845	02849	02853	
1068	02857	02861	02865	02869	02873	02877	02882	02886	02890	02894	
1069	02898	02902	02906	02910	02914	02918	02922	02926	02930	02934	
1070	02938	02942	02946	02951	02955	02959	02963	02967	02971	02975	4
1071	02979	02983	02987	02991	02995	02999	03003	03007	03011	03015	1 0
1072	03019	03024	03028	03032	03036	03040	03044	03048	03052	03056	2 1
1073	03060	03064	03068	03072	03076	03080	03084	03088	03092	03096	3 1
1074	03100	03104	03109	03113	03117	03121	03125	03129	03133	03137	4 2
1075	03141	03145	03149	03153	03157	03161	03165	03169	03173	03177	5 2
1076	03181	03185	03189	03193	03197	03201	03205	03209	03214	03218	6 2
1077	03222	03226	03230	03234	03238	03242	03246	03250	03254	03258	7 3
1078	03262	03266	03270	03274	03278	03282	03286	03290	03294	03298	8 3
1079	03302	03306	03310	03314	03318	03322	03326	03330	03334	03338	9 4
1080	03342	03346	03350	03354	03358	03362	03366	03371	03375	03379	
1081	03383	03387	03391	03395	03399	03403	03407	03411	03415	03419	
1082	03423	03427	03431	03435	03439	03443	03447	03451	03455	03459	
1083	03463	03467	03471	03475	03479	03483	03487	03491	03495	03499	
1084	03503	03507	03511	03515	03519	03523	03527	03531	03535	03539	
1085	03543	03547	03551	03555	03559	03563	03567	03571	03575	03579	
1086	03583	03587	03591	03595	03599	03603	03607	03611	03615	03619	
1087	03623	03627	03631	03635	03639	03643	03647	03651	03655	03659	
1088	03663	03667	03671	03675	03679	03683	03687	03691	03695	03699	
1089	03703	03707	03711	03715	03719	03723	03727	03731	03735	03739	
1090	03743	03747	03751	03755	03759	03763	03767	03771	03775	03778	
1091	03782	03786	03790	03794	03798	03802	03806	03810	03814	03818	
1092	03822	03826	03830	03834	03838	03842	03846	03850	03854	03858	
1093	03862	03866	03870	03874	03878	03882	03886	03890	03894	03898	
1094	03902	03906	03910	03914	03918	03922	03926	03930	03933	03937	
1095	03941	03945	03949	03953	03957	03961	03965	03969	03973	03977	
1096	03981	03985	03989	03993	03997	04001	04005	04009	04013	04017	
1097	04021	04025	04029	04033	04036	04040	04044	04048	04052	04056	
1098	04060	04064	04068	04072	04076	04080	04084	04088	04092	04096	
1099	04100	04104	04108	04112	04116	04120	04123	04127	04131	04135	
1100	04139	04143	04147	04151	04155	04159	04163	04167	04171	04175	

Error greater than	Probability	Difference	Error greater than	Probability	Difference	Error greater than	Probability
0.0	1.00000	5378	2.5	0.09175	1226	5.2	4.53×10^{-4}
0.1	0.94622	5353	2.6	.07949	1090	5.4	2.70×10^{-4}
0.2	.89269	5304	2.7	.06859	964	5.6	1.59×10^{-4}
0.3	.83965	5233	2.8	.05895	849	5.8	9.15×10^{-5}
0.4	.78732	5139	2.9	.05046	744	6.0	5.19×10^{-5}
0.5	0.73593	5023	3.0	0.04302	648	6.2	2.9×10^{-5}
0.6	.68570	4887	3.1	.03654	564	6.4	1.6×10^{-5}
0.7	.63683	4735	3.2	.03090	487	6.6	8.5×10^{-6}
0.8	.58948	4566	3.3	.02603	420	6.8	4.5×10^{-8}
0.9	.54382	4382	3.4	.02183	359	7.0	2.3×10^{-6}
*1.0	0.50000	4188	3.5	0.01824	306	7.2	1.2×10^{-6}
1.1	.45812	3983	3.6	.01518	261	7.4	6.0×10^{-7}
1.2	.41829	3771	3.7	.01257	219	7.6	3.0×10^{-7}
1.3	.38058	3556	3.8	.01038	185	7.8	1.4×10^{-7}
1.4	.34502	3335	3.9	.00853	155	8.0	6.8×10^{-8}
1.5	0.31167	3116	4.0	0.00698	129	9.0	1.3×10^{-9}
1.6	.28051	2898	4.1	.00569	108	10	1.5×10^{-11}
1.7	.25153	2681	4.2	.00461	88	20	$2. \times 10^{-41}$
1.8	.22472	2471	4.3	.00373	73	30	5×10^{-91}
1.9	.20001	2267	4.4	.00300	60	40	$3. \times 10^{-160}$
2.0	0.17734	2069	4.5	0.00240	48	50	$3. \times 10^{-249}$
2.1	.15665	1881	4.6	.00192	40	60	$4. \times 10^{-358}$
2.2	.13784	1702	4.7	.00152	31	70	$1. \times 10^{-486}$
2.3	.12082	1532	4.8	.00121	26	80	$1. \times 10^{-634}$
2.4	.10550	1375	4.9	.00095	20	90	$1. \times 10^{-802}$
2.5	0.09175		5.0	0.00075		100	$1. \times 10^{-990}$

* *1.0 = "Probable Error".*

Name	Symbol	Atomic Weight	Density at 0^0 and 76 cm	Hardness	Breaking Strength	Young's Modulus	Resilience of Volume	Coefficient Expansion Cubical, 0^0 100^0, 76 cm	Melting Point	Boiling Point	Specific Heat, 0^0 100^0, 76 cm	Latent Heat Melting	Heat Conductivity	Electrical Conductivity	Thermo-Electric Heights	Electro-Chemical Equiv.
Multiply . . .	by	....	....	....	10^9	10^{12}	10^{12}	.....	....	....	....	....	...	.001	100	.001
Aluminum . .	Al	27.1	2.6	3 —	2.	0.7	0.5?	.000070	700	..	.212	..	.35	.33	— 0.8	...
Antimony . .	Sb	120.	6.7	3 +	..	..	..	.000034	440	1200	.050	..	.04	.03	+24.	0.938
Arsenic . . .	As	74.9	5.7	3 +	..	..	..	.000018	S	450	.083 c.	..	..	.03	+14.	...
Barium . . .	Ba	136.8	3.8	..	..	..	..	...	1300?	..	..	..	..	..	..	...
Beryllium* .	Be	9.1	2.1	..	..	..	..	...	900?	..	.42	..	..	..	..	...
Bismuth. . .	Bi	208.	9.8	2 +	..	..	..	.000040	270	1200	.030	13	.02	.008	—55 c.	...
Boron . . .	B	10.9	2.5?	..	..	..	..	...	a.	..	.25	..	..	..	..	...
Bromine . .	Br	79.77	3.1 l	..	..	..	..	.00104 l	—7	61	.084	16	..	..	..	...
Cadmium . .	Cd	111.8	8.6	2 +	..	..	..	.000094	318	800	.055	14	.22	.14 +	+ 3	8.280
Caesium. . .	Cs	132.7	1.9	..	..	..	..	...	27	..	..	..	..	..	..	...
Calcium . . .	Ca	40.0	1.6	1 +	..	..	..	...	700?	..	.18	..	..	.13	+15	...
Carbon . . .	C	11.97	2. G.	1 — G.	..	..	..	.00002 G.	I.	V.	.2 G.	..	.01 G.	.000	..	...
Cerium . . .	Ce	141.	6.6	..	..	..	..	...	700	..	.045	..	..	..	..	...
Chlorine . .	Cl	35.37	1.3 l	..	..	..	..	...	..	—33.6	.121 g.	..	..	..	..	3.671
Chromium . .	Cr	52.3	6.7	8 ?	..	..	..	...	2000	..	.1	...	..	..	..	...
Cobalt . . .	Co	58.7	8.5	6 ?	..	..	..	.000037	1500	..	.107	...	..	.10	—22	...
Columbium † .	Cb	94.	7.2	..	..	..	..	...	..	..	..	..	..	..	..	...
Copper . . .	Cu	63.2	8.9	3 —	4.	1.2	1.6	.000051	1100	..	.093	30?	.9 +	.60	+ 4	3.280 C.
Didymium . .	D	145.	6.5	..	..	..	..	...	900?	..	.046	..	..	..	..	...
Erbium . . .	E	167.	..	..	..	..	..	...	..	..	..	..	..	..	..	...
Fluorine. . .	F	19.0	..	..	..	..	..	...	..	..	..	..	..	..	..	...
Gallium . . .	Ga	69.4	5.9	..	..	..	..	...	30	..	.08	19	..	..	..	...

Abbreviations: a, melts in the voltaic arc; c, crystallized; C, cupric; g, gaseous; G, graphite; I, infusible; l, liquid; s, solid; S, sublimes without melting; V, vaporized in the voltaic arc. * Same as Glucinum. † Same as Niobium.

Name	Symbol	Antomic Weight	Density at 0° and 76 cm	Hardness	Breaking Strength	Young's Modulus	Resilience of Volume	Coefficient Expansion Cubical, 0° 100°, 76 cm	Melting Point	Boiling Point, 76 cm	Specific Heat, 0° 100°, 76 cm	Latent Heat Melting	Heat Conductivity	Electrical Conductivity	Thermo-Electric Heights	Electro-Chemical Equiv.
Multiply . .	by	....	....	....	10^9	10^{12}	10^{12}	......	....	....	....	....	....	.001	100	.001
Glucinum* . .	G	9.1	2.1	..	..	..	..	....	900 ?	..	.42	..	..	..	..	...
Gold	Au	196.2	19.3	3 —	3.—	0.8	0.6	.000044	1100	..	.032	..	.6	.47	+ 2	6.789
Hydrogen .	H	1.0000	0.03 l	..	..	..	..	.00367 g.	b.	b.	3.41 g.	..	.000 g.	..	..	.1038
Indium . .	In	113.4	7.3	3 ?	..	..	..	.00014	176	700 ?	.057	..	..	..	..	...
Iodine . .	I	126.55	4.95	..	..	..	..	....	110	200	.054	12	..	..	..	13.13
Iridium . .	Ir	193.	22.	..	..	..	..	.000021	2200	..	.032	..	..	..	+ 2.5	...
Iron . . .	Fe	55.9	7.8	5 —	6.	1.9	1.5	.000036	1600	..	.113	35 ?	.16	.10	+ 17	1.934 F.
Lanthanum .	La	139.	6.1	..	..	..	..	....	700 ?	..	.045	..	..	..	..	...
Lead . . .	Pb	206.4	11.3 +	2 —	.1—.3	0.1+	..	.000088	330	1500	.032	5.6	.08	.052	0	10.71
Lithium . .	Li	7.01	0.59	..	..	..	..	....	180	..	.94	..	..	.11	+ 14 ?	...
Magnesium .	Mg	24.0	1.7	2 +	..	..	..	.000083	650 ?	1100 ?	.25	..	.4	.3	+ 2	...
Manganese .	Mn	54.6	7 — 8	9 ?	..	..	..	....	1800 ?	..	.12	..	..	..	..	...
Mercury .	Hg	199.8	14.19 s.	..	..	..	0.5	.000182	—39	357	.032 s.	2.8	.018	.0106	— 0.4	1.037 M.
Molybdenum	Mo	95.7	8.6	1 +	..	..	..	....	1600 ?	..	.067	..	..	..	..	...
Nickel . . .	Ni	58.3	8.9	4 +	..	..	..	.000038	1500	..	.109	..	.11	08	— 20.	3.03
Niobium† .	Nb	94.	7.	..	..	..	..	....	—	..	—	..	..	..	..	...
Nitrogen . .	N	14.01	0.4 l	..	..	..	..	.00367 g.	b.	b.	.244 g.	..	.000 g.	..	..	.485
Osmium . .	Os	198.	22.	..	..	..	..	.000020	2200 ?	..	.031	..	..	..	..	...
Oxygen . .	O	15.96	0.7 l	..	..	..	..	.00367 g.	b.	b.	.218 g.	..	.000 g.	..	..	.828
Palladium .	Pd	106.0	11—12	3 +	..	1.0	.7 ?	.000035	1700	..	.059	36	.07	.073	— 7	...
Phosphorus .	P	30.96	1.8	..	..	..	..	0 to .00004	44.3	289	.20.	5	..	.000	+30R.	...
Platinum . .	Pt	195.0	21.5	4 +	3.+	1.6	1.1 ?	.000027	1900	..	.032	27	.09	.10	0 ±	...

Abbreviations: b, below — 100°; F, Ferric; g, gas; l, liquid under 300 atmospheres; M, mercuric; R, red; s, solid.
* Same as Beryllium. † Same as Columbium.

Name	Symbol	Atomic Weight	Density at 0^0 and 76 cm	Hardness	Breaking Strength	Young's Modulus	Resilience of Volume	Coefficient Expansion Cubical, 0^0 100^0, 76 cm	Melting Point	Boiling Point, 76cm	Specific Heat, 0^0 100^0, 76 cm	Latent Heat Melting	Heat Conductivity	Electrical Conductivity	Thermo-Electric Heights	Electro-Chemical Equiv.
Multiply . .	by	....	....	..	10^9	10^{12}	10^{12}	....	....	....	....	....	....	.001	100	.001
Potassium .	K	39.03	0.87	0+	..	..	..	.00025	60	725	.17	..	..	.12	—13.?	4.051
Rhodium .	Rh	104.1	11.—12.	..	..	..	..	.000026	2000	..	.058	..	..	..	..	..
Rubidium .	Rb	85.2	1.5	..	..	..	..	...	38	..	—	..	..	..	..	..
Ruthenium .	Ru	104.	12.	..	..	..	..	.000030	2000?	..	.061	..	..	..	..	..
Selenium .	Se	79.	4.8 c.	3—	..	..	..	.0002—.001	217	680	.084 c.	..	..	.000	+800	..
Silicon . .	Si	28.1	2.3 c.	..	..	..	..	.00002	1200?	..	.18 c.	..	..	..	..	..
Silver . . .	Ag	107.7	10.5	2+	3.	0.7	0.5?	.000058	1000	..	.056+	21.	1.1	.64	+2.+	11.18
Sodium . .	Na	23.00	0.98	..	..	..	..	.00021	95	900	.30	..	0.4	.2	—6.?	2.387
Strontium .	Sr	87.4	2.5	..	..	..	..	...	700?	..	—	..	..	.04	+9.?	..
Sulphur . .	S	31.98	2.0	2+	..	..	..	.0003	114	448	.17	9.4	..	..	..	..
Tantalum .	Ta	182.	10.+	..	..	..	..	...	—	..	—	..	..	..	..	..
Tellurium .	Te	128.	6.4 c.	2+	..	..	..	.00005–.0001	450	..	.048 c.	..	..	.000	+500	..
Thallium .	Tl	204.	11.8	2—	..	..	..	.000094	290	700?	.034	..	..	.05+	..	..
Thorium. .	Th	232.	11.?	..	..	..	..	...	..	..	—	..	..	..	..	..
Tin. . . .	Sn	118.	7.2+	2	.2—.4	0.4	0.3?	.000069	230	1500	.055	14.	.15	.09	0.±	3.06S.
Titanium .	Ti	50.	—	8 ?	..	..	..	...	..	..	—	..	..	..	..	..
Tungsten* .	W	183.7	17.—19.	3+	..	..	..	...	1800?	..	.034	..	..	..	..	..
Uranium. .	U	240.	18.5	..	..	..	..	...	800?	..	.062	..	..	..	..	..
Vanadium .	V	51.2	5.+	..	..	..	..	...	—	..	—	..	..	..	..	..
Wolfram* .	W	183.7	17.—19.	..	..	..	..	...	1800?	..	.034	..	..	..	..	..
Yttrium . .	Y	90.	—	..	..	..	..	...	—	..	—	..	..	..	..	..
Zinc . . .	Zn	64.9	7.0+	3+	.2—.5	0.9	0.5?	.000088	420	1000	.093+	28.	.29	.18	+3.	3.37
Zirconium .	Zr	90.	4.	..	..	..	..	...	1400	..	.066	..	..	..	..	..

c, crystallized.   S, Stannic.   * Tungsten and Wolfram the same.

Name (Commercial Materials)	Density	Hardness	Breaking Strength	Resistance to Crushing	Resistance to Shearing	Simple Rigidity	Young's Modulus	Resilience of Volume	Coefficient Expansion Cubical 0^0—100^0	Melting Point	Boiling Point 76 cm	Specific Heat 0^0—100^0	Latent Heat Melting	Heat Con-ductivity	Electrical Con-ductivity	Thermo-Electric Heights
Multiply by	tr	..	10^9	10^9	10^9	10^{12}	10^{12}	10^{12}	......	....	...	....	....	....	.001	100
Aluminum	2.7	3—	2.0	..	..	.25	.7	.5?	.00007	700?	..	.21	..	.3+	.3+	—1.?
Antimony	6.7	3+	..	..	..	..	..	..	...	440	1200	.05	..	.04	.03+	+6.
Bamboo	0.4	..	.044	..	..	..	..	..	...	..	..	..	..	..	..	..
Bismuth pressed . .	9.8	2+	..	..	..	..	..	..	.00004	270	1200	.03	13.	.02—	.008	—97.
Brass* (cast)	8.3+	3+	1.2	.7	..	.24	.6+	.4?	.00005+	900?	..	.094—	..	.2—.3	.1—.2	..
" (hard drawn)	8.5	3+	4.—9.	..	..	.37	1.0	1.0	.000057	..	..	.09+	..	.2—.3	.1—.2	..
Bricks & cement .	1.7	..	.02	.04—.1	..	..	..	..	...	..	..	.2+	..	.003	..	..
Bronze†	8.8	3+	2.5	..	..	.26	.6+	.4?	.000054	900?	..	..	..	..	..	..
Copper (cast)	8.8	3	1.3	4.?	..	.4	1.1	1.6?	.00005	1100?	..	..	30	.8	.6	0±?
" (hard drawn)	8.8+	..	3.—5.	..	..	.45	1.2	1.7	.000051	..	..	..	30?	.8?	.6+	0±?
Cork	0.24	..	..	..	..	..	..	..	...	..	..	..	..	.0005	..	..
Cotton	..	..	..	..	..	..	..	..	...	..	..	..	..	.0001	..	..
Felt	..	..	..	..	..	..	..	..	...	..	..	..	..	.0002	..	..
German silver . .	8.5	3+	..	..	..	..	..	..	.000055	..	..	..	..	.09	.03—.06	—12
Glass (crown)	2.5+	..	.3—.6	..	..	.2	.6+	.4?	.000025	400	..	.19	..	.001±	.0000	..
" (flint)	3.5—	..	..	..	..	.24	.6	.4	.000025	..	..	.19?	..	.001+	.0000	..
Gold (75%)	15?	..	1.—3.	..	..	..	..	..	.000046	1050	..	..	..	..	..	..
Granite	2.7+	..	..	.4—.8	..	..	..	..	.000026	..	..	..	..	.004	..	..
Hair	..	..	..	..	..	..	..	..	...	..	..	..	..	.0001	..	..
Hemp	..	..	.5	..	..	..	..	..	...	..	..	..	..	.0001?	..	..
India Rubber . . .	0.95	..	..	..	..	..	..	..	...	..	..	..	..	.0004	..	..

* 72% Copper, 28% Zinc.

† 86% Copper, 10% Tin, 4% Zinc.

Name	Density	Hardness	Breaking Strength	Resistance to Crushing	Resistance to Shearing	Simple Rigidity	Young's Modulus	Resilience of Volume	Coefficient Expansion Cubical, 0° 100°, 76 cm	Melting Point	Boiling Point, 76 cm	Specific Heat, 0° 100°, 76 cm	Latent Heat Melting	Heat Conductivity	Electrical Conductivity	Thermo-Electric Heights
Multiply . . .	by	..	10^9	10^9	10^9	10^{12}	10^{12}	10^{12}	.....	...	..	...	....	....	.001	100.
Iron (grey cast)	7.5+	6?	1.?	8.	2.	.4—.5	1.1—1.3	1.0	.000033	1200	..	.13?	25?	..	..	...
« (white cast)	7+	8?	1.?	8.	2.	.4—.5	1.1—1.3	1.0	.000036	1100	..	.13	34?	..	..	...
« (wrought) .	7.8	4+	4.—7.	3.	3.+	0.75	1.9+	1.5	.000037	1600	..	.11	..	.16	.07—.1	+18.
Lead (pressed)	11.3	2—	0.2	.5?	..	0.02+	.07—.13	..	.000088	300+	1500	..	6.	.08	.05	0?
Leather . . .	...	..	0.3	..	..	..	..	..	...	..	..	..	..	...	..	...
Marble. . . .	2.7	3	..	.4	..	..	..	..	.00002+?	..	..	.21	..	.002	..	...
Paper	..	..	..	..	..	..	..	..	...	..	..	..	..	.0001	..	...
Platinum . . .	20?	..	3.+	..	..	0.6?	1.6+	1.1?	.000027	1800?	..	.03+	27?	.09	.07—.1	—1.+3.
Sand (with air spaces) . . .	1.45	7	..	..	..	..	..	..	...	..	..	.19	..	.0004	..	...
Silk.	..	..	50?	..	..	..	..	..	...	..	..	..	..	..	..	...
Silver (Sterling)	..	..	2.—3.?	..	..	0.3?	0.7?	0.5?	.00006	900	..	..	21?	..	..	...
Slate	2.8	..	0.7+	..	..	..	..	..	...	..	..	..	..	.004?	..	...
Steel (cast) . .	7.8+	..	8.—10.	..	..	0.8+	2.2	1.3?	.000034	1400	..	.12	..	.14	..	+11.
« (tempered)	7.8+	9?	7.+	..	..	1.0?	2.7?	1.8?	.000037	..	..	.12?	..	..	..	...
« (wire) . .	7.9?	..	10.—20.	..	..	..	2.8?	1.9?	.000037	..	..	.12?	..	..	..	...
Tin (pressed)	7.3	2?	.2—.4	..	..	0.15	0.4	0.3	.000068	..	..	.05+	14?	..	.08	...
Vulcanite . .	1.0+	..	..	..	..	..	..	..	.0002?	..	..	..	..	.0003	..	...
Wood (hard) .	.6—1.	..	1.0+	.6+	.8±	0.010?	0.10	..	.00002—	..	..	.6+	..	.0006	..	...
« (soft) .	.4—.7?	..	0.8—	.4+	.05—.12	0.007?	0.09±	..	.00001+	..	..	.6—	..	.0004	..	...
Wool	..	..	..	..	..	..	..	..	...	..	..	..	..	.0001—	..	...
Zinc (rolled) .	7.0	3+?	.5—1.5	..	..	0.4	0.9	0.6	.000089	400	1000	..	28?	.3—?	.16	+3.

Note: Annealing generally increases electrical conductivity, but greatly diminishes breaking strength (10—30%). Powdering reduces heat conductivity of most substances to about .0002.

Name	Symbol	Density	Hardness	Coefficient Expansion cubic, 0—100°	Melting Point	Boiling Point, 76 cm	Specific Heat	Latent Heat Melting	Solubility at 20° in %	Solubility at 100° in %
Acid										
,, Acetic	$HC_2H_3O_2$	1.1	.	..	16	117	.46	44	100	100
,, Oxalic	$H_2C_2O_4.2H_2O$	1.6	.	..	.	.	.	.	12	90?
,, Phosphoric	H_3PO_4	1.9	.	..	40	.	.	26	.	100
,, Phosphorous	H_3PO_3	1.7	.	..	72	.	.	37	.	100
,, ,, Hypo-	H_3PO_2	1.5	.	..	17	.	.	36	.	100
,, Sulphuric (hyd.)	$H_2SO_4.H_2O$	1.8	.	..	10	.	.32	.	100	100
,, Tartaric	$H_2C_4H_4O_6$	1.8	.	..	135	.	.29	.	58	77
Arsenate of										
,, Lead	$Pb_3As_2O_8$	.	.	..	.	.	.073	.	0	0
,, Potassium	KH_2AsO_4	2.9	.	..	.	.	.175	.	.	.
,, ,, (anhyd.)	$KAsO_3$	.	.	..	.	.	.156	.	70	.
Borate of										
,, Lead	PbB_2O_4	.	.	..	.	.	.090	.	0	.
,, ,, Bi-	PbB_4O_7	.	.	..	.	.	.114	.	0+	.
,, Potassium	KBO_2	.	.	..	.	.	.205	.	sol	.
,, ,, Bi-	$K_2B_4O_7$	.	.	..	.	.	.220	.	sol	sol
,, Sodium	$NaBO_2$	.	.	..	.	.	.257	.	47	.
,, ,, Bi-	$Na_2B_4O_7$	2.4	.	..	600	.	.233	.	4	36
,, (Borax)	$Na_2B_4O_7.10H_2O$	1.7	.	..	.	.	.385	.	7	67
Bromide of										
,, Lead	$PbBr_2$	6.6	.	..	500	.	.053	.	0?	0+
,, Potassium	KBr	2.7	.	.000126	700	.	.113	.	39	51
,, Silver	$AgBr$	6.3	.	.000104	430	.	.074	.	0	0
Carbonate of										
,, Barium	$BaCO_3$	4.3	.	..	.	.	.110	.	0	0
,, Calcium	$CaCO_3$	2.7	1+	..	.	.	.210	.	0	0
,, Iron	$FeCO_3$	3.8	4	..	.	.	.193	.	0	0
,, Lead	$PbCO_3$	6.5	.	..	.	.	.079	.	0	0
,, Potassium	K_2CO_3	2.3	.	..	850	.	.211	.	51	62
,, Sodium	Na_2CO_3	2.5	.	..	850	.	.26	.	20	33
,, ,, (acid)	$NaHCO_3$	2.2	.	..	.	.	.	.	9	.
,, Strontium	$SrCO_3$	3.6	.	..	.	.	.148	.	0	0
Chloral	$C_2H_3Cl_2O_2$	1.8	.	..	50	97	.	33	sol	.
Chlorate of										
,, Barium	$BaCl_2O_6.H_2O$	3.2	.	..	400	.	.157	.	29	59
,, Potassium	$KClO_3$	2.3	.	..	350	.	.20	.	7	38
,, ,, Per-	$KClO_4$	2.5	.	..	600	.	.19	.	.	.
,, Sodium	$NaClO_3$	2.3	.	..	300	.	.	.	50	70
Chloride of										
,, Ammonium	H_4NCl	1.5	2—	.000188	sub	400	.38	.	27	42
,, Barium	$BaCl_2$	3.8	.	..	.	.	.090	.	26	37
,, ,, (crystals)	$BaCl_2.2H_2O$	3.0	.	..	.	.	.171	.	31	44
,, Calcium	$CaCl_2$	2.2	.	..	720	.	.164	.	42	60
,, ,, (crystals)	$CaCl_2.6H_2O$	1.6	.	.0006	29	.	.4+	40	83	100
,, Carbon	C_2Cl_6	2.0	.	..	187	187	.2	.	0+	.
,, Copper	Cu_2Cl_2	3.5	.	..	434	1000	.138	.	0	.
,, Iron	$FeCl_2$	2.5	.	..	300	.	.	.	47	.

Name	Symbol	Density	Hardness	Coefficient Expansion Cubic 0—100°	Melting Point	Boiling Point, 76 cm	Specific Heat 0—100°	Latent Heat Melting	Solubility at 20° in %	Solubility at 100° in %
Chloride of										
,, Lead . . .	$PbCl_2$	5.8	.	..	500	900	.067	.	1	5
,, Lithium . .	$LiCl$	2.0	.	..	600	.	.282	.	45	57
,, Magnesium .	$MgCl_2$	2.2	.	..	700	.	.194	.	70?	.
,, Mercury . .	$HgCl_2$	5.4	.	..	290	300	.067	.	7	35
,, ,, (calomel)	Hg_2Cl_2 . . .	7.1	.	..	.	.	.052	.	0	.
,, Potassium .	KCl	2.0	.	.000114	730	.	.172	.	26	36
,, & platinum .	K_2PtCl_6 . . .	3.5	.	..	.	.	.113	.	1	5
,, Rubidium .	$RbCl$	2.2	.	..	.	.	.112	.	.	.
,, Silver . . .	$AgCl$	5.6	.	.00010	450	.	.091	.	0	0
,, Sodium . .	$NaCl$	2.1	2	.000121	775	.	.214	.	27	28
,, Strontium .	$SrCl_2$	3.0	.	..	850	.	.120	.	35	50
,, Tin	$SnCl_2$	.	.	..	250	625	.102	.	67?	.
,, ,, (crystals)	$SnCl_2.2H_2O$.	2.7	.	..	.	.	.	.	80?	.
,, Zinc . . .	$ZnCl_2$	2.8	.	..	260	700	.136	.	80	.
Chromate of										
,, Lead . . .	$PbCrO_4$ . . .	5.9	.	..	.	.	.090	.	0	.
,, Potassium .	K_2CrO_4 . . .	2.7	.	..	.	.	.187	.	39	45
,, ,, Bi- . . .	$K_2Cr_2O_7$ . . .	2.7	.	..	.	.	.168	.	11	50
,, Sodium . .	Na_2CrO_4 . . .	2.7	.	..	.	.	.	.	.	.
,, ,, Bi- . . .	$Na_2Cr_2O_7$ . .	.	.	..	.	.	.	.	60?	.
Cyanide of										
,, Mercury . .	HgC_2N_2 . . .	4.0	.	..	.	.	.100	.	12	35
,, Potassium .	KCN	1.5	.	..	.	.	.	.	.	55
,, ,, Ferri- . .	$K_6Fe_2C_{12}N_{12}$.	1.8	.	..	.	.	.233	.	30	44
,, ,, Ferro . .	$K_4FeC_6N_6.3H_2O$	1.9	.	..	.	.	.280	.	25	50
Fluoride of										
,, Calcium . .	CaF_2	3.2	4	.00004?	900	.	.212	.	0	0
Hyposulphite of										
,, Barium . .	$BaS_2O_3.H_2O$.	3.4	.	..	.	.	anh. .163	.	0+	.
,, Lead . . .	PbS_2O_3 . . .	.	.	..	.	.	.092	.	0+	.
,, Potassium .	$K_2S_2O_3$ . . .	.	.	..	.	.	.197	.	sol	.
,, Sodium . .	$Na_2S_2O_3$ . . .	.	.	..	.	.	.221	.	41	100
,, ,, (crystals)	$Na_2S_2O_3.5H_2O$	1.7	.	.00013	30?	.	.445	.38	64	.
Iodide of										
,, Copper . .	Cu_2I_2	4.4	.	..	600	760	.069	.	.	.
,, Lead . . .	PbI_2	6.2	.	.000101	380	900	.043	.	0.1	0.5
,, Mercury . .	HgI_2	6.1	.	.000072	250	350	.042	.	5?	.
,, ,, (mercurous)	Hg_2I_2	9.7	.	..	290	310	.039	.	0	.
,, Potassium .	KI	3.1	.	.000128	640	.	.082	.	59	67?
,, Silver . . .	AgI	5.7	.	—.000004	530	.	.062	.	0	0
,, Sodium . .	NaI	3.6	.	..	630	.	.088	.	64	76
Naphthalene	$C_{10}H_8$	1.2	.	..	80	215	. .	36	0	0+
Nitrate of										
,, Ammonium .	H_4NNO_3 . . .	1.7	.	..	150	.	.455	.	67?	.
,, Barium . .	BaN_2O_6 . . .	3.2	.	..	600	.	.150	.	8	26
,, Lead . . .	PbN_2O_6 . . .	4.4	.	..	.	.	.114	.	36	58
,, Potassium .	KNO_3	2.1	2	..	340	.	.235	49	24	71

Name	Symbol	Density	Hardness	Coefficient Expansion Cubic 0—100°	Melting Point	Boiling Point, 76 cm	Specific Heat 0—100°	Latent Heat Melting	Solubility at 20° in %	Solubility at 100° in %
Nitrate of										
„ Silver . . .	$AgNO_3$	4.3	.	. .	210	.	.144	.	70	90
„ Sodium . .	$NaNo_3$	2.2	2—	. .	310	.	.270	65	47	64
„ Strontium .	SrN_2O_6	2.9	.	. .	650	.	.181	.	42	50
Oxalate of										
„ Potassium .	$K_2C_2O_4.H_2O$.	.	.	. .	.	.	.236	.	25	40
„ „ Tetr- . .	$KH_3C_4O_8.2H_2O$.	.	.	. .	.	.	.283	.	5	.
Oxide of										
„ Aluminum .	Al_2O_3	3.9	9	. .	.	.	.198	.	0	0
„ Antimony .	Sb_2O_3	5.5	.	. .	.	.	.093	.	0+	0+
„ Arsenic . .	As_2O_3	3.7	.	. .	.	.	.128	.	2	8?
„ Bismuth . .	Bi_2O_3	8.1	.	. .	.	.	.061	.	0	.
„ Boron . . .	B_2O_3	1.8	.	. .	580	.	.237	.	2	.
„ Calcium . .	CaO	3.1	.	. .	.	.	.	.	.1+	.1—
„ „ (hydrate)	CaO_2H_2	2.1	.	. .	.	.	.	.	.1+	0.1
„ Chromium	Cr_2O_3	5.0	.	. .	.		.177	.	0	.
„ Copper . .	CuO	6.4	.	. .	.	.	.135	.	0	0
„ „ (cuprous)	Cu_2O	6.0	4—	. .	.	.	.111	.	0	0
„ Iron . . .	Fe_2O_3	5.2	5+	.00004	.	.	.16	.	0	0
„ Lead . . .	PbO	9.3	.	. .	.	.	.051	.	0+	.
„ Magnesium .	MgO	3.3	.	. .	.	.	.244	.	0	0
„ „ (hydrate)	MgO_2H_2	.	.	. .	.	.	.312	.	0+	.
„ Manganese .	MnO	5.1	.	. .	.	.	.157	.	0	.
„ „ Per- . .	MnO_2	5.0	2+	. .	.	.	.159	.	0	.
„ Mercury . .	HgO	11.	.	. .	.	.	.052	.	0	0+
„ Molybdenum	MoO_3	4.4	.	. .	.	.	.154	.	0.2	0.1
„ Nitrogen . .	N_2O_5	.	.	. .	30	.	.	77	80?	.
„ Potassium .	K_2O	2.6	.	. .	.	47	.	.	50	.
„ „ (hydrate)	KOH	2.0	.	. .	.	.	.	.	67	.
„ Silicon . .	SiO_2	2.2	.	.00004	.	.	.19	.	0	0
„ Sodium . .	Na_2O	.	.	. .	.	.	. .	.	40	70
„ „ (hydrate)	$NaOH$	2.1	.	. .	.	.	. .	.	60	.
„ Tin	SnO_2	6.9	6+	.00002	.	.	.091	.	0	.
„ Titanium . .	TiO_2	4.	.	.00003	.	.	.172	.	0	.
„ Tungsten . .	WO_3	6.8	.	. .	.	.	.085	.	0	.
„ Zinc . . .	ZnO	5.7	.	. .	.	.	.125	.	0	0
Phosphate of						.			(?)	
„ Calcium . .	CaP_2O_6	.	.	. .	.		.199	.	0+	.
„ Lead . . .	$Pb_2P_2O_7$	.	.	. .	.	.	.082	.	0	.
„ Potassium .	$K_4P_2O_7$	.	.	. .	.	.	.191	.	sol	.
„ „ (acid) .	KH_2PO_4	2.3	.	. .	.	.	.208	.	sol	.
„ Sodium . .	$Na_4P_2O_7$	2.4	.	. .	900	.	.228	.	5	25
„ „ (acid) . .	$Na_2HPO_4.12H_2O$	1.5			36	.	.454	.	20	
Silicate of										
„ Al etc. (clay)	$Al_2Si_2O_7.2H_2O$,etc.	2.	.	.00002?	.	.	.2+	.	0	0
„ Calcium . .	$CaSiO_3$	4.5	.	. .	.	.	.178	.	0	0
„ Zirconium .	$ZrSiO_4$	.	7+	. .	.	.	.132	.	0	0

Name	Symbol	Density	Hardness	Coefficient Expansion cubic 0—100°	Melting Point	Boiling Point, 76 cm.	Specific Heat 0—100°	Latent Heat Melting	Solubility at 20° in %	Solubility at 100° in %
Sulphate of										
„ Ammonium	$(H_4N)_2SO_4$	1.8	.	. .	140	.	.350	.	43	50
„ Barium	$BaSO_4$	4.4	.	.00006	.	.	.110	.	0	0
„ Calcium	$CaSO_4$	3.0	3	. .	.	.	.19	.	0.2	0.2
„ „ (hydrat.)	$CaSO_4.2H_2O$	2.3	2—	. .	.	.	.26	.	0.2	0.2
„ Cobalt	$CoSO_4.7H_2O$	1.9	.	. .	.	.	.343	.	48	.
„ Copper	$CuSO_4$	3.6	.	. .	.	.	.184	.	19	43
„ „ (crystals)	$CuSO_4.5H_2O$	2.3	2+	. .	.	.	.30	.	30	67
„ Iron	$FeSO_4.7H_2O$	1.9	2	. .	.	.	.350	.	50	80
„ Lead	$PbSO_4$	6.3	.	. .	.	.	.083	.	0	0
„ Magnesium	$MgSO_4$	2.7	.	. .	.	.	.225	.	26	40
„ „ (hydrat.)	$MgSO_4.7H_2O$	1.7	.	. .	.	.	.38	.	55	87
„ Manganese	$MnSO_4$	3.0	.	. .	.	.	.18	.	31	.
„ „ (hydrat.)	$MnSO_4.5H_2O$	2.1	.	. .	.	.	.33	.	50	.
„ Nickel	$NiSO_4$	.	.	. .	.	.	.216	.	28	.
„ „ (hydrat.)	$NiSO_4.7H_2O$	2.0	.	. .	.	.	.341	.	52	.
„ Potassium	K_2SO_4	2.6	.	. .	.	.	.193	.	10	21
„ „ (acid)	$KHSO_4$	2.3	.	. .	205	.	.244	.	32	53
„ „ & Al (alum)	$K_2Al_2S_4O_{16}.24H_2O$	1.7	2+	. .	.	.	.371	.	13	78
„ „ & Cr.	$K_2Cr_2S_4O_{16}.24H_2O$	1.8	.	. .	.	.	.324	.	14	—
„ Sodium	Na_2SO_4	2.7	.	. .	900	.	.230	.	30?	25?
„ „ (crystals)	$Na_2SO_4.10H_2O$	1.5	.	. .	.	.	.	.	60?	.
„ Strontium	$SrSO_4$	3.7	.	.00006	.	.	.140	.	0	0
„ Zinc	$ZnSO_4$	3.5	.	. .	.	.	.174	.	35	51
„ „ (hydrat.)	$ZnSO_4.7H_2O$	2.0	2+	. .	.	.	.34	.	62	87
Sulphide of										
„ Antimony	SbS_3	4.5	.	. .	.	.	.084	.	0	.
„ Bismuth	Bi_2S_3	7.4	.	. .	.	.	.060	.	0	.
„ Copper	CuS	4.0	.	. .	.	.	.	.	0	.
„ „ (cuprous)	Cu_2S	5.6	3—	. .	.	.	.121	.	0	.
„ „ & iron	$CuFeS_2$	4.2	4—	. .	.	.	.131	.	0	.
„ Iron	FeS	4.8	.	. .	.	.	.136	.	0	.
„ „ Bi-	FeS_2	5.0	6+	.00003	.	.	.128	.	0	.
„ Lead	PbS	7.5	2+	.00007	.	.	.050	.	0	.
„ Mercury	HgS	7.9	2+	. .	.	.	.051	.	0	.
„ Nickel	NiS	4.6	.	. .	.	.	.128	.	0	.
„ Potassium	K_2S	2.1	.	. .	.	.	.	.	50?	.
„ Silver	Ag_2S	7.2	2+	. .	.	.	.075	.	0	.
„ Tin	SnS	5.0	.	. .	.	.	.084	.	0	.
„ „ Bi-	SnS_2	4.5	.	. .	.	.	.119	.	0	.
„ Zinc	ZnS	4.1	4—	.000036	.	.	.122	.	0	.
Talc	$3\,MgO,4SiO_2.H_2O$	2.7	1	. . .	.	.	.	.	0	.
Tartrate of										
„ Potass. (acid)	$KHC_4H_4O_6$	.	.	. .	.	.	.	.	0.6	6
„ „ & sodium	$KNaC_4H_4O_6.4H_2O$	.	.	. .	50	.	.33	.	50?	

Name	Symbol	Density	Hardness	Coefficient Expansion cubical, 0^0-100^0	Melting Point	Boiling Point	Specific Heat	Latent Heat of Melting	Heat Conductivity	Specific Inductive Capacity	Index of Refraction (D) Minimum	Index Refract. Medium (D) or Ordinary	Index of Refraction (D) Maximum	Index of Dispersion A—H
Agate	SiO_2	2.6	7	..	..	..	.19?	..	..	..	..	1.540	..	..
Albumen	$C_{44}H_{70}N_{11}O_{14}$ (?) etc.	1.+	..	..	..	..	..	..	..	..	..	1.36	..	..
Alum	$Al_2K_2S_4O_{16}.24H_2O$	1.7	2.+	..	..	..	.371	..	..	..	..	1.456	..	.018
" chrome	$Cr_2K_2S_4O_{16}.24H_2O$	..	..	..	..	..	.324	..	..	..	..	1.481	..	..
" iron	$Fe_2K_2S_4O_{16}.24H_2O$	..	..	..	..	..	..	..	..	..	..	1.482	..	..
" " & ammonium	$Fe_2(H_4N)_2S_4O_{16}.24H_2O$	..	..	..	..	..	..	..	..	..	..	1.485	..	..
" selenium (?)*	$Al_2K_2Se_4O_{16}.24H_2O$?	1.97	..	..	..	..	..	..	..	..	..	1.480	..	..
" thallium	$Tl_2K_2S_4O_{16}.24H_2O$	..	..	..	..	..	..	..	..	..	..	1.489	..	..
Amber	$C_{10}H_{16}O$ (?)	1.+	2.+	..	..	..	..	..	..	..	..	1.532	..	..
Amethyst	SiO_2	2.7	7	..	..	..	.19?	..	..	..	..	1.544	1.553	..
Anatase	TiO_2	3.8	..	..	..	..	..	..	..	..	2.496	2.535	..	..
Anglesite	$PbSO_4$	6.3	..	..	..	..	..	..	..	..	1.877	1.882	1.894	.065?
Apatite	$Ca_5P_3O_{12}F+Ca_5P_3O_{12}Cl$	3.2	5	..	..	..	..	..	..	..	1.638	1.642	..	..
Aragonite	$CaCO_3$	2.94	4.+	.000065	..	..	.205	..	..	..	1.530	1.682	1.686	.033
Blende (zinc)	ZnS	4.0	4.—	..	..	..	..	..	..	..	..	2.369	..	..
Borax	$Na_2B_4O_7.10H_2O$	1.7	..	..	..	..	.385	..	..	..	1.446	1.468	1.471	..
Bromide of Potassium	KBr	2.7	..	.00013?	700	..	.113	..	..	..	..	1.559	..	..
" " Silver	$AgBr$	6.5	..	00010	430	..	.074	..	..	..	..	2.253	..	.20?
Calc spar †	$CaCO_3$	2.71	3	.000018	..	..	..	..	..	..	1.486	1.658	..	.033
Camphor	$C_{10}H_{16}O$ (?)	1.—	..	..	175	205	..	..	..	..	..	..	..	..
Canada Balsam	..	1.07	..	..	..	..	..	..	..	..	..	1.53	..	.02?

* See Landolt and Börnstein, Table 95.

† Same as Iceland Spar.

Name	Symbol	Density	Hardness	Coefficient Expansion cubical, 0°—100°	Melting Point	Boiling Point	Specific Heat	Latent Heat of Melting	Heat Conductivity	Specific Inductive Capacity	Index of Refraction (D) Minimum	Index Refract. Medium (D) or Ordinary	Index of Refraction (D) Maximum	Index of Dispersion A—H
Caoutchouc	. . . .	0.95	..	...	..	..	..	..	..	2.2	...	...	...	..
Carbonate of Lead	$PbCO_3$	6.5	..	...	..	..	.079	..	..	..	1.804	2.076	2.078	.10?
Celestine	$SrSO_4$	3.9	3+	.000061	..	..	.140	..	..	..	1.622	1.624	1.631	.021?
Chlorate of Sodium	$NaClO_3$	2.29	..	...	300	..	..	..	..	..	...	1.515	...	..
Chloride of Silver	$AgCl$	5.6	..	.00010	450	..	.091	..	..	..	...	2.061	...	.12?
Chromate Magnesium	$MgCrO_4$	..	..	...	..	..	..	..	..	..	1.521	1.550	1.568	..
" Potassium	K_2CrO_4	2.7	..	...	..	..	.189	..	..	..	...	1.725	...	..
Chrysoberyl	$BeAl_2O_4$	3.7	8+	...	..	..	..	..	..	..	...	...	...	..
Chrysolite	$MgSiO_3$ (?)	3.4	7—	...	..	..	..	..	..	..	...	...	...	..
Coal*	C, 75—95 %	1.3	2+	.00006	..	..	.3?	..	.001?	..	...	...	...	..
Cyanide Potas., Ferri-	$K_6Fe_2C_{12}N_{12}$	1.8	..	...	..	..	.23	..	..	..	1.566	1.569	1.583	..
" " Ferro-	$K_4FeC_6N_6.3H_2O$	1.9	..	...	..	..	.28	..	..	..	...	1.575	1.581	..
Diamond	C (?)	3.5	10	.000004	..	..	.16	..	..	..	...	2.5	...	.06
Ebonite	. . . .	..	..	...	..	..	..	..	..	2—3	...	...	...	..
Emerald	$Be_3Al_2Si_6O_{18}$	2.7	8	...	..	..	..	..	..	..	...	1.58?	...	..
Feldspar ("Adular")	SiO_2, K_2O, etc.	2.5	6	...	..	..	..	..	..	..	...	...	...	..
Fluor Spar	CaF_2	3.2	4	.000060	900	..	.21	..	.010	..	...	1.434	...	.011
Garnet	SiO_2, Al_2O_3, CaO, etc.	3.8	7±	...	..	..	..	..	..	..	...	...	...	..
	SiO_2 | PbO | K_2O | Na_2O													
Glass (crown) . . . %	59 | 9 | 21 | 3	2.5	..	.000023	..	..	.19	..	..	6+	...	1.53	...	.023
" (flint)	55 | 37 | 6 | 1	3.6	..	.000023	..	..	.19?	..	..	7+	...	1.61	...	.043

* The density of coal varies from 1.2 to 1.5; that of coal with air spaces varies from .8 to 1.1.

Name	Symbol	Density	Hardness	Coefficient Expansion Cubical 0°—100°	Melting Point	Boiling Point, 76 cm	Specific Heat 0°—100°	Latent Heat of Melting	Heat Conductivity	Specific Inductive Capacity	Index of Refraction (D) Minimum	Index Refract Medium (D) or Ordinary	Index of Refraction (D) Maximum	Index of Dispersion A—H
Heavy Spar	$Ba\,SO_4$	4.48	3+	.000058	..	..	.110	..	..	..	1.636	1.638	1.648	.021
Ice*	H_2O	.917	1.5	.0001?*	0	100	.50	79	.002	..	..	1.310	1.311	.013?
Iodide of Ammonium	H_4NI	2.4	..	...	..	..	..	..	..	..	..	1.703	..	..
" " Potassium	KI	3.07	..	.00013	650	..	.082	..	..	..	..	1.667	..	..
" " Silver	AgI	5.6	..	−.000004	..	..	.062	..	..	..	..	2.182	..	.30?
Ivory	. . . .	1.9	..	...	..	..	..	..	..	..	..	1.539	1.541	..
Mica	$K_2O, Al_2O_3, Si\,O_2$, etc.	2.8	2+	...	..	..	.2?	..	..	..	1.561	1.594	1.600	..
Nitrate of Barium	$Ba\,N_2O_6$	3.2	..	...	..	..	..	..	..	..	..	1.571	..	.08?
" " Lead	$Pb\,N_2O_6$	4.4	..	...	..	..	.114	..	..	..	..	1.782	..	..
" " Potassium	KNO_3	2.0	2	...	340	..	.235	..	..	..	1.335	1.506	1.506	..
" " Sodium	$Na\,NO_3$	2.2	..	...	320	..	.270	..	..	..	1.337	1.586	..	..
Paraffine	$C_{20}?\,H_{42}?$ + etc.	0.90	..	.001+	55	400	..	..	..	3	..	..	..	..
Phosphorus	P	1.8	..	.0003	44	289	.20	5.0	..	..	..	2.14	..	.20?
Quartz	$Si\,O_2$	2.65	7	.000040	..	..	.186	..	.005	..	..	1.544	1.553	.019
Realgar	As_2S_2	3.5	..	...	..	..	..	..	..	..	..	2.4?	..	..
Resin	. . . .	1.07	..	.0002	..	..	..	..	.0003	2	..	1.55?	..	..
Rochelle Salt	$KNa\,C_4H_4O_6 . 4\,H_2O$	..	..	...	..	..	..	..	..	..	1.491	1.495	1.498	..
Rock Salt	$Na\,Cl$	2.2	2	.00012	800	..	.219	..	.011	..	..	1.544	..	.031
Ruby	Al_2O_3	4.0	9	...	..	..	.22?	..	..	..	..	1.78?	..	..
Sapphire	Al_2O_3	3.8	9	...	..	..	.217	..	..	..	..	1.79?	..	..
Selenite†	$Ca\,SO_4 . 2\,H_2O$	2.3	2−	...	..	..	.26	..	..	..	1.521	1.523	1.530	.014

* The coefficient of expansion of ice is quoted by Landolt and Börnstein as **negative** according to Schumacher, **positive** according to four other observers.

† Gypsum.

Name	Symbol	Density	Hardness	Coefficient Expansion Cubical 0°—100°	Melting Point	Boiling Point, 76 cm	Specific Heat 0°—100°	Latent Heat of Melting	Heat Conductivity	Specific Inductive Capacity	Index of Refraction (D) Minimum	Index Refract. Medium (D) or Ordinary	Index of Refraction (D) Maximum	Index of Dispersion A—H
Selenium (crystals)	Se	4.6	..	.0002	217	620	.084	..	...	...	...	2.98	...	1.3?
Shellac	...	1.+	..	...	..	...	...	..	...	3	...	...	...	...
Spermaceti	...	0.91	..	...	44	...	...	..	...	1.5?	...	1.54	...	...
Spinel	$Mg\,Al_2\,O_4$	3.8	8	...	..	...	...	..	...	...	...	1.715	...	...
Sugar (crystals)	$C_{12}\,H_{22}\,O_{11}$	1.59	..	...	170	...	.30	..	...	...	1.537	1.565	1.570	.017?
Sulphate* of Copper	$Cu\,SO_4.5H_2\,O$	2.27	2+	...	..	...	.30	..	...	...	1.515	1.538	1.545	.024?
,, Magnesium	$Mg\,SO_4.7\,H_2\,O$	1.7	..	...	..	...	.38	..	...	...	1.432	1.455	1.461	...
,, Nickel	$Ni\,SO_4.7\,H_2\,O$	2.0	..	...	..	...	.34	..	...	...	1.467	1.489	1.492	...
,, Potassium	$K_2\,SO_4$	2.6	..	...	..	...	.193	..	...	...	1.493	1.495	1.498	...
,, Zinc	$Zn\,SO_4.7\,H_2\,O$	2.0	..	...	..	...	.33	..	...	...	1.457	1.480	1.484	...
Sulphur	S	2.0	..	.0002	..	...	.17	9.4	...	3+	1.951	2.038	2.241	...
Tallow	...	0.95	..	...	40	...	...	..	...	...	...	1.49	...	...
Tartar Emetic	$K(SbO)C_4\,H_4\,O_6.H_2\,O$	..	..	...	..	...	...	..	...	...	1.620	1.636	1.638	...
Tartaric Acid	$H_2\,C_4\,H_4\,O_6$	1.75	..	...	135	...	.29	..	...	...	1.495	1.535	1.605	...
Thallium (prisms)	Tl (?)	11.8	..	.0001	290	700?	.034	..	...	...	...	1.752	...	.07?
Topaz	$Al_{12}\,Si_6\,O_{25}\,F_{10}$ (?)	3.5	8	...	..	...	...	..	...	...	1.612	1.614	1.621	.017
Tourmaline	$Al_2\,O_3$, $Si\,O_2$, etc.	3.0	7+	...	..	...	...	..	...	...	1.626	1.648	...	...
Varnishes	...	1.1	..	...	..	...	...	..	...	...	...	1.53	...	...
Vulcanite	...	1.±	..	...	..	...	...	..	...	3	...	...	...	...
Wax	...	0.96	..	.001	63	...	.5	42	.0001	2?	...	1.54	...	...
Zircon	$Zr\,Si\,O_4$	4.4	7+	...	..	...	.132	..	...	...	...	1.92	1.97	...

* See Alums.

Name	Symbol	Density (0°)	Viscosity (20°)	Surface Tension (20°)	Resilience of Volume	Coefficient Expansion at 0°	Freezing Point	Boiling Point, 76 cm	Critical Temperature	Critical Pressure	Pressure of Vapor (20°)	Specific Heat, 0°—100°	Latent Heat Vaporization	Heat Conductivity	Spec. Induct Capacity	Index of Refraction (D)	Index of Dispersion A—H.
Multiply by	. . . .	..	..	..	10^{12}	...	..	..	..	10^6	10^6	..	..	....	..	..	..
Acetate of Amyl . .	$C_5H_{11}OC_2H_3O$ . .	0.896	.0071	..	..	.00115	..	150	..	..	..	..	..	.00030	..	1.404	.017
„ „ Ethyl . .	$C_2H_5OC_2H_3O$ . .	0.909	.0035	..	..	.00127	...	75	245	43	..	.56	100	.00035	..	1.373	.016
„ „ Isobutyl .	$(CH_3)_2(C_2H_3O)_2$ . .	0.817	..	..	..	.00103	..	108	296	..	..	..	..	...	..	..	..
„ „ Methyl . .	$CH_3OC_2H_3O$ . . .	0.956	.0032	..	..	.00129	..	58	233	59	..	.51	110	.00038	..	1.361	.015
„ „ Propyl . .	$C_3H_7OC_2H_3O$ . . .	0.90	.0046	..	..	.00111	..	..	282	..	..	..	..	.00033	..	1.384	.016
Acetone	$(CH_3)_2CO$	0.814	.0031	..	..	.00135	..	56	233	53	.240	.53	126	...	..	1.36	.017
Acid* Acetic. . . .	$HOC_2H_3O_2$	1.080	.0101	..	..	.00105	.17	117	222	..	.025	.53	120	.00047	..	1.374	.016
„ „ (anhyd.)	$(C_2H_3O)_2O$	1.097	..	..	..	.00105	..	138	..	..	..	..	66	...	..	1.390	.017
„ Butyric . . .	HOC_4H_7O	0.98	.013	..	..	.00103	..	160	..	..	.010	..	115	.00036	..	1.398	.017
„ „ Iso-† .	$HO(CH_3)_2CHCO$.	0.96	.01 ?	..	..	...	..	154	†..	..	..	..	..	.00034	..	1.393	.017
„ Formic . . .	$HOCHO$	1.23	..	..	..	.00099	5	105	..	..	.042	..	115	.00065	..	1.371	.018
„ Nitric	HNO_3	1.56	..	..	.031?	.00111	—47	..	..	..	..	..	115	...	..	..	..
„ Propionic . .	HOC_3H_5O	1.016	.0094	..	..	.00110	..	142	340	..	.011	..	..	.00039	.	1.387	.017
„ Sulphuric . .	H_2SO_4	1.84	..	..	.032?	.00059	10	330?	..	..	..	.34	122	.00077	..	1.43?	.017?
„ Valeric . . .	HOC_5H_9O	0.959	..	..	..	.00105	..	184	..	..	..	..	104	.00032	..	..	..
„ „ Iso-. .	$HO(CH_3)_2C_2H_3CO$	0.950	..	..	..	...	..	173	..	..	.008	..	..	.00031	..	1.404	.018
Alcohol, Amyl . . .	$C_5H_{11}OH$	0.83	.04 ?	..	..	...	..	136	..	..	..	.65	121	.00033	..	1.41?	.018?
„ „ Benzyl	C_7H_7OH	1.063	..	..	..	.00079	..	207	..	..	..	..	..	...	..	1.540	.044
„ „ Butyl .	C_4H_9OH	0.826	.023	..	..	...	..	115	..	..	..	..	..	.00034	..	1.399	.017
„ (common)†† Ethyl .	C_2H_5OH	0.810	..	25.4	.012	.00106	..	78.2	236	63	.058	.65	206	.00042	..	1.36	.015
„ (wood) § Methyl	CH_3OH	0.817	..	..	..	.00114	..	66	..	..	.118	.65	264	.00050	..	1.33	.013
„ „ Propyl.	C_3H_7OH	0.81	.020	..	..	...	..	97	..	..	..	..	..	.00037	..	1.385	.016

* See Hydrochloric Acid. † Isobutyric acid boils at 205° acc. to Lossen. †† Ordinary (grain) alcohol. § Wood spirit.

Name	Symbol	Density (0°)	Viscosity (20°)	Surface Tension (20°)	Resilience of Volume	Coefficient Expansion at 0°	Freezing Point	Boiling Point, 76 cm	Critical Temperature	Critical Pressure	Pressure of Vapor (20°)	Specific Heat, 0°–100°	Latent Heat Vaporizatin	Heat Conductivity	Spec. Induct Capacity	Index of Refraction (D)	Index of Dispersion A–H
Multiply	...	..	...	..	10^{12}	....	..	..	..	10^6	10^6	..	....	....	..	..	..
Aldehyde	CH_3CHO	0.805	...	..	..	.00160	..	22	..	..	..	..	136	...	..	1.332	.014
Aniline	$C_6H_5NH_2$	1.03	...	..	..	.00082	—8	183	..	..	..	..	..	.00041	..	1.58	.06+
Benzene	C_6H_6	0.899	.0052	..	.016	.00118	4	80	286	70?	.100	.44	92	.00033	2.2	1.500	.043
Benzoate of Ethyl	$C_2H_5OC_7H_5O$	1.066	...	..	..	.00093	..	213	..	..	..	..	..	...	..	1.506	.042
„ „ Methyl	$CH_3OC_7H_5O$	1.107	...	..	..	.00089	..	200	..	..	..	..	..	...	..	1.517	.045
Bromide of Amyl	$C_5H_{11}Br$	1.24	...	..	..	.00105	..	129	..	..	..	..	48	.00024	..	1.44?	.023
„ „ Antimony	$SbBr_3$	3.6	...	..	..	.00058	92	275	..	..	..	..	..	...	..	...	..
„ „ Ethyl	C_2H_5Br	1.473	.0031	..	..	.00134	..	40	236	..	.516	.22	62	.00025	..	1.424	.024
„ „ Ethylene	$C_2H_4Br_2$	2.17	...	..	..	.00099	10	131	..	..	.013	.17	44	...	..	1.538	.036
„ „ Methyl	CH_3Br	1.664	...	..	..	.00142	..	13	..	..	..	..	..	...	..	...	..
„ „ Phosphorus	PBr_3	2.925	...	..	..	.00084	..	175	..	..	..	..	..	...	..	...	..
„ „ Propyl	C_3H_7Br	1.38	...	..	..	...	..	71	..	..	..	..	..	.00026	..	1.434	.023
„ „ Silicon	$SiBr_4$	2.813	...	..	..	.00095	—14	150	..	..	..	..	..	...	..	...	..
Bromine	Br	3.19	...	74?	..	.00104	—7	61	..	..	..	.11	46	...	..	...	..
Bromobenzene	C_6H_5Br	1.52	.0096	..	..	...	..	155	..	..	..	..	..	.00027	..	1.560	.050
Butyrate of Ethyl	$C_2H_5OC_4H_7O$	0.903	.0053	..	..	.00119	..	117	304	..	..	..	..	.00032	..	1.396	.017
„ „ Methyl	$CH_3OC_4H_7O$	0.91	...	..	..	.00122	..	101	..	..	..	..	87	.00034	..	1.389	.017
„ Propyl	$C_3H_7OC_4H_7O$	...	.0070	..	..	.00099	..	..	333	..	..	..	..	...	..	...	..
Carbonate of Ethyl	$(C_2H_5)_2CO_3$	1.000	...	..	..	.00117	..	126	..	..	..	..	..	...	..	1.385	.016
Chloral	CCl_3CHO	1.53	...	..	..	.00095	—75	99	..	..	..	..	54	...	..	1.456	.024
„ hydrate	$CCl_3CHO.H_2O$	...	...	..	..	...	50	98	..	..	..	..	132	...	..	...	..

Name	Symbol	Density (0°)	Viscosity (20°)	Surface Tension (20°)	Resilience of Volume	Coefficient Expansion at 0°	Freezing Point	Boiling Point, 76 cm	Critical Temperature	Critical Pressure	Pressure of Vapor (20°)	Specific Heat, 0°–100°	Latent Heat Vaporization	Heat Conductivity	Spec. Induct Capacity	Index of Refraction (D)	Index of Dispersion A–H.
Multiply by	. . . .	..	..	..	10^{12}	...	..	..	..	10^{6}	10^{6}	..	..	....	..	..	..
Chloride of Acetyl .	C_2H_3OCl . . .	1.130	..	..	..	.00131	..	53	..	..	..	..	..	...	..	1.390	.020
,, Allyl	C_3H_5Cl . . .	0.95	.0028	..	..	.00129	..	45	240	..	..	..	..	...	..	1.415	.025
,, Amyl	$C_5H_{11}Cl$ . . .	0.891	..	..	..	.00117	..	102	..	..	..	..	56	.00028	..	...	..
,, Antimony . .	$SbCl_3$	2.68	..	..	..	.00080	73	224	..	..	..	..	..	...	..	...	..
,, Arsenic . . .	$AsCl_3$	2.205	..	..	..	.00098	..	130	..	..	..	.18	40?	...	..	...	..
,, Boron	BCl_3	1.35	..	..	..	...	..	18	..	..	..	..	..	...	..	...	..
,, Carbon, Tetra-	CCl_4	1.630	.0080	..	..	.00118	—25	77	285	60	.121	..	45	.00025	..	1.461	.024
,, ,, Proto-	C_2Cl_4	1.649	..	..	..	...	..	122	..	..	..	.20	..	...	..	...	..
,, Ethyl	C_2H_5Cl	0.921	..	..	..	.0015?	..	11	183	53	.132	.43	90	...	..	...	..
,, Ethylene . . .	$(CH_2Cl)_2$ . . .	1.280	.007?	..	..	.00112	..	85	283	..	..	.31	..	...	..	1.444	.021
,, Ethylidene . .	CH_3CHCl_2 . .	1.20	.0040	..	..	.0012	..	59	255	..	..	..	67	...	..	1.417	.020
,, Isobutyl . . .	$(CH_3)_2C_2H_3Cl$.	0.895	.0038	..	..	...	..	68	..	..	..	..	..	.00028	..	...	..
,, Phosphorus .	PCl_3	1.612	..	..	..	.00109	..	77	285	..	.134	.20	51	...	..	...	..
,, Propyl	C_3H_7Cl	0.91	.0060	..	..	...	..	50	..	..	..	..	..	.00028	..	1.389	.017
,, Silicon . . .	$SiCl_4$	1.524	..	..	..	.00129	..	59	..	..	.260	.19	..	...	..	...	..
,, Sulphur . . .	S_2Cl_2	1.706	..	..	..	.00096	..	140	..	..	..	.20	49	...	..	...	..
,, Tin, Tetra- .	$SnCl_4$	2.27	..	..	..	.00113	..	115	..	..	..	.14	31	...	..	...	..
,, Titanium . .	$TiCl_4$	1.761	..	..	..	.00094	..	136	..	..	..	.18	..	...	..	...	..
Chlorobenzene . . .	C_6H_5Cl	1.12	..	..	..	...	—45	132	..	..	..	..	..	.00030	..	1.525	.044
Chloroform	$CHCl_3$	1.525	.0045	30	..	.0012?	—70	61	260	56	.215	.24	61	.00029	..	1.446	.022
Cyanide of Ethyl . .	C_2H_5CN . . .	0.801	..	..	..	...	..	97	..	..	..	.63	..	...	..	...	..
,, ,, Methyl .	CH_3CN	0.835	..	..	..	.00121	..	75	..	..	..	..	..	...	..	...	..

Name	Symbol	Density (0°)	Viscosity (20°)	Surface Tension (20°)	Resilience of Volume	Coefficient Expansion at 0°	Freezing Point	Boiling Point, 76 cm	Critical Temperature	Critical Pressure	Pressure of Vapor (20°)	Specific Heat, 0°–100°	Latent Heat Vaporization	Heat Conductivity	Spec. Induct Capacity	Index of Refraction	Index of Dispersion A—H.
Multiply by		..	..	..	10^{12}	...	..	..	..	10^6	10^6	..	..	...	..	..	..
Cyanide of Phenyl	C_6H_5CN	1.023	..	..	..	.00093	—17	191	..	..	..	..	..	...	..	..	..
Diethylamine	$(C_2H_5)_2NH$	...	..	..	..	.00136	..	57	220	39	..	..	..	...	..	..	..
Ether	$(C_2H_5)_2O$	0.73	.0019	20?	.009	.00148	..	35	193	38	.578	.54	91	.00030	3.3	1.353	.015
Formate of Ethyl	C_2H_5OCHO	0.94	.0032	..	..	.00134	..	54	230	50	..	..	105	.00038	..	1.36	.016
„ „ Isoamyl	$(CH_3)_2C_3H_5OCHO$	..	..	..	..	.00099	..	..	305	..	..	..	..	...	..	..	..
„ „ Methyl	CH_3OCHO	0.998	..	..	..	.00140	..	33	..	..	..	..	114	...	..	..	..
„ „ Propyl	C_3H_7OCHO	0.919	.0042	..	..	.00118	..	82	267	..	..	..	..	.00036	..	..	..
Glycerine	$C_3H_5O_3H_3$	1.270	..	..	.040	.0005	17	290	..	..	..	..	..	.00067	..	1.473	.019
Hydrochloric acid	HCl	0.90	..	..	..	...	low	—80	51	..	..	..	...	...	..	..	..
Iodide of Amyl	$C_5H_{11}I$	1.544	..	..	..	.00096	..	155	..	..	..	..	47	.00020	..	1.49?	.032?
„ „ Ethyl	C_2H_5I	1.975	..	..	..	.00114	..	71	..	..	.147	.17	47	.00022	..	1.513	.041
„ „ Isobutyl	$(CH_3)_2C_2H_3I$	1.64	.0069	..	..	...	..	121	..	..	..	..	..	...	..	1.496	.035
„ „ Methyl	CH_3I	2.20	.0041	..	..	.00120	..	44	..	..	..	..	46	.00021	..	1.530	.047
„ „ Propyl	C_3H_7I	1.78	.0060	..	..	...	..	103	..	..	..	..	..	.00022	..	1.505	.038
Iodine (melted)	I	..	..	..	..	...	110	200	..	..	..	..	24	...	..	..	..
Mercury *†	Hg	13.596	..	540	.5	.00018	—39	350	..	..	†	.034	62	...	..	..	..
Nitrobenzene	$C_6H_5NO_2$	1.21	.015?	..	..	.00083	3	210	..	..	..	.35	..	...	..	1.553	.063
Nitroglycerine	$C_3H_5(NO_3)_3$	1.6+	..	..	..	...	10	185	..	..	..	..	..	...	..	..	..
Oil,†† Bitter almond	C_6H_5CHO	1.064	..	..	..	.00094	..	179	..	..	..	..	..	...	..	1.546	.061
„ Olive	...	0.92	..	35	.021	.00080	..	..	..	..	..	..	..	...	3+	1.47	..
„ Rape	...	0.92	..	..	.021	...	..	..	..	..	..	..	..	...	2+	1.47	..
„ Sperm	...	0.92	..	..	..	...	..	..	..	..	..	..	..	...	3	1 46	..

* See Table 24. † See Table 13, C. †† Oils, see also Petroleum and Turpentine. Oil of Vitriol, see Acid, sulphuric

Name	Symbol	Density (0°)	Viscosity (20°)	Surface Tension (20°)	Resilience of Volume	Coefficient Expansion at 0°	Freezing Point	Boiling Point 76 cm	Critical Temperature	Critical Pressure	Pressure of Vapor (20°)	Specific Heat 0°–100°	Latent Heat Vaporization	Heat Conductivity	Spec. Induct. Capacity	Index of Refraction (D)	Index of Dispersion A–H
Multiply by	. . . .	. .	. .	.	10^{12}	. .	.	. .	.	10^6	10^6	. .	.	. . .	.	. .	. .
Oxalate of Ethyl	$(C_2H_5)_2O_2C_2O_2$	1.102	. .	.	.	.00107	.	185	.	.	. .	. .	73	. . .	.	1.410	.018
" " Methyl	$(CH_3)_2O_2C_2O_2$	1.16	. .	.	.	.00108	50	161	.	.	. .	. .	.	. . .	.	. .	. .
Petroleum (light)	$C_8?H_{18}?$ etc.	0.7	. .	.	.	. .	.	110?	.	.	. .	. .	76?	. . .	2	1.4+	. .
" (heavy)	$C_{13}?H_{28}?$ etc.	0.8+	. .	32	.016	.00090	.	220?	.	.	. .	. .	.	. . .	2	1.45	. .
Phenol	C_6H_5OH	1.08	. .	.	.	.00067	38	186	.	.	. .	. .	.	. . .	.	1.550	.049
Phosphorus (melted)	P	1.76?	. .	.	.	. .	44.3	289	.	.	. .	. .	.	. . .	.	2.08	.16?
Propionate of Ethyl	$C_2H_5OC_3H_5O$	0.923	.0045	.	.	.00129	.	98	280	.	. .	. .	.	. . .	.	. .	. .
" " Isobutyl	$(CH_3)_2C_2H_3OC_3H_5O$	0.893	.0068	.	.	.00101	.	137	320	.	. .	. .	.	. . .	.	. .	. .
" " Methyl	$CH_3OC_3H_5O$	0.92	.0040	.	.	.00120	.	98	263	.	. .	. .	.	. . .	.	. .	. .
" " Propyl	$C_3H_7OC_3H_5O$	0.902	.0059	.	.	.00103	.	123	305	.	. .	. .	.	. . .	.	. .	. .
Salicylate of Methyl	$CH_3OC_7H_5O_2$	1.20	. .	.	.	.00084	.	224	.	.	. .	. .	.	. . .	.	1.537	.060
Succinate of Ethyl	$(C_2H_5)_2O_2C_4H_4O_2$	1.05	. .	.	.	.00101	.	217	.	.	. .	. .	.	. . .	.	. .	. .
Sulphide Carbon. Bi-	CS_2	1.29	. .	32	.018	.00114	.	47	272	76	.397	.24	90	.00034	2.1	1.63	.091
" Ethyl	$(C_2H_5)_2S$	0.825	.0034	.	.	.00120	.	91?	.	.	. .	.48	.	.00033	.	1.442	.025?
Sulphur (melted)	S	. .	. .	.	.	. .	114	448	.	.	. .	. .	362	. . .	.	. .	. .
Toluene	$C_6H_5CH_3$	0.882	.0047	.	.	. .	.	111	320	.	. .	. .	.	.00031	.	1.496	.041
Turpentine	$C_{10}H_{16}$	0.88	. .	29	.017	.00071	—10	160	.	.	.006	.46	70	.00026	2.1	1.47	.024
Valeraldehyde. Iso-	$(CH_3)_2C_2H_3CHO$	0.822	. .	.	.	.00119	.	93	.	.	. .	. .	.	. . .	.	1.39	.018?
Valerate of Amyl	$C_5H_{11}OC_5H_9O$	0.87	. .	.	.	.00103	.	189	.	.	. .	. .	.	. . .	.	1.412	.018
" " Ethyl	$C_2H_5OC_5H_9O$	0.88	.0061	.	.	. .	.	133	.	.	. .	. .	.	.00031	.	1.397	.017
" " Methyl	$CH_3OC_5H_9O$	0.90	. .	.	.	.00112	.	116	.	.	. .	. .	.	.00032	.	1.395	.017
Water*†∓§	H_2O	1.000	.0140	80	.021	†	0	100∓	400?	.	.023§	1.005	536	.00170	.	1.333	.014

* See Table 25. † See Table 23. ∓ See Table 14. § See Table 13, C

Note. Density of blood, 1.060; milk, 1.032; sea water, 1.026. Resilience of sea water, $.022 \times 10^{12}$.

Name	Symbol	Sp.Gr.ref.to Hydrogen	Density, 0°, 1,000,000 dynes per sq. cm.	Resilience of Volume, 0°, p=10⁶	Coefficient Expansion 0—100°, 76 cm.	Temperature of Solidification	Temp. of Condensation 76 cm.	Critical Temperature	Critical Pressure	Specif.Heat Constant Pressure	Specif.Heat Const. Vol.	Latent Heat Condensation	Heat Conductivity	Specific Inductive Capacity	Index of Refraction 0°, 76 cm. Line D	Index of Dispersion A—H	Solubility % in Water, 20°, 76 cm.
Multiply by.		..	...	10^6	...	..	..	..	10^6	..	..	..	.001	...	...	10^{-6}	..
Acetate of Ethyl	$C_2H_5OC_2H_3O$	44.3	...	..	...	..	75	245	43	.35	.3?	100	..	...	...	..	..
Acetylene	C_2H_2	13.3	.00117	..	...	..	low?	37	..	..	..	..	..	...	...	..	..
Acetone	$(CH_3)_2CO$	29.2	solid	..	...	17	117	233	53	.35	.3?	126	..	...	...	..	..
Air*	77% N, 23% O	14.43	.001276	.999	.00367	..	low	—158	15	.238	.169	..	.055	1.0015	1.000293	7.5	0022
Alcohol, Ethyl**	C_2H_5OH	23.3	liquid	..	...	..	78.2	236	63	.45	.4?	206	..	...	1.00087?	35?	..
" Methyl†	CH_3OH	16.2	liquid	..	...	..	66	..	..	.45	.4?	264	..	...	...	..	..
Aldehyde	CH_3CHO	22.1	liquid	..	...	..	22	..	..	..	..	136	..	...	...	..	..
Ammoniacal Gas	H_3N	8.55	.000759	..	...	—75	—38	..	..	.50	.38	295	.050	...	1.000385	..	33
Arsenic	As_4	153	solid	..	...	subl.	450	..	..	..	..	..	..	...	1.0011?	..	..
Arsen d'Hydrogen	H_3As	38.9	.0035	..	...	..	—58	..	..	..	..	..	..	...	...	..	..
Benzene	C_6H_6	40.0	solid	..	...	+4	80	286	70	.34	.3?	92	..	...	...	..	..
Bromide of Ethyl	C_2H_5Br	..	...	..	...	..	40	236	..	.16	.1?	62	..	...	...	..	..
" " Ethylene	$C_2H_4Br_2$	..	solid	..	...	10	131	..	..	..	..	44	..	...	...	..	..
Bromine	Br_2	79.5	liquid	..	...	—7	61	..	..	055	.042	46	..	...	...	..	..
Carbonic Dioxide††	CO_2	22.1	.001951	.992	.00371	††	—80	31	78	.20	.15	48	.033	1.0023	1.000454	11	.18
" Oxide	CO	13.9	.001218	.999	.00367	..	low	..	..	.24	.17	..	.054	...	1.000335	12	.0028
Chloride Arsenic	$AsCl_3$	90.9	...	..	...	..	130	..	..	.11	.1?	40?	..	...	...	..	..
" Boron	BCl_3	49.?	...	..	...	..	18	..	..	..	..	..	..	...	...	..	..
" Carbon, Tetra-§	CCl_4	78.1	liquid	..	...	—25	79	285	60	..	..	45?	..	...	...	..	..
" Ethyl	C_2H_5Cl	32.0	...	..	...	..	11	183	53	.27	.24?	90	..	...	...	..	..
" Ethylene §§	$C_2H_4Cl_2$	49.7	...	..	...	..	85	283	..	.23	.2?	..	..	...	...	..	..
" Iodine	ICl	..	solid	..	...	27	101	..	..	.051	.039	..	..	...	...	..	..

* Air contains 79% N and 21% O by volume.
†† Carbonic Acid Gas. Solid at —58° acc. to Faraday.
** Ordinary (grain) alcohol.
§ Formerly called Bi-chloride.
† Wood Spirit.
§§ Dutch Liquid

Name	Symbol	Sp. Gr. ref. to Hydrogen	Density 0° 1.000.000 dynes per sq. cm.	Resilience of Volume 0°, p=10⁶	Coefficient Expansion 0°—100°, 76 cm	Temperature of Solidification	Temp. of Condensation 76 cm.	Critical Temperature	Critical Pressure	Specif. Heat Constant Pressure	Specif. Heat Const. Vol.	Latent Heat Condensation	Heat Conductivity	Specific Inductive Capacity	Index of Refraction 0°, 76 cm. Line D	Index of Dispersion A—H	Solubility % in Water 20°, 76 cm.
Multiply by	. . . .	..	...	10^6	...	...	...	...	10^6	..	..	..	.001	...	...	10^{-6}	..
Chloride Methyl . . .	CH_3Cl . . .	25.0	...	..	...	...	22	...	..	..	..	..	..	...	...	..	..
" Phosphorus .	PCl_3	70.3	...	..	...	...	77	285	..	.13	.1?	51	..	...	...	..	..
" Silicon . . .	$SiCl_4$ . . .	85.7	...	..	...	...	59	...	..	.13	.1?	..	..	...	...	..	..
" Silver . . .	$AgCl$ . . .	..	solid	..	...	453	...	...	..	..	..	..	..	...	...	..	..
" Sulphur . .	S_2Cl_2 . . .	68.0	...	..	...	...	140	...	..	..	..	49	..	...	...	..	..
" Tin, Stannic*	$SnCl_4$ . . .	133	...	..	...	...	115	...	..	.094	.08?	31	..	...	...	..	..
" Titanium . .	$TiCl_4$ . . .	98.7	...	..	...	...	136	...	..	.13	.1?	..	..	...	...	..	..
Chlorine	Cl_2	35.6	.003091	..	...	...	—34	...	..	12	.09	..	..	...	1.000772	..	.7
Chloroform	$CHCl_3$ . . .	..	liquid	..	...	—70	61	260	56	.14	.1?	61	..	...	1.00144	53	..
Coal-gas	H_2, CH_4, etc. .	6?	.0005?	..	...	...	low?	...	..	..	..	..	.15?	1.0019	...	..	..
Cyanide Ethyl	C_2H_5CN . .	..	...	..	...	...	97	...	..	.43	..	..	..	...	...	..	..
Cyanogen	$(CN)_2$ . . .	26.1	.00230	.987	.00388	—35	—21?	...	..	..	..	..	..	...	1.000822	40	..
Ethane†	C_2H_6 . . .	..	gas	..	...	...	low?	...	..	..	..	..	..	...	...	..	006
Ether	$(C_2H_5)_2O$ . .	37.3	liquid	..	...	...	35	193	38	.43	.4?	91	..	...	1.00152?	53	..
Ethylene††	C_2H_4	14.1	.001253	..	...	...	—110	+5	44	.39	.32	..	.041	...	1.000678	..	.02
" Oxide . . .	C_2H_4O . . .	20.5	...	..	...	...	13	...	..	..	..	139	..	...	...	..	..
Fluoride Boron . . .	BF_3	34.2	.0030	..	...	...	low?	...	..	..	..	..	..	...	...	..	..
" Silicon . . .	SiF_4	51.9	.0046	..	...	...	low?	...	..	..	..	..	..	...	...	..	..
Hydriodic Acid Gas. .	HI	64.1	.0057	..	...	—51	—40?	...	..	.055	.039	..	..	...	...	..	..
Hydrobromic " " . .	HBr	39.1	.0035	..	...	—87	—60?	...	..	.082	.057	..	..	...	...	..	..
Hydrochloric " " . .	HCl	18.0	.0017	..	...	—110	—80	51	..	.190	.135	..	..	...	...	..	..

* Tetra-formerly called Bi-chloride. † Bicarburetted Hydrogen. †† Olefiant Gas.

Name	Symbol	Sp.Gr.ref.to Hydrogen	Density, 0°, 1,000,000 dynes per sq. cm.	Resilience of Volume, 0°, p=10⁶	Coefficient Expansion 0–100°, 76 cm.	Temperature of Solidification	Temp. of Condensation 76 cm.	Critical Temperature	Critical Pressure	Specif.Heat Constant Pressure	Specif.Heat Const. Vol.	Latent Heat Condensation	Heat Conductivity	Specific Inductive Capacity	Index of Refraction 0°, 76 cm. Line D	Index of Dispersion A–H	Solubility % in Water, 20°, 76 cm.
Multiply by.		..	...	10^6	...	..	..	..	10^6	..	..	..	.001	...	...	10^{-6}	...
Hydrocyanic Acid	HCN	13.7	liquid	..	...	—15	26	...	..	..	..	..	..	...	...	..	..
Hydrofluoric "	HF	..	liquid	..	...	—34	20	...	..	..	..	..	..	...	...	..	..
Hydrogen	H_2	1.000	.0000884	1.001	.00366	low.	low.	—174	100	3.40	2.40	..	.39	1.0013	1.000139	2.8	.0002
Iodine	I_2	126	solid	..	...	110	200	...	..	.033	.025	24	..	...	...	..	..
Mercury*	Hg	100.6	liquid	..	...	—39	350	...	..	..	..	62	..	...	1.00056	..	..
Methane**	CH_4	8.0+	.000717	..	...	..	low.	—76	47	.59	.47?	..	.076	...	...	..	.002
Nitric Anhydride	N_2O_5	..	solid	..	...	30	47	...	..	..	..	45	..	...	...	..	..
Nitric Oxide	NO	15.0	.001325	..	...	..	low	...	..	.22	.16	..	.052	...	1.000298	..	.09
Nitrous Oxide†	N_2O	22.0	.001943	.988	.00372	—100	—90	...	..	.21	.16	101	.035	...	1.000516	30	.—
Nitrogen	N_2	14.03	.001239	.999	.00367	..	low	—124	43	.24	.17	..	.054	...	1.000298	9.6	.0020
Oxygen	O_2	15.95	.001411	..	.00367	..	low	—105	49	.22	.16	..	.056	...	1.000271	8.3	.0040
Phosphorus	P_4	63.8	solid	..	...	44.3	289	...	..	..	..	..	..	...	1.0014?	..	...
Phosph'd Hydrogen	H_3P	17.5	.0015+	..	...	..	low?	...	..	..	..	..	..	...	...	..	...
Sulphide Carbon, Bi-	CS_2	38.1	liquid	..	...	..	47	272	76	.16?	.13?	90	..	...	1.0015?	150?	...
" Ethyl	$(C_2H_5)_2S$	...	...	..	...	..	91	...	..	.40	.38?	..	..	...	...	..	...
Sulphur††	S_6 (at 450°)	95.5	solid	..	...	114	448	...	..	..	..	362	..	...	1.0016?	..	...
Sulph'd Hydrogen	H_2S	17.2	.0015	..	...	—86	—62?	...	..	.24	.18?	..	..	...	1.00064?	..	.44
Sulphuric Anhydride	SO_3	39.9	solid	..	...	16	46+	...	..	..	..	147	..	...	...	..	..
Sulphurous " §	SO_2	32.	.002699	.984	.00390	—78	—9	155	80	.15	.11?	92	..	1.0052	1.00070	..	..
Turpentine	$C_{10}H_{16}$	69.	liquid	..	...	—10	160	...	..	.5	..	70	..	...	...	..	9.
Water§§	H_2O	9.00	freezes	..	...	0	100	400?	..	.48	.37?	536	..	...	...	..	..

* See Table 13, C. ** Marsh Gas. † Laughing Gas.
†† See Table 13, C. § Sulphurous Acid Gas. §§ Aqueous Vapor, or Steam. See Tables 13, C–D.

13, A. Maximum Pressure of Vapors at Different Temperatures (−70° to +120°) in Megadynes per sq. c.

Name	Symbol	−70°	−60°	−50°	−40°	−30°	−20°	−10°	0°	+10°	+20°	+30°	+40°	+50°	+60°	+70°	+80°	+90°	+100	+110	+120
Acetylene	C_2H_2	..	..	..	..	..	12?	20?	30?	45?	60?	80?	..	..	..	..	..	..	..	..	..
Ammonia	H_3N	..	..	..	..	1.2	1.9	2.8	4.2	6.0	8.5	12	15	20	26	33	41	51	62	..	..
Arsen'd Hydrogen	H_3As	..	0.9	1.5	2	3	5	7	9	12	..	..	..	..	..	..	..	..	..	..	..
Carbonic Dioxide	CO_2	2	4	7	10	15	20	27	36	47	60	75	92	..	..	..	..	..	..	..	..
Chloride Boron ..	BCl_3	..	..	..	..	..	.23	.34	.51	.75	1.1	1.5	2.1	2.7	3.6	4.5	5.7	..	..	..	..
„ Phosphorus	PCl_3	..	..	..	..	..	..	..	.05	.08	.13	.21	.31	.46	.65	.90	..	..	..	..	..
„ Silicon	$SiCl_4$	..	..	..	..	..	.03	.06	.11	.17	.26	.39	.57	.81	1.1	..	..	..	..	..	..
Chlorine	Cl_2	..	..	..	..	1?	2?	2?	3?	4?	4?	..	..	..	..	..	..	..	..	..	..
Cyanogen	C_2N_2	..	..	..	..	..	1.2	1.7	2.4	3.3	4.4	6	8	..	..	..	..	..	..	..	..
Ethane	C_2H_6	..	..	..	..	..	..	..	46	(at	4°)	..	..	..	..	..	..	..	..	..	..
Ethylene	C_2H_4	5	7	10	14	19	25	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Fluoride Boron ..	BF_3	5	8	13	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Hydriodic Acid ..	HI	..	..	..	..	..	2.8	3.3	4.0	5	7	..	..	..	..	..	..	..	..	..	..
Hydrochloric „ ..	HCl	2	3	5	7+	10	14	29	27	36	45	55	68	85	..	..	..	..	..	..	..
Methylether	$CH_3OC_2H_5$	..	..	..	..	.77	1.2	1.7	2.5	3.4	4.8	6.4	..	..	..	..	..	..	..	..	..
Nitrous Oxide ...	N_2O	3	5	8	11	16	22	29	36	45	56	69	84	..	..	..	..	..	..	..	..
Sulph'd Hydrogen	H_2S	1.1	1.5	2	3	4	6	8	11	14	19	24	30	37	45	54	..	..	..	..	..
Sulphurous Anh.	SO_2	..	..	..	..	0.4	0.6	1.0	1.5	2.3	3.3	4.6	6.2	8.3	11	14	18	22	28	..	42

13 B. Maximum Pressure of Vapors at Different Temperatures (0°–190°) in Megadynes per sq. cm.

Name	Symbol	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	100	110	120	130	140	150	160	170	180	190
Acetone	$(CH_3)_2CO$	..	..	.24	.38	.56	.81	1.2	1.6	2.2	2.9	3.7	4.8	6.1	7.6	9.3	..	..	..	..	..
Acid Acetic . .	HOC_2H_3O	.012	.016	.025	.039	.059	.088	.13	.19	.27	.39	.54	.76	1.04	1.42	1.91	..	..	..	..	..
„ Butyric . .	HOC_4H_7O	..	.007	.010	.014	.019	.027	.038	.053	.074	.10	.14	.20	.27	.38	.52	.70	96	1.31	1.8	..
„ Formic . .	$HOCHO$	..	.025	.042	.069	.11	.17	.26	.37	.53	.74	1.02	..	..	..	..	..	..	..	..	..
„ Propionic .	HOC_3H_5O	..	.007	.011	.017	.026	.037	.057	.084	.12	.18	.26	.36	.52	.73	1.02	1.42	..	..	..	..
„ Valeric, Iso-	HOC_5H_9O	..	.006	.008	.012	.016	.022	.030	.041	.056	.077	.11	.14	.20	.26	.36	.49	.66	.88	1.19	1.6
Alcohol Ethyl. .	C_2H_5OH	.017	.032	.059	.10	.18	.29	.47	.72	1.08	1.6	2.3	3.2	4.3	5.8	7.6	9.8	..	..	..	..
„ Methyl .	CH_3OH	.036	.067	.12	.20	.33	.51	.77	1.14	1.7	2.3	3.2	4.4	5.8	7.6	9.8	13	..	..	..	..
Benzene	C_6H_6	.034	.060	.10	.16	.25	.36	.52	.73	1.00	1.4	1.8	2.3	3.0	3.8	4.7	5.8	7.0	8.5	..	..
Bromide Ethyl .	C_2H_5Br	.022	.035	.052	.075	.11	.14	.20	.27	.35	.45	.58	.72	.89	1.08	1.3	..	..	..	..	..
„ Ethylene	$C_2H_4Br_2$	.005	.009	.014	.023	.037	.057	.088	.13	.19	.28	.39	.54	.73	.97	1.28	1.6	2.1	2.6	3.3	4.0
Chloride Carbon.	CCl_4	.044	.075	.12	.19	.29	.42	.60	.83	1.12	1.5	2.0	2.5	3.2	4.0	5.0	6.1	7.3	8.9	11	13
„ Ethyl .	C_2H_5Cl	.62	.92	1.3	1.9	2.6	3.4	4.5	5.9	7.5	9.4	12	15	18	21	26	31	36	43	..	..
„ Methyl .	CH_3Cl	2.5	3.5	4.9	6.6	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Chloroform . .	$CHCl_3$	..	..	.21	.33	.48	.72	1.01	1.4	1.9	2.5	3.2	4.2	5.2	6.5	8.0	9.7	12	..	..	..
Ether	$(C_2H_5)_2O$	.25	.38	.58	.85	1.2	1.7	2.3	3.1	4.0	5.2	6.6	8.3	10	..	..	..	..	..	..	..
Iodide Ethyl . .	C_2H_5I	.056	.093	.15	.23	.34	.49	.68	..	..	..	..	..	..	..	..	..	..	..	..	..
Sulphide Carbon, Bi-	CS_2	.17	.27	.40	.58	.82	1.14	1.6	2.1	2.7	3.5	4.4	5.5	6.9	8.4	10	12	..	..	..	..
Turpentine . . .	$C_{10}H_{16}$	.003	.004	.006	.009	.014	.023	.035	.054	.082	.12	.18	.25	.34	.47	.62	.81	1.03	1.3	1.6	2.0

13C. Pressure of the Vapors of Mercury, Sulphur and Water in Megadynes per sq. cm (0° – 600°).

Temperature	0°	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190°
Mercury Hg	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0004	.0006	.0010	.0015	.0022	.0033	.0045	.007	.010	.013	.018
Temperature	200°	210	220	230	240	250	260	270	280	290	300	310	320	330	340	350	360	370	380	390°
Mercury Hg	.025	.034	.045	.060	.078	.101	.129	.165	.208	.260	.323	.400	.492	.602	.732	.885	1.06	1.27	1.52	1.80
Temperature	400°	410	420	430	440	450	460	470	480	490	500	510	520	530	540	550	560	570	580	590°
Mercury Hg	2.12	2.48	2.90	3.38	3.91	4.51	5.19	5.93	6.77	7.68	8.97	9.81	11.1							
Sulphur S	.439	.529	.630	.748	.884	1.04	1.22	1.42	1.64	1.90	2.18	2.50	2.84	3.23	3.65	4.12	4.63	5.17		
Temperature	0°	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190°
Aqueous Vapor from Sulphuric Acid: 85 % $H_2SO_4.H_2O$		.0001	.0002	.0003																
73 % $H_2SO_4.2H_2O$		.0007	.0011	.0020																
52 % $H_2SO_4.5H_2O$		.0040	.0077	.0142																
33 % $H_2SO_4.11H_2O$		.0085	.0161	.0295																
Water H_2O	.006	.012	.023	.042	.073	.123	.198	.310	.472	.701	1.014	1.44	2.0	2.7	3.6	4.8	6.2	8.0	10.1	12.6

13D. Density of Steam saturated at Different Temperatures.

Temperature	0°	10	20	30	40	50	60	70	80	90	100°
Density	.000005	.000009	.000017	.000030	.000051	.000083	.000131	.000199	.000296	.000428	.000606
Temperature	100°	110	120	130	140	150	160	170	180	190	200°
Density	.000606	.000840	.00114	.00153	.00201	.00260	.00333	.00421	.00526	.00650	.00796

14. Boiling Points of Water at Different Pressures (g = 980.61).

Barometric Pressure, cm.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
68	96.92	96.96	97.00	97.05	97.09	97.13	97.17	97.21	97.25	97.29
69	97.33	97.36	97.40	97.44	97.48	97.52	97.56	97.60	97.64	97.68
70	97.72	97.76	97.80	97.84	97.88	97.92	97.96	98.00	98.03	98.07
71	98.11	98.15	98.19	98.23	98.27	98.31	98.34	98.38	98.42	98.46
72	98.50	98.54	98.58	98.61	98.65	98.69	98.73	98.77	98.80	98.84
73	98.88	98.92	98.96	98.99	99.03	99.07	99.11	99.14	99.18	99.22
74	99.26	99.30	99.33	99.37	99.41	99.44	99.48	99.52	99.56	99.59
75	99.63	99.67	99.71	99.74	99.78	99.82	99.85	99.89	99.93	99.96
76	100.00	100.04	100.07	100.11	100.15	100.18	100.22	100.26	100.29	100.33
77	100.36	100.40	100.44	100.47	100.51	100.55	100.58	100.62	100.65	100.69

14A. Dew Points corresponding to Different Degrees of Temperature and Relative Humidity.

Temperature of the Air.	Relative Humidity of the Air.									
	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
0°	..	..	−16	−12	−9	−7	−5	−3	−1	0°
1	..	..	15	11	8	6	4	2	0	+1
2	..	−19	14	10	7	5	3	−1	+1	2
3	..	18	13	9	6	4	2	0	2	3
4°	..	−17	−12	−8	−6	−3	−1	+1	+3	+4
5°	..	−16	−11	−7	−5	−2	0	+2	+3	+5°
6	..	15	10	7	4	−1	+1	3	4	6
7	..	15	9	6	3	0	2	4	5	7
8	..	14	−9	5	2	+1	3	5	6	8
9°	..	−13	−8	−4	−1	+2	+4	+6	+7	+9°
10°	..	−12	−7	−3	0	+3	+5	+7	+8	+10°
11	..	11	6	2	+1	3	6	8	9	11
12	−19	10	5	−1	2	4	7	9	10	12
13	18	10	4	0	3	5	8	10	11	13
14°	−17	−9	−3	+1	+4	+6	+9	+11	+12	+14°
15°	−17	−8	−3	+2	+5	+7	+10	+12	+13	+15°
16	16	7	2	2	6	8	11	13	14	16
17	15	6	−1	3	6	9	11	14	15	17
18	14	5	0	4	7	10	12	14	16	18
19°	−13	−5	+1	+5	+8	+11	+13	+15	+17	+19°
20°	−13	−4	+2	+6	+9	+12	+14	+16	+18	+20°
21	12	3	3	7	10	13	15	17	19	21
22	11	2	4	8	11	14	16	18	20	22
23	10	−1	4	9	12	15	17	19	21	23
24°	−10	0	+5	+10	+13	+16	+18	+20	+22	+24°
25°	−9	0	+6	+10	+14	+17	+19	+21	+23	+25°
26	8	+1	7	11	15	18	20	22	24	26
27	7	2	8	12	16	19	21	23	25	27
28	7	3	9	13	17	20	22	24	26	28
29°	−6	+4	+10	+14	+18	+20	+23	+25	+27	+29°
30°	−5	+5	+11	+15	+18	+21	+24	+26	+28	+30°
31	4	5	11	16	19	22	25	27	29	31
32	3	6	12	17	20	23	26	28	30	32
33	3	7	13	18	21	24	27	29	31	33
34	2	8	14	18	22	25	28	30	32	34
35°	−1	+9	+15	+19	+23	+26	+29	+31	+33	+35°

15. Hygrometric Table, showing at a given temperature (T), the maximum pressure (P) of aqueous vapor in mercurial centimetres, the maximum density (D) of aqueous vapor, and the factor (F) by which the difference between a wet and a dry bulb thermometer must be multiplied to find the difference between the dew-point and the temperature (T) of the air.

T	P	D	F	T	P	D	F
—10°	0.22	.0000023	8.8	+10°	0.91	.0000093	2.1
—9	.23	25	8.5	11	0.98	.0000100	2.0
—8	.25	27	8.2	12	1.04	106	2.0
—7	.27	29	7.9	13	1.11	112	2.0
—6	.29	32	7.6	14	1.19	120	1.9
—5°	0.32	.0000034	7.3	+15°	1.27	.0000128	1.9
—4	.34	37	6.8	16	1.35	135	1.9
—3	.37	40	6.0	17	1.44	144	1.9
—2	.39	42	5.0	18	1.53	152	1.8
—1	.42	45	4.1	19	1.63	162	1.8
0°	0.46	.0000049	3.3	+20°	1.74	.0000172	1.8
+1	.49	52	2.9	21	1.85	182	1 8
+2	.53	56	2 6	22	1.96	193	1.7
+3	.57	60	2.5	23	2.09	204	1.7
+4	.61	64	2.4	24	2.22	216	1 7
+5°	0.65	.0000068	2.3	+25°	2.35	.0000229	1.7
+6	.70	73	2.2	26	2.50	242	1.7
+7	.75	77	2.2	27	2.65	256	1.7
+8	.80	82	2.1	28	2.81	270	1 7
+9	.85	87	2.1	29	2.97	285	1.7
10°	.91	.0000093	2.1	+30°	3.15	.0000301	1.6

15 A. Specific Heat of Moist Air under Constant Pressure (76 cm.)

Dew-Point	Specific Heat.	Dew-Point	Specific Heat.	Dew-Point.	Specific Heat.
— ∞°	.2383	—11°	.2387	+12°	.2404
—33	.2383	—10	.2387	13	.2405
—32	.2384	— 9	.2388	14	.2407
—31	.2384	— 8	.2388	15	.2408
—30	.2384	— 7	.2388	16	.2410
—29	.2384	— 6	.2389	17	.2412
—28	.2384	— 5	.2389	18	.2414
—27	.2384	— 4	.2390	19	.2416
—26	.2384	— 3	.2390	20	2418
—25	.2384	— 2	.2391	21	.2420
—24	.2384	— 1	.2392	22	.2423
—23	.2384	0	.2392	23	.2425
—22	.2385	+ 1	.2393	24	.2428
—21	.2385	2	.2394	25	.2430
—20	.2385	3	.2394	26	.2433
—19	.2285	4	.2395	27	.2436
—18	.2385	5	.2396	28	.2440
—17	.2385	6	.2397	29	.2443
—16	.2386	7	.2398	30	.2447
—15	.2386	8	.2399	31	.2451
—14	.2386	9	.2400	32	.2455
—13	.2386	10	.2401	33	.2459
—12°	.2387	11°	.2403	100°	.4805

15, B. Velocity of Sound in centimetres per second through Atmospheric Air at Different Temperatures and under Different Conditions of Relative Humidity.

Temperature of the Air. / Relative Humidity	0 %	20 %	40 %	60 %	80 %	100 %
0°	33,220	33,225	33,231	33,236	33,242	33,247
1°	33,281	33,286	33,292	33,298	33,304	33,310
2°	33,341	33,347	33,353	33,360	33,367	33,373
3°	33,402	33,408	33,415	33,422	33,429	33,436
4°	33,462	33,469	33,476	33,484	33,491	33,499
5°	33,523	33,530	33,538	33,546	33,554	33,562
6°	33,583	33,591	33,600	33,608	33,617	33,625
7°	33,643	33,652	33,661	33,670	33,679	33,689
8°	33,703	33,713	33,722	33,732	33,742	33,752
9°	33,763	33,773	33,784	33,794	33,805	33,815
10°	33,823	33,834	33,845	33,856	33,867	33,879
11°	33,882	33,894	33,906	33,918	33,930	33,942
12°	33,942	33,955	33,967	33,980	33,993	34,006
13°	34,001	34,015	34,029	34,043	34,056	34,070
14°	34,060	34,075	34,090	34,105	34,119	34,134
15°	34,120	34,136	34,151	34,167	34,183	34,198
16°	34,179	34,196	34,213	34,229	34,246	34,263
17°	34,238	34,256	34,274	34,292	34,310	34,328
18°	34,297	34,316	34,335	34,354	34,374	34,393
19°	34,356	34,376	34,397	34,417	34,438	34,458
20°	34,415	34,436	34,458	34,480	34,502	34,524
21°	34,474	34,496	34,520	34,543	34,566	34,589
22°	34,532	34,557	34,581	34,606	34,630	34,655
23°	34,590	34,617	34,643	34,669	34,695	34,722
24°	34,649	34,677	34,705	34,732	34,761	34,789
25°	34,707	34,737	34,766	34,796	34,826	34,856
26°	34,765	34,797	34,828	34,860	34,892	34,924
27°	34,823	34,857	34,890	34,924	34,958	34,992
28°	34,881	34,917	34,953	34,988	35,025	35,061
29°	34,939	34,977	35,015	35,053	35,092	35,130
30°	34,997	35,037	35,077	35,118	35,158	35,199
31°	35,055	35,097	35,139	35,182	35,225	35,269
32°	35,113	35,157	35,202	35,247	35,293	35,340
33°	35,170	35,218	35,265	35,313	35,362	35,412

15, C. Coefficients of Interdiffusion of Gases. (C. G. S.)*

	Air	Carbonic Oxide CO	Hydrogen H_2	Methane CH_4	Nitrous Oxide N_2O	Oxygen O_2	Sulphurous Anhydride SO_2
Carbonic Dioxide CO_2	.1423		.5614	.1586	.0982	.1409	
Hydrogen H_2		.6422				.7214	.4800
Oxygen O_2 . .		.1802	.7214				

* See Maxwell's Theory of Heat, 4th Ed. page 332. (Everett Art. 131.)

Reduction of Inches to Centimetres.

Inches.	0	1	2	3	4	5	6	7	8	9
28.0	71.119	.145	.170	.196	.221	.246	.272	.297	.323	.348
28.1	71.373	.399	.424	.450	.475	.500	.526	.551	.577	.602
28.2	71.627	.653	.678	.704	.729	.754	.780	.805	.831	.856
28.3	71.881	.907	.932	.958	.983	*008	*034	*059	*085	*110
28.4	72.135	.161	.186	.212	.237	.262	.288	.313	.339	.364
28.5	72.389	.415	.440	.466	.491	.516	.542	.567	.593	.618
28.6	72.643	.669	.694	.720	.745	.770	.796	.821	.847	.872
28.7	72.897	.923	.948	.974	.999	*024	*050	*075	*101	*126
28.8	73.151	.177	.202	.228	.253	.278	.304	.329	.355	.380
28.9	73.405	.431	.456	.482	.507	.532	.558	.583	.609	.634
29.0	73.659	.685	.710	.736	.761	.786	.812	.837	.863	.888
29.1	73.913	.939	.964	.990	*015	*040	*066	*091	*117	*142
29.2	74.167	.193	.218	.244	.269	.294	.320	.345	.371	.396
29.3	74.421	.447	.472	.498	.523	.548	.574	.599	.625	.650
29.4	74.675	.701	.726	.752	.777	.802	.828	.853	.879	.904
29.5	74.929	.955	.980	*006	*031	*056	*082	*107	*133	*158
29.6	75.183	.209	.234	.260	.285	.310	.336	.361	.387	.412
29.7	75.437	.463	.488	.514	.539	.564	.590	.615	.641	.666
29.8	75.691	.717	.742	.768	.793	.818	.844	.869	.895	.920
29.9	75.945	.971	.996	*022	*047	*072	*098	*123	*149	*174
30.0	76.199	.225	.250	.276	.301	.326	.352	.377	.403	.428
30.1	76.453	.479	.504	.530	.555	.580	.606	.631	.657	.682
30.2	76.707	.733	.758	.784	.809	.834	.860	.885	.911	.936
30.3	76.961	.987	*012	*038	*063	*088	*114	*139	*165	*190
30.4	77.215	.241	.266	.292	.317	.342	.368	.393	.419	.444
30.5	77.469	.495	.520	.546	.571	.596	.622	.647	.673	.698
30.6	77.723	.749	.774	.800	.825	.850	.876	.901	.927	.952
30.7	77.977	*003	*028	*053	*079	*104	*130	*155	*180	*206
30.8	78.231	.257	.282	.307	.333	.358	.384	.409	.434	.460
30.9	78.485	.511	.536	.561	.587	.612	.638	.663	.688	.714
31.0	78.739	.765	.790	.815	.841	.866	.892	.917	.942	.968
31.1	78.993	*019	*044	*069	*095	*120	*146	*171	*196	*222
31.2	79.247	.273	.298	.323	.349	.374	.400	.425	.450	.476
31.3	79.501	.527	.552	.577	.603	.628	.654	.679	.704	.730
31.4	79.755	.781	.806	.831	.857	.882	.908	.933	.958	.984
31.5	80.009	.035	.060	.085	.111	.136	.162	.187	.212	.238
Dif. In.	.001	.002	.003	.004	.005	.006	.007	.008	.009	.010
Dif. Cm.	.003	.005	.008	.010	.013	.015	.018	.020	.023	.025

* The star indicates that the number of whole centimetres is to be read from the line underneath it.

16 A. Reduction of Mercurial Centimetres to Megadynes per sq. cm. g=980.

cm	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Dif.
70	0.9327	0.9340	0.9354	0.9367	0.9380	0.9393	0.9407	0.9420	0.9433	0.9447	13.3
71	.9460	.9473	.9487	.9500	.9513	.9527	.9540	.9553	.9567	.9580	1 1
72	.9593	.9607	.9620	.9633	.9647	.9660	.9673	.9687	.9700	.9713	2 3 3 4
73	.9727	.9740	.9753	.9767	.9780	.9793	.9807	.9820	.9833	.9847	4 5
74	.9860	.9873	.9886	.9900	.9913	.9926	.9940	.9953	.9966	.9980	5 7
75	.9993	1.0006	1.0020	1.0033	1.0046	1.0060	1.0073	1.0086	1.0100	1.0113	6 8 7 9
76	1.0126	1.0140	1.0153	1.0166	1.0180	1.0193	1.0206	1.0220	1.0233	1.0246	8 11
77	1.0260	1.0273	1.0286	1.0300	1.0313	1.0326	1.0339	1.0353	1.0366	1.0379	9 12

16 B. Reduction of Mercurial Centimetres to Megadynes per sq. cm. g=981.

cm	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Dif.
70	0.9336	0.9350	0.9363	0.9376	0.9390	0.9403	0.9416	0.9430	0.9443	0.9456	13.3
71	.9470	.9483	.9496	.9510	.9523	.9536	.9550	.9563	.9576	.9590	1 1
72	.9603	.9616	.9630	.9643	.9656	.9670	.9683	.9696	.9710	.9723	2 3 3 4
73	.9737	.9750	.9763	.9777	.9790	.9803	.9817	.9830	.9843	.9857	4 5
74	.9870	.9883	.9897	.9910	.9923	.9937	.9950	.9963	.9977	.9990	5 7
75	1.0003	1.0017	1.0030	1.0043	1.0057	1.0070	1.0083	1.0097	1.0110	1.0123	6 8 7 9
76	1.0137	1.0150	1.0163	1.0177	1.0190	1.0203	1.0217	1.0230	1.0243	1.0257	8 11
77	1.0270	1.0283	1.0297	1.0310	1.0323	1.0337	1.0350	1.0363	1.0377	1.0390	9 12

17. Elevation in Metres above the Sea Level corresponding to Different Barometric Pressures at 10° Centigrade (g=980.6).

cm	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
60	1959	1945	1931	1918	1904	1890	1876	1863	1849	1836
61	1822	1808	1795	1782	1768	1754	1741	1727	1714	1701
62	1687	1674	1660	1647	1634	1621	1607	1594	1581	1568
63	1555	1541	1528	1515	1502	1489	1476	1463	1450	1437
64	1424	1411	1398	1385	1372	1360	1347	1334	1321	1308
65	1295	1283	1270	1257	1245	1232	1219	1207	1194	1182
66	1169	1157	1144	1131	1119	1107	1094	1082	1069	1057
67	1044	1032	1020	1007	995	983	971	958	946	934
68	922	910	897	885	873	861	849	837	825	813
69	801	789	777	765	753	741	729	717	705	693
70	681	670	658	646	634	623	611	599	587	576
71	564	552	541	529	517	506	494	483	471	460
72	448	437	425	414	402	391	379	368	356	345
73	334	322	311	300	288	277	266	255	243	232
74	221	210	199	187	176	165	154	143	132	121
75	110	99	88	77	66	55	44	33	22	11
76	0	−11	−22	−33	−43	−54	−65	−76	−87	−98
77	−108	−119	−130	−141	−151	−162	−173	−183	−194	−205
78	−215	−226	−236	−247	−258	−268	−279	−289	−300	−310

17 A. Correction for Temperature in 17.

Mean Temp.	Subtr. %	Mean Temp.	Add %	Mean Temp.	Add %
0°	3.5	10	0.0	20	3.5
1	3.2	11	0.4	21	3.9
2	2.8	12	0.7	22	4.2
3	2.5	13	1.1	23	4.6
4	2.1	14	1.4	24	5.0
5	1.8	15	1.8	25	5.3
6	1.4	16	2.1	26	5.7
7	1.1	17	2.5	27	6.0
8	0.7	18	2.8	28	6.4
9°	0.4	19	3.2	29	6.7

17 B. Correction for Humidity in 17.

Dew-Point	Add %	Dew-Point	Add %	Dew-Point	Add %
−∞	0.0	+10	0.5	+20	0.9
−20	0.0	11	0.5	21	0.9
−15	0.1	12	0.5	22	1.0
−10	0.1	13	0.6	23	1.1
−5	0.2	14	0.6	24	1.1
0	0.2	15	0.6	25	1.2
+2	0.3	16	0.7	26	1.3
+4	0.3	17	0.7	27	1.3
+6	0.3	18	0.8	28	1.4
+8	0.4	19	0.8	29	1.5

18 a. Reduction of Mercurial Columns to 0°. Corrections for Expansion to be subtracted.

Temperature	Length in centimetres of the Mercurial Column measured by a Brass Scale.									Correction for glass scale
	70	71	72	73	74	75	76	77	78	
	cm	cm	cm	cm	cm	cm	cm	cm	cm	
0°	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	011	011	012	012	012	012	012	012	013	001
2	023	023	023	024	024	024	024	025	025	002
3	034	034	035	035	036	036	037	037	038	002
4	045	046	046	047	048	048	049	050	050	003
5	0.056	0.057	0.058	0.059	0.060	0.060	0.061	0.062	0.063	0.004
6	068	069	069	071	072	072	073	074	075	005
7	079	080	081	082	083	085	086	087	088	006
8	090	092	093	094	095	097	098	099	101	006
9	102	103	104	106	107	109	110	112	113	007
10	0.113	0.114	0.116	0.118	0.119	0.121	0.122	0.124	0.126	0.008
11	124	126	128	129	131	133	135	137	138	009
12	135	137	139	141	143	145	147	149	151	009
13	147	149	151	153	155	157	159	161	164	010
14	158	160	163	165	167	169	172	174	176	011
15	0.169	0.172	0.174	0.177	0.179	0.181	0.184	0.186	0.189	0.012
16	181	183	186	188	191	194	196	199	201	013
17	192	195	197	200	203	206	208	211	214	013
18	203	206	209	212	215	218	221	224	227	014
19	215	218	221	224	227	230	233	236	239	015
20	0.226	0.229	0.232	0.236	0.239	0.242	0.245	0.248	0.252	0.016
21	237	241	244	247	251	254	258	261	264	017
22	249	252	256	259	263	266	270	273	277	017
23	260	264	267	271	275	278	282	286	290	018
24	271	275	279	283	287	291	294	298	302	019
25	0.283	0.287	0.291	0.295	0.299	0.303	0.307	0.311	0.315	0.020
26	294	298	302	306	311	315	319	323	327	021
27	305	310	314	318	323	327	331	336	340	021
28	317	321	326	330	335	339	344	348	353	022
29	328	333	337	342	347	351	356	361	365	023
30	0.339	0.344	0.349	0.354	0.359	0.363	0.368	0.373	378	024

18 b. Correction for the Capillarity of Mercurial Columns to be added.

Internal Diameter of Tube	Height of Meniscus unknown	Height of Meniscus in Centimetres							
		0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18
0.1 cm	.9?								
0.2	.46								
0.3	.29								
0.4	.26	0.083	0.122	0.154	0.198	0.237	...	...	...
0.5	.15	.047	.065	.086	.119	.145	0.180	...	...
0.6	.11	.027	.041	.056	.078	.098	.121	0.143	...
0.7	.09	.018	.028	.040	.053	.067	.082	.097	0.113
0.8	.07		.020	.029	.038	.046	.056	.065	.077
0.9	.05		.015	.021	.028	.033	.040	.046	.052
1.0	.04			.015	.020	.025	.029	.033	.037
1.1	.03			.010	.014	.018	.021	.024	.027
1.2	.03			.007	.010	.013	.015	.018	.019
1.3	.02			.004	.007	.010	.012	.013	.014

18 c. Correction for the Pressure of Mercurial Vapor to be added.

Temperature	0°	5°	10°	15°	20°	25°	30°	35°	40°
Add cm.	0.001?	.001?	.002?	.002?	.002?	.002?	.003?	.003?	.004?

18, d. Factors for the Reduction of the Density of a Gas to 76 cm.

Pressure cm	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Dif.
70	1.0857	1.0842	1.0826	1.0811	1.0795	1.0780	1.0765	1.0750	1.0734	1.0719	
71	1.0704	1.0689	1.0674	1.0659	1.0644	1.0629	1.0615	1.0600	1.0585	1.0570	15
72	1.0556	1.0541	1.0526	1.0512	1.0497	1.0483	1.0468	1.0454	1.0440	1.0425	
73	1.0411	1.0397	1.0383	1.0368	1.0354	1.0340	1.0326	1.0312	1.0298	1.0284	
74	1.0270	1.0256	1.0243	1.0229	1.0215	1.0201	1.0188	1.0174	1.0160	1.0147	14
75	1.0133	1.0120	1.0106	1.0093	1.0080	1.0066	1.0053	1.0040	1.0026	1.0013	
76	1.0000	.9987	.9973	.9961	.9948	.9935	.9922	.9909	.9896	.9883	13
77	.9870	.9857	.9845	.9832	.9819	.9806	.9794	.9781	.9769	.9756	

18, e. Factors for the Reduction of the Density of a Gas to 0° Centigrade.

Temperature	+0°	+1°	+2°	+3°	+4°	+5°	+6°	+7°	+8°	+9°	Dif.
0°	1.0000	1.0037	1.0073	1.0110	1.0147	1.0184	1.0220	1.0257	1.0294	1.0330	36.7
10	1.0367	1.0404	1.0440	1.0477	1.0514	1.0551	1.0587	1.0624	1.0661	1.0697	1 4
20	1.0734	1.0771	1.0807	1.0844	1.0881	1.0918	1.0954	1.0991	1.1028	1.1064	2 7
30	1.1101	1.1138	1.1174	1.1211	1.1248	1.1285	1.1321	1.1358	1.1395	1.1431	3 11
40	1.1468	1.1505	1.1541	1.1578	1.1615	1.1652	1.1688	1.1725	1.1762	1.1798	4 15
50	1.1835	1.1872	1.1908	1.1945	1.1982	1.2019	1.2055	1.2092	1.2129	1.2165	5 18
60	1.2202	1.2239	1.2275	1.2312	1.2349	1.2386	1.2422	1.2459	1.2496	1.2532	6 22
70	1.2569	1.2606	1.2642	1.2679	1.2716	1.2753	1.2789	1.2826	1.2863	1.2899	7 26
80	1.2936	1.2973	1.3009	1.3046	1.3083	1.3120	1.3156	1.3193	1.3230	1.3266	8 29
90	1.3303	1.3340	1.3376	1.3413	1.3450	1.3487	1.3523	1.3560	1.3597	1.3633	9 33
100°	1.3670	1.3707	1.3743	1.3780	1.3817	1.3854	1.3890	1.3927	1.3964	1.4000	

18, f. Factors for the Reduction of the Volume of a Gas to 76 cm.

Pressure cm	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Dif.
70	0.9211	0.9224	0.9237	0.9250	0.9263	0.9276	0.9289	0.9303	0.9316	0.9329	13. 16
71	.9342	.9355	.9368	.9382	.9395	.9408	.9421	.9434	.9447	.9461	1
72	.9474	.9487	.9500	.9513	.9526	.9539	.9553	.9566	.9579	.9592	1 1 2 3
73	.9605	.9618	.9632	.9645	.9658	.9671	.9684	.9697	.9711	.9724	3 4
74	.9737	.9750	.9763	.9776	.9789	.9803	.9816	.9829	.9842	.9855	4 5 5 7
75	.9868	.9882	.9895	.9908	.9921	.9934	.9947	.9961	.9974	.9987	6 8
76	1.0000	1.0013	1.0026	1.0039	1.0053	1.0066	1.0079	1.0092	1.0105	1.0118	7 9 8 11
77	1.0132	1.0145	1.0158	1.0171	1.0184	1.0197	1.0211	1.0224	1.0237	1.0250	9 12

18, g. Factors for the Reduction of the Volume of a Gas to 0° Centigrade.

Temperature		Temperature		Temperature		Temperature		Temperature		Temperature		Temperature	
0°	1.0000	5°	0.9820	10°	0.9646	15	0.9478	20°	0.9316	25°	0.9160	30°	0.9008
1	0.9963	6	.9785	11	.9612	16	.9445	21	.9285	26	.9129	31	.8978
2	.9927	7	.9750	12	.9578	17	.9413	22	.9253	27	.9098	32	.8949
3	.9891	8	.9715	13	.9545	18	.9380	23	.9222	28	.9068	33	.8920
4	.9855	9	.9680	14	.9511	19	.9348	24	.9190	29	.9038	34	.8891
Dif.	36		35		34		33		32		31		29

19. Weight in grams of 1 cubic centimetre of dry air.

Barometric pressure ($g = 980.6$)

Temperature of the Air.	72 cm	73 cm	74 cm	75 cm	76 cm	77 cm	Diff. per cm.	
0°	.001225	.001242	.001259	.001276	.001293	.001310	17	
1	1220	1237	1254	1271	1288	1305	.1	2
2	1216	1233	1249	1267	1283	1300	.2	3
3	1212	1228	1245	1262	1279	1296	.3	5
4	1207	1224	1241	1257	1274	1290	.4	7
5°	.001203	.001219	.001236	.001253	.001270	.001286	.5	8
6	1198	1215	1232	1248	1265	1282	.6	10
7	1194	1211	1227	1244	1260	1277	.7	12
8	1190	1206	1223	1239	1256	1272	.8	14
9	1186	1202	1219	1235	1251	1268	.9	15
10°	.001181	.001198	.001214	.001231	.001247	.001263	16	
11	1177	1194	1210	1226	1243	1259	.1	2
12	1173	1189	1206	1222	1238	1255	.2	3
13	1169	1185	1202	1218	1234	1250	.3	5
14	1165	1181	1197	1214	1230	1246	.4	6
15°	.001161	.001177	.001193	.001209	.001225	.001242	.5	8
16	1157	1173	1189	1205	1221	1237	.6	10
17	1153	1169	1185	1201	1217	1233	.7	11
18	1149	1165	1181	1197	1213	1229	.8	13
19	1145	1161	1177	1193	1209	1224	.9	14
20°	.001141	.001157	.001173	.001189	.001204	.001220	15	
21	1137	1153	1169	1185	1200	1216	.1	2
22	1133	1149	1165	1181	1196	1212	.2	3
23	1130	1145	1161	1177	1192	1208	.3	4
24	1126	1141	1157	1173	1188	1204	.4	6
25°	.001122	.001138	.001153	.001169	.001184	.001200	.5	7
26	1118	1134	1149	1165	1180	1196	.6	9
27	1114	1130	1145	1161	1176	1192	.7	10
28	1110	1126	1142	1157	1172	1188	.8	12
29	1107	1122	1138	1153	1169	1184	.9	13
30°	.001103	.001119	.001134	.001149	.001165	.001180		

20. Correction for Moisture in Table 19.

Dew-Point	Subtract	Dew-Point	Subtract	Dew-Point	Subtract	Dew-Point	Subtract
—10°	.000,001	0°	.000,003	+10°	.000,006	+20°	.000,010
—8	.000,002	+2	.000,003	+12	.000,006	+22	.000,012
—6	.000,002	+4	.000,004	+14	.000,007	+24	.000,013
—4	.000,002	+6	.000,004	+16	.000,008	+26	.000,015
—2	.000,003	+8	.000,005	+18	.000,009	+28	.000,016

20A. Weight in grams of air displaced by 1 gram of brass of density 8.4.

Density of Air	.00110	.00112	.00114	.00116	.00118	.00120
Weight Displaced	.000131	.000133	.000136	.000138	.000140	.000143
Density of Air	.00120	.00122	.00124	.00126	.00128	.00130
Weight Displaced	.000143	.000145	.000148	.000150	.000152	.000155

21. Factors for the Reduction of Apparent Weighings in Air with Brass Weights to Vacuo.

Density of the Substance Weighed.	Density of the Air. .00115	.00120	.00125	Density of the Substance Weighed.	Density of the Air. .00115	.00120	.00125
0.70	1.00151	1.00157	1.00164	2.0	1.00044	1.00046	1.00048
0.75	„ 140	„ 146	„ 152	2.5	„ 32	„ 34	„ 35
0.80	„ 130	„ 136	„ 141	3.0	„ 25	„ 26	„ 27
0.85	„ 122	„ 127	„ 132	3.5	„ 19	„ 20	„ 21
0.90	„ 114	„ 119	„ 124	4.0	„ 15	„ 16	„ 16
0.95	„ 107	„ 112	„ 117	4.5	„ 12	„ 12	„ 13
1.0	1.00101	1.00106	1.00110	5.0	1.00009	1.00010	1.00010
1.1	1.00091	1.00095	1.00099	6.0	1.00005	1.00006	1.00006
1.2	„ 82	„ 86	„ 89	7.0	„ 3	„ 3	„ 3
1.3	„ 75	„ 78	„ 81	8.0	„ 1	„ 1	„ 1
1.4	„ 68	„ 71	„ 74	9.0	0.99999	0.99999	0.99999
1.5	1.00063	1.00066	1.00068	10	0.99998	0.99998	0.99998
1.6	„ 58	„ 61	„ 63	12	„ 6	„ 6	„ 5
1.7	„ 54	„ 56	„ 59	14	„ 5	„ 4	„ 4
1.8	„ 50	„ 52	„ 55	16	„ 3	„ 3	„ 3
1.9	„ 47	„ 49	„ 51	18	„ 3	„ 2	„ 2
2.0	1.00044	1.00046	1.00048	20	0.99992	0.99992	0.99991

Apparent Specific Volume of Water.

22. Space in cubic centimetres occupied by a quantity of Water weighing apparently 1 gram when counterpoised in Air with Brass Weights of the Density 8.4.

Temperature of the Water.	Density of the Air .00110	.00115	.00120	.00125	.00130
0°	1.00109	1.00113	1.00117	1.00122	1.00126
1	„ 103	„ 107	„ 112	„ 116	„ 121
2	1.00099	„ 103	„ 108	„ 112	„ 116
3	„ 97	„ 101	„ 106	„ 110	„ 114
4	„ 96	„ 100	„ 105	„ 109	„ 114
5°	1.00097	1.00101	1.00106	1.00110	1.00114
6	„ 99	„ 103	„ 108	„ 112	„ 117
7	1.00103	„ 107	„ 111	„ 116	„ 120
8	„ 108	„ 112	„ 116	„ 121	„ 125
9	„ 114	„ 118	„ 123	„ 127	„ 131
10°	1.00122	1.00126	1.00131	1.00135	1.00139
11	„ 131	„ 135	„ 140	„ 144	„ 148
12	„ 141	„ 146	„ 150	„ 155	„ 159
13	„ 153	„ 158	„ 162	„ 166	„ 171
14	„ 166	„ 171	„ 175	„ 179	„ 184
15°	1.00180	1.00185	1.00189	1.00194	1.00198
16	„ 196	„ 200	„ 205	„ 209	„ 214
17	„ 212	„ 217	„ 221	„ 225	„ 230
18	„ 231	„ 235	„ 239	„ 244	„ 248
19	„ 250	„ 254	„ 258	„ 263	„ 267
20°	1.00270	1.00275	1.00279	1.00283	1.00288
21	„ 291	„ 295	„ 300	„ 304	„ 309
22	„ 313	„ 318	„ 322	„ 326	„ 331
23	„ 336	„ 340	„ 344	„ 349	„ 353
24	„ 360	„ 364	„ 368	„ 373	„ 377
25°	1.00384	1.00389	1.00393	1.00398	1.00402
26	„ 410	„ 414	„ 419	„ 423	„ 428
27	„ 437	„ 441	„ 445	„ 450	„ 454
28	„ 464	„ 468	„ 473	„ 477	„ 482
29	„ 492	„ 497	„ 501	„ 505	„ 510
30°	1.00521	1.00525	1.00530	1.00534	1.00538

23. Space in cu. cm. occupied by a quantity of Water weighing 1 gram in Vacuo.

0°	1.00012	Dif.	25°	1.00287	Dif.	50°	1.01194	Dif.	75°	1.02565	Dif
1	1.00006	−4	26	1.00313	28	51	1.01242	48	76	1.02629	64
2	1.00002	−4	27	1.00339	26	52	1.01291	49	77	1.02693	64
3	1.00000	−2	28	1.00367	28	53	1.01340	49	78	1.02757	64
4	0.99999	−1	29	1.00395	28	54	1.01389	49	79	1.02821	64
5°	1.00000	+1	30°	1.00424	29	55°	1.01438	49	80°	1.02886	65
6	1.00002	2	31	1.00454	30	56	1.01487	49	81	1.02951	65
7	1.00006	4	32	1.00485	31	57	1.01536	49	82	1.03017	66
8	1.00011	5	33	1.00517	32	58	1.01586	50	83	1.03084	67
9	1.00017	6	34	1.00550	33	59	1.01637	51	84	1.03152	68
10°	1.00025	8	35°	1.00585	35	60°	1.01690	53	85°	1.03220	68
11	1.00034	9	36	1.00620	35	61	1.01743	53	86	1.032[illegible]	68
12	1.00044	10	37	1.00656	36	62	1.01797	54	87	1.03357	69
13	1.00056	12	38	1.00693	37	63	1.01851	54	88	1.03426	69
14	1.00069	13	39	1.00731	38	64	1.01907	56	89	1.03496	70
15°	1.00083	14	40°	1.00769	38	65°	1.01963	56	90°	1.03566	70
16	1.00099	16	41	1.00808	39	66	1.02020	57	91	1.03637	71
17	1.00115	16	42	1.00848	40	67	1.02077	57	92	1.03709	72
18	1.00133	18	43	1.00888	40	68	1.02136	59	93	1.03781	72
19	1.00152	19	44	1.00928	40	69	1.02195	59	94	1.03855	74
20°	1.00173	21	45°	1.00970	42	70°	1.02255	60	95°	1.03930	75
21	1.00194	21	46	1.01013	43	71	1.02315	60	96	1.04005	75
22	1.00216	22	47	1.01056	43	72	1.02377	62	97	1.04081	76
23	1.00238	22	48	1.01101	45	73	1.02439	62	98	1.04157	76
24	1.00262	24	49	1.01147	46	74	1.02502	63	99	1.04234	77
25°	1.00287	25	50°	1.01194	47	75°	1.02565	63	100°	1.04311	77

23, A. Space in cu. cm. occupied by 1 gram of Mercury.

0°	0.073,551	10°	0.073,684	20°	0.073,816	Dif.		
1	.073,564	11	.073,697	21	.073,830		13	14
2	.073,578	12	.073,710	22	.073,843	.1	1	1
3	.073,591	13	.073,723	23	.073,856	.2	3	3
4	.073,604	14	.073,737	24	.073,870	.3	4	4
5°	0.073,617	15°	0.073,750	25°	0.073,883	.4	5	6
6	.073,631	16	.073.763	26	.073,896	.5	7	7
7	.073,644	17	.073,776	27	073,910	.6	8	8
8	.073,657	18	.073,790	28	.073.923	.7	9	10
9	.073,670	19	.073,803	29	.073,936	.8	10	11
10°	0.073,684	20°	0.073.816	30°	0.073,950	.9	12	13

23, B. Space in cu. cm. occupied by a quantity of Mercury weighing apparently 1 gram when balanced by Brass Weights of Density 8.4 in Air of Density .0012.

0°	0.073.547	10°	0.073,680	20°	0.073,812	Dif.		
1	.073,560	11	.073,693	21	.073,826		13	14
2	.073 574	12	.073,706	22	.073,839	.1	1	1
3	.073,587	13	.073,719	23	.073,852	.2	3	3
4	.073,600	14	.073.733	24	.073,866	.3	4	4
5°	0.073,613	15°	0.073,746	25°	0.073,879	.4	5	6
6	.073,627	16	.073,759	26	.073,892	.5	7	7
7	.073,640	17	.073,772	27	.073,906	.6	8	8
8	.073,653	18	.073,786	28	.073,919	.7	9	10
9	.073,666	19	.073.799	29	.073,932	.8	10	11
10°	0.073,680	20°	0.073,812	30°	0.073,946	.9	12	13

Density of Water, Mercury and Glycerine.

24. Density of Mercury at different temperatures.

0°	13.596	90°	13.377	180°	13.162	270°	12.948
10°	13.572	100°	13.353	190°	13.138	280°	12.924
20°	13.547	110°	13.329	200°	13.114	290°	12.900
30°	13.523	120°	13.305	210°	13.091	300°	12.876
40°	13.498	130°	13.281	220°	13.067	310°	12.853
50°	13.474	140°	13.257	230°	13.043	320°	12.829
60°	13.450	150°	13.233	240°	13.019	330°	12.805
70°	13.426	160°	13.210	250°	12.995	340°	12.781
80°	13.401	170°	13.186	260°	12.972	350°	12.757

25. Density of Water at different temperatures.

0°	0.99988	25°	0.99714	50°	0.98819	75°	0.97497
1	94	26	.99687	51	772	76	437
2	98	27	61	52	725	77	376
3	1.00000	28	34	53	677	78	315
4	01	29	06	54	629	79	254
5°	1.00000	30°	0.99578	55°	0.98582	80°	0.97193
6	0.99998	31	548	56	534	81	131
7	94	32	518	57	486	82	069
8	89	33	486	58	437	83	006
9	83	34	453	59	388	84	.96942
10°	0.99975	35°	0.99419	60°	0.98338	85°	0.96878
11	66	36	384	61	286	86	814
12	56	37	348	62	234	87	750
13	44	38	311	63	181	88	686
14	31	39	274	64	127	89	621
15°	0.99916	40°	0.99236	65°	0.98073	90°	0.96554
16	01	41	198	66	018	91	488
17	.99885	42	158	67	.97963	92	421
18	67	43	118	68	907	93	354
19	48	44	078	69	850	94	286
20°	0.99828	45°	0.99037	70°	0.97793	95°	0.96216
21	07	46	.98976	71	735	96	146
22	.99785	47	954	72	676	97	076
23	62	48	910	73	617	98	005
24	39	49	865	74	557	99	.95934
25°	0.99714	50°	0.98819	75°	0.97497	100°	0.95863

26. Density of Commercial Glycerine (0°—30°).

0°	1.269	5°	1.266	10°	1.263	15°	1.260	20°	1.257	25°	1.253
1°	1.268	6°	1.265	11°	1.262	16°	1.259	21°	1.256	26°	1.253
2°	1.268	7°	1.265	12°	1.262	17°	1.258	22°	1.255	27°	1.252
3°	1.267	8°	1.264	13°	1.261	18°	1.258	23°	1.255	28°	1.252
4°	1.267	9°	1.263	14°	1.260	19°	1.257	24°	1.254	29°	1.251
5°	1.266	10°	1.263	15°	1.260	20°	1.257	25°	1.253	30°	1.251

w't	VOL. 15°	15°	16°	17°	18°	19°	20°	21°	22°
0	0.00	·9992	·9990	·9988	·9987	·9985	·9983	·9981	·9979
1	1.26	·9971	·9969	·9967	·9966	·9964	·9962	·9960	·9958
2	2.51	·9953	·9951	·9949	·9947	·9945	·9943	·9942	·9940
3	3.75	·9936	·9934	·9932	·9930	·9928	·9926	·9924	·9922
4	5.00	·9920	·9918	·9916	·9914	·9912	·9909	·9907	·9905
5	6.24	·9903	·9901	·9899	·9897	·9895	·9892	·9890	·9888
6	7.47	·9887	·9885	·9883	·9880	·9878	·9876	·9874	·9872
7	8.70	·9871	·9869	·9866	·9864	·9861	·9859	·9857	·9855
8	9.93	·9856	·9854	·9851	·9849	·9846	·9844	·9842	·9839
9	11.16	·9842	·9839	·9837	·9834	·9832	·9829	·9827	·9824
10	12.38	·9828	·9825	·9823	·9820	·9817	·9815	·9813	·9810
11	13.59	·9814	·9811	·9809	·9806	·9803	·9800	·9798	·9795
12	14.81	·9801	·9798	·9795	·9793	·9790	·9787	·9784	·9781
13	16.03	·9789	·9786	·9783	·9780	·9777	·9774	·9772	·9769
14	17.24	·9777	·9774	·9771	·9768	·9765	·9762	·9759	·9756
15	18.45	·9765	·9762	·9759	·9755	·9752	·9749	·9746	·9743
16	19.65	·9753	·9750	·9746	·9743	·9740	·9736	·9733	·9730
17	20.85	·9741	·9738	·9734	·9731	·9727	·9724	·9721	·9717
18	22.05	·9729	·9725	·9722	·9718	·9715	·9711	·9708	·9704
19	23.25	·9718	·9714	·9711	·9707	·9703	·9699	·9696	·9692
20	24.45	·9707	·9703	·9699	·9695	·9691	·9687	·9683	·9679
21	25.64	·9695	·9691	·9687	·9683	·9679	·9674	·9670	·9666
22	26.83	·9683	·9679	·9674	·9670	·9666	·9661	·9657	·9653
23	28.01	·9671	·9666	·9662	·9657	·9653	·9648	·9644	·9639
24	29.19	·9659	·9654	·9650	·9645	·9640	·9635	·9631	·9626
25	30.37	·9647	·9642	·9637	·9632	·9627	·9621	·9617	·9612
26	31.54	·9633	·9628	·9623	·9618	·9613	·9607	·9602	·9597
27	32.71	·9619	·9614	·9608	·9603	·9598	·9592	·9587	·9582
28	33.86	·9604	·9599	·9593	·9588	·9583	·9577	·9571	·9566
29	35.02	·9589	·9583	·9578	·9572	·9567	·9561	·9555	·9549
30	36.17	·9573	·9567	·9561	·9556	·9550	·9544	·9538	·9532
31	37.30	·9556	·9550	·9544	·9538	·9532	·9526	·9520	·9514
32	38.44	·9539	·9533	·9527	·9521	·9515	·9508	·9502	·9496
33	39.57	·9522	·9516	·9509	·9503	·9497	·9490	·9484	·9478
34	40.69	·9504	·9498	·9491	·9485	·9479	·9472	·9466	·9459
35	41.81	·9486	·9479	·9473	·9466	·9460	·9453	·9447	·9440
36	42.92	·9467	·9460	·9454	·9447	·9440	·9433	·9427	·9420
37	44.02	·9448	·9441	·9434	·9428	·9421	·9414	·9407	·9400
38	45.12	·9429	·9422	·9415	·9408	·9401	·9394	·9388	·9381
39	46.21	·9410	·9403	·9396	·9389	·9382	·9375	·9368	·9361
40	47.30	·9390	·9383	·9376	·9369	·9362	·9354	·9347	·9340
41	48.38	·9370	·9363	·9356	·9348	·9341	·9334	·9327	·9320
42	49.45	·9349	·9342	·9334	·9327	·9320	·9312	·9305	·9298
43	50.51	·9328	·9321	·9313	·9306	·9298	·9291	·9284	·9276
44	51.57	·9307	·9299	·9292	·9284	·9277	·9269	·9262	·9254
45	52.62	·9286	·9278	·9271	·9263	·9256	·9248	·9240	·9233
46	53.67	·9265	·9257	·9250	·9242	·9234	·9226	·9219	·9211
47	54.71	·9244	·9236	·9229	·9221	·9213	·9205	·9198	·9190
48	55.75	·9223	·9215	·9207	·9200	·9192	·9184	·9176	·9168
49	56.78	·9201	·9193	·9185	·9178	·9170	·9162	·9154	·9146
50	57.80	·9179	·9171	·9163	·9155	·9147	·9139	·9132	·9124

W't	Vol. 15°	15°	16°	17°	18°	19°	20°	21°	22°
50	57.80	.9179	.9171	.9163	.9155	.9147	.9139	.9132	.9124
51	58.81	.9157	.9149	.9141	.9133	.9125	.9117	.9110	.9102
52	59.82	.9135	.9127	.9119	.9111	.9103	.9095	.9087	.9079
53	60.82	.9113	.9105	.9097	.9089	.9081	.9073	.9065	.9057
54	61.82	.9091	.9083	.9075	.9067	.9059	.9050	.9042	.9034
55	62.81	.9069	.9061	.9053	.9045	.9037	.9028	.9020	.9012
56	63.79	.9046	.9038	.9030	.9022	.9013	.9005	.8997	.8989
57	64.77	.9023	.9015	.9007	.8998	.8990	.8982	.8974	.8966
58	65.74	.9000	.8992	.8984	.8975	.8967	.8959	.8951	.8943
59	66.70	.8977	.8969	.8961	.8952	.8944	.8936	.8928	.8920
60	67.65	.8954	.8946	.8938	.8929	.8921	.8913	.8905	.8897
61	68.60	.8931	.8923	.8914	.8906	.8898	.8890	.8882	.8873
62	69.55	.8908	.8900	.8891	.8883	.8875	.8867	.8859	.8850
63	70.49	.8885	.8877	.8868	.8860	.8852	.8844	.8836	.8827
64	71.42	.8862	.8854	.8845	.8837	.8829	.8821	.8813	.8804
65	72.34	.8838	.8830	.8821	.8813	.8805	.8797	.8789	.8780
66	73.26	.8815	.8807	.8798	.8790	.8782	.8773	.8765	.8756
67	74.18	.8792	.8784	.8775	.8767	.8759	.8750	.8742	.8733
68	75.08	.8768	.8760	.8751	.8743	.8735	.8726	.8718	.8709
69	75.98	.8744	.8736	.8727	.8719	.8711	.8702	.8694	.8685
70	76.88	.8721	.8713	.8704	.8696	.8688	.8679	.8671	.8662
71	77.77	.8698	.8689	.8681	.8672	.8664	.8655	.8647	.8638
72	78.65	.8674	.8665	.8657	.8648	.8640	.8631	.8623	.8614
73	79.51	.8649	.8640	.8632	.8623	.8615	.8606	.8598	.8589
74	80.37	.8625	.8616	.8608	.8599	.8591	.8582	.8574	.8565
75	81.23	.8601	.8592	.8584	.8575	.8567	.8558	.8550	.8541
76	82.08	.8576	.8567	.8559	.8550	.8542	.8533	.8525	.8516
77	82.92	.8552	.8543	.8535	.8526	.8518	.8509	.8501	.8492
78	83.76	.8528	.8519	.8511	.8502	.8494	.8485	.8476	.8468
79	84.59	.8503	.8494	.8486	.8477	.8469	.8460	.8451	.8443
80	85.41	.8478	.8469	.8461	.8452	.8444	.8435	.8426	.8418
81	86.22	.8453	.8444	.8436	.8427	.8419	.8410	.8401	.8393
82	87.03	.8428	.8419	.8411	.8402	.8394	.8385	.8376	.8368
83	87.84	.8404	.8395	.8387	.8378	.8370	.8361	.8352	.8344
84	88.63	.8379	.8370	.8362	.8353	.8345	.8336	.8327	.8319
85	89.42	.8354	.8345	.8337	.8328	.8320	.8311	.8302	.8294
86	90.20	.8329	.8320	.8312	.8303	.8295	.8286	.8277	.8269
87	90.97	.8303	.8294	.8286	.8277	.8269	.8260	.8251	.8243
88	91.72	.8277	.8268	.8260	.8251	.8243	.8234	.8225	.8217
89	92.47	.8251	.8242	.8234	.8225	.8217	.8208	.8199	.8191
90	93.22	.8225	.8216	.8208	.8199	.8190	.8181	.8173	.8164
91	93.96	.8199	.8190	.8182	.8173	.8164	.8155	.8147	.8138
92	94.68	.8172	.8163	.8155	.8146	.8137	.8128	.8120	.8111
93	95.39	.8145	.8136	.8128	.8119	.8110	.8101	.8093	.8084
94	96.09	.8118	.8109	.8101	.8092	.8083	.8074	.8066	.8057
95	96.78	.8090	.8081	.8073	.8064	.8055	.8046	.8038	.8029
96	97.45	.8061	.8052	.8044	.8035	.8026	.8017	.8009	.8000
97	98.11	.8032	.8023	.8015	.8006	.7997	.7988	.7980	.7971
98	98.75	.8002	.7993	.7985	.7976	.7967	.7958	.7950	.7941
99	99.38	.7972	.7963	.7955	.7946	.7937	.7928	.7920	.7911
100	100.00	.7941	.7932	.7924	.7915	.7906	.7897	.7889	.7880

Per Cent %	Acetic Acid $C_2H_4O_2$	Nitric Acid. HNO_3	Phosphoric Acid. H_3PO_4	Sulphuric Acid. H_2SO_4	Tart. Acid. $C_4H_6O_6$	Alcohol sol. in Ether.	Methyl Alcohol. CH_4O	Hydrate of Sodium Na OH.	Hydrate of Potassium KOH	Glycerine $C_3H_8O_3$	Sugar (Cane) $C_{12}H_{22}O_{11}$
0	0.999	0.999	0.999	0.999	0.999	0.719	0.999	0.999	0.999	0.999	0.999
2	1.002	1.010	1.010	1.010	1.008	.721	.993	1.02	1.02	1.004	1.007
4	1.005	1.022	1.021	1.024	1.017	.723	.989	1.04	1.03	1.009	1.015
6	1.008	1.035	1.032	1.039	1.026	.724	.985	1.06	1.05	1.014	1.023
8	1.011	1.047	1.044	1.053	1.036	.726	.982	1.09	1.07	1.019	1.031
10	1.014	1.059	1.056	1.068	1.045	0.728	0.980	1.11	1.09	1.024	1.039
12	1.017	1.071	1.068	1.084	1.055	.729	.978	1.13	1.11	1.030	1.047
14	1.020	1.083	1.080	1.099	1.065	.731	.976	1.16	1.12	1.035	1.056
16	1.023	1.096	1.093	1.114	1.075	.733	.974	1.18	1.14	1.040	1.065
18	1.026	1.108	1.106	1.129	1.085	.734	.972	1.20	1.16	1.045	1.073
20	1.028	1.121	1.119	1.144	1.095	0.736	0.970	1.22	1.18	1.050	1.082
22	1.031	1.134	1.132	1.160	1.106	.738	.968	1.24	1.20	1.055	1.091
24	1.034	1.147	1.146	1.175	1.116	.739	.965	1.27	1.22	1.061	1.100
26	1.036	1.160	1.159	1.191	1.127	.741	.963	1.29	1.24	1.066	1.110
28	1.039	1.173	1.174	1.207	1.138	.743	.961	1.31	1.26	1.071	1.119
30	1.041	1.186	1.188	1.224	1.149	0.745	0.959	1.33	1.29	1.076	1.129
32	1.044	1.199	1.204	1.240	1.160	.746	.957	1.35	1.31	1.081	1.138
34	1.046	1.213	1.218	1.257	1.171	.748	.955	1.37	1.33	1.086	1.148
36	1.048	1.226	1.233	1.274	1.182	.750	.953	1.39	1.36	1.092	1.158
38	1.050	1.239	1.248	1.290	1.193	.751	.950	1.41	1.39	1.097	1.168
40	1.052	1.252	1.264	1.306	1.205	0.753	0.947	1.44	1.41	1.102	1.178
42	1.054	1.265	1.280	1.323	1.217	.754	.945	1.46	1.43	1.107	1.189
44	1.056	1.278	1.297	1.340	1.229	.756	.943	1.48	1.45	1.112	1.199
46	1.058	1.292	1.313	1.361	1.240	.757	.940	1.50	1.48	1.117	1.210
48	1.060	1.305	1.330	1.380	1.253	.759	.938	1.52	1.51	1.122	1.221
50	1.062	1.318	1.348	1.399	1.26	0.760	0.935	1.54	1.53	1.127	1.232
52	1.063	1.330	1.365	1.418	1.28	.761	.932	1.56	1.56	1.132	1.243
54	1.065	1.342	1.383	1.438	1.29	.763	.929	1.58	1.58	1.137	1.254
56	1.066	1.353	1.401	1.459	1.30	.764	.926	1.60	1.61	1.143	1.266
58	1.067	1.364	1.420	1.480	1.32	.765	.923	1.62	1.64	1.148	1.277
60	1.069	1.375	1.439	1.502		0.766	0.919	1.64	1.66	1.153	1.289
62	1.070	1.386		1.525		.767	.915	1.66	1.69	1.158	1.301
64	1.071	1.396		1.540		.769	.911	1.68	1.72	1.163	1.313
66	1.072	1.405		1.568		.770	.905	1.70	1.75	1.168	1.325
68	1.073	1.414		1.591		.771	.900	1.73	1.77	1.173	1.337
70	1.073	1.423		1.615		0.773	0.896	1.75	1.79	1.178	1.350
72	1.074	1.431		1.638		.774	.890			1.183	1.363
74	1.074	1.438		1.662		.775	.885			1.188	1.375
76	1.075	1.445		1.686		.777	.880			1.193	
78	1.075	1.453		1.710		.778	.873			1.198	
80	1.075	1.460		1.734		0.779	0.868	1.8?	2.0?	1.203	
82	1.075	1.467		1.758		.781	.862			1.209	
84	1.074	1.474		1.774		.782	.857			1.214	
86	1.074	1.481		1.791		.784	.851			1.220	
88	1.073	1.488		1.807		.785	.846			1.225	
90	1.071	1.495		1.819		0.786	0.840	1.9?	2.1?	1.231	
92	1.070	1.502		1.829		.788	.835			1.237	
94	1.069	1.509		1.836		.789	.829			1.242	
96	1.064	1.516		1.840		.791	.823			1.248	
98	1.060	1.523		1.841		.793	.817			1.254	
100	1.055	1.530		1.839		0.794	0.810	2.0?	2.2?	1.260	

Density of Solutions at 15°.

Per Cent. %	Hydrochloric Acid HCl	Ammonia Gas H_3N	Carbonate of Potas. K_2CO_3	Chloride of Calcium $CaCl_2$	Chloride of Zinc $ZnCl_2$	Hypo Sodium $Na_2S_2O_3 \cdot 5H_2O$	Nitrate of Copper CuN_2O_6	Nitrate of Sodium $NaNO_3$	Sulph. Iron $FeSO_4 \cdot 7H_2O$	Sulph. Magnes. $MgSO_4 \cdot 7H_2O$	Sulph. Zinc. $ZnSO_4 \cdot 7H_2O$
0	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
2	1.009	.990	1.017	1.016	1.019	1.010	1.011	1.012	1.010	1.009	1.012
4	1.019	.982	1.036	1.033	1.036	1.020	1.024	1.025	1.020	1.018	1.023
6	1.029	.974	1.054	1.051	1.052	1.031	1.038	1.039	1.031	1.028	1.034
8	1.039	.966	1.073	1.068	1.071	1.041	1.052	1.053	1.042	1.038	1.046
10	1.049	0.958	1.092	1.086	1.090	1.052	1.065	1.067	1.053	1.048	1.058
12	1.059	.951	1.111	1.105	1.109	1.063	1.077	1.082	1.064	1.058	1.072
14	1.069	.944	1.131	1.123	1.127	1.074	1.091	1.096	1.076	1.068	1.084
16	1.079	.937	1.151	1.142	1.145	1.085	1.105	1.110	1.087	1.078	1.096
18	1.089	.930	1.171	1.162	1.164	1.097	1.119	1.125	1.099	1.088	1.109
20	1.100	0.924	1.192	1.181	1.185	1.108	1.134	1.141	1.111	1.099	1.123
22	1.110	.918	1.213	1.202	1.206	1.120	1.151	1.157	1.124	1.109	1.136
24	1.120	.912	1.234	1.222	1.227	1.131	1.171	1.173	1.136	1.120	1.149
26	1.130	.907	1.256	1.244	1.248	1.143	1.191	1.189	1.148	1.131	1.163
28	1.140	.902	1.278	1.265	1.269	1.155	1.211	1.206	1.160	1.142	1.178
30	1.150	0.897	1.300	1.287	1.290	1.167	1.231	1.223	1.173	1.153	1.193
32	1.160	.892	1.323	1.310	1.315	1.179	1.250	1.240	1.186	1.164	1.208
34	1.170	.887	1.346	1.332	1.339	1.191	1.270	1.258	1.199	1.175	1.223
36	1.180	.883	1.370	1.355	1.365	1.204	1.290	1.276	1.212	1.186	1.239
38	1.190		1.394	1.379	1.391	1.216	1.310	1.295	1.225	1.198	1.254
40	1.200		1.418	1.402	1.419	1.229	1.331	1.315	1.238	1.210	1.270
42			1.442		1.445	1.242	1.352	1.335		1.222	1.287
44			1.467		1.472	1.255	1.374	1.355		1.234	1.303
46			1.492		1.499	1.268	1.397	1.375		1.246	1.319
48			1.518		1.532	1.281	1.420	1.396		1.259	1.336
50			1.543		1.565	1.294	1.443	1.417		1.271	1.352
52			1.570		1.599		1.468			1.284	1.369
54					1.633		1.493			1.297	1.387
56					1.668		1.519				1.405
58					1.703						1.424
60					1.739						1.444

Per Cent. %	Acet. Lead $PbC_4H_6O_4 \cdot 3H_2O$	Carbonate of Sodium Na_2CO_3	Chloride of Ammonium H_4NCl	Chloride of Magnesium $MgCl_2$	Chloride of Sodium NaCl	Chloride of Potassium KCl	Bichromate of Potas. $K_2Cr_2O_7$	Nitrate of Potassium KNO_3	Sulph. Copper $CuSO_4 \cdot 5H_2O$	Sulph. Sodium $Na_2SO_4 \cdot 10H_2O$	Sulphurous Anhydride SO_2
0	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	1·999
2	1.013	1.021	1.005	1.016	1.014	1.012	1.01	1.011	1.012	1.007	1·004
4	1.027	1.042	1.012	1.033	1.028	1.025	1.03	1.023	1.024	1.015	1·009
6	1.042	1.063	1.018	1.050	1.043	1.038	1.04	1.035	1.037	1.023	1·015
8	1.057	1.084	1.024	1.067	1.058	1.052	1.05+	1.048	1.051	1.031	1·021
10	1.072	1.106	1.030	1.085	1.072	1.065	1.07	1.061	1.064	1.039	1.027
12	1.088	1.128	1.036	1.103	1.088	1.079	1.08	1.074	1.078	1.047	1.033
14	1.105	1.150	1.042	1.121	1.103	1.093	1.10	1.088	1.091	1.055	1.040
16	1.121	1.175	1.047	1.140	1.118	1.107		1.102	1.105	1.063	1.047
18	1.132	1.200	1.053	1.158	1.134	1.121		1.116	1.120	1.072	1.054
20	1.155		1.058	1.177	1.150	1 135		1.131	1.134	1.080	1.062
22	1.173		1.064	1.197	1.167	1.150		1.146	1.149	1.088	
24	1.192		1.069	1.217	1.183	1.165		1.161	1.165	1.096	
26			1.075	1.237	1.200				1.181	1.105	
28				1.258					1.197	1.113	
30				1.278					1.214	1.122	

Per Cent. %	Hydrochloric Acid. HCl	Nitric Acid HNO_3	Sulphuric Acid H_2SO_4	Alcohol C_2H_6O	Ammonia Gas. H_3N	Carbonate Potassium K_2CO_3	Carbonate Sodium Na_2CO_3	Chloride Calcium $CaCl_2$	Chloride Sodium NaCl	Hydrate Potassium KOH	Hydrate Sodium NaOH
0	100	100	100	100	100	100	100	100	100	100	100
2	102	..	101	98	92	100	100	100	100	100	100
4	103	..	101	96	86	100	100	101	101	100	100
6	105	..	102	94	79	101	100	101	101	101	101
8	107	..	102	93	71	101	101	101	101	101	101
10	109	..	103	91	65	101	101	101	102	101	101
12	111	..	103	90	59	101	101	102	102	101	102
14	109	..	104	89	53	101	101	102	103	101	102
16	106	..	104	88	47	101	101	103	103	102	103
18	102	..	105	87	41	102	102	103	104	103	103
20	88	104	105	86	36	102	102	104	105	103	104
22	73?	104	106	86	30	102	102	105	105	104	105
24	59?	105	106	85	25	103	103	106	106	105	106
26	48?	106	107	85	20	103	103	107	107	106	107
28	..	107	108	84	15	104	104	108	108	107	108
30	..	108	109	84	10	104	104	109	..	109	110
32	..	109	111	84	5	105	105	111	..	110	112
34	..	110	112	83	0	106	105	113	..	112	114
36	..	110	114	83	—5?	107	..	114	..	114	116
38	..	111	116	83	..	108	..	116	..	116	118
40	..	112	118	83	..	109	..	118	..	119	120
42	..	113	120	82	..	110	..	120	..	122	123
44	..	114	122	82	..	111	..	122	..	125	125
46	..	115	124	82	..	112	..	124	..	128	128
48	15?	116	126	82	..	113	..	126	..	131	131
50	..	117	128	82	..	114	..	128	..	134	134
52	..	118	130	81	..	115	..	131	..	137	137
54	..	119	133	81	..	116	..	134	..	140	140
56	..	119	137	81	..	..	..	138	..	143	143
58	..	119	139	81	..	..	..	141	..	146	146
60	..	120	142	81	..	..	..	144	..	149	149
62	..	120	145	81	..	..	..	148	..	..	..
64	..	120	150	81	..	..	..	152	..	..	..
66	..	119	156	80	..	..	..	156	..	..	..
68	..	116	163	80	..	..	..	160	..	..	..
70	..	..	170	80	..	..	..	164	..	..	..
72	..	..	176	80	..	..	..	169	..	..	..
74	..	..	183	80	..	..	..	173	..	..	..
76	..	..	190	80	..	..	..	178	..	..	..
78	..	..	198	80	..	..	..	..	..	..	..
80	..	100?	206	79	..	..	..	..	..	..	..
82	..	..	214	79	..	..	..	..	..	..	316?
84	..	..	225	79	..	..	..	..	..	..	..
86	..	..	236	79	..	..	..	..	..	316?	..
88	..	..	248	79	..	..	..	..	..	..	..
90	..	..	260	79	..	..	..	..	..	..	..
92	..	..	274	79	..	..	..	..	..	..	..
94	..	..	288	78	..	..	..	..	..	..	..
96	..	40?	303	78	..	..	..	..	..	..	..
98	..	..	318	78	..	..	..	..	..	red	red
100	..	..	333	78	..	..	..	..	..	heat	heat

Name	Symbol	Per Cent by Weight 0	1	2	3	4	5	6	8	10	12	14	16	18	20	25	30	35	40	45	50	60	70	80	90	100
Acid Acetic	$HC_2H_3O_2$	1.00	.99	.99	.99	.98	.98	.98	.97	.96	.95	.95	.94	.93	.92	.90	.88	.86	.83	.81	**.78**	.73	.68	**.62**	.56	**.50**
" Hydrochloric	HCl	1.00	.98	**.96**	.95	.94	.93	.91	.88	.85	.82	.79	**.76**	.	.	.	.	.	.	.	.	.	.	.	.	.
" Nitric	HNO_3	1.00	1.00	.99	**.98**	.97	.95	**.93**	.91	.90	.88	.87	.85	.84	.82	**.78**	.	.	.	.	.	.	.	.	.	.
" Sulphuric	H_2SO_4	1.00	.99	**.98**	.97	**.97**	.95	**.95**	.93	.92	.90	.89	.87	.86	.84	.80	.76	.72	.68	.64	.60	**.53**	.47	.42	.38	**.33**
" Tartaric	$H_2C_4H_4O_6$	1.00	1.00	.99	.99	**.98**	.98	.97	.95	.93	.92	**.91**	.90	.89	.88	.85	.82	.80	.77	**.74**	.	.	.	.	.	.
Alcohol. Ethyl	C_2H_5OH	1.00	1.00	1.01	1.01	1.01	1.02	1.02	1.03	**1.03**	1.03	1.04	1.04	1.05	**1.05**	1.04	**1.03**	1.00	**.98**	.	.	.	.	.	.	**.65**
" Methyl	CH_3OH	1.00	1.00	1.01	1.02	1.03	1.04	1.04	1.05	1.06	**1.07**	1.07	1.07	1.07	**1.07**	1.03	**.98**	.	.	.	.	.	.	.	.	**.65**
Ammonia	H_3N	1.00	1.00	**.999**	.999	**.999**	.998	**.997**	.995	.99	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
Carbonate Sodium	Na_2CO_3	1.00	.99	.97	**.95**	.94	.93	.92	.91	**.89**	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
Chloride Ammonium	H_4NCl	1.00	.99	**.98**	.96	.95	**.94**	.94	.93	.92	.90	.88	.86	.85	.83	.79	**.75**	.	.	.	.	.	.	.	.	.
" Calcium	$CaCl_2$	1.00	.99	.98	.97	**.96**	.94	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
" Potassium	KCl	1.00	.99	**.97**	.96	.94	.93	.92	**.90**	.88	.86	.84	.82	.80	**.78**	.73	.	.	.	.	.	.	.	.	.	.
" Sodium	$NaCl$	1.00	**.98**	.97	.96	.95	.94	**.93**	.91	.89	.87	.86	.84	.82	.81	**.79**	.	.	.	.	.	.	.	.	.	.
Hydrate Potassium	KOH	1.00	.98	**.97**	**.95**	.94	.93	.92	.90	**.87**	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
" Sodium	$NaOH$	1.00	**.98**	.97	.95	**.94**	.94	.93	.92	.91	.90	.89	.88	.87	.86	**.83**	.	.	.	.	.	.	.	.	.	.
Nitrate Ammonium	H_4NNO_3	1.00	.99	.98	.97	**.96**	.95	.94	**.92**	.91	.89	.88	.87	.86	.85	.82	.79	.76	.73	**.70**	.	.	.	.	.	.
" Potassium	KNO_3	1.00	.99	.97	**.96**	.95	.94	.93	.92	**.90**	.88	.87	.85	**.83**	.	.	.	.	.	.	.	.	.	.	.	.
" Sodium	$NaNO_3$	1.00	.99	**.98**	.97	.96	.95	.94	.93	**.91**	.89	.87	.85	.83	.82	.80	**.78**	.76	.	.	.	.	.	.	.	.
Sugar	$C_{12}H_{22}O_{11}$	1.00	.99	.99	.98	**.98**	.97	.96	.95	.94	.93	.92	**.91**	.90	.89	.86	.83	.81	.78	**.75**	.	.	.	.	.	.
Sulphate Ammonium	$(H_4N)_2SO_4$	1.00	.99	.98	**.97**	.95	.94	**.93**	.92	.90	.88	.86	.85	.83	**.82**	.	.	.	.	.	.	.	.	.	.	.
" Copper	$CuSO_4$	1.00	.99	.98	.96	**.95**	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
" Iron	$FeSO_4$	1.00	.99	.98	.96	**.95**	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
" Magnesium	$MgSO_4$	1.00	.99	.97	**.95**	.94	.93	**.92**	.90	.88	.86	.84	.82	.80	.78	**.74**	.	.	.	.	.	.	.	.	.	.
" Sodium	Na_2SO_4	1.00	.99	.98	.97	**.96**	.94	.93	**.92**	**.90**	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
" Zinc	$ZnSO_4$	1.00	.99	.98	.96	**.95**	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Most of the numbers in this table were obtained by interpolation. Those nearest observed values are printed in heavy type.

Name	Symbol	Per Cent by Weight. 0	1	2	3	4	5	6	8	10	12	14	16	18	20	25	30	35	40	45	50	60	70	80	90	100
Acid, Acetic . . .	$HC_2H_3O_2$	**0**	**.05**	07	.08	.10	.11	.12	.14	**.15**	.15	.15	.16	.16	**.16**	.16	.15	.13	.11	.09	**.07**	.05	.04	.02	.01	**0.0**
„ Hydrochloric	HCl	**0**	8	16	24	32	**39**	44	53	**62**	69	73	74	75	75	72	**66**	59	**51**							
„ Nitric . . .	HNO_3	**0**	6	11	16	21	26	**31**	39	46	**53**	59	64	69	73	**76**	77	76	**73**	68	63	**51**	40	27		
„ Oxalic . . .	$H_2C_2O_4$	**0**	1	3	4	**5**	6	6	**8**																	
„ Phosphoric .	H_3PO_4	**0**	1	2	2	3	3	4	5	6	7	8	9	10	**11**	13	15	17	18	19	**20**	18	**14**	10	**6**	
„ Sulphuric .	H_2SO_4	**0**	**4**	8	12	16	**20**	25	32	**39**	45	51	**57**	61	**65**	**71**	**73**	**72**	**68**	62	**54**	**37**	**21**	**11**	**10**	**8**
„ Tartaric . .	$H_2C_4H_4O_6$	**0**	0.1	0.2	0.3	0.4	**0.6**	0.7	0.8	0.8	0.9	0.9	0.9	1.0	**1.0**	1.0	**1.0**	0.9	**0.8**	0.6	**0.5**					
Bromide Potassium	KBr	**0**	1	2	3	4	**5**	6	8	10	12	14	16	18	**19**	24	29	**34**								
Carbonate „ .	K_2CO_3	**0**	1	2	3	4	**6**	7	9	12	14	16	18	20	21	**22**	22	21	19	17	**15**					
„ Sodium	Na_2CO_3	**0**	1	2	3	4	**4**	5	6	**7**	7	**8**														
Chloride Ammon. .	H_4NCl	**0**	2	4	6	8	**9**	11	14	18	21	**24**	27	30	33	**40**										
„ Potassium	KCl	**0**	1	3	4	6	**7**	9	11	14	16	**19**	20	22	24	**28**										
„ Sodium .	$NaCl$	**0**	1	3	4	5	**7**	8	10	11	13	14	16	17	**19**	**21**										
„ Zinc . .	$ZnCl_2$	**0**	1	2	3	4	**5**	6	6	7	8	8	8	9	9	9	**9**	8	7	6	5	**4**				
Hydrate Potas. .	KOH	**0**	3	6	9	12	**16**	19	25	**31**	37	**42**	46	49	**51**	54	**54**	**51**	**45**							
Iodide „ .	KI	**0**	1	2	2	3	**3**	4	6	7	9	10	11	13	**15**	19	23	27	31	34	**38**	46				
Nitrate Ammon. .	H_4NNO_3	**0**	1	2	3	4	**6**	7	9	11	13	15	17	19	21	25	**28**	30	32	34	**36**					
„ Copper .	CuN_2O_6	**0**	1	2	3	3	**4**	4	5	6	6	7	8	8	9	**11**	11	**10**								
„ Potassium.	KNO_3	**0**	1	2	3	4	**4**	5	6	8	9	**11**	13	14	**15**											
„ Silver . .	$AgNO_3$	**0**	1	1	2	2	**3**	3	4	5	6	7	8	9	**9**	10	12	14	16	18	20	**21**				
„ Sodium .	$NaNO_3$	**0**	1	2	3	4	**4**	5	6	7	8	10	11	12	**13**	15	**16**									
Sulphate Ammon. .	$(H_4N)_2SO_4$	**0**	1	2	3	4	**5**	6	8	9	11	13	15	16	**18**	20	**22**									
„ Copper . .	$CuSO_4$	**0**	0	1	2	2	**2**	2	3	**3**	3	4	4	**4**												
„ Magnesium	$MgSO_4$	**0**	1	1	2	2	**2**	3	4	**4**	4	**5**	5	5	**5**	**4**										
„ Potassium .	K_2SO_4	**0**	1	1	2	3	**4**	5	7	**8**																
„ Sodium .	Na_2SO_4	**0**	1	1	2	3	**4**	5	6	**7**	8	**8**	9													
„ Zinc . . .	$ZnSO_4$	**0**	0	1	1	2	**2**	2	3	**3**	3	**4**	4	**4**	**4.3**	4.4	**4.1**	3								

The numbers in this table must be multiplied by 0.000,000,000,01 (10^{-11}) to reduce them to the C.G.S. System. The conductivity of the solutions named above diminishes as the temperature rises at the rate of about 2 % per degree, with the exception of sulphuric and phosphoric acids, in which the rate varies from 1 to 4% according to the strength of the solution. The conductivities were determined at about 18°.

31. B. Refractive and Dispersive Indices of Solutions at about 18°.

Name and Symbol	Per Cent.	Index of Refract.	Index of Dispers.	Name and Symbol	Per Cent.	Index of Refract.	Index of Dispers.
Acid, acetic $HC_2H_3O_2$	0	1.333	.014	Chloride Amm. H_4NCl	10	1.351	.016
" " "	20	1.348	..	" " "	20	1.370	.018
" " "	40	1.362	..	" Calcium $CaCl_2$	20	1.384	.019
" " "	60	1.371	..	" " "	40	1.441	.023
" " "	80	1.378	..	" Sodium $NaCl$	10	1.350	.016
" " "	100	1.374	.017	" " "	20	1.368	.018
" Hydrochloric, HCl	35	1.413	.023	" Zinc $ZnCl_2$	20	1.370	.018
" Nitric, HNO_3	50	1.401	.024	" " "	40	1.410	.021
" Sulphuric, H_2SO_4	0	1.333	.014	Hydrate Potas. KOH	40	1.403	.018
" " "	20	1.358	..	" Sodium, $NaOH$	10	1.359	.016
" " "	40	1.382	.016	" " "	20	1.384	.018
" " "	60	1.410	..	" " "	30	1.404	.020
" " "	80	1.434	.018	Nitrate Sodium $NaNO_3$	20	1.355	.017
" " "	100	1.434?	.017?	" " "	40	1.380	.021
Alcohol, C_2H_5OH	0	1.333	.014	Sugar, $C_{12}H_{22}O_{11}$	10	1.348	.015
" "	40	1.358	.015?	" "	20	1.364	.016
" "	100	1.360	.015?	" "	30	1.381	.017

31. C. Table for preparing Mixtures of any Desired Strength.

Per Cent. of A	No. Parts of A to 100 of B	Per Cent. of B	Per Cent. of A	No. Parts of A to 100 of B	Per Cent. of B	Per Cent. of A	No. Parts of A to 100 of B	Per Cent. of B	Per Cent. of A	No. Parts of A to 100 of B	Per Cent. of B	Per Cent. of A	No. Parts of A to 100 of B	Per Cent. of B
0	0.000	100	20	25.000	80	40	66.667	60	60	150.00	40	80	400 00	20
1	1.010	99	21	26.582	79	41	69.492	59	61	156.41	39	81	426.32	19
2	2.041	98	22	28.205	78	42	72.414	58	62	163.16	38	82	455.56	18
3	3.093	97	23	29.870	77	43	75.439	57	63	170.27	37	83	488.24	17
4	4.167	96	24	31.579	76	44	78.571	56	64	177.78	36	84	525.00	16
5	5.263	95	25	33.333	75	45	81.818	55	65	185.71	35	85	566 67	15
6	6.383	94	26	35.135	74	46	85.185	54	66	194.12	34	86	614.28	14
7	7.527	93	27	36.986	73	47	88.679	53	67	203.03	33	87	669.23	13
8	8.696	92	28	38.889	72	48	92.308	52	68	212.50	32	88	733.33	12
9	9.890	91	29	40.845	71	49	96.078	51	69	222.58	31	89	809.09	11
10	11.111	90	30	42.857	70	50	100.00	50	70	233.33	30	90	900.00	10
11	12.360	89	31	44.928	69	51	104.08	49	71	244.83	29	91	1,011.1	9
12	13.636	88	32	47.059	68	52	108.33	48	72	257.14	28	92	1,150.0	8
13	14.943	87	33	49.254	67	53	112.77	47	73	270.37	27	93	1,328.6	7
14	16.279	86	34	51.515	66	54	117.39	46	74	284.62	26	94	1,566.7	6
15	17.647	85	35	53.846	65	55	122.22	45	75	300.00	25	95	1,900.0	5
16	19.048	84	36	56.250	64	56	127.27	44	76	316.67	24	96	2,400.0	4
17	20.482	83	37	58.730	63	57	132.56	43	77	334 78	23	97	3,233.3	3
18	21.951	82	38	61.290	62	58	138.10	42	78	354.55	22	98	4,900.0	2
19	23.457	81	39	63.934	61	59	143.90	41	79	376.19	21	99	9,900 0	1
20	25.000	80	40	66.667	60	60	150.00	40	80	400 00	20	100	∞	0

31. D. Coefficients of Diffusion of Saline Solutions in Water.

Hydrochloric Acid	.000, 0100	Sulphate of Potassium .	.000, 0037
Hydrate of Potassium . .	.000, 0070	Sulphate of Sodium . .	.000, 0030
Sulphuric Acid	.000, 0052	Sulphate of Magnesium .	.000, 0020
Nitrate of Potassium . .	.000, 0052	Sugar	.000, 0019
Common Salt	.000, 0046	Gum Arabic	.000, 0010
Nitrate of Sodium . . .	.000, 0040	Albumen	.000, 0002
. . . .	. . . .	Caramel	.000, 0001

31, E. Rotation in degrees of the Plane of Polarization for the Fraunhofer Lines A—H, produced by passing through 100 cm, of various solutions, containing in each case 1 gram of a given substance in 100 cu. cm.

Name and Symbol of Substance		A	B	C	D	E	F	G	H
Acid. Malic. $H_2C_4H_4O_5$+aq	—	..	..	..	0.3				
„ Tartaric $H_2C_4H_4O_6$+aq.	+	..	..	1.2	1.5	1.9	2.0		
Camphor $C_{10}H_{16}O$+alcohol.	+	..	..	3.0	4.2	6.1	8.0		
Cholesterine $C_{26}H_{44}O$+ether	—	..	2.0	2.6	3.2	4.0	4.9	6.2	
Cinchonidine $C_{20}H_{24}N_2O$+alcohol.	—	..	..	..	11				
Cinchonine $C_{20}H_{24}N_2O$+alcohol.	+	..	..	..	24				
Conchinine $C_{20}H_{24}N_2O_2 2\frac{1}{2}H_2O$+alcohol	+	..	..	..	24				
Glycocol CH_2NH_2COOH+alcohol.	+	..	..	2.2	2.9	3.8	4.9	5.7	
Malate of Ammon $(H_4N)_2C_4H_4O_5$+aq.	—	..	..	..	2.2				
„ „ Lithium $Li_2C_4H_4O_5$+aq.	—	..	..	..	1.3				
„ „ Potassium $K_2C_4H_4O_5$+aq.	—	..	..	..	0.7				
„ „ Sodium $Na_2C_4H_4O_5$+aq.	—	..	..	..	1.0				
Morphine chl. $C_{17}H_{19}NO_3HCl.3H_2O$+aq.	—	..	..	..	10				
„ sulph. $2C_{17}H_{19}NO_3.H_2SO_4.5H_2O$+aq.	—	..	..	..	10				
Quinine hydr. $C_{20}H_{24}N_2O_2.3H_2O$+alcohol	—	..	..	..	15				
„ sulph. $C_{20}H_{24}N_2O_2.H_2SO_4\,7H_2O$+aq.	—	..	..	..	16				
Salicine $C_{13}H_{18}O_7$+aq.	—	..	..	..	6.5				
Santonid, para-; $C_{15}H_{18}O_3$+alcohol.	+		58	66	89	126	167		
Santonine $C_{15}H_{18}O_3$+alcohol.	—		11	12	16	22	26		
Sugar (Cane-) $C_{12}H_{22}O_{11}$+aq.	+	3.8	4.8	5.3	6.65	8.5	10.1	13.2	15.7
„ grape „	+	..	..	..	5				
„ milk „	+	..	..	..	5				
„ maltose „	+	..	..	..	14				
„ lactose „	+	..	..	..	8				
„ inverted „	—	..	..	..	2.9				

31, F. Rotation in degrees of the Plane of Polarization for Fraunhofer Lines A—H produced by plates of various substances 1 cm. thick.

Name and Symbol of Substance		A	B	C	D	E	F	G	H
Benzil, $C_{14}H_{10}O_2$		..	..	..	248				
Bromate of Sodium $NaBrO_3$		..	..	..	28				
Chlorate „ $NaClO_3$		..	24	25	32	40	46	59	69
Cinnabar, HgS		..	3000	?					
Diacetylphenolphthaleine		..	168	?..	197	246	?		
Ethylenediaminesulphate		..	..	..	155				
Guanidine Carbonate		..	123	?..	146	178	?		
Hyposulphate of Calcium $CaS_2O_6.4H_2O$		..	..	..		21			
„ Lead $PbS_2O_6.4H_2O$		..	..	41	55	73	89		
„ Potassium $K_2S_2O_6.2H_2O$		..	..	62	84	105	123		
„ Strontium $SrS_2O_6.4H_2O$		..	..	..	..	16			
Iodate sodium, per- $NaIO_4$		..	..	194	233	285	342	471	
Nicotine (liquid) $C_{10}H_{14}N_2$	—	..	..	..	16				
Quartz (ordinary right handed) SiO_2	+	127	157	173	217	275	32[illegible]	425	511
Strychnine (sulphate) $2C_{21}H_{22}N_2O_2.H_2SO_4$		..	108	?					
Tartaric Ether (liquid) $(C_2H_5)_2C_4H_4O_6$	+	..	..	..	0.8				
Turpentine right handed $C_{10}H_{16}$	+	..	..	..	14.1				
„ (liquid) left handed $C_{10}H_{16}$	—	..	..	..	37.0				

31, G. Rotation of the Plane of Polarization caused by a Unit Magnetic Field (C. G. S.) in Unit Thicknesses of Different Substances.

Bisulphide of Carbon (sodium light)	0°.0070	Water (white light)	0°.0001
" " " (thallium ")	0°.0086	Coal gas	0°.000,000,2

Note. In these, and in nearly all cases, the rotation is with the current producing the magnetic field. A solution of ferric chloride in methyl alcohol is mentioned as one of the exceptions to this rule (Deschanel, § 839).

31, H. Magnetic Moment of 1 cu. cm. of various substances (C. G. S.)

Name of Substance	Magnetization induced by Unit Field	Maximum Magnetization	Maximum Permanent Magnetization	Name of Substance	Magnetization induced by Unit Field
Iron	300?	1400	..	Nickel Oxide	+0.1?
Steel	70?	1400	< 800	Water	−0.01?
Cobalt	300?	800?	..	Bismuth	−0.01?
Nickel	140?	500	..	Phosphorus	−0.004?
Iron Oxide	0.2?	..	..		

31, I. Coefficients of Friction (f) for water corresponding to Velocities (v) in centimetres per second (From Weisbach).

v	f	v	f	v	f	v	f	v	f
0	∞	100	.00299	200	.00264	300	.00249	400	.00239
10	.00554	110	.00293	210	.00262	310	.00248	410	.00239
20	.00445	120	.00288	220	.00260	320	.00247	420	.00238
30	.00396	130	.00284	230	.00258	330	.00246	430	.00238
40	.00368	140	.00280	240	.00256	340	.00245	440	.00236
50	.00347	150	.00276	250	.00255	350	.00244	450	.00236
60	.00333	160	.00274	260	.00254	360	.00243	460	.00235
70	.00321	170	.00271	270	.00253	370	.00242	470	.00235
80	.00312	180	.00269	280	.00251	380	.00241	480	.00234
90	.00305	190	.00266	290	.00250	390	.00240	490	.00234
100	.00299	200	.00264	300	.00249	400	.00239	500	.00233

31, J. Coefficients of Friction of Solids on Solids.

	Oak	Hard Wood	India Rubber	Leather	Hemp.	Bronze	Iron	Cast Iron
Oak	.2—.5	.38	...	.30	.52	.48	..	.49
" soaped	.16	..	...	...	...	.16	..	.19
Bronze	.48	..	...	...	...	.20	.18	.21
Iron (cast, smooth)	.49	..	.56	.2	.08	.2	.2—.4	.2—.4
" " wet	.24	..	.36	.36	..	.31	..	.31
" " greased	.08	..	.20	.15	..	.15	...	.15

31, K. Action of Plates (1 cm thick and bounded by plane surfaces) upon normally incident Radiant Heat.

	Substance	Reflects	Absorbs	Transmits		Substance	Reflects	Absorbs	Transmits
White Heat	Lampblack	0%	100%	0%	White Heat	Water	4%	86%	10%
	India Ink	5	95	0		Aqueous Solutions	4	86	10
	Ice	4	91	5?		Alcohol	5	82	13
	Alum	6	86	8?		Ether	5	75	20
	White Lead	44?	56	0		Oils	6	73	21
	Glass	8	62	30		Chloroform	6	69?	25?
	Shellac	8	47	45		Turpentine	6	64	30
	Polished Metals	80?	20?	0		Bisulphide Carbon	12	35	53
	Rock Salt	8	0	92		Mercury	75?	25?	0
Steam Heat	Lampblack	0	100	0	Steam Heat	Shellac	8	72	20
	White Lead	0	100	0		Mercury	75?	25?	0
	Ice	4	96	0		Polished Metals	80?	20?	0
	Alum	6	94	0		Rock Salt	8	0?	92
	Glass	8	92	0		Vapors at 1 cm	0	0—10?	90?—100
	India Ink	10	90	0		Perm Gases 76 cm	0	0—.02?	99.98+

31, L. Estimates of the number of Units of Heat radiated in 1 sec. by 1 sq. cm. blackened surface in space at 0°.

Temp.	Rad.	Temp.	Rad.	Temp.	Rad.	Temp.	Rad.	Temp.	Rad.	Temp.	Rad.	Temp.	Rad.
−273°	−.019?	+100°	+.012?*	500°	.1?	900°	.5?	1300°	2?**	1700°	5?	2500°	60?†*
−200°	−.015?	200°	.028?*	600°	.2?†	1000°	.7? //	1400°	2?**	1800°	7?††	3000°	270?†*
−100°	−.009?	300°	.05?*	700°	.2?§*	1100°	.9? //	1500°	3?	1900°	10?	3500°	1200?§§
0°	.000	400°	.08?*	800°	.3?§	1200°	1.2?†§	1600°	4?	2000°	13?	4000°	5400?§§

* Dark. † Dull red. §* "Red Heat". § Cherry Red. // Orange. †§ Yellow ** "White Heat". †† Flame. †* Voltaic Arc Light. §§ Sunlight.

32a. Heats of Combustion in Oxygen.

Name of Substance Consumed.	Chemical Reaction involving 16.0 grams of Oxygen in each case.	Grams of Substance consumed.	Grams of Product formed.	Units of Heat developed.	Units of Heat per gram consumed.	Megergs per milligram consumed.	Electromotive force in volts.
Acetylene	$2\,C_2H_2 + 5\,O_2 = 4\,CO_2 + 2\,H_2O$	5.2	21.2	62,000	12,000	500	
Alcohol	$C_2H_6O + 3\,O_2 = 2\,CO_2 + 3\,H_2O$	7.7	23.7	54,000	7,000	290	
Arsenic	$As_4 + 3\,O_2 = 2\,As_2O_3$	50.0	66.0	51,500	1,030	43	
"	$As_4 + 5\,O_2 = 2\,As_2O_5$	30.0	46.0	44,000	1,460	61	
Barium	$2\,Ba + O_2 = 2\,BaO$	136.8	152.8	130,000	950	40	
Bismuth	$Bi_4 + 3\,O_2 = 2\,Bi_2O_3$	138.7	154.7	13,300	96	4.0	
Calcium	$2\,Ca + O_2 = 2\,CaO$	39.9	55.9	130,000	3,280	138	
Carbon	$C + O_2 = CO_2$	6.0	22.0	48,000	8,000	334	
Carbonic Oxide	$2\,CO + O_2 = 2\,CO_2$	28.0	44.0	67,000	2,400	100	
Chlorine	$2\,Cl_2 + O_2 = 2\,Cl_2O$	70.7	86.7	− 18,000	− 250	−10	
Copper	$4\,Cu + O_2 = 2\,Cu_2O$	126.2	142.2	40,000	320	13	
"	$2\,Cu + O_2 = 2\,CuO$	63.1	79.1	38,000	600	25	
Ethane	$2\,C_2H_6 + 7\,O_2 = 4\,CO_2 + 6\,H_2O$	4.3	20.3	54,000	12,500	520	
Ether	$C_4H_{10}O + 6\,O_2 = 4\,CO_2 + 5\,H_2O$	6.2	22.2	56,000	9,000	375	
Ethylene	$2\,C_2H_4 + 6\,O_2 = 4\,CO_2 + 4\,H_2O$	4.7	20.6	56,000	12,000	500	
Hydrogen	$2\,H_2 + O_2 = 2\,H_2O$	2.0	18.0	69,000	34,500	1440	1.49
Iodine	$2\,I_2 + 5\,O_2 = 2\,I_2O_5$	50.6	66.6	9,000	177	7.4	
Iron	$2\,Fe + O_2 = 2\,FeO$	55.9	71.9	75,000	1350	56	
"	$3\,Fe + 2\,O_2 = Fe_3O_4$	41.9	57.9	66,000	1575	66	
Lead	$2\,Pb + O_2 = 2\,PbO$	206.4	222.4	50,000	243	10	
Magnesium	$2\,Mg + O_2 = 2\,MgO$	24.0	40.0	146,000	6,100	255	
Mercury	$4\,Hg + O_2 = 2\,Hg_2O$	399.6	415.6	42,000	105	4.4	
"	$2\,Hg + O_2 = 2\,HgO$	199.8	215.8	30,000	153	64	
Methane	$CH_4 + 2\,O_2 = CO_2 + 2\,H_2O$	4.0	20.0	52,000	13,100	550	
Nitrogen	$2\,N_2 + O_2 = 2\,N_2O$	28.0	44.0	18,000	− 650	−27	
"	$N_2 + O_2 = 2\,NO$	14.0	30.0	−22,000	−1550	−65	
"	$N_2 + 2\,O_2 = 2\,NO_2$	7.0	23.0	− 1,000	− 150	-6.3	
Phosphorus	$P_4 + 5\,O_2 = 2\,P_2O_5$	12.4	28.3	71,000	5750	240	
Potassium	$2\,K_2 + O_2 = 2\,K_2O$	78.1	94.1	136,000	1745	73	
Selenium	$Se_2 + 2\,O_2 = 2\,SeO_2$	39.4	55.4	29,000	730	31	
Silver	$2\,Ag_2 + O_2 = 2\,Ag_2O$	215.4	231.4	5,800	27	1.1	
Sodium	$2\,Na_2 + O_2 = 2\,Na_2O$	46.0	62.0	152,000	3,300	138	
Spermaceti	— — — — — — —	—	—	—	10,300	—	
Stearine	— — — — — — —	—	—	—	9,700	—	
Strontium	$2\,Sr + O_2 = 2\,SrO$	87.3	103.3	131,000	1,500	63	
Sulphide Carbon, Bi-	$CS_2 + 3\,O_2 = CO_2 + 2\,SO_2$	12.7	28.6	43,000	3,400	142	
Sulphur	$S_2 + 2\,O_2 = 2\,SO_2$	16.0	32.0	36,000	2,250	94	
Thallium	$2\,Tl_2 + O_2 = 2\,Tl_2O$	408.0	424.0	42,400	104	4.3	
Tin	$2\,Sn + O_2 = 2\,SnO$	118.0	134.0	68,000	575	24	
"	$2\,Sn + 2\,O_2 = 2\,SnO_2$	59.0	75.0	72,600	1,230	51	
Turpentine	$C_{10}H_{16} + 14\,O_2 = 10\,CO_2 + 8\,H_2O$	4.9	20.8	52,000	10,700	446	
Wax	— — — — — — —	—	—	—	10,500	—	
Wood	(50% C)	—	—	—	4,000	—	
Zinc	$2\,Zn + O_2 = 2\,ZnO$	64.9	80.9	84,400	1,300	54	

32b Heats of Combustion in Chlorine.

Name of Substance Consumed.	Chemical Reaction involving 70.7 grams of Chlorine in each case.	Grams of Substance consumed.	Grams of Product formed	Units of Heat developed.	Units of Heat per gram consumed.	Megergs per milligram consumed.	Electromotive force in volts.
Antimony	$Sb_4 + 6\,Cl_2 = 4\,SbCl_3$	80.7	151.4	57,000	707	30	1.23
Arsenic	$As_4 + 6\,Cl_2 = 4\,AsCl_3$	49.9	120.6	50,000	994	42	1.08
Copper	$Cu + Cl_2 = CuCl_2$	63.1	133.8	61,000	960	40	1.32
Hydrogen	$H_2 + Cl_2 = 2\,HCl$	2.0	72.7	47,000	23,500	980	1.01
Iron	$2\,Fe + 3\,Cl_2 = Fe_2Cl_6$	37.3	108.0	65,000	1,750	73	1.40
Potassium	$K_2 + Cl_2 = 2\,KCl$	78.1	148.8	207,000	2,650	110	4.48
Tin	$Sn + 2\,Cl_2 = SnCl_4$	59.0	129.7	64,000	1,080	45	1.38
Zinc	$Zn + Cl_2 = ZnCl_2$	64.9	135.6	99,000	1,530	64	2.15

33. Heats of Combination.

Name of Substance Acted upon.	Chemical Reaction involving 16.0 grams of Oxygen or its equivalent.	Grams of Substance consumed.	Grams of Product formed.	Units of Heat developed.	Units of Heat per gram consumed.	Megergs per milligram consumed.	Electromotive force in volts.
Copper	$2\,Cu + O_2 + 2\,SO_3 + Aq. = 2\,CuSO_4 \cdot Aq.$	63.1	159.1	54,200	860	36	1.17
Nitric Oxide	$4\,NO + O_2 + 2\,H_2O + Aq. = 4\,HNO_3 \cdot Aq.$	60.0	94.0	36,300	600	25	0.78
Nitrous Acid	$2\,HNO_2 \cdot Aq. + O_2 = 2\,HNO_3 \cdot Aq.$	47.0	63.0	18,300	390	16	0.40
Zinc	$2\,Zn + O_2 + 2\,SO_3 + Aq. = 2\,ZnSO_4 \cdot Aq.$	64.9	160.9	108,500	1,670	70	2.35

34. Contact differences of Potential in Volts.*

	60% Sulph. Zinc $ZnSO_4.7H_2O.Aq.$	45% " "	20% " "	12% Alum. $Al_2K_2S_4O_{16}.24H_2O.Aq.$	29% Sulph.Copper $Cu.SO_4.5H_2O.Aq.$	Amalgamated Zinc	Zinc	Lead	Tin	Iron	Brass	Copper	Platinum	Carbon	Mercury	Water (pure)	Nitric Acid (concentrated)
Chloride Ammon.† 25%							—.637	—.189	—.364	—.652	—.348	—.396	.057				
Salt, NaCl, Aq. 24%							—.565	—.267	—.334	—.605	—.435	—.475	—.856				
Sulphate of Copper $CuSO_4.5H_2O$, Aq. 29%	095	.102										.070					
Sulphate of Copper $CuSO_4.5H_2O$, Aq. 14%			.090									.103					
Amalgamated Zinc	284					0	—.144	—.357	—.463	—.744	—.822	—.894	—1.125	—1.208		—.100	
Zinc	430	.444	.238	.536		.144	0	—.210	—.281	—.600	—.679	—.750	—.981	—1.096		.0±?	
Lead				.139		.357	.210	0	—.099	—.401	—.472	—.542	—.771	—.858		—.171	
Tin				.225		.463	.281	.099	0	—.313	—.372	—.456	—.690	—.795		—.177	
Iron				.653		.744	.600	.401	.313	0	—.064	—.146	—.369	—.485	—.502	—.148	
Brass				.014		.822	.679	.472	.372	.064	0	—.087	—.287	—.414		—.231	
Copper				.127	—.070	.894	.750	.542	.456	.146	.087	0	—.238	—.370	—.308	—.2	
Platinum				—.246		1.125	.981	.771	.690	.369	.287	.238	0	—.113	—.156	—.3	—.672
Carbon						1.208	1.096	.858	.795	.485	.414	.370	.113	0	—.092	—.1	
Mercury										.502		.308	.156	.092	0		
Water (pure)	.18				.0?	.100	.0±	.171	.177	.148	.231	.2	.3	.1		0	
Sulphuric Acid H_2SO_4, Aq. — trace							—.241										.078
5%							—.344										
9%						—.358								—.035			
17%						—.429											
83%								—.120	—.25		.016			.2			
100%	1.699			1.456	1.269	.848		1.				1.113	1.5	.7		1.298	
Sulphate of Mercury§															.475		

* Each number in this table represents the potential in volts of one substance (above the number) when brought in contact with a second substance (at the left of the number) at the potential zero. † $H_4NCl + Aq.$ § $3\,HgSO_4 + 2\,H_2O + Aq = Hg_3O_2SO_4 + 2\,H_2SO_4 + Aq.$

35. Electromotive Force of Voltaic cells.

Name of Cell	Negative or Dissolving Pole	Solution next Negative Pole.	Solution next Positive Pole.	Positive Pole	Temperature	Electromotive Force in Volte
[Beetz]	Potas.Amalg.	Hydrate of Potassium.	Permanganates(withMnO_2)*	Sulphur?†	18°?	3.
Bunsen	Amalg. Zinc	30% Sulphuric Acid.	Pure Nitric Acid	Carbon	„	1.96
„	„	„	60% „	„	„	1.89
„	„	„	Bichromates	„	„	1.87
Clark	„	Sulphate of Zinc and	Sulphate of Mercury (paste)	Mercury	15°	1.435
„	„	„ „ „ „	„ „ „ „	„	24°	1.425
Daniell I.	„	60% Sulphuric Acid.	30% Sulphate of Copper	Copper	18°?	1.08
„ „	„	30% „ „	„ „ „ „	„	„	0 98
„ II.	Zinc	1% Sulphate of Zinc.	„ „ „ „	„	„	1.12
„ „	„	5% „ „ „	„ „ „ „	„	„	1.14
„ „	„	10% „ „ „	„ „ „ „	„	„	1.12
„ „	„	20% „ „ „	„ „ „ „	„	„	1.11
„ III.	„	30% Sulphuric Acid.	50% Nitrate of Copper	„	„	1.00
Davy	Amalg. Zinc	25% Chloride of Sodium.	Sulphate of Mercury (paste)	Carbon	„	1.41
Grove	„	30% Sulphuric Acid.	Nitric Acid	Platinum	„	1.93
Leclanché	„	25% Chloride of Ammonium.	Powdered oxides Manganese	Carbon	„	1.32
Silver Chloride	Zinc	„ „ „	Silver Chloride	Silver	„	1.03

* Ganot, § 814. † Daniell, page 553. § Or water. Ganot § 812.

36. Electromotive Force in Volts and Striking Distance in Millimetres.

mm.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0	500	1000	1470	1920	2360	2780	3190	3580	3960
1	4340	4700	5050	5400	5740	6070	6390	6700	7000	7300
2	7600	7890	8170	8450	8730	9000	9270	9540	9810	10070
3	10320	10580	10830	11080	11320	11560	11800	12040	12270	12500

The values in this table are subject to a probable error of about 100 volts.

37a. Specific Electrical Resistances of Conductors at 0°.

Name of Substance	Resistance of a centimetre-cube in microhms.*	Resistance in ohms of a wire 1 m. long 1 mm. diam.	Resistance in ohms of a wire 1 m. long weighing 1 g.	Per Cent of Increase per degree centigrade
Silver, annealed	1.50*	0.019	0.16	0.377
„ hard drawn . . .	1.60	0.020	0.17	..
Copper, annealed. . . .	1.58	0.020	0.14	0.388
„ hard drawn . .	1.61	0.020	0.14	..
Gold, annealed	2.0	0.026	0.39	0.365
„ hard drawn . . .	2.1	0.026	0.41	..
Aluminum annealed . .	2.8	0.036	0.07	..
Brass.	5.5	0.070	0.46	..
Zinc, pressed	5.6	0.073	0.40	0.365
Platinum, annealed. . .	9.0	0.115	1.90	..
Iron, annealed	9.5	0.121	0.74	..
Gold(2) Silver(1) alloy .	10.9	0.138	1.65	0.065
Nickel, annealed	12.4	0.157	1.10	..
Tin, pressed	13.0	0.166	0.95	0.365
Lead, „	19.0	0.242	2.16	0.387
German Silver	20.8	0.265	1.77	0.044
Platinum(2) Silver(1) alloy	24.0	0.306	3.3?	0.031
Antimony	35.2	..	2.36	0.389
Mercury (liquid)	94.2	..	..	0.072
Bismuth	130.0	1.656	12.7	0.354
Electric Light Carbons . (about)	6000.	..	..	..

* These results must be multiplied by 1000 to reduce them to the C. G. S. system

37b. Specific Electrical Resistances of Insulators.

Name of Substance	Resist. in Ohms of a centimetre-cube †	% increase per °.
Selenium	(about) 60,000.	+ 1.
Gutta Percha	„ 7,000,000,000,000,000.	— 10?
Shellac	„ 9,000,000,000,000,000.	..
Ebonite	„ 30,000,000,000,000,000.	..
Paraffine	„ 30,000,000,000,000,000.	..
Glass	Greater than any above.	great, negative.
Air and other Gases.	Practically Infinite.	..

† These results must be multiplied by 1,000,000,000 to reduce them to the C. G. S. System

38. Specific Electrical Resistance in Ohms, of a Centimetre-cube of different Electrolytes (see Table 31).

Per Cent. %	Hydrochloric Acid, HCl	Nitric Acid. HNO_3	Sulphuric Acid H_2SO_4	Sulphate of Copper $CuSO_4$	Sulphate of Zinc $ZnSO_4$	Chloride Ammonium H_4NCl	Chloride Sodium NaCl
5	2.6	3.8	5.0	56.0	55.0	11.6	16.0
10	1.6	2 2	2.6	33.0	33.0	6.0	9.0
15	1 4	1 6	1.9	25.0	26.0	4.0	6.5
20	1.3	1 4	1.5	20.0	23.0	3.2	5.5
25	1.4	1.3	1.4	..	22.5	2.8	5.0
30	1.5	1.3	1 4	..	25 0		
35	1.7	1.3	1.4		30.0		
40	2.0	1 4	1.5				
45		1.5	1.6				
50		1.6	1 9				
60		2.0	2.7				
70		2.5	4.8				
80		3.7	9.0				
90			10.0				
100			12.5				

Note. The results in this table must be multiplied by 1,000,000,000 to reduce them to the C. G. S. System. They are intended to be accurate at about 18°, but are subject to a probable error of about 10%. See Table 31.

39 – Fahrenheit and Centigrade Thermometers.

C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.
−125	−193.0	0	32.0	25	77.0	50	122.0	75	167.0	100	212.0	225	437.0	350	662
120	184.0	1	33.8	26	78.8	51	123.8	76	168.8	105	221.0	230	446.0	400	752
115	175.0	2	35.6	27	80.6	52	125.6	77	170.6	110	230.0	235	455.0	450	842
110	166.0	3	37.4	28	82.4	53	127.4	78	172.4	115	239.0	240	464.0	500	932
105	157.0	4	39.2	29	84.2	54	129.2	79	174.2	120	248.0	245	473.0	550	1022
100	148.0	5	41.0	30	86.0	55	131.0	80	176.0	125	257.0	250	482.0	600	1112
95	139.0	6	42.8	31	87.8	56	132.8	81	177.8	130	266.0	255	491.0	650	1202
90	130.0	7	44.6	32	89.6	57	134.6	82	179.6	135	275.0	260	500.0	700	1292
85	121.0	8	46.4	33	91.4	58	136.4	83	181.4	140	284.0	265	509.0	750	1382
80	112.0	9	48.2	34	93.2	59	138.2	84	183.2	145	293.0	270	518.0	800	1472
75	103.0	10	50.0	35	95.0	60	140.0	85	185.0	150	302.0	275	527.0	850	1562
70	94.0	11	51.8	36	96.8	61	141.8	86	186.8	155	311.0	280	536.0	900	1652
65	85.0	12	53.6	37	98.6	62	143.6	87	188.6	160	320.0	285	545.0	950	1742
60	76.0	13	55.4	38	100.4	63	145.4	88	190.4	165	329.0	290	554.0	1000	1832
55	67.0	14	57.2	39	102.2	64	147.2	89	192.2	170	338.0	295	563.0	1050	1922
50	58.0	15	59.0	40	104.0	65	149.0	90	194.0	175	347.0	300	572.0	1100	2012
45	49.0	16	60.8	41	105.8	66	150.8	91	195.8	180	356.0	305	581.0	1200	2192
40	40.0	17	62.6	42	107.6	67	152.6	92	197.6	185	365.0	310	590.0	1300	2372
35	31.0	18	64.4	43	109.4	68	154.4	93	199.4	190	374.0	315	599.0	1400	2552
30	22.0	19	66.2	44	111.2	69	156.2	94	201.2	195	383.0	320	608.0	1500	2732
25	13.0	20	68.0	45	113.0	70	158.0	95	203.0	200	392.0	325	617.0	1600	2912
20	−4.0	21	69.8	46	114.8	71	159.8	96	204.8	205	401.0	330	626.0	1700	3092
15	+5.0	22	71.6	47	116.6	72	161.6	97	206.6	210	410.0	335	635.0	1800	3272
10	+14.0	23	73.4	48	118.4	73	163.4	98	208.4	215	419.0	340	644.0	1900	3452
5	+23.0	24	75.2	49	120.2	74	165.2	99	210.2	220	428.0	345	653.0	2000	3632
−0	+32.0	25	77.0	50	122.0	75	167.0	100	212.0	225	437.0	350	662.0	2100	3812

40. Hydrometer Scales.

Reading	Baumé heavy liquids	Baumé light liquids	Beck heavy liq.	Beck light liq.	Cartier	Twaddell.
0	1.000		1.000	1.000		1.000
5	1.035		1.030	.971		1.025
10	1.073	1.000	1.062	.944		1.050
15	1.114	.967	1.097	.919	.970	1.075
20	1.158	.936	1.133	.895	.936	1.100
25	1.205	.907	1.172	.872	.905	1.125
30	1.257	.880	1.214	.850	.876	1.150
35	1.313	.854	1.259	.829	.849	1.175
40	1.375	.830	1.308	.810	.824	1.200
45	1.442	.807	1.360	.791		1.225
50	1.517	.785	1.417	.773		1.250
55	1.599	.764	1.478	.756		1.275
60	1.691	.745	1.545	.739		1.300
65	1.795		1.619	.723		1.325
70	1.912		1.700	.708		1.350
75	2.045		1.790			1.375
80						1.400
100						1.500

41. Wave-lengths in Air.

Fraunhofer Line	Designation	Element	Color	Bunsen's scale	Kirchoff's scale	Wave-Length cm.
—	Kα	K	—	17	383	.00007685
A	—	—	—	18	404	.00007605
B	—	—	Red	28	593	.00006870
—	Liα	Li	—	32	645	.00006708
C	Hα	H	—	34	694	.00006563
D_1	Na	Na	Yellow	50	1003	.00005896
D_2					1007	.00005890
—	—	Tl	Green	68	—	.00005350
E	—	—	—	71	1523	.00005270
F	Hβ	H	—	90	2080	.00004862
—	Srδ	Sr	Blue	105	2386	.00004607
f	Hγ	H	—	127	—	.00004341
G	—	—	—	128	2854	.00004309
g	—	Ca	—	135	2870	.00004227
—	Hδ	H	Violet	151	—	.00004102
—	Kβ	K	—	153	—	.00004060
H_1	H	Ca	—	162	—	.00003969
H_2	—	—	—	166	—	.00003934

42. a. English Board of Trade (Imperial) Wire Gauge.

No. of Wire on Gauge	Diameter of Wire in cm.	No.	Diam.	No.	Diam.	No.	Diam.	No.	Diam.	No.	Diam
		1	0.762	11	0.295	21	.0813	31	.0295	41	.0112
		2	.701	12	.261	22	.0711	32	.0274	42	.0102
		3	.610	13	.234	23	.0610	33	.0254	43	.0091
7/0	1.270	4	.581	14	.203	24	.0559	34	.0234	44	.0081
6/0	1.179	5	.538	15	.183	25	.0508	35	.0213	45	.0071
5/0	1.097	6	.488	16	.163	26	.0457	36	.0193	46	.0061
4/0	1.016	7	.447	17	.142	27	.0417	37	.0173	47	.0051
3/0	.945	8	.406	18	.122	28	.0376	38	.0152	48	.0041
2/0	.884	9	.366	19	.102	29	.0345	39	.0132	49	.0031
0	.823	10	0.325	20	0.091	30	.0315	40	.0122	50	.0025

42 b. Birmingham Wire Gauge (B. W. G.)

No. of Wire on Gauge	Diameter of Wire in cm	No.	Diam.	No.	Diam.	No.	Diam.	No.	Diam.
		1	0.80	10	0.35	19	0.110	28	0.037
		2	.74	11	32	20	.091	29	.034
		3	.68	12	.28	21	.083	30	.031
		4	.62	13	.25	22	.073	31	.026
		5	.57	14	.22	23	.065	32	.023
0000	1.2	6	.53	15	.19	24	.057	33	.021
000	1.1	7	.48	16	.17	25	.051	34	.018
00	1.0	8	.43	17	.15	26	.046	35	.013
0	0.9	9	0.39	18	0.13	27	0.041	36	0.010

43. Musical Pitch (Tempered Scale—complete Vibrations per second).

Physical Pitch	32 foot Octave	16 foot Octave	Great Octave	Little Octave	2 foot Octave	1 foot Octave	6 inch Octave	3 inch Octave	Concert Pitch (approx.)
C	16.0	32.0	64.0	128.0	256.0**	512.0	1024	2048	
	16.5	32.9	65.9	131.8	263.5**	527.0	1054	2108	
C#	17.0	33.9	67.8	135.6	271.2**	542.4	1085	2170	C
	17.4	34.9	69.8	139.6	279.2	558.3	1117	2233	
D	18.0	35.9	71.8*	143.7	287.4†	574.7	1149	2298	C#
	18.5	37.0	73.9*	147.9	295.8†	591.5	1183	2366	
D#	19.0	38.1	76.1*	152.2	304.4†	608.9	1218	2436	D
	19.6	39.2	78.3	156.7	313.4	626.7	1253	2507	
E	20.2	40.3	80.6	161.3	322.5	645.1	1290	2580	D#
	20.7	41.5	83.0	166.0	332.0	664.0	1328	2656	
F	21.4	42.7	85.4	170.9	341.7	683.4	1367	2734	E
	22.0	44.0	88.0	175.9	351.7	703.5	1407	2814	
F#	22.6	45.3	90.5	181.0	362.0	724.1	1448	2896	F
	23.3	46.6	93.2	186.3	372.6	745.3	1491	2981	
G	24.0	47.9	95.9	191.8	383.6	767.1§	1534	3068	F#
	24.7	49.4	98.7	197.4	394.8	789.6§	1579	3158	
G#	25.4	50.8	101.6	203.2	406.4	812.8§	1626	3251	G
	26.1	52.3	104.6	209.1	418.3	836.6	1673	3346	
A	26.9	53.8	107.6	215.3	430.5††	861.1	1722	3444	G#
	27.7	55.4	110.8	221.6	443.2††	886.3	1773	3545	
A#	28.5	57.0	114.0	228.1	456.1††	912.3	1825	3649	A
	29.3	58.7	117.4	234.8	469.5	939.0	1878	3756	
B	30.2	60.4	120.8	241.6	483.3	966.5	1933	3866	A#
	31.1	62.2	124.4	248.7	497.4	994.8	1990	3979	
	32.0	64.0	128.0	256.0	512.0	1024.0	2048	4096	B

Note. The Paris Conservatoire standard of pitch, recently adopted by the International Congress at Vienna, is 435 vibrations per second for the note A of the treble staff. This gives C=261 on the natural scale. American instruments tuned to "Concert Pitch" give C = 270 +.

* Lowest D of Bass Voice. ** Middle C of Piano. † Lowest D of Flute. †† Violin A. § Highest G of Treble Voice.

44 A. Reduction of Minutes (′) and seconds (″) to thousandths of a degree ($\frac{1}{1000}$)°.

Min.	°	Sec.	Thousandths of degrees
0′	°.0	0″.0	00
		3″.6	01
6′	°.1	7″.2	02
		10″.8	03
12′	°.2	14″.4	04
		18″.0	05
18′	°.3	21″.6	06
		25″.2	07
24′	°.4	28″.8	08
		32″.4	09
30′	°.5	36″.0	10
		39″.6	11
36′	°.6	43″.2	12
		46″.8	13
42′	°.7	50″.4	14
		54″.0	15
48′	°.8	57″.6	16
		61″.2	17
54′	°.9		

Min.	°	Sec.	Thousandths of degrees
1′	°.0	1″.2	17
		4″.8	18
7′	°.1	8″.4	19
		12″.0	20
13′	°.2	15″.6	21
		19″.2	22
19′	°.3	22″.8	23
		26″.4	24
25′	°.4	30″.0	25
		33″.6	26
31′	°.5	37″.2	27
		40″.8	28
37′	°.6	44″.4	29
		48″.0	30
43′	°.7	51″.6	31
		55″.2	32
49′	°.8	58″.8	33
		62″.4	34
55′	°.9		

Min.	°	Sec.	Thousandths of degrees
2′	°.0	2″.4	34
		6″.0	35
8′	°.1	9″.6	36
		13″.2	37
14′	°.2	16″.8	38
		20″.4	39
20′	°.3	24″.0	40
		27″.6	41
26′	°.4	31″.2	42
		34″.8	43
32′	°.5	38″.4	44
		42″.0	45
38′	°.6	45″.6	46
		49″.2	47
44′	°.7	52″.8	48
		56″.4	49
50′	°.8	60″.0	50
56′	°.9		

Min.	°	Sec.	Thousandths of degrees
3′	°.0	0″.0	50
		3″.6	51
9′	°.1	7″.2	52
		10″.8	53
15′	°.2	14″.4	54
		18″.0	55
21′	°.3	21″.6	56
		25″.2	57
27′	°.4	28″.8	58
		32″.4	59
33′	°.5	36″.0	60
		39″.6	61
39′	°.6	43″.2	62
		46″.8	63
45′	°.7	50″.4	64
		54″.0	65
51′	°.8	57″.6	66
		61″.2	67
57′	°.9		

Min.	°	Sec.	Thousandths of degrees
4′	°.0	1″.2	67
		4″.8	68
10′	°.1	8″.4	69
		12″.0	70
16′	°.2	15″.6	71
		19″.2	72
22′	°.3	22″.8	73
		26″.4	74
28′	°.4	30″.0	75
		33″.6	76
34′	°.5	37″.2	77
		40″.8	78
40′	°.6	44″.4	79
		48″.0	80
46′	°.7	51″.6	81
		55″.2	82
52′	°.8	58″.8	83
		62″.4	84
58′	°.9		

Min.	°	Sec.	Thousandths of degrees
5′	°.0	2″.4	84
		6″.0	85
11′	°.1	9″.6	86
		13″.2	87
17′	°.2	16″.8	88
		20″.4	89
23′	°.3	24″.0	90
		27″.6	91
29′	°.4	31″.2	92
		34″.8	93
35′	°.5	38″.4	94
		42″.0	95
41′	°.6	45″.6	96
		49″.2	97
47′	°.7	52″.8	98
		56″.4	99
53′	°.8	60″.0	100
59′	°.9		

Dif.	0″.2	0″.4	0″.6	0″.8	1″.0	1″.2	1″.4	1″.6	1″.8	2″.0	2″.2	2″.4	2″.6	2″.8	3″.0	3″.2	3″.4	3″.6
	6	11	17	22	28	33	39	44	50	56	61	67	72	78	83	89	94	100

44 B. Correction to be added to Dates in Different years to compare them with Tables for 1891.

Jan. Feb.	h. m. s.	days	Mch.-Dec.	h. m. s.	days	year	h. m. s.	days	year	h. m. s.	days	year	h. m. s.	days
1892	−5 48 49	−.242	1892	+18 11 11	+.758	1893	+12 22 22	+.516	1894	+6 33 33	+.273	1895	+0 44 44	+.031
1896	−5 4 5	−.211	1896	+18 55 55	+.789	1897	+13 7 6	+.547	1898	+7 18 17	+.304	1899	+1 29 28	+.062

44 C. Gain of Sidereal Time during different Intervals of Mean Solar Time.

Days	Gain	Days	Gain	Days	Gain	Days	Gain	Hours	Gain	Hours	Gain	Minutes	Gain	Minutes	Gain
½	1m. 58s.	3½	13m. 48s.	6½	25m. 38s.	9½	37m. 27s.	1	0m. 10s.	7	1m. 9s.	0—3	0 sec.	34—39	6 sec.
1	3 57	4	15 46	7	27 36	10	39 26	2	0 20	8	1 19	4—9	1	40—45	7
1½	5 55	4½	17 45	7½	29 34	10½	41 24	3	0 30	9	1 29	10—15	2	46—51	8
2	7 53	5	19 43	8	31 32	11	43 22	4	0 39	10	1 39	16—21	3	52—57	9
2½	9 51	5½	21 41	8½	33 31	11½	45 20	5	0 49	11	1 48	22—27	4	58—63	10
3	11 50	6	23 39	9	35 29	12	47 19	6	0 59	12	1 58	28—33	5	64—70	11

44 D. Sidereal Time at Greenwich mean Noon on Different Dates in the year 1891.

Jan.	0 18h. 39m. 27s.	March	0 22h. 32m. 4s.	May	0 2h. 32m. 34s.	July	0 6h. 33m. 4s.	Sept.	0 10h. 37m. 31s.	Nov.	0 14h. 31m. 4s.
	10 19 18 53		10 23 11 30		10 3 12 0		10 7 12 30		10 11 16 56		10 15 17 26
	20 19 58 19		20 23 50 55		20 3 51 25		20 7 51 55		20 11 56 22		20 15 56 51
Feb.	0 20 41 41	April	0 0 34 17	June	0 4 34 47	Aug.	0 8 35 17	Oct.	0 12 35 47	Dec.	0 16 36 17
	10 21 21 6		10 1 13 43		10 5 14 13		10 9 14 43		10 13 15 13		10 17 15 43
	20 22 0 32		20 1 53 9		20 5 53 38		20 9 54 8		20 13 54 48		20 17 55 8

44 E. Semidiameter of the Sun in Thousandths of a Degree for the First Day of Each Month in 1891.

Jan. 0.272; Feb. 0.271; Mar. 0.269; Apr. 0.267; May 0.265; June 0.263; July 0.263; Aug. 0.263; Sept. 0.265; Oct. 0.267; Nov. 0.269; Dec. 0.271.

44, F. Declination of the Sun in Degrees at Greenwich Mean Noon for 1891.

Day	Jan.	Feb.	March	April	May	June	July	August	Sept.	Oct.	Nov.	Dec.	Day
	−	−	±	+	+	+	+	+	±	−	−	−	
0	23.091	17.378	−7.958	4.157	14.772	21.920	23.191	18.293	+8.664	2.791	14.117	21.661	0
1	23.011	17.096	7.579	4.543	15.077	22.060	23.127	18.044	8.303	3.180	14.440	21.820	1
2	22.925	16.809	7.199	4.928	15.377	22.193	23.057	17.790	7.938	3.568	14.759	21.972	2
3	22.831	16.516	6.816	5.312	15.673	22.319	22.980	17.531	7.572	3.956	15.074	22.118	3
4	22.729	16.219	6.432	5.694	15.966	22.439	22.897	17.268	7.203	4.343	15.385	22.255	4
5	22.619	15.918	−6.046	6.074	16.253	22.553	22.806	16.999	+6.833	4.729	15.692	22.386	5
6	22.502	15.611	5.659	6.453	16.536	22.660	22.710	16.726	6.461	5.114	15.994	22.509	6
7	22.378	15.301	5.271	6.830	16.815	22.760	22.606	16.449	6.087	5.498	16.292	22.626	7
8	22.246	14.986	4.881	7.205	17.089	22.854	22.496	16.167	5.711	5.881	16.585	22.734	8
9	22.108	14.666	4.491	7.578	17.358	22.941	22.380	15.881	5.334	6.262	16.873	22.836	9
10	21.961	14.343	−4.099	7.949	17.622	23.021	22.257	15.590	+4.955	6.643	17.157	22.929	10
11	21.808	14.016	3.707	8.318	17.882	23.095	22.128	15.296	4.575	7.021	17.436	23.015	11
12	21.648	13.684	3.314	8.684	18.136	23.162	21.993	14.997	4.194	7.398	17.710	23.094	12
13	21.481	13.350	2.920	9.048	18.386	23.222	21.851	14.694	3.811	7.773	17.978	23.165	13
14	21.306	13.011	2.525	9.409	18.630	23.275	21.703	14.388	3.427	8.147	18.242	23.228	14
15	21.125	12.669	−2.131	9.768	18.869	23.321	21.548	14.078	+3.043	8.518	18.500	23.283	15
16	20.938	12.324	1.736	10.124	19.102	23.360	21.388	13.763	2.657	8.888	18.752	23.331	16
17	20.743	11.975	1.341	10.477	19.331	23.393	21.221	13.446	2.271	9.255	18.999	23.371	17
18	20.542	11.623	0.945	10.828	19.553	23.419	21.049	13.124	1.883	9.620	19.241	23.403	18
19	20.335	11.269	0.550	11.175	19.770	23.437	20.871	12.800	1.495	9.983	19.476	23.428	19
20	20.121	10.911	−0.155	11.519	19.982	23.449	20.687	12.472	+1.107	10.343	19.706	23.444	20
21	19.901	10.551	+0.240	11.860	20.188	23.454	20.497	12.141	0.718	10.701	19.930	23.453	21
22	19.675	10.188	0.635	12.198	20.388	23.452	20.301	11.806	+0.328	11.056	20.148	23.454	22
23	19.443	9.822	1.029	12.532	20.582	23.444	20.100	11.468	−0.061	11.409	20.359	23.447	23
24	19.204	9.454	1.423	12.863	20.771	23.428	19.893	11.128	0.451	11.758	20.565	23.432	24
25	18.960	9.083	+1.816	13.190	20.953	23.405	19.680	10.784	−0.841	12.105	20.764	23.409	25
26	18.710	8.710	2.208	13.514	21.130	23.376	19.463	10.437	1.232	12.449	20.957	23.379	26
27	18.455	8.335	2.600	13.834	21.300	23.340	19.239	10.088	1.622	12.789	21.143	23.341	27
28	18.194	7.958	2.991	14.151	21.464	23.297	19.011	9.736	2.012	13.127	21.323	23.294	28
29	17.927	[7.579]	3.380	14.463	21.623	23.247	18.777	9.381	2.401	13.460	21.495	23.241	29
30	17.655		3.769	14.772	21.775	23.191	18.538	9.024	2.791	13.791	21.661	23.179	30
31	17.378		4.157		21.920		18.293	8.664		14.117		23.109	31

44, G. Eqnation of Time in Minntes and Seconds at Greenwich Mean Noon for 1891.

Day	Jan.	Feb.	March	April	May	June	July	August	Sept.	Oct.	Nov.	Dec.	Day
	+	+	+	±	−	±	+	+	±	−	−	±	
	m s	m s	m s	m s	m s	m s	m s	m s	m s	m s	m s	m s	
0	3 16	13 40	12 45	+4 17	2 51	−2 35	3 20	6 10	+0 15	9 59	16 18	−11 14	0
1	3 45	13 48	12 33	3 59	2 59	2 27	3 32	6 7	−0 4	10 18	16 20	10 52	1
2	4 13	13 56	12 21	3 41	3 6	2 18	3 44	6 3	0 23	10 37	16 21	10 29	2
3	4 41	14 2	12 8	3 23	3 13	2 8	3 55	5 59	0 42	10 56	16 21	10 5	3
4	5 8	14 8	11 55	3 5	3 19	1 58	4 6	5 54	1 1	11 14	16 20	9 41	4
5	5 35	14 13	11 42	+2 48	3 25	−1 48	4 17	5 48	−1 21	11 32	16 19	− 9 17	5
6	6 2	14 18	11 28	2 30	3 30	1 37	4 27	5 42	1 41	11 50	16 16	8 51	6
7	6 28	14 21	11 14	2 13	3 34	1 26	4 37	5 35	2 1	12 7	16 13	8 26	7
8	6 54	14 24	10 59	1 56	3 38	1 15	4 47	5 28	2 21	12 24	16 9	7 59	8
9	7 19	14 26	10 44	1 39	3 41	1 3	4 56	5 20	2 42	12 40	16 5	7 33	9
10	7 44	14 27	10 29	+1 23	3 44	−0 51	5 5	5 12	−3 3	12 56	15 59	− 7 6	10
11	8 8	14 28	10 13	1 7	3 46	0 39	5 13	5 3	3 23	13 12	15 53	6 38	11
12	8 32	14 27	9 57	0 51	3 48	0 27	5 21	4 53	3 44	13 27	15 45	6 10	12
13	8 54	14 26	9 40	0 35	3 49	0 15	5 28	4 43	4 5	13 41	15 37	5 42	13
14	9 17	14 24	9 24	0 20	3 49	− 0 2	5 35	4 32	4 27	13 55	15 28	5 13	14
15	9 38	14 22	9 7	+0 5	3 49	+0 10	5 42	4 21	−4 48	14 9	15 19	− 4 45	15
16	9 59	14 18	8 50	−0 10	3 48	0 23	5 48	4 9	5 9	14 22	15 8	4 16	16
17	10 19	14 14	8 32	0 24	3 47	0 36	5 53	3 56	5 30	14 34	14 57	3 46	17
18	10 38	14 9	8 15	0 38	3 45	0 49	5 58	3 43	5 52	14 46	14 44	3 17	18
19	10 57	14 3	7 57	0 52	3 43	1 2	6 2	3 30	6 13	14 58	14 31	2 47	19
20	11 15	13 57	7 39	−1 5	3 40	+1 15	6 6	3 16	−6 34	15 8	14 17	− 2 18	20
21	11 32	13 50	7 21	1 18	3 37	1 28	6 9	3 1	6 55	15 18	14 3	1 48	21
22	11 48	13 43	7 3	1 30	3 33	1 40	6 12	2 46	7 16	15 27	13 47	1 18	22
23	12 4	13 34	6 44	1 42	3 29	1 53	6 14	2 31	7 37	15 36	13 31	0 48	23
24	12 19	13 26	6 26	1 53	3 24	2 6	6 15	2 15	7 58	15 44	13 13	− 0 18	24
25	12 33	13 16	6 8	−2 4	3 18	+2 19	6 16	1 59	−8 19	15 51	12 55	+ 0 12	25
26	12 46	13 6	5 49	2 15	3 12	2 31	6 17	1 43	8 39	15 57	12 37	0 42	26
27	12 58	12 56	5 31	2 25	3 06	2 44	6 16	1 26	8 59	16 3	12 17	1 12	27
28	13 10	12 45	5 12	2 34	2 59	2 56	6 16	1 9	9 19	16 8	11 57	1 41	28
29	13 21	[12 33]	4 54	2 43	2 52	3 8	6 14	0 51	9 39	16 12	11 36	2 11	29
30	13 31		4 35	2 51	2 44	3 20	6 12	0 33	9 59	16 15	11 14	2 40	30
31	13 40		4 17		2 35		6 10	0 15		16 18		3 9	31

44, H. Solar System.

Names	Time of Sidereal Revolution in Mean Solar Days	Relative distance from Sun Earth =1	Relative Mass Earth =1	Distance in Mega-Kilom. 10^{11} cm	Diameter in Megametres 10^8 cm	Mass in tetra-Mega-Kilos 10^{24} grams	Mean Density g. per cu. cm.
Sun . . .	...	...	320.000	...	1.392	2,000,000	1.4
Mercury .	87.97	.387	0.07?	58.	4.8	0.4?	6.?
Venus . .	224.70	.723	0 8?	108.	12.2	5.?	6.?
Earth . .	365.26	1.000	1.00	149.	12.74	6.1	5.6
Moon . .	27.32	.0026*	0.012	*0.39	3.48	0.07	3.4
Mars . . .	686.98	1.524	0.11	227.	8.	0.7	4.
Jupiter . .	4332.53	5.203	310.	777.	142.	1900.	1.3
Saturn . .	10759.22	9.539	93.	1424.	119.	570.	0.7
Uranus .	30686.82	19.18	14.	2864.	50.	85.	1.3
Neptune .	60126.71	30.05	17.	4487.	60.	100.	0.9

* Distance from the Earth.

45. Mean Position of Fixed Stars, Jan. 0 1891.

Names	Designation	Magnitude	Right Ascension	Yearly Change	Declination	Yearly Change
			h m s	s	°	°
Sirrah	α Andromedae	2	0 2 45.2	+3.09	+28.489	+.0055
Polaris	α Ursae Minoris	2	1 18 53.4	2.36	+88.727	+.0053
—	α Arietis	2	2 1 1.7	3.37	+22.947	+.0048
Aldebaran	α Tauri	1	4 29 39.9	3.44	+16.289	+.0021
Capella	α Aurigae	1	5 8 38.2	4.43	+45.886	+.0011
Rigel	β Orionis	1	5 9 17.9	2.88	— 8.328	+.0012
Beteigeuze	α Orionis	1	5 49 16.2	3.25	+ 7.386	+.0003
Canopus	α Argus	1	6 21 31.9	1.33	—52.636	—.0005
Sirius	α Canis Majoris	1	6 40 20.6	2.64	—16.567	—.0013
Castor	α^2 Geminorum	2-1	7 27 38.7	3.84	+32.127	—.0021
Procyon	α Canis Minoris	1	7 33 35.7	3.14	+ 5.504	—.0025
Pollux	β Geminorum	1-2	7 38 38.7	3.68	+28.289	—.0023
Regulus	α Leonis	1-2	10 2 34.0	3.20	+12.500	—.0049
Denebola	β Leonis	2	11 43 30.0	3.06	+15.181	—.0056
—	α Crucis	1	12 20 32.6	3.30	—62.495	—.0056
Spica	α Virginis	1	13 19 27.0	3.15	—10.592	—.0053
—	β Centauri	1	13 56 8.0	4.18	—59.847	—.0049
Arcturus	α Bootis	1	14 10 41.3	2.73	+19.750	—.0053
—	α^2 Centauri	1	14 32 12.5	4.04	—60.383	—.0042
Antares	α Scorpii	1-2	16 22 43.4	3.67	—26.189	—.0023
Vega	α Lyrae	1	18 33 14.8	2.03	+38.683	+.0009
Altair	α Aquilae	1-2	19 45 27.9	2.93	+ 8.581	+.0026
Deneb	α Cygni	2-1	20 37 42.9	2.04	+44.891	+.0035
Formalhaut	α Piscis Aust.	1-2	22 51 37.6	3.32	—30.200	+.0053
Markab	α Pegasi	2	22 59 19.8	+2.98	+14.619	+.0054

Note. The yearly precession of the equinoxes is about 50″.25, or 0°.00245+. The mean (not apparent) obliquity of the ecliptic for 1891 is about 23°, 27′, 13″, or 27°.453. The mean obliquity decreases annually by 0″.8, or 0°.0002.

46. Latitudes and Longitudes Measured from Greenwich.

Place		Latitude °		Longitude h m s		Elevation Metres
Aberdeen	O	57.149	N	0 8 23	W	
Amsterdam	T	52.371	N	0 19 39	E	
Antwerp	T	51.221	N	0 17 37	E	
Athens	O	37.972	N	1 34 55	E	
Baltimore	T	39.298	N	5 6 28	W	55
Belfast		54.66	N	0 23 ..	W	
Berlin	O	52.505	N	0 53 35	E	40
Bonn		50.729	N	0 28 23	E	50
Boston	T	42.358	N	4 44 15	W	63
Brussels		50.853	N	0 17 29	E	90
Calcutta	T	22.557	N	5 53 19	E	39
Cambridge U. S.	O	42.380	N	4 44 31	W	
Cambridge Eng.	O	52.215	N	0 0 23	E	
Cape of Good Hope	O	33.934	S	1 13 55	E	
Christiania	O	59.912	N	0 42 54	E	42
Copenhagen	O	55.687	N	0 50 19	E	53
Cork	T	51.90	N	0 33 51	W	
Dublin	O	53.387	N	0 25 21	W	T 24
Edinboro	O	55.956	N	0 12 43	W	T 139
Geneva		46.200	N	0 24 37	E	
Genoa	T	44.419	N	0 35 41	E	
Glasgow	O	55.879	N	0 17 11	W	
Göttingen		51.530	N	0 39 46	E	130
Greenwich	O	51.477	N	0 0 0		T 64
Heidelburg		49.40	N	0 34 32	E	100
Leipzic		51.335	N	0 49 34	E	100
Lisbon	O	38.705	N	0 36 34	W	
Liverpool	O	53.401	N	0 12 17	W	
Magnetic Pole		77.83	N	4 14	W	
London		51.514	N	0 0 23	W	50
Madrid	O	40.408	N	0 14 45	W	663
Manchester		53.48	N	0 9 ..	W	
Melbourne		37.831	S	9 39 54	E	
Montreal	T	45.52	N	4 54 13	W	44
Munich		48.146	N	0 46 26	E	525
Naples	O	40.863	N	0 57 1	W	
New Orleans	T	29.963	N	6 0 14	W	43
New York	O	40.730	N	4 55 57	W	T 86
Paris	O	48.836	N	0 9 21	E	60
Philadelphia	T	39.953	N	5 0 39	W	50
Quebec	O	46.805	N	4 44 49	W	T 108
Queenstown	T	51.85	N	0 33 6	W	
Rio de Janeiro	O	22.907	S	2 52 41	W	T 69
Rome		41.898	N	0 49 54	E	29
Rotterdam	T	51.908	N	0 17 55	E	28
San Francisco	O	37.790	N	8 9 43	W	T 111
Savannah	T	32.081	N	5 24 21	W	42
St. John (N.S.)	T	45.262	N	4 24 15	W	38
St Petersburg	O	59.942	N	2 1 14	E	11
Stockholm	O	59.343	N	1 12 14	E	20
Strassburg	O	48.582	N	0 31 2	E	150
Sydney	O	33.861	S	10 4 50	E	T 65
Triest	O	45.643	N	0 55 2	E	T 17
Venice	O	45.430	N	0 49 25	E	
Vienna		48.210	N	1 5 32	E	182
Washington	O	38.894	N	5 8 12	W	T 63
Wellington	T	41.288	S	11 39 11	E	T 12

[*Note.* T = Time Signal. O = Observatory.

47. Acceleration of Gravity in Different Latitudes (cm. per sec. per sec.).*

Lat.	+0°	+1°	+2°	+3°	+4°	+5°	+6°	+7°	+8°	+9°	Dif
0°	978.10	978.10	978.11	978.12	978.13	978.14	978.16	978.18	978.20	978.23	1
10°	978.25	978.29	978.32	978.36	978.40	978.44	978.48	978.53	978.58	978.63	4
20°	978.69	978.75	978.81	978.87	978.93	979.00	979.06	979.13	979.21	979.28	7
30°	979.35	979.43	979.51	979.59	979.67	979.75	979.83	979.92	980.00	980.09	8
40°	980.17	980.26	980.34	980.43	980.52	980.61	980.69	980.78	980.86	980.95	9
50°	981.04	981.13	981.21	981.30	981.38	981.46	981.54	981.62	981.70	981.78	8
60°	981.86	981.93	982.01	982.08	982.15	982.21	982.28	982.34	982.41	982.47	7
70°	982.52	982.58	982.63	982.68	982.73	982.77	982.82	982.86	982.89	982.93	4
80°	982.96	982.99	983.01	983.03	983.05	983.07	983.08	983.09	983.10	983.11	1

48. Length of Seconds-Pendulum in Different Latitudes (cm.).*

Lat.	+0°	+1°	+2°	+3°	+4°	+5°	+6°	+7°	+8°	+9°	Dif
0°	99.103	99.103	99.103	99.104	99.105	99.106	99.108	99.110	99.112	99.115	1
10°	99.118	99.121	99.125	99.128	99.132	99.137	99.141	99.146	99.151	99.156	4
20°	99.162	99.168	99.174	99.180	99.187	99.193	99.200	99.207	99.214	99.222	7
30°	99.229	99.237	99.245	99.253	99.261	99.269	99.278	99.286	99.295	99.303	8
40°	99.312	99.321	99.330	99.338	99.347	99.356	99.365	99.374	99.383	99.391	9
50°	99.400	99.409	99.418	99.426	99.435	99.443	99.451	99.459	99.467	99.475	8
60°	99.483	99.491	99.498	99.505	99.512	99.519	99.526	99.532	99.539	99.545	7
70°	99.550	99.556	99.561	99.566	99.571	99.576	99.580	99.584	99.588	99.591	4
80°	99.594	99.597	99.600	99.602	99.604	99.606	99.607	99.608	99.609	99.610	1

* These values are calculated for the sea level. A deduction of 0.03 % should be made for each kilometre of elevation above the ground and a deduction of 0.02 % should be made for each kilometre of elevation of the ground above the sea.

49a. Reduction of Measures to and from the C. G. S. System.

		Equivalent	Logarithm	Reciprocal
Lengths in centimetres				
1 inch	=	2.53997	0.40483	.393705
1 link = 7.92 in.	=	20.1165	1.30355	.0497103
1 foot = 12 in.	=	30.4796	1.48401	.0328088
1 yard = 3 ft.	=	91.4389	1.96113	.0109363
1 fathom = 6 ft.	=	182.878	2.26216	.00546813
1 rod = $16^1/_2$ ft.	=	502.914	2.70149	.00198841
1 chain = 100 links = 66 ft.	=	2011.65	3.30355	.000497103
1 statute mile = 5280 ft.	=	160,932	5.20664	6.21378×10^{-6}
1 nautical mile	=	185,200(?)	5.2676	5.40×10^{-6}
Areas in square centimetres				
1 square inch	=	6.4514	0.80966	.15500
1 square foot = 144 sq. in.	=	929.01	2.96802	.0010764
1 square yard = 9 sq. ft.	=	8361.1	3.92226	.00011960
1 acre = 43,560 sq. ft.	=	4.0468×10^7	7.60711	2.4711×10^{-8}
1 square mile = 640 acres	=	2.5899×10^{10}	10.41329	3.8611×10^{-11}
Volumes in cubic centimetres				
1 cubic inch	=	16.386	1.21449	.061026
1 cubic foot = 1728 cu. in.	=	28316	4.45203	3.5316×10^{-5}
1 cubic yard = 27 cu. ft.	=	764526	5.88339	1.3080×10^{-6}
1 U. S. pint = 1.043 lbs. water	=	473	2.6750	.002114
1 U. S. quart = 2 pints	=	946	2.9760	.001057
1 dry quart	=	1101	3.0418	.000908
1 U. S. gallon = 231 cu. in. = 4 qts	=	3785	3.5781	.0002642
1 imperial gallon = 10 lbs. water	=	4541	3.6572	.0002202
Masses in grams				
1 grain	=	.0647987	$\bar{2}.81157$	15.4324
1 ounce (Avoirdupois) = $^1/_{16}$ lb.	=	28.3494	1.45254	.0352741
1 ounce (Troy) = 480 grains	=	31.1034	1.49281	.0321509
1 pound (Troy) = 12 oz. Troy	=	373.240	2.57199	.00267924
1 pound (Avoir) = 7000 grains	=	453.590	2.65666	2.20463×10^{-3}
1 English ton = 2240 lbs.	=	1.01604×10^6	6.00691	9.84210×10^{-7}
Times in mean solar seconds				
1 year (tropical) = 365.24222 days	=	31,556,928	7.49809	3.16888×10^{-8}
1 sidereal year = 365.25637 days	=	31,558,150	7.49811	3.16875×10^{-8}
1 (mean solar) day	=	86,400	4.93651	.000011574074
1 hour	=	3,600	3.55630	.00027777778
1 minute	=	60	1.77815	.016666667
1 so-called sidereal second	=	0.9972695666	$\bar{1}.99881$	1.0027379091
1 true sidereal second	=	0.9972696721	$\bar{1}.99881$	1.0027378030
Velocities in centimetres per second				
1 kilometre per hour	=	27.7778	1.44370	.0360000
1 foot per second	=	30.4796	1.48401	.0328088
1 mile per hour	=	44.7033	1.65034	.0223696
1 nautical mile per hour	=	51.44	1.7113	.01944
1 kilometre per minute	=	1666.67	3.22185	.0006000000
1 mile per minute	=	2682.20	3.42849	.000372827
Accelerations in cm. per sec. per. sec.				
1 foot per sec. per sec.	=	30.4796	1.48401	.0328088
Densities in grams per cu. cm.				
1 grain per cubic inch	=	.0039544	$\bar{3}.59708$	252.88
1 lb. per cubic foot	=	.016019	$\bar{2}.20463$	62.426
Heat Units in ergs.				
1 unit of heat = 1 gram-degree C.	=	4.17×10^7	7.620	2.40×10^{-8}
1 lb.-degree Fahrenheit	=	1.051×10^{10}	10.022	9.52×10^{-11}
1 lb.-degree Centigrade	=	1.89×10^{10}	10.277	5.29×10^{-11}
1 Calorie = 1000 g°	=	4.17×10^{10}	10.620	2.40×10^{-11}

49 b. Continuation. Reduction of Measures to and from the C. G. S. System.

Values marked with an asterisk (*) are independent of the acceleration of gravity (g).

	Equivalent $g = 980$	Equivalent $g = 981$	Logarithm $g = 980$	Logarithm $g = 981$	Reciprocal $g = 980$	Reciprocal $g = 981$
Force in dynes						
1 grain weight, in dynes . . . . . . =	63.50	63.57	1.80279	1.80324	.01575	.01573
1 gram " " " . . . . . . =	980.	981.	2.99123	2.99167	.0010204	.0010194
1 oz. (avoir) " " " . . . . . . =	2.778×10^{4}	2.781×10^{4}	4.44377	4.44421	3.599×10^{-5}	3.596×10^{-5}
1 lb. (avoir) " " " . . . . . . =	4.445×10^{5}	4.450×10^{5}	4.64789	5.64833	2.250×10^{-6}	2.247×10^{-6}
1 kilogram " " " . . . . . . =	980,000	981,000	5.99123	5.99167	1.0204×10^{-6}	1.0194×10^{-6}
1 tonne " " " . . . . . . =	980,000,000	981,000,000	8.99123	8.99167	1.0204×10^{-9}	1.0194×10^{-9}
1 English ton " " " . . . . . . =	9.957×10^{8}	9.967×10^{8}	8.99814	8.99858	1.004×10^{-9}	1.003×10^{-9}
1 poundal* " " . . . . . . =	13825*		4.14067*		7.2331×10^{-5}*	
Pressure in dynes per sq. cm.						
1 lb. per sq. ft. in dynes per sq. cm. =	478.5	479.0	2.67987	2.68031	.002090	.002088
1 gram per sq. cm. " " " " =	980	981	2.99123	2.99167	.0010204	.0010194
1 kilo. per sq. decim. " " " " =	9,800	9,810	3.99123	3.99167	.00010204	.00010194
1 cm. mercury at 0° " " " " =	13,324	13,338	4.12464	4.12508	.00007505	.00007498
1 lb. per sq. in. " " " " =	68,902	68,973	4.83823	4.83868	1.451×10^{-5}	$1.45[illegible]\times10^{-5}$
1 kilo per sq. cm. " " " " =	980,000	981,000	5.99123	5.99167	1.0204×10^{-6}	1.0194×10^{-6}
1 atmosphere 30 in. mercury at 62½° F. =	1,012,200	1,013,200	6.00526	6.00571	9.880×10^{-7}	9.870×10^{-7}
" 76 cm. " " 0° C. =	1,012,630	1,013,660	6.00545	6.00589	9.875×10^{-7}	9.865×10^{-7}
" 30 in. " " 0° C. =	1,015,300	1,016,300	6.00659	6.00703	9.849×10^{-7}	9.839×10^{-7}
1 kilo per sq. mm. . . . . =	98,000,000	98,100,000	7.99123	7.99167	1.0204×10^{-8}	1.0194×10^{-8}
1 megadyne per sq. cm.* . . . . =	1,000,000*		6.00000*		1.000000×10^{-6}*	
Work in ergs						
1 gram-centimetre in ergs . . . . =	980	981	2.99123	2.99167	1.0204×10^{-3}	1.0194×10^{-3}
1 kilogram metre " " . . . . =	98,000,000	98,100,000	7.99123	7.99167	1.0204×10^{-8}	1.0194×10^{-8}
1 foot-pound " " . . . . =	13,550,000	13,560,000	7.13190	7.13234	7.381×10^{-8}	7.373×10^{-8}
1 foot-poundal* " " . . . . =	421390*		5.62468*		2.3731×10^{-6}*	
1 joule* . . . . =	10,000,000*		7.00000*		.0000001*	
Power in ergs per second						
1 horse-power {33,000 ft. lbs. per min. . =	7.452×10^{9}	7.459×10^{9}	9.87226	9.87270	1.342×10^{-10}	1.341×10^{-10}
1 horse-power {75 kilogrammetres per sec. =	7.350×10^{9}	7.357×10^{9}	9.86629	9.86673	1.361×10^{-10}	1.359×10^{-10}
1 man's power (approx) in ergs per sec. =	1×10^{9}		9.0		1×10^{-9}	
1 unit of heat per sec. " " " " =	4.17×10^{7}*		7.620*		2.40×10^{-8}*	
1 watt in ergs per sec. . . . . =	1×10^{7}*		7.00000*		1×10^{-7}*	

50. Numbers Frequently Required in Calculation.

	Number	Logarithm	Reciprocal.
Mathematical Constants.			
Ratio of circumference to diameter $\pi =$	3.1415927	.049715	.3183099
Square of Ditto $\pi^2 =$	9.8696044	0.99430	.1013212
Square Root of Ditto $\sqrt{\pi} =$	1.7724539	0.24857	.5641896
Square Root of 2 $\sqrt{2} =$	1.4142136	0.15051	.7071068
Square Root of 3 $\sqrt{3} =$	1.7320508	0.23856	.5773503
Square Root of 10 $\sqrt{10} =$	3.1622777	0.50000	.3162278
Logarithmic Base $e =$	2.7182818	0.43429	.3678794
Logarithmic Modulus $M =$	0.4342945	$\bar{1}.63778$	2.302585
1 degree in circular measure $1^\circ =$	.01745329	$\bar{2}.24188$	57.29578
1 minute in circular measure $1' =$	.00029089	$\bar{4}.46373$	3437.747
1 second in circular measure $1'' =$	.00000485	$\bar{6}.68557$	206264.8
Probable error (mean error = 1)	0.67449	$\bar{1}.82898$	1.4826
Astronomical Constants.			
Sidereal time in Mean time	0.9972696	$\bar{1}.99881$	1.0027379
1 (tropical) year — See Table 49 — in days	365.24222	2.56258	0.0027379
Annual precession of equinoxes (50''.25) in days	0.01415	$\bar{2}.1509$	70.6
Aberration constant (20''45) in degrees	0.00568	$\bar{3}.754+$	176.
Sun's mean angular semidiameter (16'2'') in degrees	0.267	$\bar{1}.427$	3.74
Solar parallax (8''.83?)	.00245	$\bar{3}.390$	408.
Earths equatorial radius in kilom.	6378.	3.8047	.0001568
Earths polar " " "	6356.+	3.8032	.0001573
Gravity. Attraction between two unit masses (1 g) at unit distance (1 cm.) in dynes	6.5×10^{-8}	$\bar{8}.813$	1.54×10^{7}
Seconds-pendulum (lat 45°) in cm.	99.356	1.99719	.010065
1 gram (lat. 45°) in dynes	980.61	2.99149	.0010198
Atmospheric mean molecular weight	28.86	1.4603	.03465
" Pressure (76 cm. Paris) in megadynes per sq. cm.	1.01360	0.00587	.98658
Density of Air, (0°, 76 cm. Paris)	0.0012932	$\bar{3}.11167$	773.3
" " " (0°, 1 megadyne per sq. cm.)	0.0012759	$\bar{3}.10581$	783.8
" " Hydrogen (0°, 76 cm. Paris)	0.00008957	$\bar{5}.95216$	11164.
" " " (0°, 1 megadyne per sq. cm.)	0.00008837	$\bar{5}.94630$	11316.
" " Water at 4°	1.00001	0.000004	0.99999
" " " apparent at 21°	0.997	$\bar{1}.9987$	1.003
" " Crown glass, about	2.5	0.400	0.40
" " Brass "	8.4	0.924	.119
" " Mercury at 19°	13.550	1.13194	.073800
Sound Velocity in dry air at 0° in cm. per sec. 33,220 or	33,200	4.521	.000301
1 mean semitone involves ratio $\sqrt[12]{2}$	1.059463	0.02509	.94387
Light. Velocity in cm. per sec	3.00×10^{10}	10.477	3.33×10^{-11}
" sodium, wave length in air cm.)	.0005893	$\bar{5}.77033$	16970
Refractive Index of water for ditto	1.333	0.1248	.750
Dispersive " " " " "	0.014+	2.15	70
Rotation of ditto by Quartz plate 1 cm. thick	217°	2.3365	.00461
Candle power of 1 sq. cm. melted platinum (2 Carcels)	20	1.30	.05
Heat. Conductivity of Copper C. G. S.	0.9+	$\bar{1}.96$	1.1
Coefficient of Expansion of glass (cubical)	.000025	$\bar{5}.40$	40,000.
" " " " steel (linear)	.000012	$\bar{5}.08$	83,000.
" " " " brass "	.000019	$\bar{5}.28$	53,000.
" " " " mercury (cubical 20°)	.000180+	$\bar{4}.255$	55,000
" " " " gases "	.00367	$\bar{3}.564+$	273
Latent Heat of Water	79	1.900	.0127
" " " Steam (100°, 76 cm.)	536	2.729	.00187
Specific Heat of Brass	0.091	$\bar{2}.973$	10.6
" " " Glass	0.19	$\bar{1}.279$	5.3
" " " Water (0°—100°)	1.005	0.002	.995
" " " Air (0°—100°, 76 cm) 0.2383 or	0.238	$\bar{1}.377$	4.20
" " " Gases, (ratio of 2 Sp. Hts)	1.408	0.1486	.710
Mechanical Equivalent of			
1 unit of heat (1 g°) in ergs, 4.166×10^{7} or	4.17×10^{7}	7.620	2.40×10^{-8}
1 kilogrammetre (lat. 45°)	9.8061×10^{7}	7.99149	1.0198×10^{-8}
1 foot-pound (" ")	1.3557×10^{7}	7.13217	7.376×10^{-8}
Electro-Chemical equivalent of Hydrogen in grams per ampère per sec.	.00001038	$\bar{5}.0162$	9634
1 Electrostatic Unit of E. M. F. in volts	300	2.477	.00333
Electromotive Force of Daniell cell in volts	1.0 to 1.2	0.00 to 0.08	1.0 to 0.8
Internal Resistance of Quart " " " ohms	1 to 2	0.0 to 0.3	1.0 to .5
1 B. A. unit in legal ohms	0.9889	$\bar{1}.9952$	1.011
1 Siemens " " "	0.9434	$\bar{1}.9747$	1.060
Specific Electrical Resistance of Mercury (C. G. S.)	0.942×10^{5}	4.974	1.061×10^{-5}
Magnetic susceptibility of iron	300?	2.5?	.003?
Total Intensity of Earths Field	.3 to .7	$\bar{1}.5$ to $\bar{1}.8$	3 to 1.5

PHYSICAL MEASUREMENT.

Part Fourth.

APPENDICES AND EXAMPLES FOR THE USE OF TEACHERS.

APPENDIX I.

THE LABORATORY.

THE first requisite for a course in elementary Physical Measurement is a well lighted and uniformly heated room, with the ordinary precautions to secure good ventilation. Experience has shown that these advantages cannot practically be obtained in basements, however suitable the latter may be for certain scientific purposes. The first floor of a building, properly supported by brick pillars to prevent vibration, has undoubted advantages for a course of measurements. The use of iron in construction should be in so far as possible avoided on account of its magnetic influence.

If the room above a physical laboratory is to be occupied, there should be an empty space between the floor of that room and the laboratory ceiling.

The latter should *not* be supported by a rod or rods connecting it with the former, but by separate beams or trusses reaching to the walls. Under these conditions only will the vibrations of the upper floor be cut off.

Sometimes, to avoid annoyance from this source, the laboratory is placed in the upper story of a building. The advantages of skylights as a method of illumination, and rafters for purposes of suspension, have been justly urged. They are, however, offset by many practical objections, among which may be mentioned the danger of leakage and the accumulation of dust, both occurring at inconvenient altitudes. Rafters are, moreover, not good points of suspension, since the roof of a building is very sensitive to the wind, and to other sources of vibration. For these reasons, and on account of economy in heating, attics are undesirable for the purposes of physical measurement.

The best possible place in a large building for laboratory work is that usually set aside for lecture purposes; namely, a two or three story room reaching from the first floor to the attic floor, and situated either in an L, or at one end of the building, so as to be lighted from three sides. The attio should be used solely for the storage of apparatus, or as a means of reaching different points in the laboratory ceiling, where suspensions, for instance, may be needed. In the absence of a two-story room, a staircase may perhaps be utilized for long suspensions when necessary (Exp. 65).

The laboratory should be situated as far as convenient from public travel, but not too far from the work-shop. It is hardly desirable that a powerful engine of any sort should be placed under the same roof as the laboratory. Even a smoothly running gas engine may interfere with delicate measurements. The neighborhood of dynamo and even telegraph wires should be avoided, and particularly vertical portions of such wires. Powerful currents, if admitted at all to the laboratory, should come and go through parallel wires (see ¶ 193, 8).

The conditions here named are not so formidable as they may perhaps appear. A common square wooden building with an attic and basement, situated in the middle of a field, has advantages for the purpose of physical measurement which some of our best institutions do not possess.

It is well to have the laboratory ventilated by registers in the floor and ceiling. A large number of small registers in the floor is likely to cause less draught than one or two large registers. It is best, on account of dust, to heat the air before it enters the room, by steam. Furnace heat is, however, not objectionable if the pipes, furnace-chamber, and air-boxes are perfectly tight, so that no dust from the ashes can enter them. The floor, walls, and ceiling, should be finished in paint, wax, oil, shellac, or varnish, so that they may easily be kept clean. Other precautions against dust will be spoken of later on.

The window-sills, if of the ordinary pattern, should be at least 2 ft. 10 in. from the floor, in order that work-tables or benches may be placed in front of them without cutting off the light. There should be, moreover, no obstacle — such as steam-pipes — to prevent such tables or benches from being set close to the walls. This is not only a matter of convenience in preventing small objects from falling behind the tables, but also in some cases the only means of making tables steady enough for delicate experiments. In some cases, such tables have to be supported by pillars reaching to the basement; but this will not be necessary for elementary work.

It is well, on at least one side of the room, not exposed to the sun, to have a wooden bench, 2 feet wide, 2 in. thick, and 33 in. high, fitted permanently to the wall, and if necessary, into the window spaces. A small platform or balcony facing southward will be found convenient for experiments requiring *direct sunlight.*

The windows on the sunny sides of the room should have blinds giving free access to air. White window curtains, closing from below upward, are a convenience, but by no means a necessity. Wooden shutters, or curtains of enamelled cloth almost entirely opaque to light, are needed on all the windows to obtain the best results in photometry, and are absolutely indispensable if experiments in photography (not included in this course) are to be made.[1]

The expense of arranging a laboratory so that it may be darkened is very much less than that of con-

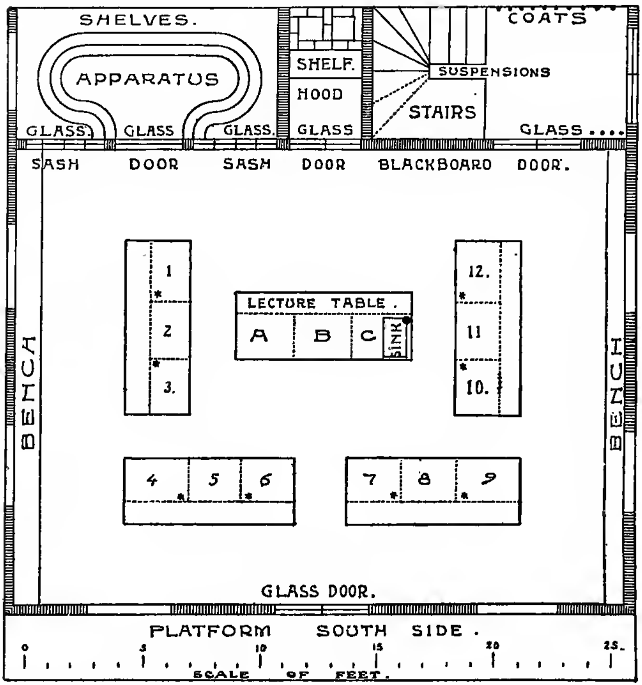
[1] For the latter purpose, it would be well to have yellow glass set into one or two of the windows.

structing the customary "dark room" for photometric and photographic experiments, and has the advantage of furnishing plenty of air and space to the students. This arrangement is, however, practicable only when, as in the course of experiments which the author has planned, *all* the students are to work together at a given time upon a given class of experiments.

There is no need, under these circumstances, of having a separate "weighing room" or a separate room for electrical measurements. Ordinary ballances and galvanometers, provided with glass cases, may generally be left in place without danger of injury. It may occasionally be desirable, during experiments with certain corrosive acids, to shut up all the finer apparatus in a cabinet, and for this reason such a cabinet should be provided. The cabinet should have glass sides and doors if possible, and plenty of shelves. It should occupy about $\frac{1}{20}$ as much floor space as is necessary altogether for the accommodation of students (see plan, page 906).

The furnace flue should be built side by side with one or more ventilating flues in a chimney next one wall of the room. A small closet or "hood" should be built round the chimney (see plan). This hood is intended as a place to store batteries and chemicals from which noxious odors may arise. There should be a continual current of air passing into it from the room, and out through an opening near the top into one of the ventilating flues. It is convenient, but not necessary, to make this closet large enough to work in. A large wooden box placed in front of an open fireplace is an excellent substitute for a hood.

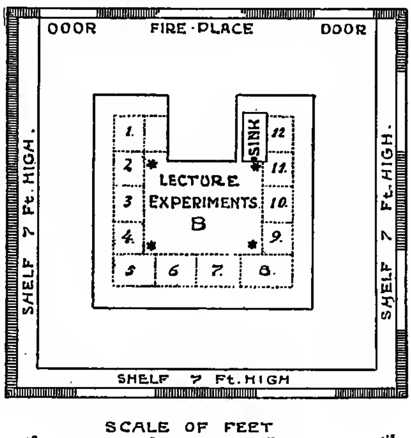
There will be needed for lecture purposes, or for purposes of demonstration, a table of considerable size, about 34 in. high, not too far from one of the



PLAN OF LABORATORY FOR SECTIONS OF 12 STUDENTS EACH.

walls where a blackboard may be placed, and in a position where it may be seen from all parts of the room (see plan). This table should be supplied with water, drainage, and gas, which can, in the absence of electricity, be used for both heating and lighting purposes. If steam is used in the building, a steampipe leading to the table may also be a convenience. The table should be furnished with drawers and cupboards, in which the lecturer may keep the greater part of the instruments which he will need during the course (see plan, A, B, and C, page 906).

Other tables, 30–34 in. high,[1] also furnished with drawers and cupboards for the use of students (see plan, 1–12) may be conveniently arranged in a hollow



Plan of Laboratory for Sections of six Students each.

square, so that the students working at them may face the lecturer. If the room is very small, all these tables may be built into one. The author has made use of a table (Fig. 2) 9 × 9 ft. square in lecturing to a class of 10 or 12 students, and in directing the

[1] 30 inches if chairs are to be used by the students, 34 inches if the students are to work standing, or to make use of high stools.

work of at least 6 students at one time. Around the outside of the table were 12 drawers with small closets beneath them, so that each student had a special place for apparatus. Some such arrangement is necessary, to bring into play in the minds of students that sense of at least temporary ownership which lends interest to the study, as well as to the preservation of the instruments intrusted to them.

In addition to the table or tables already mentioned, a considerable number of common tables or benches may (if space permits) be utilized for instruments which are to remain permanently in place; as, for instance, ordinary balances (Exp. 6), optical benches (Exp. 41), electric micrometers (Exp. 65), astatic galvanometers (Exp. 76), and "dip" apparatus (Exp. 77). These details will depend largely upon the system adopted for carrying on the course.

APPENDIX II.

APPARATUS.

THE apparatus which will be required for the course of 100 experiments described in this book, and for a few outside experiments needed for the purpose of illustration, is catalogued below under 20 heads, following the order in which the apparatus is wanted. The author has selected in the case of alternative methods, such apparatus as he himself has found most convenient to employ.

1. **For general use,** — first, measures of length: (*a*) a *Metre Rod*, graduated in millimetres either on wood, brass, or steel. Wooden rods have the smallest coefficient of expansion, are satisfactory, and cost only about 25 cts.; (*b*) a *Measure* graduated in *cm.* on cloth or steel tape 10 metres long (Fig. 224), cloth, costing about 75 cts.; (*c*) a *Gauge* (with vernier), reading to $\frac{1}{10}$ *mm.*, with a shaft 10 or 15 *cm.* long, made to order for about $1.00 in Paris (Fig. 2); and (*d*) a *Long Gauge* with vernier reading to $\frac{1}{10}$ *mm.*, with a shaft 40 or 50 *cm.* long (Fig. 222), made to order for about $1.50 in Paris; one of these is enough for 5 or 10 students. Other instruments for the measurement of length will be found under 5.

Second, for the measurement of weight: (*e*) a *Balance* (20 *kilo* capacity) with weights (Fig. 188), costing \$5.00 or \$10.00; one instrument enough for 5 or 10 students; (*f*) a *Balance* (flat pan) sensitive to $\frac{1}{10}$ gram (Fig. 1), costing \$3.00 or \$4.00; (*g*) *Weights* (iron) for ditto, cost about 50 cts.; (*cg.*) *Weights* (1 *cg.* to 100 *g.*), costing from \$2.00 to \$5.00 (*c*, Fig. 4). Other instruments for the measurement of weight will be found under 3.

Third, for the measurement of time, (*h*) a *Clock* with a wooden seconds pendulum (Fig. 152), to which a break-circuit may be attached, so as to re-enforce the ticks by electrical means. The second-hand can be made also to close a circuit, so as to give the signals (needed for the determination of rates of cooling) once or twice a minute. Such a clock (without connections), made by the Seth Thomas Co., costs about \$20.00. One is of course enough for all students.

Fourth, (*i*) a *Barometer* (aneroid), costing \$5.00 or \$10.00 (4, Fig. 53); one for the whole class; (*j*) one *Hygrodeik*[1] (Fig. 13), for the whole class, \$5.00 or \$10.00; and (*k*) a *Thermometer* (Fig. 51), 0° to 100° Centigrade, graduated on glass stem, costing about \$1.00 in Europe, and \$3.00 in the United States, — if possible, one thermometer for each student.

Finally, (*l*) a *Lens* or magnifying glass (Fig. 34), costing about 50 cts.; (*m*) a spirit *Level*, costing

[1] A substitute for a hygrodeik consists of a pair of thermometers, one of which has its bulb surrounded with wet lamp-wicking. The readings of these thermometers are to be interpreted by Table 15.

about the same; and (*n*) some wooden *Blocks*, on which to mount apparatus.

Total cost, $40.00 to $60.00.

2. **Preliminary Experiments** (I. to IV.): (*a*) a wooden *Block* (about 10 *cm.* cube); (*b*) a similar *Block hollowed* with an auger 5 *cm.* in diameter to a depth of 8 *cm.*, then closed by a wooden plug even with one surface; (*c*) a *small Block* about 3 *cm.* cube, — all of these blocks oiled or paraffined to make them impervious to water; (*d*) 10 or 12 *steel Balls*, such as are used in the bearings of a bicycle wheel, weighing about 30 grams (or as much as a hydrometer can float), and costing about $1.00; (*e*) a *Brush* of camel's-hair; (*f*) and a *Nicholson's Hydrometer* (Fig. 4, page 8), costing from $3.00 to $5.00.

A gauge, balance, weights, thermometer, &c., have already been mentioned (1, (*c*), (*f*), (*g*), (*cg*), (*k*), &c.). In the rest of this list the names of instruments once mentioned will not in general be repeated.

Total cost, $5.00 to $7.00.

3. **The Balance** (Exps. V. to X.): (*a*) a *Balance*, sensitive to 1 *cg.*, either uncovered as in Fig. 14, page 26, or with a glass case (Fig. 15, page 28), and costing from $6.00 to $22.00. The capacity of the balance should be at least 100 grams; (*b*) a *Barodeik* (Fig. 14, page 26) consisting of a litre flask hermetically sealed, and counterpoised by pieces of window glass. Such a flask should cost not over $1.00. It is well to have one such flask and counterpoise permanently mounted on a special balance (sensitive to 1 *cg.*) with a paper scale especially constructed so as

to show the density of the air. The complete instrument ought not to cost more than $10.00; (*c*) a *Barometer tube* (Fig. 10, page 17), 80 *cm.* long, 5 *mm.* internal and 10 *mm.* external diameter (cost 25 to 50 cts.); (*d*) a *small Beaker* (*b*, Fig. 10), costing less than 25 cts.; (*e*) a *nickel-plated Cup*, holding 250 grams or more (cost 25 or 50 cts.); (*f*) a *standard Weight*, (100 *g.*), costing about $1.00; (*g*) a *glass Ball* (or marble) 4 or 5 *cm.* in diameter (about 10 cts.); (*h*) 2 *small Rings* of equal weight (to prevent ball from rolling); (*i*) a hydrostatic *Arch* (Fig. 18, page 43), made of sheet brass for less than 25 cts.

Total cost, $10.00 to $35.00.

4. **The Specific Gravity Bottle, &c.** (Exps. XI. to XVIII.): (*a*) a *Specific Gravity Bottle*, or a common "2 ounce," wide-mouth, glass-stoppered bottle (Fig. 19, page 50). The stopper should be solid so as not to enclose a bubble of air. If hollow it must be filled with paraffine or other material not acted upon by ordinary liquids; cost less than 25 cts.; (*b*) a *Densimeter* with jar (Fig. 20, page 60), costing about $2.00; (*c*) a *U Tube* with rubber couplings, and (*d*) a *Y Tube* with rubber couplings, both tubes of glass or metal, about 3 *mm.* internal and 7 *mm.* external diameter (cost about 25 cts.); (*e*) 4 straight *glass Tubes* of the same diameter and about one metre long (about $1.00); (*f*) a *rubber Tube* (for connections), 1 metre long, 6 *mm.* internal diameter (25 to 50 cts.); (*g*) an *Air Pump*, or Richards' injector; the latter can be attached to an ordinary water faucet, and gives a more or less perfect vacuum; cost about $1.00; (*h*) a small *stop-cock*

ending in 2 tubes 6 or 7 *mm.* outside diam. (about 25 cts.); (*i*) a stout *Flask* (Fig. 24, page 67), capable of resisting the atmospheric pressure (25 cts. to $1.00); and (*j*) 3 *India-rubber Stoppers* to fit the flask, with 2 holes, with 1 hole, and with no hole (cost about 10 cts. each).

Total cost, $5.00 to $7.00.

5. **Length** (Exps. XIX. to XXI.): (*a*) a *Micrometer Gauge* (Fig. 28, page 73), reading to $\frac{1}{100}$ *mm.*, made to order for about $2.00 in Paris (American instruments reading only to $\frac{1}{50}$ *mm.* cost from $4.00 to $6.00); a *Spherometer* (Fig. 38, page 83), costing about $20.00. A much cheaper instrument, accurate enough for most purposes, could undoubtedly be made by soldering the necessary appendages to a nut and screw with a millimetre thread. Such threads and instruments for cutting them, though rare in America, are common in France. (*c*) A piece of *plate Glass* about 5 *cm.* square (10 cts.). The Vernier Gauge, Balls, and Lens, have been mentioned in 1, 2, and 3.

Total cost, $5.00 to $25.00.

6 **Expansion** (Exps. XXII. to XXX.): First, thermometers of the ordinary sort (see 1, *k*), at least one for each student; then (*a*) an *Air Thermometer* (or manometer) consisting of a stout glass tube 40 *cm.* long, 2 *mm.* internal diameter, graduated in *mm.* (see Fig. 56, page 119), at a cost of about $2.00; (*b*) for purposes of illustration (only), an *air-pressure Thermometer* (Fig. 60, page 127). The bulbs *a* and *c* should be about 5 *cm.* in diameter, and should have

capacities of at least 100 and 200 *cu. cm.* respectively. The tube *b* should be graduated in *mm.* for a distance of 40 *cm.* or more, and should have an internal diameter of about 2 *mm.* A similar instrument (made by Alvergniat, Paris) cost $3.00 unmounted. Add for mounting, and for a kilogram of mercury, about $2.00. One instrument enough for class. Corrections for different readings are easily calculated, and should not exceed 1°; (*c*) a *weight Thermometer*, or test-tube drawn out to a fine point (see ¶ 240), and (*d*), a *self-registering Thermometer* (or maximum and minimum), costing $3.00 in London. One instrument enough for a class.

Next, for heating (or cooling) purposes: (*e*) a *Bunsen burner* ($1.00) (3, Fig. 53), or its equivalent; (*f*) a *steam Boiler* (Figs. 53 and 54, page 115), capable of admitting a thermometer ($1.50); (*g*) a *rubber Tube* for steam (*ad*, Fig. 46, page 90), 50 *cm.* long (about 25 cts.); (*h*) a *Steam Jacket* or tube (*di*, Fig. 56) 1 metre long, and 3 or 4 *cm.* in diameter, with corks (50 cts.); (*i*) an *ice Trough*, 1 metre long and 5 *cm.* deep, made of a strip of tin, and (*j*) a *vapor Boiler*, or stout flask of about 100 *cu. cm.* capacity, drawn out at the mouth into a tube 5 or 6 *cm.* in diameter (about 25 cts.). It may be well to surround the flask with wire netting in case of accident. [The use of this apparatus is *not* accurately represented in Fig. 64. The manometer should be raised on a block of wood and fastened there, so as not to roll, the rubber tube should slope *toward the boiler*, and the boiler should be nearly covered by the hot water.]

Finally, the special apparatus (*k*) of *Dulong and Petit*, modified as in Fig. 47, page 95 (brass, about \$4.00); one enough for 4 or 5 students, since each may take his own observations; and (*l*) the *Manometric apparatus* of ¶ 76 (Fig. 62), costing about \$1.00, exclusive of the filter-stand, and mercury. It might be well to have two bottles blown especially for this apparatus, with tubes issuing directly from the top and bottom. One instrument enough for 2 students; (*m*) A wooden *Micrometer Frame* (*bcon*, Fig. 46), with screws *f* and *j*, ought not to cost more than \$1.00; (*n*) two *Test tubes* (10 cts.), and (*o*) a *Medicine dropper* (10 cts.; see Fig. 58, page 121), with the flask and stoppers already mentioned (see 4, *i* and *j*), will be required.

Total cost \$15.00 to \$20.00.

7. **Calorimetry** (Exps. XXXI. to XXXVIII.): (*a*) a *Calorimeter* (Figs. 70, 71, 72, page 144), with an inner cup of 100 or 200 *cu. cm.* capacity (50 cts. to \$1.00); (*b*) a *Stirrer* (Fig. 50, page 107); (*c*) a *Measuring glass* (Fig. 75, page 159) of 2 or 3 *cu. cm.* capacity (25 cts.); (*d*) a *Steam Shot heater* (Fig. 79, page 179), with an inner cup of 100 or 200 *cu. cm.* capacity (about \$1.00); (*e*) a *Bottle for ice-water* (wide mouth, 250 *cu. cm.* capacity, with wire netting to restrain ice,—about 10 cts.); (*f*) a *Steam Trap* (*b*, Fig. 83, page 203), materials costing about 25 cts.; (*g*) a glass *Beaker for Calorimetry*, of 100 or 200 *cu. cm.* capacity (see ¶ 105 (1), cost less than 25 cts. Each student should have at least one thermometer (see (1)) at his disposition during these experiments.

Total cost, \$2.00 to \$3.00.

8. **Radiometry** (Exps. XXXIX. to XL.): (*a*) an *Optical Bench* (Figs. 89 and 100), consisting of a board or plank 1 or 2 *m.* long, 10 *cm.* broad, set up edgewise, with 8 grooved blocks or "sliders" to fit loosely over it. One slider is to hold a candle, another a kerosene lamp, a third a telescope, a fourth a lens, a fifth a pasteboard screen, a sixth and seventh wires (Fig. 104), and an eighth a wire netting. Each slider can be clamped by a small screw eye, and each carries a marker. A paper or wooden *mm.* scale is attached to the board. Cost of bench and sliders, \$2.00 or \$3.00; (*b*) a Bunsen *Photometer* (Fig. 94, page 224). The diaphragm (incorrectly represented in *de*, Fig. 94) consists simply of a piece of paper with an oil-spot (Figs. 91–92) mounted as in *de*, Fig. 93,—cost, possibly 50 cts.; (*c*) a *Candle*; (*d*) a small *Kerosene Lamp;* and (*e*) either a *differential Thermometer* with gauge (Fig. 86, page 216), or else a Thermopile and Galvanometer (Fig. 88). Cost of the differential thermometer (made nicely by a tinman), \$1.00 or \$2.00.

Total cost, \$4.00 or \$5.00.

9. **Focal Lengths** (Exps. XL. to XLIII.). In addition to the Optical Bench, &c., mentioned in 8, there will be needed (*a*) a small *Telescope* (with cross-hairs), which may be borrowed from the sextant or spectrometer mentioned below (in 10); (*b*) a *long-focus Lens*, the achromatic object-glass of the telescope; (*c*) a *converging Lens* (the crown glass of the combination) and (*d*) a *diverging Lens* (the flint glass of the combination). These lenses if 3 or 4 *cm.* in

diameter, would cost separately \$3.00 or \$4.00. Spectacle lenses at less than \$1.00 per dozen will answer most purposes. It is desirable to have (*e*) 3 small *Mirrors*, (slightly) convex and concave, for purposes of illustration (¶ 118), and if possible (*f*) a "*Doublet Lens*," such as is used for rectilinear photography, costing about \$15.00 (one sufficient for 4 or 5 students). The principle can be illustrated by two "meniscus" spectacle lenses mounted facing each other with a small diaphragm between them.

Total cost \$1.00 to \$20.00 (the former, if spectacle lenses are employed, or if lenses are borrowed from optical instruments elsewhere accounted for).

10. **Goniometry** (Exps. XLII. to XLVII.): (*a*) a *Sextant*[1] (Fig. 106, page 246) reading to 10″, costing \$15.00 in Liverpool; (*b*) a Babinet *Spectrometer*,[1] reading to minutes or fractions of a minute, costing about \$20.00 in Paris. A cheap spectrometer can be made with a paper scale, but such instruments are apt in the hands of students to yield unsatisfactory results. It is perhaps better to dispense with the spectrometer altogether, and to use a sextant in its place. In this case a metallic shield with (*c*) a narrow *Slit* (1 *mm.* × 10 *cm.*) will be required in certain experiments. This must be illuminated by (*d*) a *Sodium Flame* (see foot-note, page 260). (*e*) an *Artificial Horizon* (page 545); (*f*) a small *Prism*, and (*g*) a *Diffraction Grating* (page 267), complete, at a nom-

[1] It is indifferent whether the student begins with the sextant or with the spectrometer. One instrument of each kind is therefore enough for at least 2 students.

inal expense, the list of apparatus required for angular measurements.

Total cost, \$35.00 to \$80.00.

11. **Sound** (Exps. XLVIII. to LV.): (*a*) a ***Resonance Tube*** (Fig. 121, page 272), costing \$1.00 or \$2.00; (*b*) a *Monochord* (Fig. 122, page 274), costing perhaps \$2.00; (*c*) a *Bow* (*ai*, Fig. 122), costing \$1.00; (*d*) *Signalling* apparatus, for instance, two hammers and boards (¶ 137, III.); (*e*) a *Smoked glass* apparatus (Fig. 125, page 288). The dimensions may be as follows: total length ·60 *cm.*, breadth 15 *cm.*, height 30 *cm.*, cost about \$3.00 for carpenter's work; (*f*) a *Toothed Wheel* apparatus (Figs. 135 and 136, page 302), watchmaker's charge about \$1.00; (*g*) *Tuning Forks* with the following rates of vibration per second: G♯ = 51.2; A = 54; A♯ = 57; B = 60; C = 64; C = 128; A = 216; C = 256. Cost made by blacksmith out of best steel, about \$8.00 in all. Additional forks (68, 72, &c., up to 128) desirable, but not necessary. Forks A = 440 + and C = 512 + to be had of dealers in musical instruments (25 cts. each); (*h*) a *Pitch Pipe* (Fig. 273, page 554), or its equivalent (less than \$1.00); (*i*) a *Cloth Band* for torsional vibrations (page 554); and (*j*) means of stretching considerable lengths of *Wires*. Resin will be needed in these experiments. Students will do well to work in pairs upon all these experiments.

Total cost, \$15.00 to \$20.00.

12. **Kinetics** (Exps. LVI. to LX.): (*a*) a *simple Pendulum* (Fig. 150, page 316), at a nominal cost; (*b*) an *irrotational Pendulum* (Figs. 153 and 154, page 321;

scale of figures about $\frac{1}{30}$), cost, made by ordinary carpenter, tinman, and blacksmith, not over \$5.00; (*c*) a *Torsion Head*, or suspension (AB, Fig. 159 page 334), costing about \$1.00; (*d*) a *Torsion Pendulum*, or ring apparatus (BCEFD, Fig. 159), costing about \$1.00; (*e*) a *Spiral Spring* apparatus (Fig. 158, page 331), cost nominal; (*f*) a *Falling-Bodies' apparatus* (Fig. 149, page 313), materials costing less than \$1.00; and for purposes of illustration, (*g*) a pair of *billiard Balls*, suspended as in Fig. 146 (cost about \$2.00).

Total cost, \$9.00 to \$10.00.

13. **Statics** (Exps. LXI. to LXII.): (*a*) Two *Spring Balances* of 10 kilos' capacity, graduated to $\frac{1}{10}$ Kilo (Fig. 160, page 338); cost about \$2.00; (*b*) a set of *Safety-valve Weights* (Fig. 160) from 1 to 10 kilos (about \$1.00); (*c*) a *Lever* (Fig. 162, page 341), costing perhaps 25 cts.; (*d*) a *loaded Board* (Figs. 170, page 348), costing about \$1.00; and (*e*) a pair of *Triangular Supports* (*i* and *j*, Fig. 173, page 350).

Total cost, \$4.00 or \$5.00.

14. **Elasticity and Cohesion** (Exps. LXIII.–LXVII.). (*a*) Two *Steel Beams* (*ag*, Fig. 173) both 100 + *cm.* long, one 6 + *mm.* square, the other 6 + *mm.* × 12 + *mm.*, costing about \$1.00; (*b*) an *Electric Micrometer* (Fig. 173), costing with platinum points about \$20.00. Two students can use same micrometer. [The electrical connections are unnecessary. A common screw with a brass head soldered to it and graduated with a knife will answer. Fasten a small

metallic mirror to the beam and move screw until its point touches its own reflection.] (*c*) A *Torsion Apparatus* (Fig. 174, page 355), with 2 or 3 rods, costing perhaps \$3.00; (*d*) a *Torsion Balance* (Fig. 176, page 358), made at a nominal cost by additions to the Torsion Pendulum (*ae*, Fig. 176) already mentioned (12 *d*); (*e*) *Young's Modulus Apparatus* (Figs. 178 (3), and 179, page 165), costing about \$2.00, outside of the electric micrometer (see *b*); (*f*) a *Bobbin*, scooped out so as to fit over the hook of a spring balance (Fig. 180, page 367); (*g*) a *Fork* of wire for holding a film of water (page 369), and (*h*) a *Capillary Tube*, made of a broken mercury thermometer (Fig. 182, page 371). Be sure that the bore is nearly circular (not elliptical).

Total cost, \$5.00 to \$20.00.

15. **Work** (Exps. LXVIII. to LXX.): (*a*) a pine *Board* (20 × 100 *cm.*) *and Plank* (5 × 20 × 40 *cm.*, see Fig. 183, page 373) nicely planed (about \$1.00); (*b*) a *Siphon* (Fig. 185, page 378), made by a rubber tube 2 *m.* long, 3 *mm.* internal diameter (not over \$1.00); (*c*) a *Tackle* (Fig. 186), consisting of two smoothly running double blocks, of either iron or wood, to be had of dealers in shipping materials, for about \$1.00; (*d*) a *Water Motor* apparatus (Fig. 178, page 382), costing about \$30.00 for the motor ($\frac{1}{50}$ horse power), \$3.00 for the water gauge, \$1.00 for the stone jar, and \$1.00 for the supports, in addition to the cost of the spring balances mentioned in 13 *a*. One motor enough for 4 or 5 students. A much smaller motor costing about \$5.00 will answer; in this case letter balances

should be employed (see 20). (*e*) A *pasteboard Tube* 1 + *m.* long, 5 *cm.* diam. (sold for about 25 cts. by paper dealers), with corks (Fig. 192, page 390); (*f*) heavy iron *Weights* (1–10 *kilos*), costing from \$1.00 to \$2.00. Lead shot and a thermometer (1 *k*) will also be required in these experiments.

Total cost, \$10.00 to \$40.00.

16. **Magnetism** (Exps. LXXI. to LXXV.): (*a*) three *compound Magnets* (see ¶ 179, also Fig. 196, page 396), costing about \$5.00 (one or two needed at one time); (*b*) a *vibration Magnet* (Fig. 204, page 412), cost nominal; (*c*) a *Surveying Compass* (Fig. 199, page 405), costing from \$5.00 to \$15.00; (*d*) a *long-bar Magnet* (Fig. 209, page 420), 1 *m.* long, 12 *mm.* diameter (cost about \$1.00); (*e*) a *Dipping-needle* and stand (Fig. 210, page 423), costing about \$2.00; (*f*) two wooden *Blocks* 1 *cm. cube.* Iron filings and photographic paper will be required for these experiments.

Total cost, \$10.00 to \$25.00.

17. **Magneto-Electricity** (Exps. LXXVI. to LXXVII.): (*a*) a *sliding Helix* with stops and clamps (*ad*, Fig. 209, page 420), costing about \$1.00; (*b*) an *Earth Inductor* (Fig. 213, page 428, scale about $\frac{1}{20}$), containing about 100 turns of insulated copper wire (No. 18, B. W. G.) on a wooden ring 60 *cm.* in diameter, and costing from \$5.00 to \$10.00. Good results may also be obtained by a simple coil laid against a door or on a table, then suddenly turned over by hand. Cost of such a coil \$2.00 or \$3.00. (*c*) A *ballistic Galvanometer*, made by loading the needle of an astatic

galvanometer (Fig. 207, page 418, scale about $\frac{1}{10}$) with a few grams of lead at each end. Cost of the astatic galvanometer, about $15.00.

Total cost, $20.00 to $25.00.

18. **Galvanometry** (Exps. LXXVIII. to LXXXIV.): (*a*) a Single Ring (*S. R.*) *Galvanometer* (Fig. 217, page 438, see footnote page 439), cost without compass about $10.00; with compass $25.00; (*b*) Double-Ring (*D. R.*) *Galvanometer* (Fig. 225, page 448), cost about $10.00, exclusive of surveying compass (see 16 *c*): (*c*) electro-*Dynamometer* (Fig. 228, page 451, scale $\frac{1}{15}$), cost $10.00 or $15.00; (*d*) an *Ammeter* (Fig. 231, page 466), costing $5.00 or $10.00; (*e*) a *Vibration Galvanometer* (Fig. 230, page 461), which the student can himself construct at a cost of about 50 cts.; (*f*) a *Commutator* (Figs. 216, page 435, scale $\frac{1}{10}$), costing perhaps $1.00; (*g*) a *Shunt*, consisting of a piece of uninsulated German Silver wire of about 1 ohm resistance, with copper connections (Fig. 249, page 486), cost nominal: (*h*) several cheap *Keys*, 50 cts. to $2.00.

Batteries will also be needed as follows: (*i*) *Battery Materials*, that is, materials for a small Daniell cell to be set up in a small jar or tumbler (see page 463), cost about 50 cts.; (*j*) a *Daniell Battery* of 6 litre Daniell cells (Fig. 235, page 469), $6 00; (*k*) a *Bunsen Battery* (of 3 or 4 Bunsen cells, Fig. 234), $10.00; and (*l*) a *Leclanché Battery* of 1 or 2 Lechanché cells (Fig. 236), $2.00.

Total cost, $15.00 to $30.00.

19. **Electrical Resistance** (Exps. LXXXV. to XCIII.): (*a*) a *Resistance coil* (Fig. 238, page 471, scale

$\frac{1}{4}$), made to fit calorimeter 17 *a*) cost nominal; (*b*) a *Resistance box* (Fig. 242, page 476, see also R., Fig. 264, and Figs. 240, 241. Scale of Fig. 241 about $\frac{1}{5}$), cost, roughly adjusted, about $60.00; (*c*) a British Association (*B. A.*) *Bridge* (Fig. 247, page 481, scale of *cm.* in Fig.), cost of materials about $2.00; complete instrument $10.00 to $15.00; (*d*) a *differential Galvanometer*, made by using the differential connections of the astatic galvanometer already mentioned (17 *c*); if there are no differential connections, add to the two binding posts *a* and *b*, already existing, a third binding post, *c*. Connect *a* and *c* with a resistance of say 10 ohms; connect *b* and *c* with an *equal* resistance; then equal currents through *ac* and *bc* will produce no deflection. The instrument will work as a differential galvanometer in all cases where insulation between the two circuits is not required. Cost of change in connections about $1.00. It is preferable to have the galvanometer wound with a *double wire*, as stated in footnote, page 419.

Total cost, $60.00 to $75.00.

20. **Electromotive Force** (Exps. XCIV. to C.). These experiments depend chiefly upon the apparatus mentioned in 18 and 19. There is needed also (*a*) a *Thermo-Junction* (*a*, Fig. 257, page 521), cost nominal; (*b*) a *Clark Battery* (1 cell sufficient), cost of materials about $1.00; (*c*) an *Electric Motor* (Fig. 263, page 534), with a friction brake consisting of two letter balances (see Fig. 264), and (*d*) a *Revolution Counter* (Fig. 265). Cost of motor, &c., from $5.00 to $10.00.

Total cost, $7.00 to $12.00.

In addition to the list of apparatus given above, it will be found convenient to have certain supplies always on hand, together with tools and materials to repair broken apparatus. The most important items are arranged alphabetically below, with an estimate of the number or quantity required for each student.

Alcohol, $\frac{1}{2}$ pint.
Augers, assorted.
Binding screws, electric.
[Blast lamp.]
[Brackets.]
Bunsen Burner.
Candles.
Cord, 10 feet.
Corks, assorted.
Cotton cloth, $\frac{1}{8}$ yard.
Cotton waste, 1 oz..
Ether, 2 oz.
Gas ——.
Gimlets, assorted.
Glass Beakers, assorted.
" Jars.
" Mirrors, pieces.
" Plate, "
" Test-tubes, assorted.
" Thermometer, 1 extra.
" Tubing, assorted, but especially $\frac{1}{4}$ inch.
Hammers and Hatchets.
Ice (Exp. 5 and Exps. 22–36).
Ink.
Iron filings, $\frac{1}{4}$ oz.
[Iron Plate.]
Kerosene, 1 pint.
Mercury, 1 lb.
Nails, assorted.
Oil-can.
Paper, coördinate, 12 pages.
" photographic, 1 page.
Paraffine, $\frac{1}{4}$ oz.
Pins, assorted.
Resin, for bows.
Rubber couplings, 6.
" Stoppers, assorted.
" Tubing, assorted, 3 ft.
Salt, 1 lb.
Sand, 1 lb.
Saw.
Screw-drivers.
Screws, assorted.
Shot, lead.
" Zinc (or copper).
Solder and iron.
Tacks, assorted.
" double-pointed.
Tin-foil (10 sq. in.).
Vice.
Water, cold, hot, and distilled.
Wax.
Wire, Brass, fine, $\frac{1}{4}$ oz.
" Copper, assorted, 1 oz.
" German silver, assorted, $\frac{1}{4}$ oz.
" Iron, assorted, $\frac{1}{4}$ oz.
" Steel, fine, $\frac{1}{4}$ oz.
Wood, blocks to mount apparatus. boxes, boards, planks, strips, &c.

Every instrument, tool, or receptacle should bear the number of the shelf where it belongs, and in addition a label of its own by which it may be identified. It would be well to make an alphabetical list of apparatus referring to the shelf where each instrument may be found.[1] The students could then set up their own apparatus.

[1] The names of instruments printed in italics in the list will be found in the general index at the end of this book. The words under which they are indexed are those beginning with capital letters.

APPENDIX III.

EXPENSES.

THE cost of fitting up a laboratory for the purposes of elementary physical measurement does not differ essentially from that of an ordinary school-room, except that a somewhat greater space is required for the students. This is not, however, so great as is commonly supposed. The author has found no serious difficulty in the use of a room 15 feet square, for a section of six students. The room contained a table 9 feet square, arranged as shown in the plan (Appendix I.). The total expense of fitting up the room and table, including gas, plumbing, water, and a furnace pipe, together with shelves, drawers, closets, and a small "dumb-waiter" to connect with a tool-room in the basement, was about $300.00. Since the laboratory can receive three or four sections of 6 each daily, each member of a class of from 36 to 48 students can attend three exercises a week in a room of this size. The cost of permanent laboratory fittings is only a little greater than that of the ordinary desks required for a school-room, and if it is decided to introduce experimental physics at all, the necessary appropriation is generally forthcoming.

The cost of supplies in a course of physical measurement is not, as in chemistry, an important item.

An allowance of $2.00 or $3.00 per annum for each student has been found to cover the whole expense. A still smaller sum would suffice if care were taken to prevent unnecessary waste, especially of mercury, alcohol, and battery materials. The cost of gas and water is usually nominal. Heating is not included in the estimate above, and must be allowed for as in any other course of instruction.

The cost of a complete set of apparatus will be found, by adding up the separate sums in the list (Appendix II.), to vary from $300.00 to $500.00. A single copy of each instrument will generally answer for two students to work with at one time, especially when there are two or more instruments serving a given purpose, like the sextant and the spectrometer. In some experiments it is advisable for more than two students to work together; it is, however, recommended that one set of apparatus be allowed *on the average* for each pair of students present in the laboratory at a given time. Some instruments, like the clock and the barometer, may serve for a whole class of students; but it is highly desirable that each student should have his own thermometer, gauges, weights, and other comparatively inexpensive apparatus.

If we suppose a class of students to be divided into 6 sections, each having separate access to the laboratory at stated periods during the week, it is evident that a single complete set of apparatus can serve six pairs of students. A moderate laboratory fee ($10.00 per student) is therefore sufficient to pay for a complete set of apparatus in 4 or 5 years. Experience has shown that college students are willing to pay such a fee in addition to the regular charge for tuition.

It follows that 12 students can afford one complete set of apparatus, that 24 students can afford two sets, &c. In other words, the cost of reduplicating apparatus to such an extent that a large class of students working in pairs may be able without delay to follow a regular and connected course in physical measurement, comes within the limit of what the students themselves are willing to pay. There is, perhaps, no better demonstration of the fact that such a course is desirable from an economical point of view. The first criticism, however, which occurs to a practical man when he sees a whole class working simultaneously upon a given experiment, is that a great proportion of the apparatus lies idle. It may be well, therefore, to consider, from an economical point of view, certain methods of instruction in which the proportion of apparatus employed at a given time is relatively great.

"It is suggested" (see Harvard List of Advanced Physical Experiments, 1890, page 54) "that great duplication of apparatus is not necessary in a course of experiments such as is described in this" (the Harvard, 1890) "pamphlet. It is found that students differ much in their rate of working in a physical laboratory, and consequently, in more difficult experiments, the students can be working on different parts of the subject during the same hour." A good instance of this would be in the case of experiments with double and single ring galvanometers, since the same principles are involved in the two instruments. Unfortunately, certain expensive instruments, like the balance and the resistance box, are in continual use for several weeks devoted to a given subject. To avoid conflict with such instruments, it is necessary that students should be working on *different subjects* at the same time.

This leads to the consideration of a system of instruction used in many of our oldest institutions, in which the experiments that a student is to perform are determined largely by the supply of apparatus. This system deserves especial attention, since it is in some cases the only one possible, and has the merit of extreme economy as far as apparatus is concerned.

The progress of each student in this system may be watched or controlled by an "indicator board." The names of the students can be arranged across the top of the board, and the names of experiments at the left. (See the Harvard List of advanced Physical Experiments, 1890, page 54.) A long peg is then placed under each student's name opposite the experiment which he is to perform. The long peg can be moved and replaced by a short peg, to show that the experiment has been performed, and that the apparatus is free for the next student. A complete row of short pegs opposite a given experiment shows therefore that all the students have performed it. A new experiment is then prepared in its place.

This *individual system* of instruction, aside from its economy, has undoubted educational merits. A good text-book and the personal attention of an intelligent assistant are together worth more than almost any system of lectures without such aid, as far as the understanding of experiments is concerned. When, however, we consider the mutual relations between experiments, the "individual system" as it is generally practised presents numerous defects. Professor Pickering, in his Physical Manipulation, Vol. II., appendix C., recommends that at the start about thirty experiments should be prepared. Among these are Measurements of Length, Temperature, Capacity, Weight, Force, Elasticity, Acceleration, and Light. It is true that, although some of these topics naturally precede others, any one can be explained without reference to the rest. A lecturer cannot, however, begin by explaining all. As a natural result, those students who, through lack of apparatus, are obliged to perform, for instance, experiments in light before any allusion to the wave theory has been made, work under a decided disadvantage. It is better to keep a student waiting for apparatus than for explanations. For this reason the lecturer must keep at least a month in advance of the majority of the students, even when all are working upon a given class of experiments. It is impossible after such an interval to recall the details of an explanation even with the aid of copious notes. Such details therefore are generally omitted from the lectures, and when the time comes are explained separately to each student. Again, the lecturer cannot point out the just inferences to be drawn from a given experiment until every one has performed it. In the mean time, however, many of the results of observation escape from the student's memory. Facts without principles, like principles without facts, are quickly forgotten.

A system of lectures which is only one month in advance of the laboratory work has nevertheless many obvious advantages over a preparatory course of instruction, which must be taken at least one year in advance. It is a well-known fact that lectures are the most economical system of verbal instruction. The farther lectures are separated in time from the experiments to which they relate, the less can be accomplished by the lecturer, the more is thrown upon the laboratory assistant. In some institutions, courses of purely laboratory work are given. Such courses, with a limited number of students assigned to one instructor at a given time, have undoubted advantages, but are necessarily expensive. A small amount of apparatus may, it is true, be employed successively by a large number of students; but in an elementary course the cost of reduplicating apparatus is small in comparison with the cost of reduplicating instruction.

Let us now consider what happens when, as is frequently the case, 10 or 12 students without any previous preparation are assigned to a single assistant. In the first place, time will be lost in starting the men at work. It is easy, moreover, to see that the assistant cannot, in a single exercise of two hours, devote more than 10 or 12 minutes to each student. In other words, the *quantity* of his instruction is limited. In the space of time at his command it would be impossible to cover the ground even of a half hour's lecture. This amount of time ought at least to be devoted to the explanation of each experiment. The *quality* of the instruction given in this way is apt, moreover, to be unsatisfactory. When the assistant has explained a given point separately to six or eight students, he is sometimes left with the impression that the point in question has been made sufficiently clear, and subsequent explanations are, perhaps involuntarily, either curtailed or omitted. The instruction obtained, even from inexperienced assistants, is often a great aid to the student in following a course of higher instruction; but without general lectures a course in physical measurement is necessarily incomplete. One of the advantages of having a course of lectures closely connected with the laboratory work, lies in the use of illustrative experiments performed by the lecturer. It would of course be too costly to repeat such experiments for the benefit of each student at the time when he needs them, and impracticable for the student to repeat most of them himself. Students working independently are left therefore to read about these experiments in a text-book, or to recall them as best they may from notes on some past lecture in which they seemed to have no practical bearing upon their work.

The student who has just heard a lecture upon a given experiment cannot fail to perform it with better understanding; he has at least the directions for the experiment fresh in his mind, and knows how to begin work. Experience has shown that more than half of the student's difficulties can be anticipated by a short lecture. There is, as has been already pointed out, a great economy of labor in this lecture method of instruction.

It has been found that one assistant is required to give individual instruction to about six students. If the class contains 72 students, whom we will suppose to be divided into 6 sections of 12 each, two assistants must evidently be present at one time, and at least three will be required to meet all the men, allowing 24 hours of instruction per week to each assistant. Now it is found that a single assistant can direct the work of 12 men at one time, provided that they have received their main instructions beforehand. Let us suppose that two assistants take 3 sections each, and explain once for all at the beginning of each exercise, in the clearest possible terms, the details of a single experiment which all the men are simultaneously to perform. Each assistant's hours will then be reduced from 24 to 18 per week, and the labor of individual instruction, if not lightened, will at least be freed from tiresome repetition. The same salary should therefore suffice. There is accordingly, in this system of instruction, a gain of one assistant's salary. There is, on the other hand, an increase in the amount of apparatus, for 6 whole sets will now be required instead of one. The additional 5 sets will cost from $1500.00 to $2500.00. This seems a large sum to invest in apparatus; but it must be remembered that, unlike assistants' salaries, this sum is paid only once. Even if, in the course of 10 years, all the apparatus should have to be replaced, it will in the mean time, have been paid for almost twice over by the saving in a single assistant's salary.

It is obvious that instead of giving separate explanations to each section, a single lecture attended by all the sections will suffice. There is little danger that students may forget what is said in the lectures before they come into the laboratory, provided that intermediate lectures on the same subject do not intervene. This fact may be made use of, if it is desired, to effect a considerable saving in the cost of apparatus. Instead of giving a single series of lectures, the instructor may give two series, each being attended by half of the men belonging to each section of the class. It is undoubtedly possible to arrange two sets of experiments so that each may form a continuous course, and that at the same time a conflict of apparatus may be avoided. Half of the students present at a given time will accordingly be working on one set of experiments, for instance, determinations of weight, at the same time that the other half is performing an entirely different set of experiments, for instance, measurements of length. Three complete sets of apparatus will therefore be sufficient for a class of 72 students working, as suggested above, in pairs. The details of this method have not been worked out because the expense of giving a double series of lectures would ordinarily amount in a few years to more than the original cost of reduplicating the apparatus. The method, however, involving only a single repetition of a given explanation, is obviously more economical than the ordinary system of instruction, in which explanations must be repeated separately to each student, and is to be considered, when, as is too often the case, it is absolutely impossible to obtain a sufficient appropriation for apparatus.

Let us next consider a class of 24 students, for whom one instructor or assistant is in any case sufficient. Such a class would naturally be divided into 4 sections of 6 each, and would, working in pairs, be fairly supplied by 3 complete sets of apparatus at a cost of from $900.00 to $1500.00. Suppose, however, that only one set of apparatus can be had. The simplest escape from this difficulty is to divide the class into 6 sections of 4 each, and to give a double set of lectures as has just been suggested. The exercises may if necessary be cut down to one hour each. They will then occupy, with six one-hour lectures, 24 hours per week; that is, the same time as would ordinarily be required for the individual instruction of 24 students. Each student will moreover receive the same total amount (6 hours) of instruction. Half of this will be in the lecture room, the other half in the laboratory. It would of course be better if more than one hour could be allowed for the laboratory exercises; but it is thought that the student can do more in a single hour after a thorough explanation received in the lecture, than he could accomplish in two hours under the old system, allowing for waste of time in waiting for the necessary explanations.

We have seen that with a large class (of 72 students) it *pays* (through the saving in salaries) to reduplicate the apparatus, to such an extent at least that the students, working in pairs, may follow a regular course of experiments without waiting for apparatus. We have seen also that by means of a double course of lectures, the cost of apparatus may be considerably reduced ; in fact that, with a small class (24 students) no reduplication is necessary. With still smaller classes, 12 for instance, there is no need of reduplicating either the lectures or the apparatus. It appears, therefore, that considerations of expense arising from the reduplication of apparatus need not, as is commonly supposed, stand in the way of giving to any number of students a regular course of lectures and experiments.

So far we have considered only the minimum quantity of apparatus consistent with the purposes of this course. It is much easier at the present day to obtain a sufficient appropriation for instruction than for apparatus. It may be well, however, before leaving the subject of expense, to call attention to certain practical rules by which the cost of laboratory courses may be reduced to a minimum.

Let us suppose that the number of students, at first small, is doubled. There are then three ways of meeting this increase : 1st, by doubling the supply of apparatus; for in this case, the same number of lectures and laboratory hours will probably suffice; 2d, by doubling the number of sections admitted to the laboratory; and 3d, by doubling the number of lectures or the number of separate explanations caused by assigning different kinds of instruments to different students at a given time. Of these three ways, the least expensive would naturally be selected. Now let the number of students again be doubled. Once more the least expensive item would be increased. The rules of economy lead, therefore, in the end to an *equal distribution* of expenses between the three above-named methods by which the capacity of a laboratory course may be increased.

For example, if it costs $300.00 per annum to give a single course of lectures, $75.00 per annum for the supervision of each laboratory section, and $50.00 per annum for interest and repairs on each complete set of apparatus, a class of 24 students, who are to work singly, should be divided into 4 sections (costing $300.00 per annum) and supplied with 6 sets of apparatus (at an equal cost of $300.00 per annum). The total cost ($900.00 per annum, or $37.50 per student) will be found to be less than that resulting from any other arrangement of sections.

The numbers chosen above were derived from experience, and represent approximately the cost of a course of physical measurement such as has been described in this book, leaving out of consideration the question of rent. Expenses may be reduced by allowing students to work in pairs, or by an increase in the number of students; but the results compare favorably in any case with the sums charged for laboratory courses in colleges and other institutions where the individual system of instruction is still retained.[1]

[1] In the University of Berlin, the annual fee for an elementary laboratory course is from $20.00 to $25.00, in addition to a fee of $20.00 for general lectures.

APPENDIX IV.

INSTRUCTION.

THE advantages of a course of lectures closely related to the experiments which are to be performed in the laboratory, have been already pointed out. There will be needed about as many lectures ($\frac{1}{2}$ to 1 hour each) as there are exercises. If the class consists of a single section, the exercises in the laboratory should follow immediately after the lectures. It is well in any case for the assistant in charge of a section to say a few words to the students at the beginning of an exercise, especially in regard to detailed directions which they may require. It must not, however, be imagined that such directions can take the place of a general lecture.

Among the topics which would naturally be discussed in a general lecture, may be mentioned the historical development of the subject in hand, especially any incidents — like Rumford's boring the cannon — which may appeal to the imagination.

The laws and principles involved in the experiments should also be explained. The separate sections of Part III. cover ground enough for as many lectures. Various illustrative experiments will be found in well known text-books, such as those of Deschanel and Ganot. These should be shown, if possible, to the whole class of students.[1] In hydrostatics, for instance, Pascal's vases, the hydrostatic press, and even the Cartesian diver may be shown; while Dr. Hall's experiments with a pressure gauge (Harvard List of Elementary Physical Experiments, No. 5) will be found to form a valuable addition to existing methods of explanation. In connection with experiments on the pressure of air, Mariotte's tube and an absolute air-pressure thermometer (¶ 75) would naturally be shown. It is useful to illustrate the expansion of gases by heat on a large scale. Experiments with Helmholtz' resonators, singing flames, and especially the phonograph, lend interest to the subject of sound; a word about photography and color are not out of place in the study of light; there are instructive experiments with powerful magnets, such as stopping the oscillations of light metallic bodies; the study of frictional electricity is also a natural introduction to the subject of electrodynamics.

Several experiments, mentioned in Parts I. and II., are not intended to be followed as determinations. Some of these are suitable for lecture illustrations; others can be performed (if it is thought desirable) by students outside of their regular course of measurements.[2] The experiments described in ¶ 80 and

[1] It is a good idea to have the students themselves take part in so far as practicable in such experiments. Notes should in all cases be taken.

[2] The growth of a desire on the part of students to perform experiments on their own account is a certain proof of progress in their past education and a promise of success in the future. Such a desire should be encouraged in every possible way.

¶ 82 may, for instance, be performed by students. If they are not, they should be shown in the lectures.

It is a good plan to dictate to the class or to write on the blackboard exactly what observations they are to make, and what calculations are to follow. Considerable time can be saved at a small expense by having these directions printed. The "hektograph" process was used for more than a year at Harvard College. Separate sheets were furnished to the students at each exercise, and handed in at the end of the exercise. The calculations were not made until afterward. In the mean time the instructor had an opportunity of examining the results of observation, so that evident mistakes could be pointed out to the students. It is important with *large classes* of students to preserve in this way some record of their original observations (see footnote, page 947). The student is, however, naturally anxious to know whether his results are satisfactory or not, and for this reason he should be allowed to take away with him a copy of his observations.

It is hardly necessary to allude to various processes, such as impression paper and the ordinary copying press, through which, if it is desired, the student's observations may be duplicated, whether they are made in ink or in pencil. It takes only a minute or two to copy figures by hand, when a printed form is already provided. This is perhaps on the whole the most satisfactory way. The printed forms should be cut and pierced so that they may be afterward bound together. All the observations made by a given student will of course be collected by him; all the observations on a given experiment may be bound together by the instructor. It is thus easy to compare the results of different students, and to estimate the relative merits of each.

The use of printed forms is a great assistance to the instructor, for he knows exactly where to look for a given observation, and he can see at a glance if any of the necessary data have been omitted or misunderstood. At the same time, there is reason to fear that the students may fall into a mechanical way of making observations, without thinking what they are for. The student knows that he is expected to "fill in those blanks," and this he can generally do even if the reasons are not sufficiently obvious. The same objection applies to any system of instruction in which the student receives minute directions for an experiment; for it can make no essential difference whether these directions are dictated, copied, or otherwise distributed.

To test the point in question, the author has tried the following experiment. Printed forms were given to a large class of students at the end of each lecture for several months. One day, without previous notice, the lecture was closed a few minutes before the ordinary time, and each member of the class was requested to make out a form covering the observations necessary for the experiment which had been described. The determination was one which depended upon six or seven data, any one of which if missing would prevent the calculation of the result. Three-fourths of the class presented essentially perfect forms for observation, and in addition to this, the majority named three or four additional data which would be useful in making exact corrections in the result.

It may be observed that the object of lectures is to make clear to the student what his observations are for, and what there is to learn from them. If there be any doubt whether this object is fulfilled, the natural test is an examination. To ask a student to plan out his observations is practically one form of examination; but it is one which as a general thing seems to the author unwise, because a single omission on the part of the student, unless pointed out to him, may ruin the value of his subsequent determinations.

Scientific men are frequently obliged to plan out the complete details of a determination; and it is thought desirable that students, when they have had a sufficient opportunity to see how such details are arranged, should be required in certain experiments to make their own plan for observations. At the same time, the scientific man never fails to compare his work, as far as he can, with that of others. It is not at all infrequent for him to find that he has omitted some important correction. The discovery of corrections by referring to the work of others does not incapacitate him for finding them himself. On the contrary, corrections suggest corrections. In the same way, a series of experiments in which the details are carefully and minutely planned should not incapacitate the student for making a similar arrangement, but should, on the other hand, teach him how such a result may be obtained.

The principal objection to printed forms generally arises from indolent students, who see no escape from making the required number of observations. Though unnecessary in small classes, the reasonable use of printed forms is always desirable, and with a large class, greatly diminishes the labor of instruction.

The student should be taught to consider an experiment unfinished until the result has been calculated and handed in to his instructor. It will be found convenient to use cards for this purpose. The student writes his name and the name of the experiment on one side of the card, on the other side in large figures the numerical value of the result. When all the cards have been received, they may be attached in their proper place to a board bearing the names of the students at the left, and references to experiments across the top. It is thus easy to estimate at a glance both the quantity and the quality of the work performed in the laboratory.

It has been suggested (see § 30) that determinations of the properties of substances the composition of which is unknown to the student, furnish an excellent method of testing his work. Of course it will not do to give the same substance to all the students. Experiments in elementary physical measurement are divided into two distinct kinds. In one of these, the student knows what result he ought to obtain, and simply performs his experiment to test either his own skill, or the accuracy of the instruments which he employs. Experiments in "calibration" belong to this class. The other kind of determination deals with quantities of unknown magnitude, and should be attempted by the student only when he has satisfied himself by previous experiments, that he is capable of obtaining accurate results. There is, perhaps, no greater satisfaction in a course of measurement than the discovery that quantities of absolutely unknown magnitude have been correctly measured. In estimating the value of a student's work, it is well to consider only determinations of this kind.

One word of caution is, however, necessary. Most of the materials given to the students are only commercially pure, and hence yield results which may differ indefinitely from those contained in ordinary tables. It will not do, accordingly, to assume that those results which agree most closely with these tabulated values are the best. The average result obtained by the most careful workers in a class is a much better standard; but here again caution must be observed in the case of measurements where errors tend always to increase or to diminish the result. On account of air-bubbles, for instance, the largest determinations of specific gravity are generally the best; judged, however, by the average of a class, the best results would in this case be greatly underrated. The instructor may be obliged in certain cases to make determinations himself. It is generally possible, by the use of finer apparatus or different methods, to obtain results sufficiently accurate to serve as a standard for the class.

The truest estimate of the value of a student's work, next to that furnished by a written examination, is perhaps obtained by personal inspection of the student's manner of working, and by an examination of his note-book. A word or two about note-books may not be out of place. The first and most important thing is for the student to keep his observations and his calculations separate (see § 33). If, as has been suggested, the observations are made on printed forms, there is no danger of confusion in this respect. The calculations may be made on the *back* of the printed forms or on separate sheets of paper. These calculations must in all cases be preserved. The sheets on which they are made should be of the same size and shape as those employed for observations, so that the data, calculations, and results may be bound together.

If the observations are made in an ordinary note-book, the student should follow Dr. Hall's suggestions, namely, that the left-hand pages should be devoted to observations, the right-hand pages to calculations, &c. It is a great mistake to use scraps of waste paper for arithmetical work. It is frequently necessary to review such work, and if the intermediate figures are wanting, a new calculation will be involved. The figuring, moreover, often enables an instructor to see at a glance just where a mistake was made.

Entries should be kept in so far as possible in chronological order. If mistakes are discovered later on, these should be corrected in pencil or ink of a *dif-*

ferent color from that used in the original records. These original records are often found after all to be accurate, and ought not in any case to be obliterated.[1] The use of erasers in a laboratory should be strictly prohibited.

A great and not unusual fault in note-books is a lack of sufficient fulness, or rather minuteness. The student writes, for instance, "Temperature before experiment, 50°;" without giving any idea *how long* before the experiment the temperature was taken; or again, "Length by vernier gauge, 4.01 *cm.*;" without stating by *what* vernier gauge. It would be a good plan, two or three times in a course, to have the students repeat some past experiment, making use of their notes to find the same materials and instruments that they previously employed, and to have them calculate the results without reference to any text-book. This furnishes the best test of the completeness of a student's notes.

There is a tendency on the part of some students to make their notes full by repeating explanations which are given in their text-books. This is not a very serious error; but it should be pointed out that note-books are intended for facts which a text-book cannot anticipate, and *too much* theory makes it difficult to find these facts. A good note-book is charac-

[1] "The tendency of the student to regard as unquestionably wrong any observation which is not what he expected it to be, and to make his observation tally with his expectation, is doubtless familiar to most teachers, and it should be one of the important objects of this experimental course to counteract this tendency." — *Harvard List of Elementary Physical Experiments*, page 7.

terized, not by fulness of language, but by fulness of detail.

The proceedings in an experiment should be concisely stated. Observations should be arranged as systematically as time will allow. The use of tabular forms, both for observations and for results (see "Examples," Exps. 6–10), will be found in some cases of great assistance. Calculations should be neatly made but not crowded. Generous spaces should be left between experiments, or different parts of a given experiment; and these should further be distinguished by prominent headings. An example of two pages from a note-book, with the criticisms of a teacher, is given below. A summary of results is a useful addition to the description of an experiment. Examples of such summaries will be found in the next section (V.) of this Appendix.

EXPERIMENT I.

DATE. *October 1, 1888.*

APPARATUS.

[Teacher's Remark.] *You should note that the 20 gram weight was missing.*

Block of wood,	No.	2 (*a*) *i.*
Vernier gauge,	No.	1 (*c*) *i.*
Balance,	No.	1 (*f*) *i.*
Iron weights, set	No.	1 (*g*) *i.*

OBSERVATIONS.

*Weight of the wooden block.** 122.8 *grams.*

Length of block. †	*Breadth of block.* †	*Thickness of block.* †
6 90 *cm.*	6.91 *cm.*	4.32 *cm.*
6.89	6.88	4.30
6.91	6.89	4.30
6.90	6.90	4.29
6.91	6.91	4.28
6.92	6.93	4 31
6.89	6.89	4.30
6.88	6.90	4.31
6.89	6.89	4.29
6.90	6.91	4.30

[Teacher's Remark.] *Gauge crooked? Repeat this measurement.*

[Teacher's Remark.] *Which measurement was in the middle of block?*

REMARKS.

* *The grams and tenths of grams were estimated by the small movable weight belonging to the balance.*

† *The length was measured first parallel to the grain near one corner of the block, which was slightly broken, then at equal intervals across the block. The breadth was measured across the grain, beginning at the same corner. The thickness was measured three times near each side, and once nearly in the middle. The jaws of the gauge did not come quite together, and the two zeros did not come quite opposite.*

[Teacher's Remarks.] *Good.*

Extra observations :—

Barometer	75.32 *cm.*
Thermometer	$22^{\circ}.5$ *C.*
Dew Point	40° *C.*

Good practice; not needed yet.

[Later in red ink added by student.]
This must have been Fahrenheit !

Always sign your name.

NAME. ....

EXPERIMENT I.

CALCULATIONS.

Length in cm.	*Breadth in cm.*	*Thickness in cm.*
6.90	6.91	4.32
6.89	6.88	4.30
6.91	6.89	4.30
6.90	6.90	4.29
6.91	6.91	4.28
6.92	6.93	4.31
6.89	6.89	4 30
6.88	6.90	4.31
6.89	6.89	4 29
6.90	6.91	4.30
10) 68.99	10) 69.01	10) 43.00
Average 6.899	*Average* 6.901	*Average* 4.300

```
   6.899                 47.61
   6.901                 4.300
   -----                 -----
    6899                  0000
   0000                  0000
  62091                14283
 41394                19044
 ---------            ---------
 47.609999            204.72300 = volume
                                  in cu. cm.
```

Weight in grams.

```
204.7 )  122.8  ( 0.5999 = weight of
         102 35             1 cu. cm.
         ------             in grams.
          20 450
          18 423
          ------
           2 0270
           1 8423
           ------
            18470
            18423
```

Explanatory Remarks The volume of the block (204.7 *cu. cm.*) *is found by multiplying together the average length* (6.899 *cm.*), *the average breadth* (6.901 *cm.*), *and the average thickness* (4.300 *cm.*). *Since* 204.7 *cu. cm. weigh* 122.8 *grams,* 1 *cu. cm. weighs* $\frac{1}{204.7}$ *of* 122.8 *grams, that is,* 0.5999 *grams, or* 0.600 *grams nearly.*

RESULTS.

Volume of the block	204.7 *cu. cm.*
Density of the block	0.600 *g. per cu. cm.*

NOTE. — An example of a fuller summary of results, which would make the explanatory remarks above unnecessary, will be found under Exp. I. Appendix V.

APPENDIX V.

EXAMPLES OF OBSERVATIONS AND CALCULATIONS IN EXPERIMENTS 1–100, PRESENTED IN THE FORM OF A SUMMARY OF RESULTS.

In the examples below, observations are printed in italics, and designated by capital letters; calculations are designated by small letters, and printed in ordinary type. The data are taken, in so far as possible, from results actually reported by students in the Jefferson Physical Laboratory, without any change whatever. Such data are marked with an asterisk referring to the initials of the name of the student by whom they were determined. Other results were obtained by calculation. In some of these, round numbers have been chosen with a view of simplifying the arithmetical work; but care has been taken in all cases to give results which either were or might have been obtained with the apparatus described in the course of experiments, and to represent correctly the probable error of such results.

EXPERIMENT I.

A.	*Weight of wooden block*		. . . .	122.8 *g.*
B.	*Length* "	"	(mean of 10 obs.) . .	6.899 *cm.*
C.	*Breadth*	"	" " . .	6.901 *cm.*
D.	*Thickness*	"	" " . .	4.300 *cm.*
e.	Volume	"	$(B \times C \times D) =$	204.7 *cu. cm.*
f.	Density	"	$(A \div e) =$	0.600 *g. per cu. cm.*

EXPERIMENTS II.–IV.

Weights sinking Hydrometer.		*Temperatures of Water.*
*A.**	32.08 grams.	10°.7
*B.**	31.91 "	20°.
*C.**	31.58 "	29°.

d. Allowance for temperature at 20° about 0.03 *g.* for 1°.

☞ See Fig. 4, § 59.

NOTE. The hydrometer used above bore on the average about 1.48 grams more weight than that for which Fig. 4, § 59 was constructed. An allowance of about 1.48 grams must therefore be made in comparing results.

*E.** *Distance between 2 rings on hydrometer stem* . 18 *mm.*
*F.** *Weight required to sink upper ring* . . . 31.91 *g.*
*G.** *Weight required to sink lower ring* . . . . 31.88 *g.*
h. Sensitiveness of hydrometer
$\frac{1}{100}[E \div (F - G)] = 6$ *mm. per cg.*
*I.** *Weight sinking hydrometer in water at* 20°.5 *to mark on stem with* 12 *steel balls in upper pan* 7.59 *g.*
j. Weight would have been at 20° (see *d*) . . 7.605 *g.*
k. Apparent weight of balls in air $(B - j) =$ 24.305 *g.*
*L.** *Weight sinking hydrometer in water at* 20°.5 *with balls in lower pan* . . . . 10.713 *g.*
m. Weight would have been at 20* (see *d*) . 10.728 *g.*
n. Weight of balls in water $(B - m) =$ . . 21.182 *g.*
o. Weight of water displaced $(k - n = m - j) =$ 3.123 *g.*
p. Apparent specific gravity of balls
$(k \div o) = 7.78$ *g. per cu. cm.*

* S. L. B. Oct. 14, 1887.

EXPERIMENT V.

A. Height of barometric column . . . . 75.90 *cm.*
B. Vertical height of barometric column when inclined so as to halve free space, about . 75.7 *cm.*

c. Air pressure above mercury about $(A - B) =$ 0.2 *cm.*
d. Height of barometer corrected for air $(A + c) =$ 76.1 *cm.*
E. *Internal diameter of barometer about* . . . . 0.5 *cm.*
f. Temperature of the air of the room . . . . 20° C.
g. Correction for barometer with glass scale
at 20° and 76 *cm.* (Table 18 *a*) =
— 0.245 — 0.016 = ... — 0.26 *cm.*
h. Correction for capillarity, diam. 0.5 *cm.*;
height of meniscus unknown (Table 18 *b*) + 0.15 *cm.*
i. Correction for mercurial vapor at 20°
(Table 18 *c*) . . . . + 0.00 *cm.*
j. Corrected height of barometer
$(d + g^* + h + i) =$ 76.0 *cm.*
K. *Reading of Aneroid barometer* . . . . 30.00 *inches.*
l. The same reduced to *cm.* (Table 16) . . . 76.2 *cm.*
m. Correction of Aneroid barometer $(l - j) =$ — 0.2(?) *cm.*
N. *Moisture appears on cup (mean of 3 obs.) at* . + 4° C.
O. *Moisture disappears* " " . + 6° C.
p. Dew-point $\frac{1}{2}(N + O)$ . . . . + 5° C.
Q. *Dew-point indicated by hygrodeik* . . . . + 50° F.
r. The same reduced to Centigrade (Table 39) + 10° C.
s. Correction for hygrodeik at + 10° C. $(p - r) =$ — 5° C
t. Density of dry air at 20° and 76 *cm.*
(Table 19) . . . . . . 0.001204 *g. per cu. cm.*
u. Correction for moisture, dew-
point + 5° (Table 20) — 0.000004 *g. per cu. cm.*
v. Atmospheric density $(t + u^*) =$ 0.001200 *g. per cu. cm.*
W. *Density of the air indicated*
by Barodeik . . . . 0.00118 " "
x. Correction for the Barodeik
$(v - W) =$ + 0.00002 " "

* The corrections *g* and *u* being negative, are to be added algebraically, but subtracted numerically.

EXPERIMENT VI.

NOTE. Single weights are underlined in this table.

	A. Weights in left-hand pan (in grams).*	*B.* Weights in right-hand pan (in grams).*	*C.* First obd. turning-point of index.*	*D.* Second obd. turning-point of index.*	*E.* Third obd. turning-point of index.*	*f.* Mean of C and E.	*g.* Mean of D and *f.*	*h.* Differences.	*i.* Sensitiveness† to one centigram.
1.	0.00	0.00	8.0	10.1	8 1	8.05	9.1	2 5	1.3
2.	0.02	0.00	13.1	10.1	13.0	13 05	11.6		
3.	20.00	20 00	9.8	8.6	9.8	9.8	9.2	2.3	1.2
4.	20.02	20.00	13.1	10 0	13.0	13 05	11.5		
5	50.00	50.00	13.2	6.2	13.1	13.15	9.7	2.1	1.1
6.	50.02	50.00	13.6	10.2	13.4	13.5	11.8+		
7.	100 00	100.00	12.2	7.8	12.1	12.15	10.0	1.5	0.8
8.	100.02	100.00	13.0	10.1	12.9	12 95	11.5		
9.	0.00	0.00	9.8	9 2	9.8	9.8	9.5		(See figure. ‡)
10.	100.00	100.00	11.6	7.8	11.4	11.5	9.7		
11.	100.00	100.00	11.2	6.8	11.1	11.15	9.0		
12.	0.00	0.00	11.4	9.4	11.2	11.3	10.4		

j. Mean zero reading in last part of the experiment, ‡
$\frac{1}{2}(g_9 + g_{12}) = \frac{1}{2}(9.5 + 10.4) =$ . . 10.0

k. Mean reading of balance with 100 grams in each pan, $\frac{1}{2}(g_{10} + g_{11}) = \frac{1}{2}(9.7 + 9.0) =$ . . 9.4

l. Mean weight to be added to 100 *g* in left-hand pan to balance 100 g. in right-hand pan,
$(j - k) \div i_{7,8} = (10.0 - 9.4) \div 0.8 =$ 0.8 *cg.*

m. The same in grams . . . . 0.008 *g.*

n. Ratio of the balance-arms $(A_{10} + m) \div A_{10}$ or
$100.008 \div 100 =$ 1.00008

* W. B. B., Oct. 1887.

† The sensitiveness of this balance is not so great as that represented in Fig. 16, page 32.

‡ The variations in the zero-reading were unusually large.

EXPERIMENT VII.

EXAMPLE.

OBSERVATIONS.			CALCULATIONS.		
Large Weights in left-hand pan.	*Large Weights in right-hand pan.*	*Small weights, + in right hand, — in left hand pan.*	Equivalents of Weights in terms of x*.	Substitution of the value of x (1.00080 g).*	Value of Weights in grams.
$A =$ 1-gram weight	$= x$ *	— 1 mgr.	$= x$ — 1 mgr.	= 1.0008 — .001	= 1.000
$B =$ 2- " (No. 1).	$= x + A$	— 1 mgr.	$= 2x$ — 2 mgr.	= 2.0016 — .002	= 2.000
$C =$ 2- " (No. 2).	$= B$	+ 2 mgr.	$= 2x$ + 0 mgr.	= 2.0016 + .000	= 2.002
$D =$ 5- " .	$= A + B + C$	+ 1 mgr.	$= 5x$ — 2 mgr.	= 5.0040 — .002	= 5 002
$E =$ 10- " (No. 1).	$= A + B + C + D$	— 1 mgr.	$= 10x$ — 6 mgr.	= 10.0080 — .006	= 10.002
$F =$ 10- " (No. 2).	$= E$	— 2 mgr.	$= 10x$ — 8 mgr.	= 10.0080 — .008	= 10.000
$G =$ 20- "	$= E + F$	+ 0 mgr.	$= 20x$ — 14 mgr.	= 20.0160 — .014	= 20.002
$H =$ 50- "	$= A + B + C + D + E + F + G$	— 3 mgr.	$= 50x$ — 36 mgr.	= 50.0400 — .036	= 50.004
$I =$ 100- "	$= A + B + C + D + E + F + G + H$	— 4 mgr	$= 100x$ — 73 mgr.	= 100.0800 — .073	= 100 007
$J =$ 100- "	= 100 grams (standard)	+ 7 mgr.	$= 100x$ — 73 mgr.	" "	" "

* Since $100x$ — 73 *mgr.* (see J) = 400 *grams* + 7 *mgr.*,

$100x$ = 100 *grams* + 73 *mgr.* + 7 *mgr.* = 100.080 *grams.*

Hence x = (Sum of centigram weights) = 1.00080 *grams.*

EXPERIMENTS VIII.–X.

OBSERVATIONS. CALCULATIONS.

A. Contents of left-hand pan.	*B. Contents of right-hand pan.*	*C. Mean indication of pointer.*	*D.* Corrected weight. §
1.* *Glass ball* [*ring No.* 1.]	102.93 *g.* [*ring No.* 2.]	10.0	102 930 *g.*
2.* 102.93 *g.* *ring No.* 1.	*Glass ball, ring No.* 2.	9.6	102.934 *g.*
3.* *Glass ball with wire in water.* †	61.82 *g.*	9.8	61.818 *g.*
4.* 61.82 *g.*	*Glass ball with wire in water.* †	11.0	61.810 *g.*
5.* 0.19 *g.*	*Wire in water.* †	10.2	0.188 *g.*
6. *Glass ball with wire in alcohol.* ‡	69.00	11.0	69.010 *g.*
7. 69.00.	*Glass ball with wire in alcohol.* ‡	10.0	69.000 *g.*
8. 0.19 *g.*	*Wire in alcohol.* ‡	10.0	0.190 *g.*

† *E.** *Temperature of the water.* 18°.4.

‡ *F. Temperature of the alcohol.* 20°.0.

G. Indication of the Barodeik. .00120

§ In estimating the exact weight which would bring the pointer to No. 10 of the scale, an allowance was made at the rate of 10 mgr. for each whole division through which the pointer was deflected. This allowance corresponds to the mean sensitiveness of the balance determined in Exp. VI.

* G. H. C., Oct. 1887.

CALCULATIONS CONTINUED.

a. Apparent weight of ball in air

$\frac{1}{2}(D_1 + D_2) = 102.932$ *grams.*

b. Apparent weight of ball with wire in water $\frac{1}{2}(D_3 + D_4) = 61.814$ "

c. Apparent weight of wire in water $(D_5) = 0.188$ "

d. Apparent weight of ball alone in water
$(b - c) =$ 61.626 *grams.*

e. Apparent weight of water displaced
$(a - d) =$ 41.306 "

f. Apparent specific volume of water (Table 22) at 18°.4 (see E) in air of density .00120 (see G) = ... 1.00247 *cu. cm. per g.*

g. Volume of the ball at 18°.4 $(e \times f) =$ 41.408 *cu. cm.*

h. Weight of air displaced by ball
$(g \times G) =$ 0.050 *grams.*

i. Weight of air (Table 20, A) of the density .00120 (see G) displaced by 1 gram of brass 0.000143 "

j. Weight of air displaced by brass weights
$(a \times i) =$ 0.015 "

k. Correction for the buoyancy of air
$(h - j) =$ 0.035 "

l. True weight of ball in vacuo $(a + k) =$ 102.967 *grams.*
The same by Table 21, assuming density of crown glass, 2.5 (Table 10), of air, .00120 (see *G*);
1.00034 × 102.932 (see *a*) = 102.967 *grams.*

m. Density of the ball $(l \div g) =$ . . 2.487 *g. per cu. cm.*

n. Apparent weight of ball with wire in alcohol
$\frac{1}{2}(D_6 + D_7) =$ . . . . 69.005 *grams.*

o. Apparent weight of wire in alcohol $(D_8) =$ 0.190 "

p. Apparent weight of ball alone in alcohol
$(n - o) =$ 68.815 "

q. Apparent weight of alcohol displaced
$(a - p) =$ 34.117 "

r. Apparent specific gravity of the alcohol
$(q \div e) =$ 0.826 *g. per cu. cm.*

s. True weight of the ball in alcohol
$(p - pi) =$ 68.805 *grams.*

t.	True weight (in vacuo) of alcohol displaced $(l - s) =$	34.162	grams.
	The same by Table 21, for .826 (see r) and .0012 (see G); $1.00132 \times q = 1.00132 \times 34.117 = 34.162$		

u. Expansion of glass ball between 18°.4 (see E) and 20°0. (see F), assuming cubical coefficient 0.000023 (Table 10); $0.000023 \times (F - E) \times g = 0.000023 \times 1.6 \times 41.408 =$ 0.0015 *cu. cm.*

v. Volume of glass ball at 20°.0 $(g + u) =$ 41.410 "

w. Density of the alcohol at 20°.0 $(t \div v) =$ 0.8250 *g. per cu. cm.*

EXPERIMENTS XI.–XIV.

OBSERVATIONS.

A. *Density of air by Barodeik* 0.00120

B. **Weight of Specific Gravity Bottle with air* 118.37 *grams.*

C. **Weight of Sp. Gr. Bottle filled with water* 178.76 "

D. **Temperature of the water* . . . . 22°.0 C.

E. **Weight of Sp. Gr. Bottle partly filled with sand* . . . . 198.10 *grams.*

F. **The same with spaces filled with water* . 225.29 "

G. **Temperature of the water* . . . . 23°.1 C.

H. *Weight of Sp. Gr. Bottle partly filled with sulphate of copper* . . . . 185.84 *grams.*

I. *The same with spaces filled with alcohol* . 211.09 "

J. *Temperature of the alcohol* . . . . . 18°.00 C.

K. *Weight of Sp. Gr. Bottle filled with alcohol (only)* . . . . 168.31 *grams.*

L. *Temperature of the alcohol* . . . . 20°.0 C.

CALCULATIONS.

a. Apparent weight of water filling Sp. Gr. Bottle
$(C - B) =$ . . . . 60.39 *grams*

b. Apparent specific volume of water by Table 22
at 22° (see D) and .00120 (see A) . 1.00322 *cu. cm.*

c. Capacity of Sp. Gr. Bottle at 22°
$(b \times a) =$ 60.58 *cu. cm.*

---

d. Apparent weight of sand $(E - B) =$ . . 79.73 *grams.*

e. Weight of sand in vacuo by Table 21,
assuming densities 2.2 (Oxide of Silicon, Table 9 a), and .00120 (see A),
$1.00041 \times d = 1.00041 \times 79.73 = 79.76$ "

f. Apparent weight of water filling spaces
$(F - E) = 27.19$ "

g. Apparent specific volume of water by Table 22
at 23°.1 (see G), and .00120 (see A) . . 1.00346

h. Volume of the water filling spaces
$(f \times g) = 27.28$ *cu. cm.*

i. Volume of the sand $(c - h) =$ . . . . 33.30 " "

j. Density of the sand $(e - i) =$ . . 2.395 *g. per cu. cm.*

---

* F. W. B., Oct. 1887.

k. Apparent specific gravity of alcohol from Experiment 10 (see Examples 8, 9, and 10, *r*)
0.826 *g. per cu. cm.*

l. Apparent specific volume $(1 \div k) =$ 1.211 " "

m. Apparent weight of alcohol filling spaces
$(I - H) = 25.25$ *grams.*

n. Volume of this alcohol $(l \times m) =$ . . 30.58 *cu. cm.*

o. Volume of sulphate of copper $(c - n) =$ 30.00 " "

p. Apparent weight of sulphate of copper
$(H - B) = 67.47$ *grams.*

q. The same reduced to vacuo (Table 21), assuming densities 2.3 (Table 9 a) and .00120 (see A), $1.00039 \times p = 1.00039 \times 67.47 =$ 67.50 *grams.*

r. Density of the sulphate of copper
$(q \div o) = 2.25$ *g. per cu. cm.*

---

s. Apparent weight of alcohol filling Sp. Gr. Bottle
$(K - B) =$ . . . . 49.94 *grams.*

t. Weight of air (Table 20 A) of the density .00120 (see A) displaced by 1 g. of brass . 0.000143 "

u. Effective weight of the alcohol $(s - st) =$ 49.93 "

v. Weight of air filling Sp. Gr. Bottle
$(A \times c) = 0.07$ "

w. Weight of alcohol in vacuo $(u + v) =$. 50.00 "

x. Difference between capacities of Sp. Gr. Bottle at 22° (see D) and 20° (see L), assuming cubical coefficient of expansion .000023 (Table 10),
$0.000023 \times (D - L) \times c =$
$0.000023 \times 2 \times 60.58 = 0.003$ *cu. cm.*

y. Capacity of the Sp. Gr. Bottle at 20°
$(c. - x) = 60\ 58$ "

z. Density of alcohol at 20° $(w \div y) = 0.8254$ *g. per cu. cm.*
Compare value of x, Examples 8–10 = 0.8250

NOTE. The strength of the alcohol corresponding to these densities (see Table 27) varies from 87.2 to 87.4 %.

EXPERIMENT XV.

OBSERVATIONS.

1. Description of Liquids.	2. Temperatures.	3. Readings of densimeter.
A. Distilled water.	21°	1.000
B. Alcohol of Exps. 8–14.	18°	0.831
C. Glycerine (commercial).	24°	1.250
D. * *Methyl alcohol.*	21°	0.814
E. * *Saturated salt solution.*	21°	1.204
F. * *Solution of bichromate of sodium.*	20°	1.470

* C. C. B., 1887.

CORRECTIONS.

a. True density of distilled water at 21° (Table 25) 0.99807

b. Density of 87.3 % alcohol (See Examples 11–14 NOTE) at 18° (see *B* 2), by Table 27 0.8269

c. Density of commercial glycerine at 24° (see C 2) according to Table 26 . . . . . 1.254

d. Correction for densimeter in water $(a - A) =$ -0.002

e. Correction for " in alcohol $(b - B) =$ -0.004

f. Correction for " in glycerine $(c - C) =$ $+0.004$

☞ See Fig. 21, page 72.

g. Correction for reading in methyl alcohol † . . -0.004

h. Correction for reading in salt solution † . . . $+0.003$

i. Correction for reading in bichromate solution † about $+0.013$ (?)

j. Corrected density of methyl alcohol at 21° . . 0.810

k. Corrected density of salt solution at 21° . . . 1.207

l. Corrected density of bichromate solution at 20° . 1.49(?)

† Obtained by the curve on page 62; see § 59.

EXPERIMENT XVI.

FIRST METHOD.

A.	*Height of mercurial column*	5.00 *cm.*
B.	*Height of the column of water*	68.10 "
C.	*Temperature of the air*	20° C.
d.	Difference between the lengths of the columns of water and mercury $(B-A)=$ . . .	63.10 *cm.*
e.	Density of air (Tables 19, 25) about . . .	.0012
f.	Equivalent of inequality of air pressure in centimetres of water $(d \times e)=$	0.08 *cm.*
g.	Corrected length of the column of water $(B-f)=$	68.02 *cm.*
h.	Specific gravity of mercury at 20° $(g \div A)=$	13.60
i.	Density of water at 20° (Table 25)	0.99828
j.	Density of mercury at 20° $(h \times i)$	13.58

SECOND METHOD.

K.	*Height of column of glycerine*	80.0 *cm.*
L.	*Height of column of water*	100.0 *cm.*
M.	*Temperature of the air*	20° C.
n.	Difference in length of columns $(L-K)=$	20.0 *cm.*
o.	Inequality of air pressure in *cm.* of water $(n \times e)=$	0.0 *cm.*
p.	Corrected length of column of water $(L-o)=$	100.0 *cm.*
q.	Specific gravity of glycerine at 20° $(p \div K)=$	1.250
r.	Density of water at 20° (Table 25)	0.99828
s.	Density of glycerine at 20° $(q \times r)=$ . . .	1.248

EXPERIMENTS XVII.-XVIII.

A.	*Weight of Flask with coal-gas*	mean of 5 double weighings.	200.500 *g.*
B.	*Weight of Flask with air*		201.200 *g.*
C.	*Weight of Flask after exhaustion*		200.600 *g.*
D.	*Weight of Flask after admitting water* . .		700.0 *g.*
E.	*Weight of Flask completely filled with water*		1200.0 *g*
F.	*Temperature of the water*		20° C.
G.	*Barometric pressure*		75.0 *cm*

h. Apparent weight of water required to fill flask $(E - B) =$ . . . . 1000.0 *grams.*

i. Apparent weight of water equivalent in bulk to the air exhausted $(D - B) =$ 500.0 "

j. Degree of exhaustion $(i \div h) =$ . . . . 50 %.

k. Weight of air exhausted $(B - C) =$. 0.600 *grams.*

l. Specific gravity of this air $(k \div i)$ . . 0.00120

M. *Density of air according to Barodeik* = 0.00120

n. Specific volume of water (Table 22) at 20° (see *F*) and .00120 (see *b*) = 1.00279 *cu. cm. per g.*

o. Capacity of the flask at 20° $(h \times n) =$ 1,000.3 *cu. cm.*

p. Difference in weight between 1,000.3 *cu. cm.* of air and of coal-gas . . $(B - A) =$ 0.700 *grams.*

q. Difference of density $(p \div o) =$ 0.000700 *g. per cu. cm.*

r. Density of the coal-gas at 20° (see *F*) and 75 *cm.* (see *G*) $= (M - q) =$ . . 0.000500 *g. per cu. cm.*

s. Factor for reducing density from 20° to 0° (Table 18 *e*) 1.0734

t. Factor for reducing density from 75 *cm.* to 76 *cm.* (Table 18 *d*) 1.0133

u. Density of coal-gas at 0° and 76 *cm.* $r \times s \times t =$ 0.00054 *g. per cu. cm.*

EXPERIMENT XIX.

I. * *Readings of Vernier Gauge set on glass ball of Exps.* 8–10.

1.	4.300 *cm.*
2.	4.303 "
3.	4.302 "
4.	4.313 "
5.	4.313 "
6.	4.300 "
7.	4.311 "
8.	4.310 "
9.	4.315 "
10.	4.311 "

A. Average 4.3078 *cm.*

B. * *Zero-reading of gauge* 0.000 *cm.*

c. Corrected diameter $(A - B) =$ . . 4.308 "

d. Apparent weight of water displaced by the glass ball (Examples 8–10, *e*) 41.306 *grams.*

e. Apparent specific volume of water (Table 22) at 18°.4 (Examples 8–10) 1.00247 *per g. cu. cm.*

f. Volume of glass ball $(d \times e) =$. 41.408 *cu. cm*

g. Diameter of sphere (Table 3, H) with volume equal to 41.408 *cu. cm.* (see *f*) 4.293 *cm.*

h. Reduction Factor for gauge $(g \div c) =$ 0.9965

II. * *Readings of Micrometer Gauge set on steel balls of Exps.* 2–4.

1.	7.975 *revolutions.*
2.	7.990 "
3	7.968 "
4.	7.978 "
5.	7.980 "
6.	7.981 "
7.	7.931 "
8.	7.955 "
9.	7.935 "
10.	7.968 "

A. Average 7.9664 *revolutions.*

B. * *Zero-reading of gauge* 0.00035 *rev.*

c. Corrected number of revolutions $(A - B) =$ 7.9629 "

d. Apparent weight of water displaced by 1 steel ball ($\frac{1}{12}$ of *O* in Examples 2–4) . . . 0.2603 *grams.*

e. Apparent specific volume of water (Table 22) at 20°.5 (Examples 2–4) 1.00290 *cu. cm. per g.*

f. Average volume of steel balls $(d \times e) =$ 0.2610 *cu. cm.*

g. Diameter of sphere (Table 3, H.) with volume 261.0 *cu. mm.* (see *f*) . 7.929 *mm.*

h. Pitch of the micrometer screw $(g \div c) =$ 0.9957 *mm. per rev.*

* A. E. T., Nov. 29, 1887.

EXPERIMENTS XX., XXI.

READINGS OF SPHEROMETER.

	A. * *On Plane glass, 1st surface.*	*B.* * *On Lens, 1st surface.*	*C.* * *On Lens, 2d surface.*	*D.* * *On Plane Glass, 2d surface.*
1.	1.3393 *cm.*	1.1593 *cm.*	1.1595 *cm.*	1.3395 *cm.*
2.	1.3397	1.1595	1.1596	1.3393
3.	1.3397	1.1596	1.1593	1.3394
4.	1.3391	1.1593	1.1596	1.3397
5.	1.3396	1.1597	1.1594	1.3397
6.	1.3395	1.1592	1.1594	1.3396
7.	1.3395	1.1595	1.1595	1.3395
8.	1.3396	1.1595	1.1595	1.3393
9.	1.3393	1 1594	1.1596	1.3392
10.	1.3395	1.1595	1.1595	1.3392
Averages *	1.33948 *cm.*	1.15945 *cm.*	1.15949 *cm.*	1.33944 *cm.*

Oblique distances of Central Point: —

E. * *from 1st foot.*	*F.* * *from 2d foot.*	*G.* * *from 3d foot.*
1. 2.218 *cm.*	1. 2.212 *cm.*	1. 2.215 *cm.*
2. 2.22 *cm.*	2. 2.200 *cm.*	2. 2.225 *cm.*
3. 2.225 *cm.*	3. 2.210 *cm.*	3. 2.248 *cm.*

*h.** Average for plane glass $\frac{1}{2}(A + D) =$. 1.33946 *cm.*

*i.** Average for lens $\frac{1}{2}(B + C) =$ . . . . 1.15947 "

*j.** Height of spherical surface $(h - i) =$. 0.17999 *cm.*

*k.** Average distance of central point from three feet (see *E, F,* & *G.*) . . . . . 2.219 "

*l.** Mean radius of curvature,

$$\frac{1}{2}\,\frac{l^2}{j} = \frac{1}{2}\,\frac{2.219 \times 2.219}{0.17999} = \;.\;.\;.\;.\;.\; 13.68 \text{ "}$$

* C. A. B., Oct. 12 and 14, 1885.

Note. It has been assumed in these calculations that the pitch of the spherometer screw is 1.000 *mm.* per revolution. The determination of this pitch is identical with that of a micrometer. — *See Example* 19, II.

EXPERIMENT XXII.

OBSERVATIONS.

A. Temperature of brass rod surrounded by water 20°.1

B. Length of the rod at about 20° . . . . . . 1000. *mm.*

C. Reading of the micrometer . . . . 9.121 *mm.*

D. Reading of the micrometer after the admission of steam . . . . 10.643 *mm.*

E. Reading of the barometer . . . . 30.0 *inches.*

CALCULATIONS.

f. Reading of barometer, 30.0 inches (see *E*), reduced to *cm.* (Table 16) . . . 76.2 *cm.*

g. Temperature of steam at this pressure (Table 14) 100°.07

h. Increase of temperature $(g - A) =$ . . . . 80°.0

i. Expansion of rod $(D - C) =$. . . . . . 1.522 *mm.*

j. Expansion for 1° $(i \div h) =$ . . . . . . 0.01903 *mm.*

k. Expansion for 1° and for 1 *mm.* $(j \div B) =$ 0.00001903 *mm.*

l. Mean coefficient of linear expansion of brass rod between 20° and 100° in terms of its length, at 20° . . . . $k =$ 0.0000190 +

NOTE. It is assumed in these calculations that the pitch of the micrometer screw is 1.000 *mm.* per revolution. See, however, Example 19, II., in which the pitch of a similar screw is determined.

EXPERIMENT XXIII.

OBSERVATIONS.

A. Outside diameter of the tubes (mean of 4 settings on horizontal bends) . . . . 1.00 *cm.*

B. Difference of level in water-gauge due to admission of steam to left-hand jacket . . . . . . 4.03 *cm.*

C. Distance between the bends of left-hand tube when expanded by steam . . . . 99.05 *cm.*

D. Temperature of the water in right-hand jacket (mean of 3 obs. with self-registering thermometer) 18°.2 C.

E. Difference of level in water-gauge due to admission of steam to right-hand jacket . . . . . 3.98 *cm.*

F. Distance between bends of right-hand tube when expanded by steam . . . . 98.95 *cm.*

G. Temperature of the water in left-hand jacket (mean of 3 obs. with self-registering thermometer) 20°.6 C.

H. Barometric pressure . . . . 29.3 *inches.*

CALCULATIONS.

i. Barometric pressure (H) reduced to *cm.* (Table 16) . . . . 74.4 *cm.*

j. Temperature of steam condensing at this pressure (i), see Table 16 . . . . 99°.41 C.

k. Mean temperature of cold water, $\frac{1}{2}(D + G) =$ 19°.4 C.

l. Difference of temperature $(j - k) =$ . . 80°.0 C.

m. Mean length of tubes between bends, $\frac{1}{2}(C + F) =$ . . . . 99.00 *cm.*

n. Mean length of column of hot water $(m + A) =$ . . . . 100.00 *cm.*

o. Mean difference of level in gauge, $\frac{1}{2}(B + E) =$ 4.005 *cm.*

p. Mean length of column of cold water balancing the column of hot water $(n - o) =$ . . 95.995 *cm.*

q. Relative specific volume of water at 99°.4 (see j) and at 19°.4 (see k), $(n \div p) = 100.00 \div 95.995 =$ . . . 1.0417

r. Increase of specific volume per degree $(q - 1) \div l = .0417 \div 80.0 =$ . . . 0.000521

NOTE. The last result (r) represents the mean cubical coefficient of expansion of water between 19°.4 and 99°.4, in terms of its volume at 19°.4.

EXPERIMENT XXIV.

OBSERVATIONS.

*A.**	*Weight of Specific Gravity Bottle with air*	126.565 *grams.*
*B.**	*Weight of Sp. Gr. Bottle with water* .	182.657 "
*C.**	*Temperature of the water*	21° C.
*D.**	*Weight of Sp. Gr. Bottle with alcohol*	172.49 *grams.*
*E.**	*Temperature of the alcohol in D* .	16°.7 C.
*F.**	*Weight of Sp. Gr. Bottle with alcohol*	171.42 *grams.*
*G.**	*Temperature of the alcohol in F* .	39°.2 C.
*H.**	*Weight of Sp. Gr. Bottle with alcohol*	170.43 *grams.*
*I.**	*Temperature of the alcohol in H* .	59°.2 C.
*J.**	*Weight of Sp. Gr. Bottle with alcohol*	169.44 *grams.*
*K.**	*Temperature of the alcohol in J* .	76°.3 C.

CALCULATIONS.

a. Apparent weight of water filling
Sp. Gr. Bottle $(B - A) =$ . . . 56.092 *grams.*

b. Apparent specific volume of water
(Table 22) at 21° (see C), assuming
the (mean) density of air .00120
1.00300 *cu. cm. per g.*

c. Capacity of Sp. Gr. Bottle at 21°
$(l \times m) =$ 56.260 *cu. cm.*

d. Coefficient of cubical expansion of
glass (Table 10) 0.000023

Capacity of Sp. Gr. Bottle.

e. at 16°.7 (see E) $c - cd\,(C - E) =$. 56.254 *cu. cm.*

f. at 39°.2 (see G) $c + cd\,(G - C) =$. 56.284 "

g. at 59°.2 (see I) $c + cd\,(I - C) =$ . . 56.309 "

h. at 76°.3 (see K) $c + cd\,(K - C) =$. 56.332 "

Apparent weights of alcohol filling Sp. Gr. Bottle.

i. at 16°.7 (see *E*), $(D-A)=$ . . . . . 45.93 *grams.*
j. at 39°.2 (see *G*), $(F-A)=$ . . . . . 44.86 "
k. at 59°.2 (see *I*), $(H-A)=$ . . . . . 43.87 "
l. at 76°.3 (see *K*), $(J-A)=$ . . . . . 42.88 "

Apparent specific volumes of alcohol.

m. at 16°.7 (see *E*), $(e \div i)=$ . . 1.2248 *cu. cm. per g.*
n. at 39°.2 (see *G*), $(f \div j)=$ . . 1.2547 " "
o. at 59°.2 (see *I*), $(g \div k)=$ . . 1.2835 " "
p. at 76°.3 (see *K*), $(h \div l)=$ . . 1.3137 " "
q. at 0°, INFERRED
$(m-(p-m)\div(K-E)\times E)=1.200$ " "

Mean coefficient of expansion in terms of the volume at 0°.

r. from 16°.7 to 39°.2, $(n-m)\div q\div(G-E)=.00111$
s. from 39°.2 to 59°.2, $(o-n)\div q\div(I-G)=.00120$
t. from 59°.2 to 76°.3, $(p-o)\div q\div(K-I)=.00147$

* G., Feb. 18, 1886.

EXPERIMENT XXV.

A. Reading of the thermometer in melting snow* — 0°.1 C.
*b.** Correction of thermometer at 0° $(-A)=$ + 0°.1 C.
C. Reading of barometer (reduced to cm.)* . . 76.535 *cm.*
*d.** Corresponding temperature of steam (Table 14) 100°.19 C.
E. Reading of the thermometer in steam* . . . 100°.3 C.
*f.** Correction for thermometer at 100° $(d-E)=$ . . . . — 0°.1 C.
*G.** 50-*degree column reaches from* 0° *up to* . . 50°.4 C.
*h.** The same would have reached from the freezing point, — 0°.1 (see *A*) up to . . . . . . 50°.3 C.
I. The same reaches from* 100° *down to* . . . 51°.3 C.
*j.** The same would have reached from the normal boiling point, 100°.1 (see *f*) down to . 51°.4 C.

k.* Middle-point of thermometer, $\frac{1}{2}(h+j)=$ 50°.85 C.
l.* Correction of thermometer at 50
$(50° - k) =$ . . . . —0°.85 C.
M.* *25-degree column reaches from* 0° *up to* . . 24°.8 C.
n.* The same would have reached from the freezing-
point, — 0°.1 (see *A*) up to . . . . . . 24°.7 C.
O.* *The same reaches from* 50° *down to* . . . 25°.3 C.
p.* The same would have reached from the middle-
point, 50°.85 (see *k*) down to . . . . 26°.15 C.
Q.* *The same reaches from* 50° *up to* . . . . 74°.2 C.
r.* The same would have reached from the middle-
point, 50°.85 (see *k*) up to . . . . . . 75°.05 C.
S.* *The same reaches from* 100° *down to* . . . 76°.2 C.
t.* The same would have reached from the normal
boiling-point, 100°.1 (see *f*) down to . 76°.3 C.
u.* First quarter-point of thermometer,
$\frac{1}{2}(n+p)=$ . . . . 25°.42 C.
v.* Correction for the thermometer at 25°
$(25 - u) =$ . . . . — 0°.42 C.
w.* Last quarter-point of thermometer,
$\frac{1}{2}(r+t)$ . . . . $=$ 75°.67 C.
x.* Correction for thermometer at 75°
$(75 - w) =$ . . . . — 0°.67 C.

NOTE MADE BY STUDENT. "Notice how far off the higher readings are. I repeated the measurements several times to assure myself there was no mistake."

* C. A. E., Feb. 18, 1886.

EXPERIMENT XXVI.

CALIBRATION OF AIR THERMOMETER.

A. Depth of mercury in thermometer.	*B. Weight of thermometer and mercury.*	c. Weight of mercury ($B - B_1$).	*d.* Volume of mercury (.0738 × *c*).
1. 0.0 *cm.*	45.0 *grams.*	0.0 *grams.*	0.00 *cu. cm.*
2. 10.8 "	50.5 "	5.5 "	0.41 "
3. 18.1 "	55.0 "	10.0 "	0.74 "
4. 22.7 "	58.2 "	13.2 "	0.97 "
5. 29.5 "	63.5 "	18.5 "	1.36 "
6. 37.9 "	70.0 "	25.0 "	1.85 "
7. 43.0 "	74.1 "	29.1 "	2.15 "

☞ See Fig. 57, page 120.

E. Reading of the air thermometer in melting snow 27.3 *cm.*
F. Reading of the air thermometer in steam . . 36.1 *cm.*
G. Reading of the air thermometer in water . . 29.1 *cm.*
H. Reading of a mercurial thermometer in the same 18°.0 C.
i. Volume corresponding to *E* by interpolation between d_4 and d_5 (§ 59) . . . . . . 1.23 *cu. cm.*
j. Volume corresponding to *F* by interpolation 1.73 "
k. Volume corresponding to *G* " " 1.33 "
l. Temperature of the water (formula VIII., ¶ 74),
$100° \times (k - i) \div (j - i) =$ . . . . 20°. C.
m. Absolute zero of temperature (formula IX., ¶ 74),
$-100° \times i \div (j - i) =$ . . −246° C.
n. Coefficient of expansion of air (formula X., ¶ 74),
$(j - i) \div i \div 100 =$ 0041

NOTE. It is not unusual to find, as in the example, variations of calibre in a tube which would, unless corrected for, introduce errors of at least 20 % into the results. A very slight quantity of moisture (about $\frac{1}{30}$ *mgr.*) in the tube of the thermometer would account for the error (about 10 %)

in the last two results (m and n). In view of such an error, the determination of temperature (l) must be considered as a means of confirming rather than correcting the reading of the mercurial thermometer (H).

EXPERIMENT XXVII.

OBSERVATIONS.

A. Reading of air thermometer in melting snow 273° C.
B. Height of mercurial column in barometer . . 75.60 *cm.*
C. Height of mercurial column in manometer necessary to make air thermometer read 273° *in steam* 28.00 *cm.*
D. The same in water . . . . 7.00 *cm.*
E. Reading of mercurial thermometer in the water 20°.0 C.

CALCULATIONS.

f. Temperature of steam (Table 14) at 75.60 *cm.* (see *B*) . . . . 99°.85 C.
g. Temperature of the water (1st formula, ¶ 76), $f \times D \div C = 99°.85 \times 7.00 \div 28.00 =$ 24°.96 C.
h. Absolute zero of temperature (2d formula, ¶ 76), $-f \times B \div C =$
$-99°.85 \times 75.60 \div 28.00 =$ . . . —269°.6
i. The same (accepted value) . . . . —273°
j. Error of the determination $(h - i) \div h =$ 1 %. +
k. Coefficient of increase of pressure of confined air (3d formula, ¶ 76), $C \div B \div f = 28.00 \div 75.60 \div 99.85 =$ .00371
l. The same (accepted value) 00367
m. Error of the determination $(k - l) \div k =$. 1 % +
n. Correction for the mercurial thermometer at 20° $(g - E) =$ . . . . + 5°.0 C.

Note. In view of the comparatively close agreement (within 2 %) of the results in *h* and in *k* with accepted values, it may be assumed that the determination of temperature in *g* is accurate within a few tenths of a degree; hence the large correction (+ 5°.0) in *n* is justified. Since, however, so large a correction is improbable, the thermometer in question should be compared with one already calibrated (Exp. 25). Such a comparison might possibly show that Réaumur's (not the Centigrade) scale was employed. This would account for the results of observation.

The data in the example are sufficiently accurate to serve as a rough check (§ 45) upon the results of calibration (Exp. 25); but not as a means of correcting such results.

EXPERIMENT XXVIII.

A. Reading of mercurial barometer . . . . . . 75.2 *cm.*

B. Readings of Mercurial Manometer.	*C. Readings of Air Manometer.*	*d.* Volume of the air [Example 26 *d*].	*e.* Pressure of the air [$A \pm B$].	*f* Product of Volume and Pressure.
— 25.2 *cm.*	40 75 *cm.*	2 01 *cu. cm.*	50.0 *cm.*	100.2
— 15.2 "	35.08 "	1.67 "	60.0 "	100.2
— 5.2 "	30.62 "	1.43 "	70.0 "	100.1
0.0 "	29.00 "	1.33 "	75.2 "	100.0
+ 4 8 "	27.58 "	1.25 "	80.0 "	100.0
+ 14.8 "	25.02 "	1.11 "	90.0 "	99.9
+ 24.8 "	22.87 "	1.00 "	100.0 "	100.0
+ 34.8 "	21.53 "	0.91 "	110.0 "	100.1
+ 44.8 "	20.35 "	0 83 "	120.0 "	99.6
+ 54.8 "	19.02 "	0.77 "	130 0 "	100 1
+ 64.8 "	17.77 "	0.71 "	140 0 "	99.4
+ 74.8 "	16.72 "	0 66 "	150.0 "	99.0

G. Temperatures of boiling ether.	*H. Readings of Air Manometer.*	*i.* Corresponding Pressures (see *e*).
55°.0	16.91 *cm.*	148.2 *cm.*
50°.0	19.22 "	128.5 "
45°.0	21.60 "	109.5 "
40°.0	24.53 "	92.3 "
35°.0	28.20 "	78.0 "
30°.0	39.73 "	61.8 "

☞ See Fig. 65, ¶ 79.

NOTE. This example has, for simplicity, been calculated so that the products under *f* are all nearly equal to 100. The (approximate) agreement of these products follows from Mariotte's Law (§ 79), and serves as a mutual confirmation of the data under Example 26, *A* & *B*, and under Example 28, *B* & *C*, upon which these products depend.

EXPERIMENT XXIX.

OBSERVATIONS.

A.	*Barometric pressure*	76.0 *cm.*
B.	*Temperature of the warm water*	50°.0 C.
C.	*Weight of flask with warm water*	50 0 *g.*
D.	*The same after opening under ice-water*	80.0 *g.*
E.	*Weight of flask filled with water*	170.0 *g.*

CALCULATIONS.

f. Volume of moist air in the flask at 50° (see *B*) and 76 *cm.* (see *A*), $(E - C)$ nearly = 120.0 *cu. cm.*

g. Volume of (nearly) dry air in the flask at 0° and 76 *cm.* (see *A*), $(E - D)$, nearly $=$. 90.0 *cu. cm.*

h. Degree of exhaustion produced by cooling to 0° $(g \div f) =$ . . . . 75.0 %

i. Pressure of the (nearly) dry air at 0° $(h \times A) =$ 57.0*cm.*

j. The same at 50° (see *B*) $i \times (273 + B) \div 273 = i \times 323 \div 273 =$ 67.4 *cm.*

k. Additional pressure of aqueous vapor at 50° (see *B*), $(A - j) =$ . . . . 8.6 *cm.*

l. 1 *cm.* mercury (Table 49 *b*) in megadynes per *sq. cm.* $=$ 0.0133

m. *k cm.* of mercury $(kl) =$ . . . . . 0.114 *megadynes per sq. cm.*
Difference between the pressure of aqueous vapor at 50° and at 0° by Table 13, *C* . . . . 0.117 *megadynes per sq. cm.*

EXPERIMENT XXX.

[Mean of two or more observations.]

A.	*Barometric pressure*	[76.00 *cm.*]
*B.**	*Paraffine melts at*	54°–58°
*C.**	*Alcohol boils at*	79°.2
*D.**	*Chloroform boils at*	60°.6
*E.**	*Bisulphide of carbon boils at*	47°.1
*F.**	*Ether boils at*	35°.3

* S. L. B., Nov. 1887.

EXPERIMENT XXXI.

A. *Weight of empty calorimeter (inner cup only)*				100.0 *grams.*		
B. *The same nearly filled with water* . . .				180.0	"	
c. Weight of water in calorimeter $(B - A) =$				80.0	"	
D. *Time required to cool*	(1) *from*	80° *to* 70°		10 *min.*	0 *sec.*	
	(2) "	70° " 60°		12 "	0 "	
	(3) "	60° " 50°		17 "	0 "	
	Total (4) "	80° " 50°		39 "	0 "	
E. *Weight of calorimeter with a little water* .				120.0 *grams.*		
f. Weight of the water $(E - A) =$ . . .				20.0	"	
G. *Time required when shaken to cool*	(1) *from*	80° *to* 70°		3 *min.*	20 *sec.*	
	(2) "	70° " 60°		4 "	0 "	
	(3) "	60° " 50°		5 "	40 "	
	Total (4) "	80° " 50°		13 "	0 "	
H. *Time required without shaking to cool*	(1) *from*	80° *to* 70°		5 *min.*	0 *sec.*	
	(2) "	70° " 60°		6 "	0 "	
	(3) "	60° " 50°		8 "	30 "	
	Total (4) "	80° " 50°		19 "	30 "	
I. *Weight of calorimeter with turpentine* . . .				175 *grams.*		
j. Weight of turpentine $(I - A) =$ . . .				75	"	
K. *Time required to cool*	(1) *from*	80° *to* 70°		5 *min.*	0 *sec.*	
	(2) "	70° " 60°		6 "	0 "	
	(3) "	60° " 50°		8 "	30 "	
	Total (4) "	80° " 50°		19 "	30 "	
L. *Temperature of the room*				25°.0 C.		
m. Difference between the weights in *c* and in *f*				60.0 *grams.*		
n. Corresponding difference in total time of cooling $(D_4 - G_4) =$				26.0 *minutes*		
o. Total time of cooling with 20 grams $(G_4) =$				13.0	"	

p. Corresponding thermal capacity
$(m \times o \div n) =$ 30.0
q. Thermal capacity of calorimeter alone
$(p - f) =$ 10.0
r. Thermal capacity of calorimeter with
turpentine $(m \times K_4 \div n) =$ 45.0
s. Thermal capacity of turpentine alone
$(r - q) =$ 35.0
t. Specific heat of the turpentine $(s \div j) =$ 0.4

	Mean temperature within calorimeter.		No of units of heat lost in 1 minute.		The same reduced to 1° difference in temperature.	
	$u =$ above 0°.	$v =$ above L.	$w = (c+q) \times 10^3 \div D$.	$x = (f+q) \times 10^3 \div H$	$y = (w \div v)$.	$z = (x \div v)$.
(1)	75°.0	50°.0	90	60	1.80	1.20
(2)	65°.0	40°.0	75	50	1.88	1.25
(3)	55°.0	30°.0	53	35	1.77	1.17
Average	65.	40.	73	48	1.8	1.2

EXPERIMENT XXXII.

FIRST METHOD.

A. Weight of empty calorimeter (inner cup) . 100.0 *grams.*
B. Temperature of air within calorimeter . 18°.0 C.

C. Temperatures of water.	*D. Times.*
1. 40°.6 (just before pouring)	10 *m.* 0 *sec.*
2. (not stationary)	11 " 0 "
3. 37°.4	12 " 0 "
4. 37°.1	13 " 0 "
5. 36°.8	14 " 0 "

E. Weight of calorimeter with water . . 180.0 *grams.*

f. Rate of cooling per minute
$(C_5 - C_3) \div (D_5 - D_3) =$ . . . . $0°.3$ *per min.*
g. Temperature reduced to time of pouring
$(f \times (D_3 - D_1) + C_3 =$ . . . . $38°.0$ C.
h. Rise of temperature of calorimeter $(g - B) =$ $20°.0$ C.
i. Fall of temperature of the water $(C_1 - g) =$ $2°.6$ C.
j. Weight of water $(E - A) =$ . . . . . . 80.0 *grams.*
k. Units of heat given out $(i \times j) =$ . . . 208 *units.*
l. Thermal capacity of calorimeter $(k \div h) =$ 10.4

SECOND METHOD.

M. Temperature of cold water . . . . $+10°.0$ C.
N. Temperature of shot in calorimeter just before substitution of cold water . . . . $83°0$ C.
O. Resulting temperature . . . . $18°.0$ C.
P. Weight of calorimeter with water . . . . 180.0 *grams.*
q. Weight of water $(P - A) =$ . . . . . . 80.0 "
r. Rise of temperature of water $(O - M) =$ $8°.0$ C.
s. Units of heat absorbed by water $(q \times r) =$ 640
t. Fall of temperature of calorimeter $(N - O) =$ $65°$. C.
u. Thermal capacity of calorimeter $(s \div t) =$. 9.9

THIRD METHOD.

V. Volume of water displaced by thermometer when immersed to the ordinary depth . . . . 0.9 *cu. cm.*
W. Weight of (brass) stirrer . . . . 2.0 *grams.*
x. Thermal capacity of inner cup (brass) and stirrer (1st footnote, page 161), $0.094 \times (A + W) =$ 9.6
y. Thermal capacity of the thermometer (2d footnote, page 161), $0.46 \times V =$ 0.4
z. Total thermal capacity $(x + y) =$ . . . 10.0

EXPERIMENT XXXIII.

*DETERMINATION OF THE SPECIFIC HEAT OF LEAD SHOT (¶ 94, I.).

[*A.*] [*Weight of calorimeter with packing, &c., nearly filled with lead shot*] . [797.47 *grams*].
*B.** *The same without lead shot* . . . . 357.44 *grams*
*C.** *Weight of bottle with ice and water before using* . . . . 501.02 "
*D.** *Indication of the thermometer in the ice-water* + 0°.7 C.
*E.** *Temperature of air in calorimeter* . . 23°.5 C.
*F.** *Temperature of s'ot in heater* . . . . 98°. C.
*G.** *Temperature of mixture* . . . . 22°.7 C.
*H.** *Weight of calorimeter with mixture* . . 846.17 *grams.*
*I.** *Weight of bottle with ice and water after using* 452.32 "

j. { Weight of water used ($C - I$) = . 48.70 } "
{ The same ($H - A$) = ... 48.70 }

k. { Weight of lead shot ($A - B$) = . 440.03 } "
{ The same ($H - B$) $-$ ($C - I$) = 440.03 }

*l.** Change of temperature in water ($G - D$) = 22°.0 C.
*m.** No. of units of heat absorbed by water ($j \times l$) = 1071 +
*n.** Change of temperature in lead shot ($F - G$) = 75°.3 C.
o. Thermal capacity of shot ($m \div n$) = . . . 14.22
*p.** Specific heat of the lead shot ($o \div k$) = . . 0.0323 +

* J. E. W., Feb., 1886.

NOTE. The inner cup of the calorimeter employed in this determination weighed about 44 grams, and had accordingly a thermal capacity equal to about 4 grams of water. Since its temperature fell from 23°.5 to 22°.7, that is 0°.8 C., a deduction of $0.8 \times 4 = 3 +$ units should strictly be made from the number of heat units, 1071 + (see *m*), apparently given out by the shot. This would make the specific heat (in *p*) 0.0320 instead of 0.0323.

EXPERIMENT XXXIV.

First Method.

$A.$*	*Temperature of the room*	24°.3 C.
$B.$*	*Weight of bottle with kerosene*	308.9 *grams.*
$C.$*	*Weight of bottle with water*	267.1 "
$D.$*	*Temperature of kerosene*	9°.2 C.
$E.$*	*Temperature of water*	63°. C.
$F.$*	*Temperature of mixture*	24°.3 C.
$G.$*	*Weight of bottle with water remaining* . .	254.6 *grams.*
$H.$*	*Weight of bottle with kerosene remaining* .	241.5 "

i. Specific heat of kerosene referred to water, calculated as in the last example,

$$(C - G) \times (E - F) \div (B - H) \div (F - D) = 0.47 +$$

* F. S. D., Feb., 1886.

Second Method.

$J.$	*Temperature of the room*	23°.0 C.
$K.$	*Weight of lead shot*	300.0 *grams.*
$L.$	*Weight of bottle with alcohol before using* .	500 0 "
$M.$	*Temperature of the alcohol*	+ 1°.0 C.
$N.$	*Temperature of the shot*	98°. C.
$O.$	*Temperature of the mixture*	23°.0 C.
$P.$	*Weight of bottle with alcohol after using* .	450.0 *grams.*

q. Specifio heat of the lead shot (see the last example) 0.0320

r. Heat units given to alcohol, $q \times K \times (N - O) =$ 720

s. Specific heat of the alcohol

$$r \div (L - P) \div (O - M) = \ . \ . \ . \quad 0.65 +$$

NOTE. For a fuller statement of the calculations, see last example. On account of the agreement of the temperature of the mixture with that of the room, no allowance for the thermal capacity of the calorimeter is to be made.

EXPERIMENT XXXV.

I. Preliminary observations.

A. 10 *grams water at* 20° *with* 10 *grams alcohol at* 20° *gives mixture at* . . 28° C.

B. The same with water at 10° C. . . . . . . 21° C.

c. Temperature of water (estimated) which would give mixture at 20°, about . . . . . 8° C.

D. Weight of glass beaker used as inner cup of calorimeter 30.00 *g.*

E. The same with (about) 50 *grams of alcohol* . 80.00 *g.*

F. Temperature of the same 20°.0 C.

G. Temperature of cold water just before pouring, risen to . . . . 8°.0 C.

H. Temperature of mixture 20°.0 C.

I. Weight of the same in calorimeter . . . . 130.00 *g.*

j. Weight of alcohol $(E - D) =$ . . 50 00 *g.*

k. Weight of water $(I - E) =$ . . . . 50.00 *g.*

l. Change of temperature in water $(F - G) =$ 12° 0 C.

m. No. of units of heat given out $(k \times l) =$ . . 600

n. Latent heat of mixture per gram of alcohol $(m \div j) =$ . . . 12.0

II. *Preliminary observations.*

A. 10 *grams water at* 20° *with* 1 *gram nitrate of ammonium at* 20° *gives mixture at* 14° C.

B. The same with water at 30 C. 23° C.

c. Temperature of water (estimated) which would give mixture at 20° 27° C.

D. Weight of glass beaker used as inner cup of calorimeter 30.00 *g.*

E. The same with (about) 10 *grams of nitrate of ammonium* . . . . 40.00 *g.*

F. Temperature of the same 20°.0 C.

G. Temperature of water just before pouring, fallen to 27°.0 C.

H. Temperature of mixture 20°.0 C.

I. Weight of the same in calorimeter . . . . 140.00 *g.*

j. Weight of nitrate of ammonium $(E - D) =$ 10.00 *g.*

k. Weight of water $(I - E) =$ . . . 100 00 *g.*

l. Change of temperature in water $(G - F) =$ 7°.0 C.

m. No. of units of heat absorbed $(k \times l) =$. 700

n. Latent heat of solution per gram of nitrate of ammonium $m \div j =$ 70

EXPERIMENT XXXVI.

OBSERVATIONS.

A. Weight of empty calorimeter (inner cup) .	77.00 *grams.*	
B. Weight of cotton waste with ice	100.00 "	
C. Temperatures of water in calorimeter . .	*D. Times.*	
1.	41°.0 C.	20 *min.* 0 *sec.*
2.	40°.5 C.	21 " 0 "
3.	[Ice transferred to calorimeter.]	22 " 0 "
4.	13°.4 C.	23 " 0 "
5.	10°.0 C.	24 " 0 "
6.	10°.0 C.	25 " 0 "
E. Temperature of the room	23° C.	
F. Weight of cotton waste	60.00 *grams.*	
G. Weight of calorimeter with mixture . .	229.20 "	

CALCULATIONS.

h. Weight of ice used $(B - F) =$ . . . . 40.00 *grams.*

i. Weight of water used $(G - A - h) =$. 112.20 "

j. Thermal capacity of (brass) cup $(.094\ A) =$ 7.2

k. Add for thermometer and stirrer (see Examp. 3) 0.6

l. Total thermal capacity of calorimeter $(j + k) =$ 7.8

m. Thermal capacity of calorimeter with water $(i + l) =$ 120.0

n. Temperature of water reduced to time (D_3) of mixing, $C_2 - (C_1 - C_2) =$ . . . . 40°.0 C.

o. Temperature of mixture, $C_6 = C_5 =$ . . 10°.0 C.

p. Change of temperature of water $(n - o) =$ 30°.0 C.

q. Heat units absorbed $(m \times p) =$. . . . 3600

r. Heat units absorbed by 1 gram of ice $(q \div h) =$ 90.0

s. Heat units absorbed in raising 1 gram of *melted* ice to the temperature of the mixture, $o =$ 10.0

t. Heat required to melt 1 gram of ice $(r - s) =$ 80.0

EXPERIMENT XXXVII.

Observations.

*A.** *Weight of brass calorimeter (inner cup)* 76.974 *grams.*
*B.** *Weight of calorimeter, thermometer, and stirrer.* . . . . 99.850 "
*C.** *The same with water* . . . . 223.670 "
*D.** *Temperatures of the water at intervals of* 1 *minute:*

Before admitting steam.		During admission of steam.		After admission of steam.	
1.	8°.4	4.	20°.	6.	27°.7
2.	8°.5	5.	27°.8	7.	27°.2
3.	8°.6			8.	27°.0
				9.	27°.0

E. [*Overflow from trap*] . . . . [*none*]
F. [*Temperature of the room*] . . . . . [18° ?]
*G.** *Weight of calorimeter with water and condensed steam* . . . . 227.710 *grams.*
*H.** *Barometric pressure* . . . . 74.96 *cm.*

Calculations.

i. Temperature of steam at pressure in *H* (Table 14) . . . . 99°.6 C.
j. Rate of increase of temperature before admission of steam $\frac{1}{2}(D_3 - D_1) =$. 0°.1 *per min.*
k. Rate of cooling after admission of steam $\frac{1}{2}(D_9 - D_7) =$ . . . . 0°.1 "
l. Temperature of water calculated forward to the time of 4th observation $(D_3 + j) =$ . . . 8°.7 C.
m. The same calculated backward $(D_7 + 3k) =$. 27°.5 C.
n. Rise of temperature $(m - l) =$ . . . . . . 18°.8 C.
o. Thermal capacity of calorimeter (see Examp. 36, *l*) 7.67

p.	Weight of water in calorimeter $(C - B) =$.	123.82
q.	Total thermal capacity $(o + p) =$	131.49
r.	Units of heat given out $(n \times q) =$	2472
s.	Weight of steam condensed $(G - C) =$	4.040 *grams.*
t.	Units of heat per gram of steam $(r \div s) =$	611.9
u.	Units of heat given out by 1 gram of condensed steam in cooling to the temperature of the mixture $(i - m) =$	72.1
v.	Heat given out in the condensation of 1 gram of steam $(t - u) =$	540

* M. B., Feb. 1886.

EXPERIMENT XXXVIII.

*A.**	*Weight of zinc filings*	1.00 *gram.*
*B.**	*Weight of glass calorimeter*	23.92 "
*C.**	*Weight of battery solution*	96.[00] "
*D.**	*Temperature of battery solution* . . .	19°.5 C.
*E.**	*Time occupied by chemical action* . . .	7 *minutes.*
*F.**	*Resulting temperature (maximum)* . . .	43°.[0] C.
*G.**	*Temperature* 1.5 *minutes later*	42°.[0] C.
h.	Correction for cooling $\frac{1}{2} E \times (F - G) \div 1.5 =$	2°.3 C.
i.	Corrected temperature due to chemical action $(F + h) =$	45°.3 C.
j.	Specific heat of battery solution †	0.60
k.	Thermal capacity of the solution $(C \times j) =$	57.6
l.	Thermal capacity of glass calorimeter $(0.19\ B) =$	4.5
m.	Thermal capacity of thermometer and stirrer	0.6
n.	Thermal capacity of 1 gram of zinc (Table 8)	0.1

† The specific heat in *j* was taken from Table 30, for 50 % sulphuric acid, assuming that the small quantity of bichromates present would not essentially modify the result. For methods of determining the specific heat of liquids, see Exp. 32.

o.	Total thermal capacity $(k+l+m+n)=$	62.8
p.	Rise in temperature $(i-D)=$	25°.8 C.
q.	Units of heat developed $(o+p)=$. . .	1620
R.*	*Weight of zinc oxide*	1.25 *grams.*
S.*	*Weight of battery solution*	96 [00] "
T.*	*Temperature of battery solution*	17°.[0] C.
U.*	*Temperature of mixture (maximum)* . .	21°.3 C.
v.	Total thermal capacity as in o	62.8
w.	Rise of temperature $(U-T)=$	4°.3 C.
x.	Units of heat developed $(v\times w)=$ . .	270
y.	Difference between the number of units of heat developed by 1 gram of zinc and by its equivalent (1.25 grams) of zinc oxide $(q-y)=$	1350

* E. L. A., March, 1888.

EXPERIMENTS XXXIX.-XL.

OBSERVATIONS WITH THE	THERMO-PILE.*	PHOTOM-ETER.†
A. Weight of lamp before the experiment .	169. 29 *g.*	198.[0] *g.*
B. Time of lighting the lamp	2 *h.* 44 *min.*	10 *h.* 42 *min.*
C. Fixed distance of lamp from instrument	40. *cm.*	86 *cm.*
D. Weight of candle before experiment .	38.3 *g.*	26.5 *g.*
E. Time of lighting candle	2 *h.* 44 *min.*	10 *h.* 42 *min.*
F. Mean distance of candle from instrument	30.7 *cm.* ‡	72 *cm.* §
G. Time of extinguishing lamp	3 *h.* 15 *min.*	10 *h.* 58 *min.*
H. Time of extinguishing candle . . .	3 *h.* 15 *min.*	10 *h.* 58 *min.*
I. Weight of lamp in middle of experiment	165.3 *g.*	195.7 *g.*
J. Time of relighting lamp	3 *h.* 20 *min.*	11 *h.* 5 *min.*
K. Fixed distance of lamp from instrument	40 *cm.*	64.5 *cm.*
L. Weight of candle in middle of experiment	34.7 *g.*	24.1 *g.*
M. Time of relighting candle	3 *h.* 20 *min.*	11 *h.* 5 *min.*
N. Mean distance of candle from instrument	29.8 *cm.* ‡	60.5 *cm.* §

* F. W. A., March, 1888. † W. B. M., March, 1888.
‡ Mean of 3 observations. § Single observation.

Observations with the	Thermo-pile.*	Photom-eter.†
O. Time of final extinction of lamp . .	3 *h.* 50 *min.*	11 *h.* 39 *min.*
P. Time of final extinction of candle . .	3 *h.* 50 *min.*	11 *h.* 39 *min.*
Q. Weight of lamp after the experiment .	161 5 *g.*	191.8 *g.*
R. Weight of candle after the experiment .	31.2 *g.*	20.4 *g.*
Calculations — First part of Experiment.		
a. Rate of consumption of lamp in grams per hour ‖ $(A-I) \div (G-B) =$.	7.7	8.6
b. Rate of consumption of candle in grams per hour ‖ $(D-L) \div (H-E) =$.	7.0	9.0
c. Candle power of candle $b \div 8 =$ . .	0.88	1.13
d. Candle power of lamp $C^2 \div F^2 \times c =$	1.49	1.61
e. The same reduced to 8 *g.* per hour, $8\,d \div a =$	1.55	1.50
Calculations — Second part of Experiment.		
f. Rate of consumption of lamp in grams per hour $(I-Q) \div (O-J) =$ . .	7.6	6.9
g. Rate of consumption of candle in grams per hour $(L-R) \div (P-M) =$.	7.0	6.5
h. Candle-power of candle $g \div 8 =$ . .	0.88	0.81
i. Candle-power of lamp $K^2 \div N^2 \times h =$	1.59	0.92
j. The same reduced to 8 grams per hour, $8\,i \div f =$	1.67	1.07
k. Mean relative candle-power of kerosene and paraffine under the conditions of the experiment, $\frac{1}{2}(e+j) =$ . . .	1.6 +	1.3

‖ The time of these observations is to be expressed in hours and decimal fractions of an hour.

Note. If the lamp and candle are weighed while burning, the observations B, E, J, M, O, and P, should read "time of weighing" instead of "time of lighting," "relighting," or "extinguishing." The calculations are identical, except that J and M are substituted respectively for observations G and H, which are omitted.

EXPERIMENT XLI.

*A.** *Principal focal length of lens by* ORDINARY METHOD (¶ 116, 1), *mean of 4 observations* . . . . 13.[00] *cm.*

*B.** *The same by* METHOD OF PARALLAX (¶ 116, 2) . . . . — — —

*C.** *The same by* INDIRECT METHOD (¶ 116,3) 13.287 *cm.*

*D.** *The same by* COLOR METHOD (¶ 116, 4) — — —

e. Principal focal length, mean of different methods . . . . 13.14 *cm.*

EXPERIMENT XLII.

*A.** *Nearest distance of lamp from screen consistent with perfect image* (¶ 117, 1), *mean of 4 observations* . . . . 51.68 *cm.*

*b.** Principal focal length, $\frac{1}{4}$ A = ... 12.92 *cm.*

*C.**	*Distance from lamp to lens*	Conjugate focal lengths (¶ 117, 2).	49.5 *cm.*
*D.**	*Distance from lens to screen*		17.8 *cm.*
*E.**	*Distance fram lamp to lens*		17.5 *cm.*
*F.**	*Distance from lens to screen*		49.8 *cm.*

g. Mean of smaller distances . . . . 17.65 *cm.*

h. Mean of greater distances . . . . 49.65 *cm.*

i. Principal focal length $g \times h \div (g + h) =$ 13.02 *cm.*

*J.**	*Distance from lamp to lens*	Conjugate focal lengths (¶ 117, 3).	57.8 *cm.*
*K.**	*Distance from lens to screen*		17.0 *cm.*
*L.**	*Distance from lamp to lens*		16.3 *cm.*
*M.**	*Distance from lens to screen*		58.5 *cm.*

n. Mean of smaller distances . . . . 16.65 *cm.*

o. Mean of greater distances . . . . 58.15 *cm.*

p. Principal focal length $n \times o \div (n + o) =$ 12.94 *cm.*

q. Principal focal length, mean of three methods of conjugate foci, $\frac{1}{3}(b + i + p) =$ . . . . 12.96 *cm.*

* C. A. B., March, 1886.

EXPERIMENT XLIII.

First Part.

A. Distance of farther focus from converging lens (mean of 10 *obs.)* . . . . 90.0 *cm.*

B. Distance of nearer focus from converging lens (mean of 10 *obs.)* . . . . 30.0 *cm.*

c. Principal focal length $(A \times B) \div (A - B)$. 45.0 *cm.*

Second Part.

D. Distance of farther focus from diverging lens (mean of 10 *obs.)* . . . . 90.0 *cm.*

E. Distance of nearer focus from diverging lens (mean of 10 *obs.)* . . . . 30.0 *cm.*

f. Virtual principal focal length, $(D \times E) \div (D - E) =$ . . . . . . — 45.0 *cm.*

EXPERIMENT XLIV.

A. Zero reading of sextant* . . . . $0° 4' 10''$

Readings of sextant when set on sun: —

B. Positive reading (mean of* 5 *obs.)* . $0° 35' 28''$

C. Negative reading (mean of* 5 *obs.)* . $1° + 30' 40''$

*d.** Apparent angular diameter of the sun, $\frac{1}{2}(B - C) = \frac{1}{2}(64' 48'') =$ . . . $32' 24''$

e. Apparent "semidiameter" (*d*) = . . $16' 12''$

f. The same reduced to decimal fraction of a degree (Table 44 *A*) . . . . $0°.270$

The same for March (Table 44, *E*) $0°.269$

* L. L. H., March 5th, 1886.

$G.$*	*Zero reading of sextant*	0° 4′ 10″
	Readings of sextant set on object : —	
$H.$*	*Positive reading (mean of 5 obs.)* . .	3° 22′ 34″
$I.$*	*Negative reading (mean of 5 obs.)* . .	$\bar{4}$° + 18′ 24″
$j.$*	Apparent angular diameter of the object, $\frac{1}{2}(H-I)=\frac{1}{2}(7° 4' 10'')=$ . .	3° 32′ 5″
$k.$	The same reduced to decimal fraction of a degree (Table 44 A)	3°.535
$l.$	Tangent of the angle k, (Table 5) . .	0.0618
$M.$*	*Length of the object in question*	100 *cm.*
$n.$	Distance of the object, calculated † $(M \div l)=$	1618 *cm.*
O*.	*The same by measurement (from the axis of the revolving mirror to the foot of the object)*	1623 *cm.*

* C. A. E., March, 1886.

EXPERIMENT XLV.

$A.$*	*Zero reading of sextant* ‡	2° 48′. 5
$B.$*	*Reading corresponding to 1st angle of prism*	123° 26′. 5
$C.$*	*Reading corresponding to 2d angle of prism*	121° 30′. 0
$D.$*	*Reading corresponding to 3d angle of prism*	123° 34′. [0]
$e.$	First angle, $\frac{1}{2}(B-A)=\frac{1}{2}(120° 18'. 0)=$	60° 19′. 0
$f.$	Second angle, $\frac{1}{2}(C-A)=\frac{1}{2}(118° 41'. 5)=$	59° 20′. 8
$g.$	Third angle, $\frac{1}{2}(D-A)=\frac{1}{2}(120° 45'. 5)=$	60° 22′. 8
$h.$	Sum of the three angles $(e+f+g)=$	180° 2′. 6

* A. E. T., March, 1888.

† A more accurate calculation may be made by the use of logarithmic tangents (Table 5, A). We have $\log n = \log M - \log \tan k$, $= 2.0000 - \bar{2}.7908 = 3.2902$; hence $n = 1619$ *cm.*

‡ The zero-reading of the instrument here employed was made purposely large so as to extend the limit of its negative readings (see Exp. 44). For second method, see next example.

EXPERIMENT XLVI.

A. Reading of telescope of spectrometer set on direct image of slit in collimator	180° 0′. 0
Readings of telescope set on image reflected: —	
B. by 1st face of prism	120° 0′. 0
C. by 2d face of prism	240° 0′. 0
d. Angle between 1st and 2d faces (¶ 126), $\frac{1}{2}(C - B) =$	60° 0′. 0

E. Reading of telescope set on image of slit in collimator illuminated by sodium flame and refracted by the prism angle (d) placed so as to produce a minimum deviation	140° 1′. 0
F. The same with prism rotated 180°	220° 1′. 0
g. Angle of minimum deviation $(A - E) =$	40° 1′. 0
h. The same $(F - A) =$	39° 59′. 0
i. The same (mean of *g* and *h*)	40° 0′. 0

NOTE. The reading of a sextant in this determination, see ¶ 127, II., would also be 40° (not 80°). Given a prism angle 60°, and an angle of minimum deviation 40°, the index of refraction is (see ¶ 244, and Table 4).

$\mu = \sin \frac{1}{2}(60° + 40°) \div \sin \frac{1}{2}(60°) =$

$\sin 50° \div \sin 30° = 0.7660 \div 0.5000 =$. 1.5320

EXPERIMENT XLVII.

FIRST PART.

*A.** *Zero reading of sextant* . . . . 2° 25′

Readings of sextant set upon first set of diffracted images of a thin white flame produced by a piece of linen cloth : —

*B.** *Positive reading* . . . . 2° 48′

*C.** *Negative reading* . . . . 2° 4′

d. Angular separation of images $\frac{1}{4}$ $(B - C) =$ 0° 11′.0

e. The same in decimal fraction of a degree (Table 44 *A*), $(d \div 60) =$ . . . . . 0°. 183

f. Distance between the threads (¶ 130, formula I.), 0.00006 ÷ sin 0°.183 = 0.00006 ÷ 0. 0032 = ... 0.019 *cm.*

g. No. of threads per *cm.* $(1 \div f) =$ . . . 53

*A. E. T., March, 1888.

SECOND PART.

*H.** *Zero reading of spectrometer* (*mean of* 3 *obs.*) 29° 24′ 50″

Readings on diffracted image of slit illuminated by sodium flame : —

*I.** *Positive angle* (*mean of* 4 *obs.*) . . . . 12° 4′ 15″

*J.** *Negative angle* (*mean of* 4 *obs.*) . . . 46° 43′ 30″

*k.** Angle of minimum deviation produced by diffraction grating, $\frac{1}{2}$ $(J - I) =$ . . . . 17° 19′ 37″. 5

l. The same reduced to decimal fraction of a degree (Table 44 *A*) . . . . 17°. 327

m. Distance between lines of grating as stated by manufacturer ($1 \div 12960$) = . 0.000077160 *inches.*

n. The same reduced to *cm.* (2.5400 *m*) = 0.00019599 *cm.*

o. Length of sodium light-waves in air (¶ 130, foot-note), $2 \times (n) \times \sin \frac{1}{2} (l) =$
$2 \times 0.00019599 \times 0.1506 =$. 0.0000590 + *cm.*

p. Mean of wave-lengths, D_1 & D_2 (Table 41) . . . . 0.0000589 + *cm.*

* M. B., March, 1886.

NOTE. Angles less than 25° are generally reduced with greater accuracy by a table of logarithmic sines than by a table of natural sines having the same number of places. We have by Table 4 *a* and Table 6, $\log 2 + \log n + \log \sin \frac{1}{2} l = 0.30103 + \bar{4}.29224 + \bar{1}.1779 = \bar{5}.77117$; hence $o =$ 0.00005904 *cm.* (nearly). The error in this determination (0.0000001 *cm.*) corresponds to an error of about 1′ in the observed angle of diffraction.

EXPERIMENTS XLVIII., XLIX., AND L.

OBSERVATIONS.

A. Distance between two adjacent points of minimum sound in ¶ 131, *I., with a small violin* A*-fork (mean of* 10 *obs.)* . . . . 38.0 *cm.*

B. The same with a small C*-fork* . . . . . . 32.0 *cm.*

C. Temperature of the air within resonance tube* 22°. 75 C.

D. Relative humidity of the air of the room* . . 25 %

E. Distance between nodal points in resonance tube* (¶ 132) *due to a large* A (?)*-fork* . 74.9 *cm.*

* J. E. W., December, 1885.

F.	*The same due to a middle* C–*fork*	68.0 *cm.*
G.	*Length of monochord responding to large* A–*fork* (¶ 133, III.)	80.0 *cm.*
H.	*Length of monochord responding to middle* C–*fork*	72.0 *cm.*

CALCULATIONS.

i.	Wave-length of sound due to small violin A–fork ($2\ A$) = ...	76.0 *cm.*
j.	The same due to small C–fork ($2\ B$) = . .	64.0 *cm.*
k.	Velocity of sound corresponding to atmospheric conditions in C and D (see Table 15)	34.608 *cm. per sec.*
l.	Wave-length due to large A–fork ($2\ E$) =	149.8 *cm.*
m.	Number of vibrations of large A–fork per second ($k \div l$) = ...	230.9
	The same according to instrument maker.	228.5
n.	Wave-length of middle C–fork ($2\ F$) = .	136.0 *cm.*
o.	Number of vibrations of middle C–fork per second ($k \div n$) = ...	254.5
	The same according to instrument maker	256.0
p.	Musical interval between the small A– and C–forks ($A \div B$) = ...	1.2 (nearly).
q.	Theoretical interval (6 : 5) = ...	1.20
r.	Musical interval between the large A–fork and C–fork ($E \div F$) = ...	1.10
s.	The same ($G \div H$) = ...	1.11
t.	The same according to instrument makers, 256 ÷ 228.5 = ...	1.12

EXPERIMENT LI.

Velocity of Sound — Echo Method.

*A.** *Distance between two parallel walls* (¶ 137, II.) 80.0 *metres.*

*B.** *Reading of metronome adjusted to keep time with echoes (mean of 5 observations)* 129

*C.** *Number of beats in* 100 *seconds corresponding to this reading* 215

*D.** *Temperature of outside air* . . . . 6° C.

*E.** *Relative humidity* . . . . 30 %

f. Distance traversed by sound ($2\ A$) = . . . 160 *metres.*

g. Time occupied ($100 \div C$) = ... 0.465 *sec.*

h. Velocity of sound ($f \div g$) = ... 344 *metres per sec.*

i. Velocity tabulated for conditions D and E (Table 15) . . . . 336 *metres per sec.*

* J. E. W., December, 1885.

Pendulum Method.

*J.** *Time of pendulum (on north wall of Lawrence Hall) about* . . . . 1.00 *sec.*

*K.** *Distance of signalling observer from pendulum, about* . . . . 10 *metres.*

*L.** *Distance of observer with telescope (Jarvis Field)* 350 ± 20 *metres.*

*m.** Velocity of sound
($L - K$) $\div J$ = ... 340 ± 20 *m. per sec.*

* Approximate results, recalled from memory from experiments made by students in 1881–1882, before the building of the Jefferson Physical Laboratory.

EXPERIMENT LII.

A. Number of waves traced by tuning-fork between alternate marks made by pendulum (mean of 10 *observations)* 28.27

B. Number of complete oscillations of the pendulum timed 100

C. Time occupied (mean of 10 *observations)* . . 50.0 *sec.*

d. Rate of pendulum (complete oscillations per second) $B \div C =$ 2.00

e. Rate of tuning-fork (complete oscillations per second) $A \times d =$ . . . . 56.5 +

EXPERIMENT LIII.

Note. The tuning-forks Nos. 1 and 17 are supposed to have been adjusted to an exact octave (by filing or loading one of them) before the following observations were taken.

A. No of 1st fork.	*B. No. of 2d fork.*	*C. Time of 100 beats.*	*d.* No. of beats per sec.	*e.* Totals.	*f.* Pitch of 1st fork $(e + e_{17})$.
1	2	25 *sec.*	4.0	0.0	65.6
2	3	20	5.0	4.0	69.6
3	4	22.2	4.5	9.0	74.6
4	5	30	3.3	13.5	79.1
5	6	27	3.7	16.8	82.4
6	7	20.8	4.8	20.5	86.1
7	8	26.3	3.8	25.3	90.9
8	9	23.8	4.2	29.1	94.7
9	10	25	4.0	33.3	98.9
10	11	29.5	3.4	37.3	102.9
11	12	23.2	4.3	40.7	106.3
12	13	20	5.0	45.0	110.6
13	14	22.3	4.5	50.0	115.6
14	15	27.2	3.7	54.5	120.1
15	16	28	3.6	58.2	123.8
16	17	26.5	3.8	61.8	127.4
17	1	*No beats.*	*No beats.*	65.6	131.2

EXPERIMENT LIV.

*A.**	*Time occupied by Lissajous' curves in passing through* 40 *complete cycles* (*mean of* 10 *obs.*)	18.5 *sec.*
*B.**	*Which fork must be loaded to make the figures permanent?*	The higher.
*C.**	*Number of lobes visible in the symmetrical figures* (= n, formulae, ¶ 142).	4
d.	Sign of the correction for cycles (compare B with ¶ 142)	+
e.	Number of cycles per second ($40 \div A$) = c, formulae, ¶ 142	2.16
f.	Pitch of the lower fork (Exp. 52) = p in formulae, ¶ 142	56.5 +
*g.**	Pitch of the higher fork $C \times f\,[d]\,e =$ $4 \times 56.5 + 2.16 =$	228.2 +

* M. B., December, 1885.

EXPERIMENT LV.

A.	*Number of revolutions made by a toothed wheel in* 100 *seconds when adjusted so as to show stationary waves upon a tuning-fork* (*mean of* 10 *obs.*)	473
B.	*Number of teeth*	12
c.	Pitch of the tuning-fork $\frac{1}{100} A \times B$ . .	57.0
	The same by graphical method (Exp. 52) .	56.5 +

EXPERIMENT LVI.

A. *Distance between marks made by bullet* . . . 19.2 *cm.*

B. *Time occupied by* 250 *complete oscillations of the pendulum* . . . . 200 *sec.*

c. Time occupied by pendulum after its release in reaching the middle-point of its swing, $\frac{1}{4} \times \frac{1}{250} \times B =$ . . . . 0.200 *sec.*

d. Square of time occupied (c^2) $=$ 0.0400

e. Ratio of distance through which bullet falls to square of time occupied ($A \div d$) $=$ 480

f. Acceleration of gravity ($2\,e$) $=$ 960

[Results with other pendulums reduced in the same way and tabulated as in ¶ 148.]

EXPERIMENT LVII.

A. *Distance from lower edge of bracket to top of bullet* . . . . 7.8 *cm.*

B. *The same to bottom of bullet* . . . . 9.8 *cm.*

c. Length of pendulum $\frac{1}{2}$ ($A + B$) $=$ . . . 8.8 *cm.*

D. *Time occupied by* 100 *single vibrations* . . 30.0 *sec.*

e. Time of pendulum $D \div 100 =$ . . . . . . 0.300 *sec.*

f. Square of time of pendulum (e^2) $=$ . . . 0.090

g. Ratio of length of pendulum to square of time ($c \div f$) $=$ 97.8

[Results with other pendulums reduced in the same way and tabulated as in ¶ 149.]

EXPERIMENT LVIII.

A. Diameter of rods (*ab* and *hi*, Figs. 153 and 154)	1.00 *cm.*
B. Distance between rods adjusted to	100.00 *cm.*
C. Mean interval between coincidences with seconds clock (reduced as in ¶ 152)	125 *sec.*
d. Length of pendulum $(A + B) =$ . . .	101.00 *cm.*
e. Time of pendulum $(C \div (C - 1)) =$ . .	1.00807 *sec.*
f. Acceleration of gravity corresponding to *d* and *e* (see Table, ¶ 153)	980.9

EXPERIMENT LIX.

Preliminary Observations.

A. Length of spring without load, about	50 *cm.*
B. Length of spring with bullet, about	100 *cm.*
C. Length of spring with 30 *grams*	99 + *cm.*
D. Length of spring with 31 *grams*	100 + *cm.*
E. Length of spring with 30.6 *grams, about* .	100 *cm.*
f. Weight of bullet, about	30.6 *grams.*

Observed Times of 1000 Consecutive Oscillations.

G. — With the bullet	800 *sec.*
H. — With 30.6 *grams*	801 *sec.*
I. — With 30.5 *grams*	798 *sec.*

j. Mass of the bullet,

$$30.5 + 0.1 \times (G - I) \div (H - I) = 30.57 \text{ grams.}$$

EXPERIMENT LX.

A. *Readings of the marker at intervals of 2 sec.*		*b.* Difference in 2 sec.	*c.* Mean Velocity ($b \div 2$).	*d.* Difference in 2 sec.	*e.* Acceleration ($d \div 2$).
1.	552 *mm.*				
2.	585	+ 33	+ 16.5	8.0	4 0
3.	600	+ 15	+ 7.5	10.0	5.0
4.	595	− 5	− 2.5	7.5	3.8
5.	575	− 20	− 10.0	10.0	5.0
6.	535	− 40	− 20.0		

F. *Mass of the ring* . . . . 500 *grams.*

G. *Outside diameter of ring* . . . . 20.5 *cm.*

H. *Radial thickness of ring* . . . . 0.5 *cm.*

i. Outside radius of ring, $\frac{1}{2}$ *G* . . . . . . 10.25 *cm.*

j. Mean radius $(i - \frac{1}{2} H) =$ . . . . 10.00 *cm*

k. Mean deflection of outside of ring in *cm.*
(average of $A \div 10) =$ . . . . 57.4 *cm.*

l. Mean angle of deflection in degrees,
$k \div i \times 360° \div \pi = 57.4 \div 10.25 \times 57°.3 =$ 321°

m. Acceleration of outer surface reduced to *cm.*
(average of $e \div 10) =$ 0.445 *cm. per sec. per. sec.*

n. Mean acceleration of whole mass of ring
$(m \times j \div i) =$ . . . . 0.44 "

o. Force exerted by wire on ring $(F \times n) =$ 220 *dynes.*

p. Couple exerted by wire on ring $(o \times j) =$
2200 *dyne-cm.*

q. Couple exerted per degree of twist
$(p \div l) =$ . . . . . . (nearly) 7 $\left\{ \begin{array}{l} \textit{dyne-cm.} \\ \textit{per degree.} \end{array} \right.$

Note. The marker is here supposed to be set opposite the zero of the scale carried by the ring when the ring is at rest. The length, diameter, and material of the wire should be noted.

EXPERIMENT LXI.

PRELIMINARY OBSERVATIONS.

(See Tables, pages 339 and 340, left-hand half.)

FIRST METHOD.

Readings of spring balances in kilograms corrected by first table (1), page 339.

A. — *First balance with one end of lever* . . .	0.46 *kilos.*
B. — *Second balance with other end of lever.* .	0.44 "
C. — *First balance with load on lever*	6.85 "
D. — *Second balance with load on lever* . . .	6.75 "

e. Weight of lever $(A + B) =$	0.90 "
f. Weight of lever with load $(C + D) =$ . .	13.60 "
g. Weight of load $(f - e) =$	12.70 "

SECOND, THIRD, AND FOURTH METHODS.

	Second Method.	Third Method.	Fourth Method.
A. [*Corrected*] *reading of spring balance bearing one end of lever*	+ 0.45	+ 0.30	+ 0.60 *kilos.*
B. *The same with load on lever* .	+ 6.80	— 8.20*	+ 7.80 "
c. Effect of load on lever $(B - A)$	+ 6.35	— 8.50	+ 7.20 "
D. *Distance of point where spring balance is attached from fixed point of suspension*	+ 100.0	+ 75.0	+ 25.0 *cm.*
E. *Distance of point where load is attached from fixed point of suspension*	+ 25.0	— 25 0	+ 100.0 *cm.*
f. Weight of load $(c \times D \div E) =$	25.40	25.50	1.80 *kilos.*

* *Observed reading* . . . . 7.78 *kilos.*
Correction for graduation (First Table) . + 0.10 "
Correction for inversion (180°, Second Table) + 0.32 "
Corrected reading, numerically equal to . . 8.20 "

FIFTH METHOD — OBSERVATIONS.

A.	*Corrected reading of first spring balance with load*	10.05 *kilos.*
B.	*The same for second spring balance* . .	9.95 "
C.	*Distance of point* (a) *from point* (c) (*Fig.* 166)	100.1 *cm.*
D.	*Distance of point* (b) *from point* (c) (*Fig.* 166)	99.9 *cm.*
E.	*Vertical deflection* (cd, *Fig.* 166) *with load*	10.00 *cm.*
F.	*Corrected reading of first spring balance without load*	9.60 *kilos.*
G.	*The same for second spring balance* . . .	9.60 "
H.	*Distance of point* (a) *from point* (c) (*Fig.* 167)	100.0 *cm.*
I.	*Distance of point* (b) *from point* (c) (*Fig.* 167)	99.8 *cm.*
J.	*Vertical deflection without load* (cd, *Fig.* 167)	1.04 *cm.*

CALCULATIONS.

k.	Mean reading of spring balances with load $\frac{1}{2}(A+B) =$	10.00 *kilos.*
l.	Mean length of hypothenuse with load, $\frac{1}{2}(C+D) =$	100.0 *cm.*
m.	Weight of ring with load, $2\ E \times k \div l =$	2.00 *kilos.*
n.	Mean reading of spring balances without load, $\frac{1}{2}(F+G) =$	9.60 "
o.	Mean length of hypothenuse without load, $\frac{1}{2}(H+I) =$	99.9 *cm.*
p.	Weight of ring without load $(2\ J \times n \div o) =$	0.20 *kilos.*
q.	Weight of load $(m - p) =$	1.80 "

SIXTH METHOD, ¶ 159 (6).

A. Corrected reading of spring balance pulling a point (b) in a cord (ab, Fig. 169) *to a distance (bc) from the vertical line (ac) nearly equal to* 75 *cm.* 9.65 *kilos.*

B. The same in the opposite direction (cb, Fig. 169) 9.55 "

C. Length of cord (ab, Fig. 169)	125.0 *cm.*
D. Horizontal distance (bb′, Fig. 169)	150.0 *cm.*
e. Mean deflection ($\frac{1}{2}$ *D*) = ...	75.0 *cm.*
f. Vertical distance (*ac*, Fig. 169), $\sqrt{C^2 - e^2} = \sqrt{15625 - 5625} = \sqrt{10{,}000} =$	100.0 *cm.*
g. Mean force felt by balance, $\frac{1}{2}$ $(A + B)$ . .	9.60 *kilos.*
h. Weight of load, $g \times f \div e = 9.20 \times 100.0 \div 75.0 =$	12.80 "

EXPERIMENT LXII.

A. Weight suspended from edge of board . . .	10.0 *kilos.*
B. Distance of point of suspension from triangular support (ab, Figs. 171 *and* 172) . . .	100.0 *cm.*
C. Distance of centre of gravity from triangular support (cb,′ Fig. 172)	40.0 *cm.*
d. Weight of the plank $A \times B \div C$	25.0 *kilos.*

NOTE. The position of the centre of gravity is best located in such a heavy plank by balancing the plank upon the triangular knife-edge without the weight. See however ¶ 160.

EXPERIMENT LXIII.

	A. Length of beam between supports in cm.	B. Breadth of the beam in cm.	C. Thickness of the beam in cm.	D. Deflection of the beam in cm.	E. Weight borne by the beam in grams.	f. The same in dynes (980 × E).	g. Stiffness of the beam (f ÷ D).	h. $\frac{f \times A^3}{B \times D \times C^3}$	I. Material of the beam.
1.	100.0	1.000	1.000	0.490	4,000	3.92×10^6	8.00×10^6	8.00×10^{12}	Steel No. 1.
2.	100.0	1.000	1.000	0.244	2,000	1.96 "	8.03 "	8.03 "	
3.	50.0	1.000	1.000	0.246	16,000	15.68 "	63.7 "	7.96 "	
4.	100.0	2.000	1.000	0.240	4,000	3.92 "	16.3 "	8.15 "	Steel No. 2.
5.	100.0	1.000	2.000	0.238	16,000	15.68 "	65.9 "	8.24 "	

EXPERIMENT LXIV.

Preliminary Experiments (¶ 164).

Length of rod in cm.	Diameter of rod in cm	Forces applied in kilos.	Points of application (Fig. 175.)	Length of arm in cm.	Couple exerted, kilog-cm.	Deflection produced, in degrees.	Stiffness, kilog-cm. per degree.
80	1	1	*d* & *e*	6	6	15° +	0.40 —
80	1	1	*a* & *b*	6	6	15°	0.40
80	1	1	*c* & *d*	6	6	15° —	0.40 +
80	1	1	*a* & *d*	8	8	20°	0.40
80	1	1	*a* & *c*	10	10	25° +	0.40 —
80	1	2	*d* & *e*	6	12	30°	0.40
80	1	3	*d* & *e*	6	18	45° —	0.40 +
40	1	2	*d* & *e*	6	12	15°	0.80
40	1	4	*d* & *e*	6	24	30°	0.80
80	2	6	*a* & *g*	16	96	15°	6.4
80	2	10	*a* & *g*	16	160	25° +	6.4 —

A. Reading of needle when empty torsion balance is made horizontal . . . . + 17°

B. The same with 1 *decigram in the left hand pan* + 317°

C. The same with 1 *decigram in right hand pan,*
77°, *i. e.* . . . . — 283°
D. Length of balance beam . . . . 20.4 *cm.*
E. Length of wire subject to torsion . . . . . . 100.0 *cm.*
F. Diameter of the wire . . . . 0.0300 *cm.*
g. Value of 1 gram in dynes . . . . 980 *dynes.*
h. Weight of 1 decigram in dynes ($g \div 10$) = 98.0 "
i. Length of balance arm ($\frac{1}{2}$ D) = ... 10.2 *cm.*
j. Couple exerted by decigram weight ($h \times i$) =
1000 *dyne-cm.*
k. Deflection produced by this couple, $\frac{1}{2}$ ($B - C$) = 300°
l. Coefficient of torsion of the wire ($j \div k$)=3.33 $\left\{\begin{array}{l}\textit{dyne-cm.}\\ \textit{per degree.}\end{array}\right.$

EXPERIMENT LXV.

A. Length of iron wire subject to stretching* . . 6505 *cm.*
B. Diameter of the wire (mean of* 20 *obs.)*
.0664 ± .0001 *cm.*
C. Reading of micrometer without weight (mean of*
10 *obs.)* . . . . 0.7446 ± .0001 *cm.*
D. The same with weight (mean of* 10 *obs.)*
1.2324 ± .0004 *cm.*
E. Weight added* . . . . 5,000 *grams.*
f. Deflection ($D - C$) = ... 0.4878 *cm.*
g. Value of 1 gram in dynes . . . . 980.4 *dynes.*
h. Weight reduced to dynes
($E \times g$) = ... 4.902 × 10^6 *dynes.*
i. Cross-section of the wire corresponding to diameter B, see Table 3 G . . . . . 0.003463 *sq. cm.*
j. Stress upon the wire
($h \div i$) = ... 1.416 × 10^9 *dynes per sq. cm.*
k. Strain of the wire ($f \div$ A) = ...000750

l.* Young's modulus of elasticity $(j \div k) =$ 1.89×10^{12}
*G., January, 1885.

Note. From the weight (27.00 *grams*) of 10 metres of the wire, and from the density of wrought iron (7.8, Table 9) the mean cross-section would be

$27.00 \div 7.8 \div 1000 =$ . . . . . 0.00346 *sq. cm.*

EXPERIMENT LXVI.

A.* *Diameter of steel wire* . . . . 0.02327 *cm.*

B.* *Place of breaking.*	*C*.* *Maximum reading of spring balance.*
1. *Near balance*	8.45 *kilos.*
2. *2 inches from balance*	9.61
3. *1 inch from balance*	9.30
4. *Close to balance*	9.20
5. *Close to balance*	8.50
6. *Middle*	9.68
7. *1 inch from balance*	9.66
8. *2 inches from balance*	8.95
9. *Close to balance*	8.85
10. *Middle*	8.95
d.* *Average*	9.115 *kilos.*

Note by Student. *Several bad results thrown out.*

e.* Correction of the spring balance (for zero error and graduation) for a reading of 9 kilos — 0.240 *kilos.*

f.* Correction for an inclination of 90° . . + 0.120 "

g. Value of 1 gram in dynes . . . . . 980.4 *dynes.*

h.* Corrected reading of spring balance $(d + e + f) =$ . . . . 8.995 *kilos.*

i.* The same in dynes $(h \times g) =$ . . . 8.82×10^6

j.* Cross section of wire with diameter .02327 *cm.* (see *A*), 0.000425 *sq. cm.*

k.* Breaking stress of the steel
$(i \div j) =$ . . . . . 20.8×10^9 *dynes per sq. cm.*

* J. E. W., January, 1886.

ADDITIONAL DATA.

L.	*Length of wire weighed*	100.0 *cm.*
M.	*Weight in grams*	0.335 *grams.*
n.	Weight of 1 *cm.* $(M \div L) =$	0.00335 "
o.	Length breaking under its own weight $(1000\,h \div n) =$	2.69×10^6 *cm.*

EXPERIMENT LXVII.

FIRST METHOD (¶ 169 I.).

A.	*Distance between vertical prongs of fork* . .	2.00 *cm.*
B.	*Weight required to counterpoise the fork when dipping into a beaker of water* .	1.000 *gram.*
C.	*The same with film of water (mean of 10 obs.)*	1.300 *grams.*
D.	*Temperature of the water*	20° C.
e.	Tension of film 2 *cm.* broad $(B - C) =$	0.300 *gram.*
f.	Tension of *single* surface 1 *cm.* broad $(e \div 4) =$	0.075 *gram.*
g.	Value of 1 gram in dynes	980.4 *dynes.*
h.	Surface tension of the water at 20° (see D), $(f \times g) =$	73 + *dynes per cm.*

SECOND METHOD.—CALIBRATION OF TUBE.

I.*	*Length of mercurial column*	30.13 *cm.*
J.*	*Weight of mercurial column*	7.860 *grams.*
K.	*Temperature of the room, about*	20° *C.*

l. Apparent specific volume of mercury at 20° (see K); from Table 23 $B =$ . . 0.0738

m. Volume of mercury $(J \times l) =$ . . . . . 0.580 *cu. cm.*

n. Cross-section of the tube $(m \div I) =$. 0.0192 *sq. cm.*

o. Diameter corresponding (see Table 3 G) 0.156 + *cm.*

HEIGHT OF CAPILLARY COLUMN.

P. Height to which water rises in the tube above its level outside of the tube (mean of* 5 *obs.)* 1.66 ± .01 *cm.*

q. Density of water at 20° (Table 25) 0.99828

r. Density of air at 20° (mean), Table 19 . . 0.00120

s. Weight of 1 *cu. cm.* of water at 20° in air, $(q - r) =$ 0.99708

t. Weight in air of a column of water 1.66 *cm.* long, 0.0192 *sq. cm.* in cross-section, reduced to dynes $(P \times n \times s \times g) =$ . . . . . 31.0 *dynes.*

u. Breadth of film sustaining the weight of this column = circumference of tube (Table 3, F), with diameter 0.156 *cm.* (see D) . . . . 0.490 *cm.*

v. Surface tension of the water at 20° $(t \div u) =$ 63 + *dynes per cm.*

* A. N. S., January, 1887.

EXPERIMENT LXVIII.

FIRST METHOD (¶ 171, I.).

A. Force required to draw plank with uniform velocity parallel to the fibres of the plank and of a horizontal board upon which the plank rests flatwise (mean of 10 *obs.)* . . . . 0.290 *kilo.*

B. The same with plank edgewise . . . . . . 0.310 "

C. The same with plank flatwise but bearing a load 3.50 *kilos.*

D. Weight of the plank . . . . 1.00 *kilo.*
E. Weight of the load . . . . 10.00 *kilos.*
f. Coef. of friction (1) in A $(A \div D) =$ . . . 0.29
" " (2) " B $(B \div D) =$ . . . 0.31
" " (3) " C $(C \div (D + E)) =$. 0.32

Second Method (¶ 171, II.).

G. Distance (AB, Fig. 184) *measured along horizontal surface of table from the point of contact of the under surface of board to foot of vertical measuring rod* . . . . 100.0 *cm.*

H. Height of under surface of board above horizontal surface of table at this point sufficient to make plank slide down board with uniform velocity parallel to the fibres of the plank and of the board with the plank flatwise (mean of 10 *obs.)* 30.0 *cm.*

i. Slope of under surface of board = slope of upper surface nearly = coefficient of friction $= H \div G =$ 0.30

EXPERIMENT LXIX.

Observations.

A. Circumference of the wheel of motor* . . 72.8 *cm.*

	1st	2d	3d	*4th trial.*
B. [Difference between] readings of spring balance [s]*	1.3	1.2	1.1	1.0 *kilos.*
C. No. of revolutions made by the wheel*	14	15	18	18 *rev.*
D. Duration of the experiment in seconds*	10	10	10	10 *sec.*
E. Weight in grams of the water used* .	2190	2360	2640	2514 *grams.*
F. Readings of the pressure-gauge in pounds per square inch*	20	16.5	19.5	19 *lbs. per sq. in.*

Calculations.

	1st	2d	3d	4th *trial.*	
g. Tangential force of friction reduced to megadynes (0.98 B) = . . .	1.27	1.18	1.08	0.98	*mega-dynes.*
h. Velocity of rim of wheel in *cm.* per sec. ($A \times C \div D$)=	102	109	131	131	*cm. per sec.*
i. Power utilized by motor in megergs per sec. ($g \times h$) = ...	130	129	141	128	*megergs per sec.*
j. Pressure of water reduced to megadynes per *sq. cm.* ($0.069 \times F$) =	1.38	1.14	1.35	1.31	*megad. per. sq. cm.*
k. Flow of water in *cu. cm.* per sec. ($1.00 \times E \div D$) = ...	219	236	264	251	*cu. cm. per sec.*
l. Power spent on motor in megergs per second ($j \times k$) = ...	302	269	356	329	*megergs per sec.*
m. Efficiency of motor in per cent $100 \times i \div l$ =.	43	48	40		39 %

* P. M. H., January, 1886.

EXPERIMENT LXX.

Observations.

A. *Length of pasteboard tube with corks* . . 124.0 *cm.*

B. *Thickness of corks each* . . . . 2.0 *cm.*

C. *Depth of lead shot (by difference)* . . . 20.0 *cm.*

D. *Temperature of the room* . . . . 20°.0 C.

E. *Temperature of the shot before the experiment reduced to* * . . . . 17°.0 C.

F. *Temperature of the pasteboard tube before the experiment raised to** . . . . 23°.0 C.

G. *Temperature of the shot and tube after the experiment* . . . . 23°.0 C.

H. *Number of reversals necessary to bring about this change of temperature in the shot* . . 81

CALCULATIONS.

i. Rise of temperature of the shot $(G - F) =$ 6°.0 C.

j. Distance fallen by the shot in each reversal
$(A - 2B - C) =$ . . . . 100.0 *cm.*

k. Total distance fallen by the shot $(H \times j) =$ 8100 *cm.*

l. Distance fallen per degree rise of temperature
$(k \div i) =$ . . . . 1350 *cm.*

m. Force acting upon each gram of shot . . . 980.4 *dynes.*

n. Work necessary to raise 1 gram of shot 1° in
temperature reduced to megergs
$(l \times m \div 1000000) =$ . . . . . 1.324 *megergs.*

o. Heat units necessary to raise 1 gram of lead
1° C. (See Exp. 31, and Table 8) 0.032 *units of heat.*

p. Mechanical equivalent of heat *
$(n \div o) =$ . . . . .42 { *megergs per unit of heat.*

* A simple way to cool shot to a given temperature is to mix it with colder shot from a refrigerator. The tube may be warmed to a given temperature by placing shot in it a little above that temperature. Assuming that, as in the example, the operations have been performed, so that $\frac{1}{2}(F + E) = D$, and $G = F$, the effects of cooling and thermal capacity will be eliminated (see ¶¶ 98, 102 and 104), and the probable error of the result due to other causes (see ¶ 178) ought not to exceed 5 %.

EXPERIMENTS LXXI.-LXXIV.

OBSERVATIONS — *Magnet numbered*	1.	2.	3.	
A. Distance between the poles of the magnet in centimetres . .	9.6	10 0	10.4	*cm.*
B. Weight in grams necessary to counterpoise each magnet	240.00	250.00	260.00	*grams.*
C. The same when repelled as follows: No. 1 by No. 2; No. 2 by No. 3; No. 3 by No. 1	239.36	249.30	259.33	"

OBSERVATIONS — *Magnet numbered*	1.	2.	3.
D. The same when attracted as stated	241.26	251 40	261.33 "
E. Distance between the magnets from centre to centre (in cm.) . . .	2.00	2.00	2.00 *cm.*
F. Zero-reading of torsion-apparatus	0°.0	0°.0	0°.0
G. Reading of the same when magnet points east and west . . .	+ 117°.0	+ 120°.0	+ 123°.0
H. Reading of the same when magnet points west and east	— 117°.0	— 120°.0	— 123°.0
I. Mean distance of centre of magnet from centre of compass needle, measured in east and west positions	35.0	35.0	35.0 *cm.*
J. Readings of compass needle with magnet east of needle			
(*Fig.* 200,1) *North pole deflected westward* [*N. W.*] . .	8°.8	9°.8	10°.9
(*Fig.* 200,1) *South pole deflected eastward* [*S. E.*] . .	8°.6	9°.4	10°.7
(*Fig.* 200,2) *North pole deflected eastward* [*N. E.*] . .	8°.5	9°.5	10°.6
(*Fig.* 200,2) *South pole deflected westward* [*S. W.*] . .	8°.9	9°.7	11°.0
With magnet west of needle.			
(*Fig.* 200,3) *North pole deflected westward* [*N. W.*] . .	8°.6	9°.5	10°.8
(*Fig.* 200,3) *South pole deflected eastward* [*S. E.*] . .	8°.4	9°.3	10°.4
(*Fig.* 200,4) *North pole deflected eastward* [*N. E.*] . .	8°.3	9°.2	10°.5
(*Fig.* 200,4) *South pole deflected westward* [*S. W.*] . .	8°.7	9°.6	10°.7
CALCULATIONS.			
a. Mean force felt by each magnet, $\frac{1}{2}(D - C) =$	0.950	1.050	1.000 *grams.*
b. Mean force felt by each pole, $(\frac{1}{2}a) =$	0.475	0.525	0.500 "
c. The same in dynes $(980.4 \times b) =$	466	515	490 *dynes.*
d. The same reduced to a distance of 1 *cm.* giving product of strength of poles acting, $(c \times E^2) =$	1864	2060	1960 *dynes.*

CALCULATIONS — *Magnet numbered.*	1.	2.	3.
e. Ratio of the strengths of poles in question $(d_3 \div d_2)$; $(d_1 \div d_3)$; $(d_2 \div d_1) =$ respectively . . .	0.951	0.951	1.105
f. Provisional estimate of strength of single poles, $\sqrt{d \times e} =$.	42.1	44.3	46.5 *units of magnetism.*
g. Coefficient of torsion of wire (Exp. 64) in *dyne-cm. per degree*	3.33	3.33	3.33 *dyne-cm. per degree.*
h. Mean angle of torsion observed, $\frac{1}{2}(G - H) - 90^\circ =$ . . .	27°.0	30°.0	33°.0
i. Couple exerted by or upon magnet in *dyne-cm.* $(g \times h) =$. .	90.0	100.0	110 0 *dyne-cm.*
j. Force of earth's magnetism on each pole $(i \div A) =$ . . .	9.4	10.0	10.6 *dynes.*
k. 1st estimate of the horizontal intensity of earth's magnetism $(j \div f) =$	0.223	0.226	0.228
l. Distance of nearer pole of magnet from centre of compass needle $(I - \frac{1}{2}A) =$. . .	30.2	30.0	29.8 *cm.*
m. Distance of further pole $(I + \frac{1}{2}A) =$	39.8	40.0	40.2 *cm.*
n. Field of force due to nearer pole $(f \div l^2) =$.	0.0462	0 0492	0.0524
o. Field of force due to farther pole $(f \div m^2) =$	0.0266	0.0277	0.0288
p. Resultant field of force $(n - o) =$	0.0196	0.0215	0.0236
q. Mean angle of deflection (average of J)	8°.6	9°.5	10°.7
r. Tangent of this angle (Table 5)	0.1512	0.1673	0.1890
s. 2d estimate of the horizontal intensity of the earth's magnetism $(p \div r) =$	0.130	0.129	0.125
t. The same (1st estimate — see *k*)	0.223	0.226	0.228
u. Geometric mean between the two estimates = Horizontal intensity of earth's magnetism = $\sqrt{s \times t} =$	0.170	0.171	0.169
v. "Moment" of magnets $(i \div u) =$	529	585	651
w. Strength of poles $(v \div A)$ =.	55	58	63

EXPERIMENT LXXV.

A. Number of vibrations made by a small loaded magnetic needle in* 10 *seconds under the influence of the earth's magnetism* . . . 1.5 *vibr.*

B. The same at a distance of* 1 *cm. from the middle point of a long bar magnet* . . . 8.0 *vibr.*

The same at a distance of 1 *cm. from the axis of the magnet, and at the following distances:—*

	From north end —	*Vibrations.*		*From south end —*	*Vibrations.*
*C.**	0 *cm.*	17.0	*H.**	0 *cm.*	18.0
*D.**	10 *cm.*	13.0	*I.**	10 *cm.*	14.5
*E.**	20 *cm.*	12.5	*J.**	20 *cm.*	11.0
*F.**	30 *cm.*	12.0	*K.**	30 *cm.*	7.5
*G.**	40 *cm.*	11.0	*L.**	40 *cm.*	6.0

☞ See Fig. 205, page 413, representing the squares of the numbers of vibrations.

Inferences from this Figure.

m. Distance of north pole from end of magnet . . 22 *cm.*
n. Distance of south pole from end of magnet . . 12 *cm.*
o. Distance between the poles . . . . 66 *cm.*

* C. R. E., April, 1888.

EXPERIMENT LXXVI.

A. Throws of needle of ballistic galvanometer caused by sliding a coil over the magnet through a distance of* 10 *cm., measured as follows:* —

From north end of magnet —		*Throw.*	*From south end of magnet* —		*Throw.*
*B.**	0 — 10 *cm.*	27°.8	*G.**	0 — 10 *cm.*	37° 3
*C.**	10 — 20 *cm.*	22°.1	*H.**	10 — 20 *cm.*	23°.5
*D.**	20 — 30 *cm.*	18°.0	*I.**	20 — 30 *cm.*	14°.5
*E.**	30 — 40 *cm.*	14°.2	*J.**	30 — 40 *cm.*	6° 0
*F.**	40 — 50 *cm.*	8°.5	*K.**	40 — 50 *cm.*	1°.0

☞ A figure representing the chords of the throws (see Table 3, column *d*) was here constructed by the student as explained in ¶ 189.

INFERENCES FROM THIS FIGURE.

l. Distance of north pole from end of magnet . . 19 *cm.*
m. Distance of south pole from end of magnet . . 14 *cm.*
n. { Distance between poles . . . . { 67 *cm.*
o. { The same in Exp. 75 (see *o*) . . . . { 66 *cm.*

* C. R. E., April, 1888.

EXPERIMENT LXXVII.

Throws of the needle of a ballistic galvanometer caused by revolving the coil of an earth-inductor . . . . 180°

	A. About a horizontal axis.*			*B.* About a vertical axis.*	
	1.	55°.5		1.	17°.5
	2.	55°.		2.	17°.
	3.	56°.		3.	18°.
c.	Mean	55°.5	*d.*	Mean	17°.5

e. Chord of *c* 0.931 (Table 3). *f.* Chord of *d* 0.304 (Table 3).

g. Tangent of the angle of dip $(e \div f) =$ 3.06
h. Angle of the magnetic dip (Table 3, column *b*, or Table 5) . . . . 71°.9

* C. R. E. April, 1888.

EXPERIMENTS LXXVIII.-LXXIX.

*A.** *Number of turns of wire in coil* 10

*B.** *Outside diameter of ring (mean of 8 obs.)* 37.936 *cm.*

*C.** *Depth of groove to outer surface of insulated wire (mean of 8 obs.)* . . . . 0.645 *cm.*

*D.** *Semi-diameter of wire (mean of 4 observations on 10 thicknesses)* . . . . . . 0.150 *cm.*

*E.** *Mean radius of coil* ($\frac{1}{2}$ $B - C - D$) = 18.173 *cm.*

*f.** Constant of coil

$2\,\pi \times A \div E = 2 \times 3.1416 \times 10 \div 18.173 =$ 3.457

g. Reduction factor of the galvanometer, assuming a mean value 0.170 for the horizontal intensity of the earth's magnetism (Example 74 *u*); $0.170 \div f =$ 0.0492

h. The same for measurements in ampères ($10\,g$) = 0.492

Simultaneous deflections — mean of 3 obs. in each case.

I. Double-ring galvanometer.*		*J.* Single-ring galvanometer.*	
44°.00 N. E.	43°.83 S. W.	47°.53 E. S.	47°.13 W. N.
43°.97 N. W.	44°.33 S. E.	47°.53 E. S.	47°.13 W. N.
44° 03 N. W.	44°.47 S. E.	47°.50 E. N.	47°.93 W. S.
44°.10 N. E.	43°.97 S. W.	47°.53 E. N.	48°.00 W. S.
K. Zero reading.*		*L.* Zero reading.*	
0°.0 N. E.	—0°. 25 S. W.	0°.0 E. S.	— 0°.5 W. N.
*m.** Average deflection	44°. 09	*n.** Average deflection	47°.54
o. Tangent of *m* (Table 5)	0.9688	*p.* Tangent of *n.* (Table 5)	1.0928

q. Constant of the double-ring galvanometer,

$f \times o \div p =$ . . . . 3.06 +

r. Reduction factor of the double-ring galvanometer for ampères ($h \times p \div o$) = ... 0.555

* C. A. E., April, 1888.

Note. The same student found in March the value 0.1714 for the horizontal intensity of the earth's magnetism. Substituting this value instead of 0.170, the reduction factors in h and r become 0.496 and 0.559 +.

In the abbreviations above, the first letter refers to the pointer, the second to its deflection; thus the letters N. E. indicate readings of a pointer attached to the north end of a needle when deflected eastward; while the letters E. N. refer to observations of the east end of a (transverse) pointer when deflected northward.

EXPERIMENT LXXX.

A.	*Number of turns of wire in the large coil of the dynamometer*	100
B.	*Outside diameter of the coil*	27.00 *cm*
C.	*Inside diameter of the coil*	23.00 *cm.*
d.	Mean diameter, $\frac{1}{2}(B + C) =$	25.00 *cm.*
e.	Mean radius, $\frac{1}{2}d =$	12.50 *cm.*
f.	Constant of large coil $(2\pi e \div A) =$	50.27

G.	*Number of turns in small square coil*	79.5
H.	*Outside horizontal diameter*	5.09 *cm.*
I.	*Outside vertical diameter*	5.03 *cm.*
J.	*Width of* 80 *turns of wire*	4.80 *cm.*
k.	Mean diameter of wire, $(J \div 80) =$ . . .	0.06 *cm.*
l.	Mean horizontal diameter of coil $(H - k) =$	5.03 *cm.*
m.	Mean vertical diameter of coil $(I - k) =$.	4.97 *cm.*
n.	Mean area of cross-section $(l \times m) =$.	25.00 *sq. cm.*
o.	Magnetic area of coil $(G \times n) =$ . . .	1988 *sq. cm.*
p.	Constant of Dynamometer $(f \times o) =$ . . .	99937
q.	The same for ampères, or No. of dyne-centimetres due to one ampère, $(p \div 100) =$	999 +

R. Length of dynamometer wire subject to torsion 33.3 *cm.*

s. Coefficient of torsion of a wire 100 *cm.* long of the same size and material (Exp. 64) in *dyne-cm. per degree* . . . . 3.33 { *dy. cm. per deg.*

t. Coefficient of torsion of dynamometer wire $s \times 100 \div R =$ . . . . 10.0 "

u. Reduction factor of the dynamometer for ampères $\sqrt{t \div q} =$ 0.100

V. Deflection of dynamometer (¶ 204) . . . 100°.0

W. Corresponding deflection of galvanometer 63°.4

x. Current through dynamometer,

$$u \sqrt{V} = 0.100 \sqrt{100.0} = 0.100 \times 10.0 =$$

1.00 ampères.

y. Tangent of the angle of deflection of the galvanometer (Table 5). Tan. *W.* = tan. 63°.4 = 1.997

z. Reduction factor of the galvanometer

$(x \div y) =$ . . . . 0.501 }
The same previously found (Exp. 78, *h*) . 0.492 }

NOTE. The value of the reduction factor (0.501) found in this experiment is the same as that which would have been found in Exp. 78 if the value 0.174 had been taken for the horizontal intensity of the earth's magnetism, instead of the value 0.170 found in Exp. 71–74.

EXPERIMENT LXXXI.

A. Weight of copper spiral before the experiment (mean of 3 double weighings) . . . 10.945 *grams.*

B. Duration of the experiment, 31 *m.* 30 *sec.* = 1890 *sec.*

C. Deflections of the galvanometer at intervals of 1 *min.*

1	50°.7 *E. S.*	51°.2 *W.E.*	16	51°.0 *E.N.*	51°.5 *W. N.*
2	51.2	51.8	17	51.3	51.7
3	51.4	51.8	18	51.3	51.7
4	51.4	51.8	19	51.4	51.7
5	51.4	51.8	20	51.3	51.7
6	51.3	51.8	21	51.2	51.6
7	51.4	51.8	22	51.2	51.6
8	51.4	51.9	23	51.2	51.6
9	51.4	51.9	24	51.2	51.7
10	51.4	51.9	25	51.2	51.6
11	51.3	51.8	26	51.3	51.6
12	51 4	51.9	27	51.3	51.7
13	51.4	51.9	28	51.2	51.6
14	51.3	51.8	29	51.3	51.6
15	51.4	51.9	30	51.3	51.7
Mean	51.32	51.8[0]	*Mean*	51.25	51.64

D. Weight of copper spiral after the experiment (mean of 3 *double weighings)* . . 11.348 *g.*

e. Amount of copper deposited ($D - A$) 0.403 "

f. Amount of copper deposited in 1 second ($e \div B$) = ... 0.0002132 "

g. Amount of (cupric) copper deposited in 1 second by 1 absolute unit of current (Table 8) 0.00328 "

h. Amount deposited by 1 ampère ($\frac{1}{10}\,g$) = 0.000328 "

i. Mean current in ampères ($f \div h$) = . 0.650 *ampères.*

j. Mean deflection, average of C_1 to C_{30} = 51°.50 +

k. Tangent of this angle (Table 5) . . . . . . 1.257 +

l. Reduction factor of galvanometer ($i \div k$) = 0.517

The same by dynamometer (Exp. 80, *z*) 0.501

The same by magnetic measurements (Exp. 78, *h*) 0.492

EXPERIMENT LXXXII.

☞ See Fig. 237.

	Number of vibrations observed.	Square of the No. of vibrations.	Apparent current in ampères.	The same allowing for earth's magnetism.
A. Vibrations completed in one minute by needle of vibration galvanometer under influence of earth's magnetism	9.0	81.	0.1	0.0
B. The same under the influence of currents from cells composed as follows (see ¶ 208): —				
(1) *Zinc and copper strips in dilute sulphuric acid*	50	2500	2.5	2.4
(2) *The same only zinc amalgamated*	50	2500	2.5	2.4
(3) *The same* 5 *minutes later than* (2)	45	2025	2.0	1.9
(4) *The same* 10 *minutes later than* (2)	39	1521	1.5	1.4
(5) *The same* 15 *minutes later than* (2)	31	961	1.0	0.9
(6) *The same after brushing bubbles off copper*	33	1089	1.1	1.0
(7) *The same after exposing copper* 5 *minutes to the air*	40	1600	1.6	1.5
(8) *The same after amalgamating copper*	40	1600	1.6	1.5
(9) *The same with copper in porous cup containing a solution of sulphate of copper.*	35	1225	1.2	1.1
(10) *The same* 5 *minutes later than* (9)	35	1225	1.2	1.1
(11) *The same* 10 *minutes later than* (9)	36	1296	1.3	1.2
(12) *The same* 15 *minutes later than* (9)	37	1369	1.4	1.3

C. *Weight of copper strip before obs. B* 9 . . . . . 50.0 *grams.*
D. *Weight of zinc strip before obs. B* 9 . . . . . 50.0 "
E. *Weight of copper strip after obs. B* 12 . . . . . 50.4 "
F. *Weight of zinc strip after obs. B* 12 . . . . . 49.5 "
g. Weight gained by the copper in 15 minutes $(E - C) =$ . . . . . 0.4 "
h. Weight lost by the zinc in the same time $(F - D) =$ . . . . . 0.5 "

EXPERIMENTS LXXXIII.-LXXXIV.

A. Readings of Ammeter in ampères.	*B.* Readings of 1st galvanometer.*	*C.* Readings of 2d galvanometer.*	*d.** True current in ampères.	*e.* Correction of ammeter ($d-A$).
+ 0.10	0°.0	0°.0	0.00	− 0.10
+ 4.20	62°.0	65°.0	4.27	+ 0.07
+ 7.90	73°.3	76°.6	8.06	+ 0.16

* NOTE. The reduction factor of the 1st galvanometer with 10 turns of wire is about 0.50 (see Exp. 81); that of the second is 0.56 (Exp. 79); hence with only five turns of wire the reduction factors are 1.00 and 1.12 respectively. The current in *d* is accordingly 1.00 *tan* $B + 1.12$ *tan* C.

Readings of ammeter connected with different cells for different lengths of time:—

F. TIME.	*G. Bunsen cell.*	*H. Daniell cell.*	*I. Leclanché cell.*
0 minutes.	4.55 ampères.	2.00 ampères.	3.00 ampères.
5 "	4.50 "	2.10 "	2.80 "
10 "	4.45 "	2.18 "	1.00 "
15 "	4.20 "	2.20 "	
20 "	4.00 "	2.17 "	
25 "	3.80 "	2 05 "	
30 "	3.50 "	1.90 "	

☞ See Fig. 237, page 470.

[NOTE. The results given above and in the figure were not founded upon actual observations, and are intended only to show how such observations should be made and represented.]

EXPERIMENT LXXXV.

Observations.

A. Weight of empty brass calorimeter* . 47.20 *grams.*
B. Weight of calorimeter with water* . 126.00 "

**Readings of galvanometer and thermometer at different times* : —

	C. Time.*			*D.* Galvanometer.*	*E.* Thermometer.*
1.	3 h.	14 m.	0 sec.	. . . .	23°.7
2.	"	15	0 "	0°.0 *E. S.*	
3.	"	16	0 "	. . . .	23°.8
4.	"	17	0 "	— 0°.1 *W. N.*	
5.	"	18	0 "	. . . .	23°.8
6.	3 h.	19 m.	0 sec.	Circuit made	
7.	"	"	30 "	51°.1 *E. N.*	
8.	"	20	0 "	. . . .	24°.0
9.	"	"	30 "	48°.7 "	
10.	"	21	0 "	. . . .	24° 2.
11.	"	"	30 "	48°.0 "	
12.	"	22	20 "	. . . .	24°.6
13.	"	"	30 "	47°.5 "	
14.	"	23	0 "	. . . .	24°.8
15.	"	"	30 "	47°.0 "	
16.	"	24	0 "	. . . .	25°.0
17.	"	"	30 "	46°.8 *W. S.*	
18.	"	25	0 "	. . . .	25°.3
19.	"	"	30 "	46°.5 "	
20.	"	26	0 "	. . . .	25°.5
21.	"	"	30 "	46°.3 "	
22.	"	27	0 "	. . . .	25°.7
23.	"	"	30 "	46°.2 "	
24.	"	28	0 "	. . . .	25°.9
25.	"	"	30 "	46°.0 "	
26.	3 h.	29 m.	0 sec.	Current reversed	26°.0
27.	"	"	30 "	46°.3 *E. S.*	
28.	"	30	0 "	. . . .	26°.2
29.	"	"	30 "	46°.2 "	
30.	"	31	0 "	. . . .	26°.5
31.	"	"	30 "	46°.1 "	
32.	"	32	0 "	. . . .	26°.7

	C. Time.*			*D.* Galvanometer.*	*E*. Thermometer*
33.	3 h.	32 m.	30 sec.	46°. *E. S.*	
34.	"	33	0 "		26°.9
35.	"	"	30 "	46°. "	
36.	"	34	0 "		27°.
37.	"	"	30 "	46°.2 *W. N.*	
38.	"	35	0 "		27°.2
39.	"	"	30 "	45°.8 "	
40.	"	36	0 "		27°.4
41.	"	"	30 "	45°.6 "	
42.	"	37	0 "		27°.6
43.	"	"	30 "	45°.5 "	
44.	"	38	0 "		27°.8
45.	"	"	30 "	45°.0 "	
46.	3 h.	39 m.	0 sec.	Current cut off	28°.0
47.	"	40	"		28°.0
48.	"	41	"		27°.9
49.	"	42	"		27°.9
50.	"	43	"		27°.9

* J. E. W., April, 1886.

CALCULATIONS.

f. Weight of water in calorimeter $(B - A) =$ 78.80 *g.*

g. Thermal capacity of calorimeter (¶ 90, 2; ¶ 91, III.), $0.094 \times A + 0.2 + 0.4 =$ . . . . 5.0 "

h. Total thermal capacity $(f + g) =$ . . . . 83.8 "

i. Rise of temperature observed $(E_{46} - E_5) =$ 4°.2 C.

j. Units of heat developed $(h \times i) =$ . . . . 352.

k. Length of time, $C_{46} - C_6 = 20$ *min.* 0 *sec.* = 1200 *sec.*

l. Units of heat per second $(j \div k) =$ . . . 0.293

m. Equivalent in watts (§ 15), $4.17 \times l =$. 1.22 *watts.*

n. Mean angle of deflection in *D* . . . . . . 46°.64

o. Tangent of this angle (Table 5) 1.059

p. Reduction factor of the galvanometer with 5 turns of wire (see note, Example 84) 1.00

q. Current in ampères indicated $(o \times p) =$ . . . 1.059

r. Square of this current (Table 3, *C*), $q^2 =$ . . 1.122

s. Resistance of the conductor in ohms = power in watts necessary to maintain a current of one ampère (see § 136)
$= (m \div r) = 1.22 \div 1.122 =$ . . . 1.09 *ohms*.

NOTE. In the calculation above, only the first and last observations of temperature during the action of the current were utilized. The others would have given a somewhat larger result. The student's calculation gave 1.06 *ohms* as the resistance of the coil by the method of heating. The same student found the resistance of the same coil by comparison with B. A. units (May, 1886), to be 1.00 *ohms* (see note, Example 87). The probable error in determinations conducted as in the example is about 10 %.

EXPERIMENT LXXXVI.

FIRST METHOD.

A. *Deflection of a galvanometer with Bunsen cell and unknown resistance included in the circuit* . . . . 21°.6

B. *The same with* 7 *ohms in place of unknown resistance* . . . . 20°.0

C. *The same with* 6 *ohms in place of unknown resistance* . . . . 22°.0

d. Value of the unknown resistance by interpolation,
$6 + (C - A) \div (C - B) =$ . . . . 6.2 *ohms*.

SECOND METHOD.

E. *Deflection of differential galvanometer with unknown resistance in one circuit and* 6 *ohms in the other* . . . . — 6°.0

F. *The same with* 7 *ohms in place of* 6 *ohms* + 24°.0

g. Value of the unknown resistance by interpolation,
$6 + E \div (E - F) =$
$6 + [-6.0] \div [-6.0 - 24.0] =$ . . 6.2 *ohms*.

EXPERIMENT LXXXVII.

A. *Value of known resistance (said to be equal to 1.0132 B. A. units) in legal ohms (Table 50),*
1.0132 × 0.9889 = ... 1.0021 *ohms.*

*B.** *Distances of contact from end of bridge-wire nearer the unknown resistance measured:*—

(1) *from* 0 *cm. upwards (mean of* 6 *obs.)* 50.237 *cm.*

(2) *the same with current reversed (mean of* 6 *obs.)* . . . . 50.222 *cm.*

(3) *from* 100 *cm. downward (mean of* 6 *obs.)* . . . . 50.215 *cm.*

(4) *the same with current reversed (mean of* 6 *obs.)* . . . . 50.212 *cm.*

Mean . . . . 50.222 *cm.*

*C.** *Length of the bridge-wire* . . . . . 100.00 *cm.*

d. Value of the unknown resistance,
$A \times B \div (C - B) =$
1.0021 × 50.222 ÷ 49.778 = ... 1.010 *ohms.*

* J. E. W., May, 1886.

NOTE. The value of the unknown resistance determined in this experiment includes that of the connecting wires, amounting to about 0.01 *ohm.* A deduction of 0.01 *ohm* should therefore be made in comparing this result with that obtained by the method of heating (Exp. 85).

EXPERIMENT LXXXVIII.

A. Value of known resistance used as a standard of comparison (see Example 87, A) 1.002 *ohms.*

B. Distance of contact on "bridge-wire" from end nearer unknown resistance (mean of 4 observations — see B, Example 87) . 50.00 *cm.*

C. Length of "bridge-wire" . . . . 100.00 *cm.*

D. Length of German silver wire (between connecting strips) constituting unknown resistance . . . . 100.00 *cm.*

E. Diameter of the wire (mean of 10 obs.) 0.05000 *cm.*

f. Cross section of wire with this diameter (Table 3, *G*) . . . . 0.001963 *sq. cm.*

g. Resistance of the wire $A \times B \div (C - B) =$ 1.002 *ohms.*

h. Resistance of a wire of the same material and diameter 1 *cm.* long $(g \div D) =$ . . 0.01002 *ohm.*

i. Resistance of a wire of the same material 1 *cm.* long and 1 *sq. cm.* in diameter $(h \times f) =$ 0.0000197 "

j. The same in microhms = specific resistance in microhms of a centimetre cube of the German silver 19.7

k. Compare (mean) value in Table 37 *a* . . 20.8

EXPERIMENT LXXXIX.

A. Value of known resistance used as a standard of comparison, 10 *B. A. units = in legal ohms (see Table* 50) $10 \times 0.9889 =$ 9.889 *ohms.*

B. Distance of contact on "bridge-wire" from end connected with galvanometer, mean of 4 *observations (see Example* 87, *B*) . . 37.60 *cm.*

C. Length of "bridge-wire" . . . . 100.00 *cm.*

d. Resistance of galvanometer in legal ohms, $A \times B \div (C - B)$ . . . . 5.959 *ohms.*

EXPERIMENT XC.

A. Value of known resistance used as a standard of comparison (see Example 87, *A*) 1.002 *ohms.*

B. Length of "bridge-wire" between point of contact and battery (mean of 2 *obs.)* . . 49.0 *cm.*

C. Length of "bridge-wire" . . . . 100.0 *cm.*

d. Resistance of battery in legal ohms, $A \times B \div (C - B) =$ . . . . 0.96 *ohm.*

EXPERIMENT XCI.

First Method.

A. Deflection of Single Ring tangent galvanometer* . . . . 59°.5

B. Corresponding deflection of Double-Ring tangent galvanometer with shunt (mean of 2 obs.)* . . . . 36°.05

C. Resistance of the shunt* . . . . 0.10 *ohm*

d. Reduction factor of Single Ring galvanometer (Example 80, *z*) 0.50

e. Reduction factor of Double Ring galvanometer (Example 79, note) 0.56

f. Current through the Single Ring galvanometer, $d \times$ tangent of $A =$
0.50×1.6977 (Table 5) $=$ . . 0.85 *ampère.*

g. Current through Double Ring galvanometer, $e \times$ tangent of $B = 0.56 \times 0.7279 = 0.41$ "

h. Current (by difference) through the shunt, $(f - g) =$ . . . . 0.44 "

i. Resistance of the galvanometer, $C \times h \div g = C \times 0.44 \div 41 =$ . . 0.11 *ohm.*

* F. S. D., May, 1888.

Second Method.

J. Deflection of galvanometer through a large external resistance. . . . . 20°.0

K. Value of this resistance . . . . 1000 *ohms.*

L. Deflection reduced by a shunt to . . . . 10°.0

M. Resistance of this shunt . . . . 6.00 *ohms.*

n. Resistance of the galvanometer (¶ 224, 12)
$K \times M \times (J - L) \div (L \times K + L \times M - J \times M) =$
$1000 \times 6.00 \times 10.0 \div (10.0 \times 1000) + 10.0 \times 6.00 - 20.0 \times 6.00) =$
$60{,}000 \div 9940 =$ . . . . 6.043 *ohms.*

EXPERIMENT XCII.

A. Deflection of Single Ring tangent galvanometer (mean of 2 obs.)* . . . . 62°.2

B. The same with additional resistance* . . . 35°.25

C. Value of resistance added* . . . . . . 2.00 *ohms.*

d. Resistance of battery, galvanometer, and connections (¶ 225, formula 10), $C \times$ tangent of $B \div$ (tangent of A — tangent of B) = (see Table 5) $2.00 \times 0.7067 \div (1.8967 - 0.7067)$ $= 2.00 \times 0.7067 \div 1.1900 =$ . . 1.10 + *ohms.*

e. Resistance of galvanometer (Example 91, *i*) 0.11 *ohm.*

f. Internal resistance of battery ($d - e$) = 0.99 "

* F. S. D., May, 1888.

NOTE. The electromotive force of the battery is found by multiplying the current in A ($0.50 \times$ tangent of A, see Example 80, *z*) and the total resistance in A, namely d. This gives $0.50 \times 1.8967 \times 1.10 = 1.04 +$ volts.

EXPERIMENT XCIII.

A. 1*st resistance of the shunt* (*a c* Figs. 254 and 255) *such that a Daniell battery of* 2 *cells reduces the current from a single Bunsen cell to zero* . . . . 28.0 *ohms.*

B. Corresponding resistance (*b c*) *added to Daniell or main circuit* . . . . 0.0 "

C. 2*d resistance of shunt neutralizing the current in the Bunsen or side-circuit when a new resistance is added to the main circuit* . 168.0 *ohms.*

D. Value of the resistance (*in C*) *added to the main circuit* . . . . 10.0 "

e. Resistance of the Daniell battery (formula 6, ¶ 229),

$(A \times D - B \times C) \div (C - A) =$

$(28.0 \times 10.0 - 0) \div 140 =$ . . . 2.00 "

f. Average resistance of the Daniell cells ($\frac{1}{2}\,e$) = ... 1.00 "

Resistance of 1 Daniell cell by Mance's Method (Example 90, *d*) . . . . 0.96 "

The same by Ohm's method (Example 92, *f*) . . . . 0.99 "

NOTE. Taking the electromotive force of 1 Daniell cell as 1.04 (Example 92, note), that of 2 cells will be about 2.08 *volts.* This will give through an internal battery resistance 2.00 *ohms* (see *e*) and through an external resistance of 28.0 *ohms* (see *A*) a current of $2.08 \div (28 + 2) = 0.0693 +$ ampères. The fall of potential in passing through 28.0 *ohms* is accordingly $0.0693 \times 28.0 = 1.94$ *volts,* which must be equal to the electromotive force of the Bunsen cell.

EXPERIMENT XCIV.

FIRST PART.

A. Electromotive force of a (given) Daniell cell from Exp. 92 (*see Example* 92, *note*) . 1.04 *volts*

B. Deflection of tangent galvanometer due to this and another Daniell cell in series . . 62°.4 N.E.

C. The same when the second cell is opposed to the first . . . . 1°.0 N. W.

D. Is the 2d cell stronger or weaker than the 1st? . . . . *Stronger.*

e. Electromotive force of stronger cell (see *D*) by formula 9, ¶ 231, and by Table 5,

$A \times (\tan. B + \tan. C) \div (\tan. B - \tan. C) =$

$1.04 \times (1.9128 + .0175) \div (1.9128 - .0175) =$

$1.04 \times 1.9303 \div 1.8953 \qquad = 1.06$ *volts.*

SECOND PART.

F. Deflection of tangent galvanometer due to 1 *Bunsen and* 2 *Daniell cells in sêries through an external resistance of about* 2 *ohms* 62°.0 N. E

G. The same when Bunsen cell is opposed to the two Daniell cells . . . . 5°.0 N. E.

H. Is the Bunsen cell stronger or weaker than the 2 *Daniell cells in series?* . . . . *Weaker.*

i. Electromotive force of the two Daniell cells. $(A + e) =$ . . . . 2.10 *volts.*

j. Electromotive force of the Bunsen cell, by formula 8, ¶ 231 (see *D*) and by Table 5,

$i \times (\tan. F - \tan. G) \div (\tan. F + \tan. G) =$

$2.10 \times (1.8807 - .0875) \div (1.8807 + .0875) =$

1.92 *volts.*

EXPERIMENT XCV.

A. First resistance in thermo-electric circuit in addition to that of the thermo-element, of the galvanometer, and of the connecting wires	0.0 *ohms.*
B. Corresponding resistance in circuit of Daniell cell, reducing the current to the same value as in A	1313 "
C. Second resistance in thermo-electric circuit in addition to that of the thermo-element, of the galvanometer and of the connecting wires	3.0 "
D. Corresponding resistance in circuit of Daniell cell, reducing the current to the same value as in C	2563 "
e. Increase of resistance in thermo-electric circuit corresponding to r, formula 10, ¶ 233 $(C - A) =$	3.0 "
f. Increase of resistance in Daniell circuit, corresponding to $(R_2 - R_1)$ in formula 10, ¶ 233 $(D - B) =$	1250 "
g. Electromotive force of (given) Daniell cell (Example 92, note)	1.04 *volts.*
h. Electromotive force of thermo-element (formula 10, ¶ 233) $g \times e \div f = 1.04 \times 3.0 \div 1250 =$	0.0025 "

EXPERIMENTS XCVI.–XCVII.

	A. Distances between points of contact on uniform straight wire.*	*B.* Corresponding deflections of galvanometer with 3000 ohms added resistance.*
1.	10 *cm.*	7°.
2.	20 *cm.*	19°.
3.	30 *cm.*	31°.
4.	40 *cm.*	40°.
5.	50 *cm.*	47°.
6.	60 *cm.*	52°.5
7.	70 *cm.*	57°.
8.	80 *cm.*	61°.
9.	90 *cm.*	63°.5
10.	100 *cm.*	66°.

☞ See Fig. 260, page 527.

*C.** *Deflection due to Daniell cell* . . . . 48°

*D.** *Deflection due to Leclanché cell.* . . . . . . 65°

E. *Deflection due to Bunsen cell* . . . . — — —

f. Number of *cm.* in *A* corresponding to a deflection in *B* of 48° (see *C*) by interpolation,
50 *cm.* + (48 — 47) ÷ (52.5 — 47) × 10 = 51.8 *cm.*

g. Electromotive force of the (given) Daniell cell (Example 92, note) . . . . 1.04 *volts.*

h. Number of volts per *cm.* ($g \div f$) = 0.020 *volts per cm.*

i. Number of *cm.* corresponding to deflection 65° (see *D*) of Leclanché cell by interpolation,
90 *cm.* + (65 — 63.5) ÷ (66 — 63.5) × 10 = 96.0 *cm.*

j. † Electromotive force of Leclanché cell ($h \times i$) = ... 1.9 + *volts.* †

k. Number of *cm.* corresponding to deflection of Bunsen cell . . . . ———

l. Electromotive force of Bunsen cell ($h \times k$) = ———

* S. L. B., May, 1888.

† This result (1.9 + *volts*) is far too great (see Table 35). The error was probably due to exhaustion of the Daniell cell used as a standard of comparison. A subsequent determination of one of the Daniell cells by Poggendorff's method (Exp. 99) gave 0.724 *volts* (H. F. B., May, 1888). Substituting this value for 1.04 *volts* in the example, the electromotive force of the Leclanché cell becomes 1.34 *volts.*

EXPERIMENT XCVIII.

A.* *Distance between points of contact of poles of Leclanché cell (mean of* 5 *obs.)* . . 64.46 *cm.*

B.* *Distance between points of contact of poles of Daniell cell (mean of* 5 *corresponding obs.)* . . . . 46.53 *cm.*

c. Ratio of the electromotive force of Leclanché to that of Daniell cell ($A \div B$) = . . . 1.385

d. Electromotive force of Daniell cell (Example 92, note) . . . . 1.04 *volts.*

e. Electromotive force of Leclanché cell ($c \times d$) = ... 1.42 "

* L. L. H., May, 1886

NOTE. The Leclanché cells used in the Jefferson Physical Laboratory (1885 to 1886) had electromotive forces 5 or 10 % higher than the value contained in Table 35.

EXPERIMENT XCIX.

A. Deflection of tangent galvanometer* . . . 45°.2

B. Resistance of the coil (Exps.* 85, 87) 1.002 *ohms.*

c. Reduction factor of galvanometer (Exp. 83) 1.00

d. Current in ampères ($c \times$ tangent of A), (see Table 5) . . . . 1.007 *ampères.*

e. Electromotive force of the Daniell cell ($B \times d$) = ... 1.01 *volts.*
Previous determination (Exp. 92) . . 1.04 "

* F. S. D., May, 1888.

EXPERIMENT C.

OBSERVATIONS.	*1st set of observations.*	*2d set of observations.*
A. Mean difference in megadynes between the readings of two spring balances* .	0.108	0.123 *megadynes.*
B. Number of revolutions per second, reduced from obs. lasting* 100 *sec.* .	2.65	2.27 *rev. per sec.*
C. Circumference of the pulley-wheel in centimetres*	30.3	30.3 *cm.*
D. Mean current in ampères indicated by (an ammeter or) tangent galvanometer.*	1.48	1.50 *ampères.*
E. Mean electromotive force in volts indicated (by a voltmeter or its equivalent.)*	6.3	6.2 *volts.*
CALCULATIONS.		
f. Power utilized by the motor in megergs per second ($A \times B \times C$) = .	8.7	8.5 *megergs per sec.*
g. The same in watts, $f \div 10$ = ...	0.87	0.85 *watts.*
h. Power spent upon motor in watts ($D \times E$) = ...	9.3	9.3 *watts.*
i. Efficiency of the electric motor ($g \div h$) $\times$ 100 %	9.4 %	9.1 %.

* F. S. D., May, 1888.

APPENDIX VI.

FIRST LIST OF EXPERIMENTS IN PHYSICAL MEASUREMENT, INTENDED TO COVER THE GROUND REQUIRED FOR ADMISSION TO HARVARD COLLEGE, BOTH IN ELEMENTARY AND IN ADVANCED PHYSICS.

NOTE. The experiments in this list are designated by the letter A. The abbreviations "H. U. Elem." and "H. U. Adv." refer to lists of experiments published by Harvard University, — the elementary list in October, 1889; the advanced list in June, 1890. The numbers following the abbreviations refer to the exercises in these lists to which a given experiment corresponds. Different parts of experiments are indicated by Roman numerals. Experiments marked "extra" do not correspond to any particular numbers in the lists, but are suggested as equivalents. A few experiments covering ground outside of the Harvard requirements are also included. If time is limited, these could naturally be left out, and are marked accordingly "Omit." The correspondence between this list and the two Harvard pamphlets is given at the end of the list.

Before performing the experiments the student should read the following sections in Part III.: In Chapter I., §§ 1 and 2; in Chapter II., §§ 23, 24, 30–33; in Chapter IV., §§ 50, 53–60.

HYDROSTATICS.

1 A. Find the length, breadth, and thickness in *cm.* of a block of wood, ¶ 3. Read § 5. Review §§ 1, 2, and 32, 1st paragraph. Calculate the volume in *cu. cm.* by multiplying the length, breadth, and thickness together.

Apparatus: — Block (wooden solid), and Gauge (vernier).
H. U. Elem., 7 I.

2 A. Find the weight in grams of the block used in 1 A, as in ¶ 2. Read §§ 6 and 9, also § 35. Calculate, as in ¶ 1, the density of the block.

Apparatus: — Balance (*b*); Block (wooden solid); Weights (*g*).
H. U. Elem., 7 II.

3 A. Find the density of water, or better that of a saline solution of unknown strength, by loading a block of wood until it floats or sinks indifferently (foot-note page 2); then find, as in 1 A and 2 A, the volume, weight, and density of the block. The latter is equal to the density sought. Read §§ 62, 63, and 64.

Apparatus: — Balance (*b*); Block (hollow), Gauge (vernier); Weights (*g*); Lead, shot, and (salt) water.
H. U. Elem., 10, II.

4 A. Find the specific gravity of a block of wood by flotation in water. Mark the water-line in pencil at each corner, sighting (as in Fig. 6, page 10) the under surface of the water. Measure the *average* distances, d and d', of the water-line from the upper and lower surfaces of the block. Divide d' by $d + d'$, to find the specific gravity in question. Read §§ 3, 45, and 69. See Harvard Elementary List, Ex. 9, second part.

Apparatus: — Block (wooden, hollow); Metre rod, pencil, and water.
H. U. Elem., 9, III.

5 A.* Find the weight required to sink a Nicholson's hydrometer to a given mark in water at, below, and above the temperature of the room (¶¶ 6 and 7). Plot a curve as in Fig. 7, page 12. Read § 4. Review § 59.

Apparatus: — Brush (camel's-hair); Nicholson's Hydrometer; Thermometer, and Weights (cg); Hot and cold water.

(H. U. Adv., 1.)

6 A.* Find with Nicholson's hydrometer the weight in air of some steel bicycle-balls, also that of a small wooden block, ¶ 8. Read § 43.

Apparatus: — Balls (steel); Block (small); Brush (camel's-hair); Nicholson's Hydrometer; Thermometer, and Weights (cg). H. U. Elem., 8 I., 9 I.

7 A.* Find with Nicholson's hydrometer the weight in water of objects used in 6 A (¶ 10), and calculate their apparent specific gravity (§ 66).

Apparatus same as in 6 A.

H. U. Elem., 8 II., 9 II.

NOTE. The blocks of wood must be held *down* by the lower pan (l, Fig. 9, page 15), since its weight in water is *negative*. Reverse the pan if necessary and place the block *under* it. The weight of water displaced by the block is the *difference* between the two weights required to sink the hydrometer with the block in air and in water. Divide the weight of the block by the weight of water it displaces to find its specific gravity.

* Exps. 5 A, 6 A, and 7 A may be performed with a Jolly (spring) balance instead of a Nicholson's hydrometer.

USE OF A BALANCE

8 A. Find the sensitiveness of a balance with loads of 0, 20, 50, and 100 grams in each pan (¶¶ 20–21). Plot the results, Fig. 16. Read ¶ 22, §§ 25 and 26. Review §§ 30 and 33.

Apparatus: — Balance (*a*); Weights (*cg*).

H. U. Adv., 9.

9 A. Find the ratio of the arm of a balance (¶ 23). Repeat two or three times. Reduce as in ¶ 24. Read §§ 41 and 46. Estimate probable error (§ 50).

Apparatus: — Balance (*a*); Weight (*eg*).

H. U., Extra.

10 A. Find roughly the density of air as in Exp. XVII. (¶¶ 44 and 45), calculating the degree of exhaustion. Compare the observed density with data contained in Tables 19 [and 20] for the same conditions of pressure, temperature, [and humidity]. Read § 48.

Apparatus: — Balance (*b*); Barometer (aneroid); [Hygrodeik]; Pump (Richards); Rubber Stopper (1 hole); Specific Gravity Flask; Stopcock; Thermometer, Weights (*g*)

H. U. Elem., 11.

NOTE. Rough observations of the barometer, thermometer, and hygrodeik will suffice (see Exp. 5).

11 A. Find the density of some coal-gas as in Exp. XVIII. Calculate the density of air as in No. 10 A, from observations of a barometer, thermometer, and hygrodeik (¶¶ 13, 15). Read §§ 70 and 81; see Tables 18 *d* and *e*.

Apparatus: — Balance (*a*); Rubber Stopper; Barometer (aneroid); Hygrodeik; Specific Gravity Flask; Thermometer; Weights (*cg*), and Coal-gas.

H. U. Elem., Extra

12 A. Find gross errors (if any) in the reading of a barodeik by comparing its indications with results obtained as in 10 A or 11 A. Employ the method of weighing by oscillations (¶ 20). Read §§ 49, 65, and 71.

Apparatus: — Balance (*a*), with Barodeik; Barometer (aneroid); Hygrodeik; Thermometer; Weights (*cg*).

H. U. Adv., 7.

13 A. Find the weight of a glass ball in air by a double weighing, ¶ 28. Weigh also a piece of cork coated with varnish. Read § 44. Reduce the results to vacuo by Table 21. Assume that the density of glass is 2.5.

Apparatus: — Balance (*a*); Ball (glass); Rings (small); Weights (*cg*). H. U. Adv., 8.

THE HYDROSTATIC BALANCE.

14 A. Find the weight of a glass ball in water (¶ 29). Read §§ 67 and 68. Calculate the volume and density of the ball.

Apparatus: — Arch (hydrostatic); Balance (*a*); Ball (glass); Beaker; Brush (camel's-hair); Stirrer; Thermometer; Weights (*cg*). Supplies: Wire and water.

H. U. Adv., 10, I.

15 A. Find the weight of the cork (in No. 13 A) in water by attaching a sinker to it, and weighing the sinker in water with and without cork (¶ 29). Calculate the density of the cork. Read § 34. Consider what assumptions you have made in this and in other experiments with the hydrostatic balance. Test the accuracy of one or more of these assumptions by weighing the cork in air *after* weighing it in water.

Apparatus: — Arch (hydrostatic); Balance (*a*); Beaker; Brush (camel's-hair); Cork; Sinker; Weights (*cg*). Supplies: wire and water. H. U. Adv., 12.

16 A. Find the weight of a glass ball (of No. 13 A) in alcohol at an observed temperature (¶ 30). Calculate the density of the alcohol (¶ 31).

Apparatus : — Arch (hydrostatic); Balance (*a*); Ball (glass); Beaker; Brush (camel's-hair); Stirrer; Thermometer; Weights (*cg*). Supplies: Wire and alcohol.

H. U. Adv., 11.

17 A. Find the readings of a densimeter in glycerine, water, and kerosene, and plot curve of corrections, as in Exp. XV., ¶¶ 39, 40, 41. Read § 36 (3).

Apparatus: — A Densimeter with jar containing glycerine, water, and kerosene. H. U. Adv., 16, I.

18 A. Find the density of three saline solutions by means of a densimeter, apply corrections found in 17 A. (Exp. XV., ¶¶ 39, 40, 41.)

Apparatus: — A Densimeter with three jars containing different saline solutions. H. U. Adv., 16, II.

19 A. Find the capacity of a capillary tube by means of mercury. See ¶ 169, II., and ¶ 170. Read § 39.

Apparatus: — Balance (*a*); Capillary Tube; Weights (*cg*); Mercury. H. U. Adv., 55, II.

Note. The student who wishes to take as little as possible for granted may himself determine the density of mercury, as in 21 A, before performing this experiment. The method described in 16 A is also (theoretically) possible, with, for instance, a platinum ball, which would sink in mercury. Attention is called to Tables 23 A, and 24, also to 23 B, which is intended especially to shorten calculations, in calibration by mercury.

20 A. Find the capacity of a Specific Gravity Bottle (¶ 32). Read ¶ 33.

Apparatus: — Balance (*a*); Specific Gravity Bottle; Stirrer; Thermometer; Weights (*cg*); Water.

H. U. Adv., 13.

21 A. Find the density of alcohol by the Specific Gravity Bottle, and calculate the strength of the alcohol (¶ 38). Use Table 27.

Apparatus: — Balance (*a*); Specific Gravity Bottle; Stirrer; Thermometer; Weights (*cg*); Alcohol.

H. U. [Elem. 10, I.] Adv., 15.

22 A. Find the volume of some steel balls, by the Specific Gravity Bottle. Calculate their density.

Apparatus: — Balance (*a*); Balls (steel); Specific Gravity Bottle (¶ 34); Stirrer; Thermometer; Weights (*cg*); Water. H. U. Adv., 14.

23 A. Find the volume of some crystals of sulphate of copper by the use of alcohol (¶¶ 36, 37), and calculate their density.

Apparatus: — Balance (*a*); Specific Gravity Bottle; Stirrer; Thermometer; Weights (*cg*). Supplies: Alcohol and crystallized sulphate of copper.

H. U., Extra.

24 A. Find the correction for one reading of a Vernier gauge (¶ 50, I). Read ¶ 47, but use Table 3, H. Read ¶¶ 48 and 49; also §§ 37, 72, and 73.

Apparatus: — Ball (glass); Gauge (vernier); Lens (magnifying). H. U. Adv., 2.

25 A. Find the pitch of a screw (¶ 50, II.).

Apparatus: — Balls (steel); Micrometer Gauge.

H. U. Adv., 3.

26 A. Find the constants of a spherometer (¶¶ 51 and 54).

Apparatus: — Ball (glass); Plate (glass); Spherometer.

H. U. Adv., 4.

27 A. Find the radii of curvature of 2 spherical surfaces, ¶ 55. Read ¶ 56.

Apparatus: — Lens (magnifying); Spherometer.

H. U., Extra.

Review CHAPTER V. (HYDROSTATICS).

PRESSURE.

28 A. Find the readings of a manometer under two or more different pressures (¶ 78). Find also the height of the barometric column (¶ 13). Read ¶ 77 and §§ 77, 78 and 79.

Apparatus: — Air Thermometer and Manometric Apparatus with mercury. H. U. Elem., 6.

29 A. Find the mercurial pressure required to keep air in manometer from expanding when heated from 0° to 100°. (¶ 76, as far as line 17, page 130.) Read §§ 74, 75, 76. Also ¶ 76. Calculate *e* by formula, page 131.

Apparatus: — Air Thermometer; Manometric Apparatus; Steam Boiler; Steam Jacket; Thermometer.

H. U. Elem., 25.

30 A. Find the fixed points of an Air-Thermometer (first paragraph, ¶ 73). Read § 80 and ¶ 74. Calculate *e* by formula X., page 126.

Apparatus: — Air Thermometer; Steam Boiler; Steam Jacket; Thermometer. H. U. Elem., 26.

31 A. Find the fixed, middle, and quarter points of a Mercurial Thermometer (¶¶ 66, 67, 68, 69, 70). Estimate tenths of a degree (§ 26).

Apparatus: — Beaker (for ice); Bunsen Burner; Steam Boiler; Thermometer. Supplies: Gas, ice, and water (or steam). H. U. Elem., 23 [Adv. 56].

32 A. Find the coefficient of expansion of water between about 20° and 100° (¶ 59). Read ¶¶ 60 and 61, and § 82. Review §§ 62 and 63.

Apparatus: — Expansion Apparatus with accessories. Supply of water and steam.

H. U. Adv., 53 [Elem., 10, III. or IV.].

33 A. Find the coefficient of expansion of alcohol from about 20° to 40° or 50° by the Specific Gravity Bottle. (¶¶ 62, 63).

Apparatus: — Balance (*a*); Specific Gravity Bottle; Stirrer; Thermometer; Weights (*cg*). Supplies: Alcohol and hot water. H. U., Extra.

34 A. Find the coefficient of expansion of glass by the weight thermometer (¶ 240).

Apparatus: — Balance (*a*); Bunsen Burner; Steam Boiler; Steam Jacket; Thermometer (weight); Weights (*cg*). Supplies: gas, ice, mercury, and water (or steam).
H. U. Adv., 58.

35 A. Find the coefficient of linear expansion of a brass rod from about 20° to 100° (¶ 57). Read § 83.

Apparatus: — Brass Rod; Micrometer Frame; Steam Boiler; Steam Jacket; Thermometer. H. U. Elem., 24.

36 A. Find the boiling-point of one or more liquids, and the melting-point of paraffine (¶¶ 83, 84).

Apparatus: — Stopper (1 hole); Test Tube; Thermometer. Supplies: Hot water, paraffine, alcohol, etc.
H. U. Adv., 57.

37 A. Find the temperature of the air (¶ 15) and the dew-point (¶ 16). Read ¶ 17. Obtain the relative humidity (Table 14, A) and the pressure of aqueous vapor (Table 15).

Apparatus: — Cup (nickel-plated) and Thermometer, with ice and salt. H. U., Elem. 22, II.

38 A. Find the maximum pressure of aqueous vapor at about 40° (¶ 81).

Apparatus: — Balance (*b*); Rubber Stopper; Specific Gravity Flask; Thermometer; Weights (*g*) and hot water.
H. U. Elem., 22, I.

CALORIMETRY.

39 A. Find different rates of cooling of a calorimeter (¶¶ 85, 87). Read ¶ 86, also §§ 47, 89.

Apparatus: — Calorimeter; Clock; Stirrer; Thermometer. Supply of hot water. H. U., Extra.

40 A. Find the thermal capacity of a calorimeter with thermometer and stirrer. ¶ 90 (1), I., and ¶ 90 (2); ¶ 91, I. and III. Read §§ 16, 84, 85. Review § 45.

Appa atus: — Balance (*b*); Calorimeter; Clock; Stirrer; Thermometer; Weights (*g*). Supply of hot water.

H. U. Adv., 60.

41 A. Find roughly the conductivity of sand by means of a calorimeter (¶ 241, I).

Apparatus: — Balance (*b*); Calorimeter; Clock; Stirrer; Thermometer; Weights (*g*). Supply of sand and hot water. H. U., *omit.*

42 A. Find the specific heat of lead shot (¶ 94, I.). Read §§ 86 and 90. Use Formula VII., page 194.

Apparatus: — Balance (*b*); Bottle (ice water); Calorimeter; Thermometer; Weights (*g*); Shot, ice, and water.

H. U. Elem., 27 (Adv. 62).

43 A. Find the specific heat of alcohol or turpentine by the following electrical method: * place two equal (ohm) resistance-coils (Fig. 238, ¶ 212) in two equal calorimeters (see B., Fig. 239); fill one calorimeter with w' grams of water, the other with w'' grams of alcohol; pass a current from 2 Bunsen cells in series (§ 140) through both resistance-

* This experiment is taken from the Harvard University List of Advanced Physical Experiments, 1890, Exp. No. 64. It would be well in repeating it to interchange the contents of the two calorimeters (§ 44).

coils for about 10 minutes; note the rise of temperature (t') of the water and (t'') of the alcohol. Having found the thermal capacities (c' and c'') of the two calorimeters, as in ¶ 90, 2, we may calculate the specific heat of alcohol by the formula

$$s = \frac{wt' + c' t' - c'' t''}{w'' t''}$$

To obtain acccurate results by this method, an allowance for cooling must be made in estimating the temperatures in question.

Apparatus: — Balance (*b*); Battery (2 Bunsens); 2 Resistance-coils; 2 Stirrers; 2 Thermometers; Weights (*g*). Supplies: Alcohol and water. connecting wires.

H. U. Adv., 64.

44 A. Find the latent heat of liquefaction of water as follows: Mix 1 part, by weight, of ice with 5 parts of water at about 40° in a calorimeter. Note the temperature of the water *before* pouring it into the calorimeter, and after the ice has melted. Calculate the result by formula of ¶ 102, neglecting *c*. Read ¶ 102, also §§ 87, 88, 91.

Apparatus: — Balance (*b*); Shot Heater; Stirrer: Thermometer; Weights (*g*); ice and warm water.

H. U. Elem., 28 (Adv. 63).

Note. The object of this variation from the method of ¶ 101 is to avoid considering the thermal capacity of the calorimeter.

45 A. Find the latent heat of vaporization of water essentially as in ¶ 103; but find the temperature of the water by a single observation *before* pouring it into the calorimeter, and cut off the steam when the water reaches the temperature of the room. (See note under 44 A.) Calculate the result by the formula of ¶ 104, *neglecting c*. Read ¶ 104.

Apparatus: — Balance (b); Steam Boiler; Steam Trap; Stirrer; Thermometer; Weights (g); Ice and warm water.

H. U. Elem., 29.

46 A. Find the heat of combination of zinc and nitric acid (¶¶ 105, 1, 106).

Apparatus: — Balance (a); Calorimeter, with glass lining; Clock; Stirrer; Thermometer; Weights (cg). Supplies: Zinc filings and dilute Nitric Acid.

H. U., *omit.*

47 A. Find the heat of combination of zinc oxide and nitric acid. (¶¶ 105, 2, 106.)

Apparatus same as in 46 A. Supplies: Zinc Oxide and dilute Nitric Acid. H. U., *omit.*

Review CHAPTER VI.

RADIANT HEAT AND LIGHT.

48 A. Find the candle-heat power of a kerosene lamp (¶¶ 111, 112), and calculate that of a lamp burning 8 grams of kerosene per hour (¶ 113). Read §§ 94, 95, 148.

Apparatus: — Balance (b); Candle; Clock; Galvanometer (astatic). Kerosene Lamp: Optical Bench: Thermopile: Weights (g). H. U. Adv., 99.

49 A. Find the candle-power of a kerosene lamp by Bunsen's photometer (¶ 114, 1). Read ¶ 109. Reduce the candle-power of the lamp to 8 grams per hour. Use formula and reasoning of ¶ 113.

Apparatus: — Candle; Kerosene Lamp; Optical Bench; Photometer. H. U. Elem., 34 (Adv. 32).

50 A. Find the principal focal length of a lens by two different methods (¶ 116 (1), (2)). Read § 103.

Apparatus: — Chimney (perforated); Kerosene Lamp; Lens (magnifying); Optical Bench.

H. U. Elem., 36.

51 A. Find the equivalent focal length of a compound lens as follows: Place two lights at two points H' and H'' (Fig. 7, § 104) as far as possible from the lens, and separate them so as to produce the greatest measurable distance between the images B' and B''. Measure this distance and call it d. Now substitute the lens from 50 A; focus by *moving the screen*; and let the new distance between the images be d'.

Calculate the focal length (F) of the compound lens from that (F') of the lens in 50 A, by the formula —

$$F = F' \times \frac{d}{d'}.$$

Read first two paragraphs of § 104. See Harvard List of Advanced Physical Experiments, No. 45.

Apparatus: — Candle; Kerosene Lamp. 2 Lenses ("doublet" and magnifying lens); Metre Rod; Optical Bench.

H. U. Elem., 38 (Adv. 45, 46).

52 A. Find several conjugate focal lengths of a lens (¶ 117, (1), (2), and (3)). Note the size of the images (see § 104). Calculate the principal focal length of the lens. Use formula, page 238.

Apparatus: — Chimney (perforated); Kerosene Lamp · Lens (magnifying); Metre Rod; Optical Bench.

H. U. Adv., 42 (Elem., 37, I.).

53 A. Find the virtual foci of several (nearly) plane mirrors (¶ 118). Tell which are convex and which concave, remembering that the virtual images (§ 104) of *convex* mirrors are *nearer* than the objects producing them. Read ¶ 118.

Apparatus: — Mirrors (small); Optical Bench.

H. U. Elem., 35 (Adv. 41).

54 A. Find 3 virtual foci of a long-focus converging lens (¶ 119, II.). Calculate the principal focal length.

Apparatus :— Lens (long focus), and Optical Bench.

H. U. Elem., 37, II.

55 A. Find the zero-reading (§ 32) of a sextant (¶ 123). Read § 97.

Apparatus :— A Sextant. H. U. Adv., 35, I.

56 A. Find by a sextant the angular semidiameter of the sun (¶ 124, I.).

Apparatus :— A Sextant. H. U. Adv., 35, II.

57 A. Find the three angles of a prism (¶ 125).

Apparatus :— A small Prism; Kerosene Lamp (with slit); Spectrometer (or sextant). H. U. Adv., 50.

58 A. Find the angle of minimum deviation for a ray of sodium light passing through a prism angle of known magnitude (¶¶ 126, 127). Read ¶ 128 and § 102.

Apparatus :— Prism (used in 57 A); Sodium Flame (with slit); Spectrometer (or sextant). H. U. Adv., 52.

59 A. Find the distance between the lines of a diffraction grating (¶ 130). Read ¶ 129, § 101.

Apparatus :— Diffraction Grating; Sodium Flame (with slit); Spectrometer (or sextant). H. U., Extra.

SOUND (§§ 92–96).

60 A. Find the wave-length of sound from a tuning-fork in a rubber tube (¶ 131, I.). Read § 100.

Apparatus :— Metre Rod; Rubber Tube; Tuning-Fork *Y* tube. H. U. Elem., 32.

61 A. Find the wave-length of sound from a tuning-fork in a resonance tube (¶ 132). Read §§ 98 and 99. Notice

that the lengths of the tube corresponding to a given fork are nearly propor ional to the *odd* integers 1, 3, 5, &c.

Apparatus. — Resonance Tube and Tuning-Fork (A = 220). H. U Adv., 26.

62 A. Find the pitch of a tuning-fork by the graphical method (¶ 139). Read §§ 7 and 96.

Apparatus: — Smoked Glass app.; Tuning-Fork ($c = 64$). H. U. Elem., 31.

63 A. Find the pitch of a tuning-fork by the toothed wheel (¶ 144). Read ¶ 145.

Apparatus: — Toothed Wheel apparatus; Tuning-Fork ($c = 64$). H. U., Extra.

64 A. Find the musical interval between two tuning-forks by means of a monochord (¶ 133, III.). Read ¶ 134.

Apparatus: — A Monochord and 2 Tuning-Forks (A = 216 to 220, $c = 256$). H. U. Adv., 24.

Note. A musical ear is of service in making rapidly the necessary adjustments of a monochord, but is not absolutely necessary for this experiment. Unison between the fork and string may be tested by touching the base of the fork to the end of the string. If unison exists the fork should communicate its vibration to the string.

If l is the length of the string and m its mass per unit of length, the number of vibrations (n) produced in one second by a stretching force (f) in dynes (equal to wg if w is the stretching weight) may be found by the formula —

$$n = \frac{1}{2l}\sqrt{\frac{f}{m}}.$$

Students not preparing for Harvard College may substitute ¶ 133, I. or II. for ¶ 133, III.

65 A. Find by Lissajous' curves (¶ 143) the musical interval between two C–forks 2 octaves apart, also find the

musical interval between the higher of these forks and a $G\sharp$ fork two "octaves" and a "third" below it. Read ¶¶ 134 and 142.

Apparatus :— Lens (small) ; 3 Tuning-Forks ($C = 256$, $C = 64$, $G\sharp = 51.2$) ; Kerosene Lamp for smoking, and sealing wax. H. U. Adv., 29.

Note. Instead of the forks mentioned above, two A-forks and a D-fork may be used ($A = 216$, $A = 54$, $D = 72$) or only 2 forks ($C = 64$, $C = 128$), as suggested in ¶ 143. The advantage of using three forks is that the labor and apparatus required in the next experiment may be greatly reduced.

66 A. Find the pitch of a set of forks, covering a known musical interval by the method of beats (¶ 141). Read ¶ 140.

Apparatus : — A clock and 5 tuning-forks, $G\sharp = 51.2$, $A = 54$, $A\sharp = 57$, $B = 60$, $C = 64$.

H. U. Adv., 25.

Note. If in the last experiment (No. 65 A.) an A- and a D-fork were used, 6 forks will now be required, namely : $A = 54$, $A\sharp = 57$, $B = 60$, $C = 64$, $C\sharp = 68$, and $D = 72$. If only two forks were used ($C = 64$ and $C = 128$), a set of 17 forks will be necessary to cover the interval in question.

The results of the last experiment (No. 65 A.) are reducible to the form (see ¶ 142, formula I.),

$$P = n_1 p_1 + c \ (1), \text{ and } P = n_2 p_2 + c_2 \ (2);$$

hence, substracting (2) from (1), we have

$$n_1 p_1 - n_2 p_2 = c_2 - c_1 \ (3).$$

Now from this experiment (No. 65 *A*) we find

$$p_2 - p_1 = p \ (4) \ ; \text{ whence } n_2 p_2 - n_2 p_1 = n_2 p \ (5).$$

Adding (3) and (5) we have finally

$$(n_1 - n_2)\, p_1 = n_2\, p + c_2 - c_1 \ (6),$$

where n_1 and n_2 represent the respective numbers of lobes visible when the first and when the second of two forks are compared with a third fork higher than either of them; p_1 the pitch of the first fork, p the excess of the second fork over the first, $c_2 - c_1$ the algebraic excesses of the third fork over the nearest harmonic of the first and second, respectively. The pitch of the forks chosen above is such that $n_2 - n_1 = 1$. If they are carefully tuned or loaded, c_1 and c_2 may be made nearly equal or both very small, so that in either case $c_2 - c_1$ may be neglected. After any such adjustment of pitch the observations named in Nos. 65 and 66 must of course be repeated.

67 A. Find the pitch of the note due to longitudinal vibrations in a wire (¶ 248 I) either by a pitch-pipe (Fig. 273), or (in the absence of a musical ear) by a resonance tube, ¶¶ 132, 134, II. Calculate the velocity of sound in the wire (¶ 248).

Apparatus: — A Pitch-Pipe (or Resonance Tube); Tape Measure: Wires; Cloth, Resin, etc.

H. U. Adv., 27.

68 A. Find the pitch of the note due to torsional vibrations in a wire (¶ 248, II.), either by a pitch-pipe or by a resonance tube. Calculate the velocity of these torsional vibrations in the wire.

Apparatus: — Same as in 67 A.

H. U., Extra.

69 A. Find the velocity of sound (¶ 135 (1), (2), & (3); ¶ 136, first paragraph; ¶ 137, III.). Read ¶¶ 138, 135

(4); also §§ 8, 10, 92 and 93. Use formula II, page 281.

Apparatus: — Clock; Signalling Apparatus; Tape Measure. H. U. Elem., 30.

Review CHAPTER VII

VELOCITY.

70 A. Find the velocity (v) of a bullet by a ballistic pendulum (¶ 147, (7)) as follows: Find the weight (m) of the bullet and that (M) of the pendulum; measure the length ($A\ C$) of the suspending cords. Project the bullet into the pendulum. Let the pendulum be caught by a ratchet at its furthest point. Measure the distance ($A.\ B.$ Fig. 9, § 109) through which it has swung. Read §§ 11, 12, 106 and 109. Calculate the velocity V of the pendulum by the formula

$$V = A\,B\sqrt{\frac{980}{A \cdot C}} \text{ (see § 109.)}$$

Now read §106. The impulse ft which the bullet gives the pendulum may be measured either (1) by the momentum lost by the bullet, that is, $m\ (v - V)$, or (2) by that gained by the pendulum ($M\,V$); hence $M\ V = m\ (v - V)$, or

$$v = \frac{m + M}{m} V \text{ (see ¶ 147, 7).}$$

Apparatus: — Ballistic Pendulum; Bullet (with means of projecting it); Clock; Metre Rod.

H. U. Elem., Extra.

71 A. Find the average velocity of a falling body (¶ 148). Read §§ 107, 108 and 111. Calculate the acceleration of gravity.

Apparatus: — Clock; Falling Body Apparatus; Metre Rod. H. U. Adv., 18.

72 A. Find the length of a seconds, $\frac{1}{2}$ seconds and $\frac{1}{4}$ seconds pendulum (¶ 149). Tabulate results as on page 319. Read §§ 28, 29, 40, 61, and 110.

Apparatus :— Clock; Metre Rod; Pendulum (simple).
H. U. Elem., 19 (Adv., 17).

73 A. Find the relative masses of two billiard balls as suggested on pages 312–313. Make a series of experiments all performed in exactly the same manner. Have a metre rod fixed in position, at one time so as to measure the distance $A\,A''$, at another time the distance $B\,B''$, etc.

Apparatus : — Balls (billiard) ; Metre Rod.
H. U. Elem., 20.

74 A. Find the mass of a lead bullet by the method of oscillations (¶ 154). Read ¶ 155.

Apparatus : — Clock, Spiral Spring Apparatus; Weights (*cg*) and lead bullet. H. U. Elem., 18.

FORCE AND ELASTICITY.

75 A. Find the weight in kilograms of a 28-lb. weight, (¶ 159, 1); a 56-lb. weight (¶¶ 159, 2 and 159, 3), and a 4-lb. weight (¶ 159, 4), using a lever and 1 or 2 spring balances of 10 kilos capacity.

Apparatus : — Balances (spring, 10 *k.*) ; Lever ; Weights (safety-valve) with cords. H. U. Elem., 14.

76 A. Find with 1 or 2 spring balances of 10 kilograms capacity and a system of cords, the weight in kilograms of a 4-lb. weight (¶ 159, 5), and of a 56-lb. weight (¶ 159, 6). Read § 105.

Apparatus : — Balances (spring 10 *k.*) ; Weights (safety valve) with cords. H. U. Elem., 12.

77 A. Find the weight of a board, as in ¶¶ 160 and 161. Read § 112.

Apparatus: — Plank (1 × 6 ft.); Pendulum (simple); Triangular supports; Weights (safety valve).

H. U. Elem., 17.

78 A. Find the stiffness of 5 beams by bending them (¶ 162). Read § 115.

Apparatus: — Beam (steel) Micrometer; Triangular supports; Weights (*kg*). H. U. Elem., 3.

79 A. Find the (torsional) stiffness of 2 or more rods by twisting them (¶ 164). Read §§ 13, 113 and 116.

Apparatus: — Balance (spring, 10 *k.*) and Torsion Apparatus. H. U. Elem., 4.

80 A. Find the coefficient of torsion of wire by a torsion balance (¶ 165). Review § 116.

Apparatus: — Gauge (micrometer); Metre Rod; Torsion Balance; Torsion Head; Weights (*cg*).

H. U. Elem., 15.

81 A. Find Young's Modulus of Elasticity for a wire (¶ 167). Read § 114.

Apparatus: — Gauge (micrometer); Micrometer (electric); Tape measure; Weights (*kg*); Young's Modulus Apparatus. H. U. Elem., 2 (Adv., 54).

82 A. Find the breaking strength of several wires (first paragraph, ¶ 168). Weigh a known length of the wire, and calculate what length would break under its own weight. Read ¶ 168.

Apparatus: — Balance (spring, 10 *k.*); Bobbins and Wires. H. U. Elem., 1.

83 A. Find the surface tension of water by means of the capillary tube of No. 16 A (¶ 169, II.). Read ¶ 170.

Apparatus: — Beaker; Capillary Tube; Metre Rod; Thermometer. H. U. Adv., 55, II.

84 A. Find by two methods the coefficient of friction of wood on wood (¶ 171, I., II.). Review § 105.

Apparatus: — Balance (spring, 10 *k.*); Board and Plank; Weights (*kg*). H. U. Elem, 13.

85 A. Find the efficiency of a pulley (1) for raising heavy weights and (2) for multiplying motion (¶ 173). Read §§ 14 and 117.

Apparatus: — Balance (spring, 10 *k.*); Metre Rod; Tackle; Weights (safety-valve). H. U. Elem., 21.

86 A. Find the efficiency of a Water Motor (¶ 174). Read ¶ 175, also §§ 15, 118.

Apparatus: — Balance (rough); Clock; 2 Spring Balances; Jar; Tape Measure; Water Motor (with pressure gauge) weights (*kg*). H. U., *omit.*

87 A. Find (roughly) the mechanical equivalent of heat by means of lead shot (¶ 177, first paragraph). Read ¶¶ 176 and 178.

Apparatus: — Paste-board Tube (with corks); Thermometer, and some Lead Shot. H. U. Adv., 65.

Review Chap. VIII., as far as § 119. Read §§ 119–122.

MAGNETISM.

88 A. Find the distance between the poles of a magnet by means of iron-filings, and confirm by a small compass-needle (¶ 179). Read §§ 126 and 127.

Apparatus: — Compass (vibrating); Magnet (compound); Iron Filings; Photographic paper and pencil.

H. U. Elem., 40.

89 A. Find the attraction and repulsion between two parallel magnets at a given distance (¶ 180). Estimate the strength of the poles (¶ 181). Read §§ 17 and 129.

Apparatus: — Balance (*a*); 2 Blocks (*cu. cm.*); Gauge (vernier); 3 Magnets (compound); Weights (*cg*).

H. U., Extra.

Note. In this and in following experiments, the distance between the poles of the (short) compound magnets may be called equal to $\frac{7}{10}$ the length of the magnet (see ¶ 179).

90 A. Find the couple exerted by the Earth's Magnetism upon magnets by means of torsion (¶ 182). Estimate "H." Read § 128.

Apparatus: — 3 Magnets (compound); Torsion Head and Wire tested in No. 80, A.; Wax, and Pins to serve as sights.

H. U. Adv., 68, I.

91 A. Find the deflection of a compass-needle due to a magnet of known strength (from No. 89 A) at a given distance (¶ 183). Read ¶¶ 184 and 185. Estimate "H." Calculate the true value of "H" from the estimates in Nos. 90 A and 91 A.

Apparatus: — Compass (surveying); 3 Magnets (compound); Metre Rod. H. U. Adv., 68, II.

92 A. Find the distribution of magnetism on a magnet by the method of vibrations (¶ 186). Plot a curve (Fig. 205). Estimate the distance between the poles.

Apparatus: — Clock; Magnet (vibrating needle); Magnet (long-bar); Metre Rod; Test-tube. H. U. Adv., 66.

93 A. Find the distribution of magnetism on a magnet by means of an induction coil (¶ 189). Plot the curve and estimate the distance between the poles as in No. 92 A. Read § 147, also ¶¶ 187 and 188.

Apparatus: — Galvanometer (astatic); Helix (sliding); Magnet (long-bar); Metre Rod. H. U. Adv., 69.

94 A. Find the magnetic dip by the Earth-Inductor (¶ 192), and confirm by means of a dipping needle. Read ¶¶ 190 and 191. Review § 128.

Apparatus : — Earth-Inductor ; Galvanometer (astatic, loaded so as to answer for a ballistic galvanometer), and a Level. H. U. Adv., 70.

ELECTRICAL CURRENT MEASURE,
§§ 18, 19, 130, 131.

95 A. Find the relative strength of battery currents from a 1-fluid cell under given conditions (¶ 208, (1) to (8)). Read §§ 123, 124, and ¶ 207. Reduce results as in ¶ 209, and plot them as in Fig. 237.

Apparatus : — Battery (1 Daniell); Compass (vibrating); Galvanometer. (The porous cup is to be removed from the Daniell cell.) H. U. Elem., 41.

96 A. Find the deflections of a tangent compass at the centre of a coil of wire due to currents from a Daniell cell under the conditions of ¶ 208 (9) to (12). Plot the results as in 95 A. Weigh the zinc and the copper before and after the experiment, and calculate the gain or loss of weight in each case. Read § 144. Review ¶ 209.

Apparatus : — Balance (*b*); Battery (1 Daniell); Compass (surveying); Galvanometer; Weights (*g*).
H. U. Elem., 42.

97 A. Find the constant and reduction factor of a Single-Ring Tangent Galvanometer (¶¶ 198, 199, formulæ (5) and (6)). Read §§ 18, 19, 132 and 133.

Apparatus : — Battery (6 Daniell); Galvanometer (S. R.), and connecting wire. H. U. Adv., 71, I.

98 A. Find the reduction factor of a Double-Ring Galvanometer by the method of comparison (¶ 201). Read ¶ 200.

Apparatus : — Battery (2 Daniell); 2 Commutators; 2 Galvanometers (S. R. and D. R.), and connecting wire.
H. U. Adv., 73.

99 A. Find the reduction factor of an Astatic Galvanometer by the method of comparison (¶ 201), as follows: Connect the astatic galvanometer in series with a rheostat of several thousand ohms resistance, a tangent galvanometer, and a battery. Arrange a shunt of about 1 ohm resistance so as to cut out the rheostat and astatic galvanometer. Change the resistances of the shunt and rheostat so that both galvanometers may give measurable deflections (e. g. 45°. Read § 38). Note what plugs are removed from the rheostat, also the length, diameter, and material of the shunt. Calculate the reduction factor of the combination as in the last experiment (No. 98 A).

Apparatus: — Battery (1 Daniell); 2 Galvanometers (astatic and D. R.); Gauge (micrometer); Metre Rod; Resistance Box; 1 Metre of German silver wire (about No. 25 B. W. G.). H. U. Adv., 86.

NOTE. If R, G, and S are the respective resistances of the Rheostat, Galvanometer, and Shunt, and if I is the reduction factor of the combination, the reduction factor (i) of the astatic galvanometer alone is —

$$i = I \times \frac{S}{R + G + S}.$$

The Galvanometers should be marked and the shunt laid aside for Exps. No. 101 A and 108 A, respectively; or the whole experiment (No. 99 A) may be deferred until G and S have been determined.

100 A. Find the reduction factor of a Dynamometer by comparison with a Single-Ring Galvanometer (¶ 204). Read ¶ 202, § 131.

Let C be the current in ampères indicated by the galvanometer, and a the angle of torsion in the dynamometer; then we find the reduction factor D by the formula —

$$D = \frac{C}{\sqrt{a}}.$$

Apparatus: — Battery (3 Bunsen or 6 Daniell); 2 Commutators; Dynamometer; Galvanometer (S. R.), and connecting wires. H. U. Adv., 98, I.

101 A. Find by measurement the reduction factor of a Dynamometer (¶ 203). Read §§ 134 and 135.

Use the formula

$$D = 10\sqrt{\frac{t}{KA}}.$$

Calculate the current C in No. 100 A by the formula

$$C = D\sqrt{a};$$

then find I and H, as in ¶ 204.

Apparatus: — Dynamometer; Gauge (vernier, long); Torsion Balance, and Weights (*cg*). H. U. Adv., 98, II.

102 A. Find the reduction factor of a galvanometer by the electro-chemical method (¶ 205). Calculate "H" (¶ 206). Read §§ 142 and 143.

Apparatus: — Balance (*a*); Battery (Daniell); Clock; Commutator; Galvanometer (S. R.); Weights (*cg*) and a spiral of copper wire. H. U. Adv., 71, II.

Review Chapter IX., omitting § 124.

ELECTRICAL RESISTANCE.

103 A. Find the electrical resistance of a coil of wire by the method of heating (¶¶ 212, 213). Read §§ 20, 136, and 137.

Apparatus: — Balance (*b*); Battery (2 Bunsen); Calorimeter; Resistance-Coil; Stirrer; Thermometer; Weights (*g*). H. U. Adv., 78.

104 A. Find the length of copper wire about ¼ mm. in diameter (No. 31 B. W. G.), which can be substituted for a

1-ohm coil (C) in the circuit of a Daniell cell (B) and galvanometer (G) — see Fig. 243, page 476 — without changing the deflection. Repeat with a double wire, with a German silver wire of the same diameter, and with one of twice the diameter, or 4 times the cross section (about No. 25 B. W. G.). Read ¶ 218, also § 140.

Apparatus: — Battery (1 Daniell); Compass (surveying); Galvanometer; Resistance-Coil (1 ohm); and wires as stated.

H. U. Elem., 44 (Adv., 76).

105 A. Find the (external) resistance of a circuit, as follows: First, note the deflection of the galvanometer due to each one of two equal cells, then join the cells in series (Fig. 20, § 146), and include German silver wire enough in the circuit to give the same (average) deflection as before.

Apparatus: — Battery (2 Daniell); Compass (surveying); Galvanometer with German silver wire.

H. U. Elem., 45, I. (Adv. 77, I.).

PROOF. Since the electromotive force is doubled (§ 146) and the current is the same, the total resistance must be doubled. Now the internal resistance (§ 140) is doubled, hence the external resistance must also be doubled. The resistance added is accordingly equal to the original external resistance.

106 A. Find the electrical resistance of a conductor by means of a differential galvanometer (¶ 216).

Apparatus: — Battery (1 Daniell); a Galvanometer (astatic with differential connections); the Helix of No. 93 A; a Key; and a Resistance-Box. H. U. Adv., 85.

107 A. Find gross errors (if any) in a resistance-box by means of a Wheatstone's Bridge (¶ 217). Use as a (rough) standard of comparison the resistance-coil tested in No. 103 A. Select a resistance-box in which no *gross* errors are discov-

ered, and assume in future that the resistances are accurate. Read §§ 42 and 141.

Apparatus: — B. A. Bridge; Battery (1 Daniell); Galvanometer (astatic); Resistance-Box and Resistance-Coil.

H. U. Adv., 81.

108 A. Find by Wheatstone's Bridge the resistance of the shunt used in No. 99 A, ¶ 217, and calculate the specific resistance of the material of which it is made (¶ 218). Read ¶ 219.

Apparatus: — A B. A. Bridge; Battery (1 Daniell); Galvanometer (astatic); and Shunt.

H. U. Adv., 82.

109 A. Find the resistance of a galvanometer by Thomson's method (¶ 220). Read ¶ 221.

Apparatus: — Same as in Exp. 108 A, plus a magnet.

H. U. Adv., 90.

110 A. Find the Resistance of a battery by Mance's method (¶ 222). Read ¶ 222 *a*.

Apparatus: — A B. A. Bridge; a Battery (1 Daniell); a Galvanometer (astatic); a Key; a Magnet (compound small); and a Resistance-Box. H. U. Adv., 89.

111 A. Find the mean resistance of a Daniell cell as follows: Note the deflection of each of two cells as in 105 A, and join them in multiple arc (Fig. 19, § 146). Include in the circuit enough German silver wire to give the same average deflection as before. Calculate the resistance of this wire, and multiply it by 2 to find the resistance sought.*

Proof. Since the current and electromotive force are unchanged (§ 146) the total resistance is unchanged (§ 138). The resistance added is therefore equal to the decrease in the

* It is not necessary to cut the wires in 106 A and 111 A. A greater or less length may be included between two clamps, as in ¶ 237. The wires should be kept straight, as in Fig. 249, page 486.

battery resistance caused by arranging the cells in multiple arc. Now this is half the resistance of a simple cell, therefore, etc.

Apparatus: — Battery (2 Daniell); Compass (surveying); Galvanometer, clamps and wire.

H. U. Elem. 45, II. (Adv. 77, II.).

112 A. Find the resistance of a battery by Ohm's method (¶ 225). Read § 138.

Apparatus: — A Battery (1 Daniell); Galvanometer (S. R.) and Resistance-Box. H. U. Adv., 75.

NOTE. The battery cell should be marked so that it can be identified later on.

113 A. Find the resistance of a battery by Thomson's method as follows: Connect a Daniell cell (B, Fig. 253, page 499) with an astatic galvanometer (G), through a resistance box (R), with enough plugs removed to reduce the deflection of the galvanometer to about 45°. Now connect the poles of the battery with a shunt (S) (of about 1 ohm's resistance), and find what resistance (r) in the galvanometer circuit will give the same deflection as before. Calling the respective resistances of the Resistance-Box, Galvanometer and Shunt, R, G and S, we find the battery resistance by the formula

$$B = S\,\frac{R - r}{r + G}.$$

Apparatus: — Battery (1 Daniell); Galvanometer (astatic); Resistance-Box; Shunt. H. U. Adv., 88.

114 A. Find the resistance of a battery by Beetz' method (¶ 229). Read ¶¶ 226–228.

Apparatus: — 2 Batteries (2 Daniell, 1 Leclanché); Galvanometer (astatic); 2 Keys; Resistance-Box.

H. U. Adv., 91.

ELECTROMOTIVE FORCE (Read § 139).

115 A. Find the electromotive force of a battery by the method of opposition (¶ 230 (7)). Use 5 or 6 Daniell cells and 3 Bunsen cells in series, with an astatic galvanometer and resistance-box. Estimate the electromotive force of the Daniell cells from that of the single cells tested in No. 112 A. (See ¶ 230 (2)). From this find that of the Bunsen cells. Read § 21 and § 145.

Apparatus: — Named above.

H. U. Adv. 93.

NOTE. If no number of Bunsen cells can be made to balance (approximately) any whole number of Daniell cells, notice the deflection of the galvanometer (which should be small) in two cases, and use the method of interpolation (§ 41).

116 A. Find the electromotive force of a Bunsen cell by Wiedemann's method (¶ 231).

Apparatus: — 2 Batteries (1 Bunsen, 2 Daniell); Galvanometer (S. R. or D. R.). H. U. Adv., 95.

117 A. Find corrections for a Volt Meter (¶ 231). Plot the results (Fig. 260). Read § 139.

Apparatus: — B. A. Bridge; Battery (2 Daniell); Galvanometer (astatic with extra slides); Resistance-Box.

H. U. Adv., 92.

118 A. Find the electromotive force of a Bunsen and a Leclanché cell by a volt-meter (¶ 235).

Apparatus: — Batteries (1 Bunsen, 1 Leclanché, &c.); Galvanometer (astatic); Resistance-Box.

H. U. Adv., 74.

119 A. Find the electromotive force of a Daniell cell by Poggendorff's absolute method (¶ 237).

Apparatus: — 2 Batteries (1 Daniell, 1 or 2 Bunsen); 2 Galvanometers (astatic and S. R. or D. R.); Resistance-Coil. H. U., Extra.

120 A. Find the efficiency of an electric motor (¶ 238, I).

Apparatus: — 2 Balances (spring); Battery (2 or 3 Bunsen); Clock; 2 Galvanometers (astatic and S. R. or D. R.); Motor (electric, small); Revolution Counter; Resistance-Box. H. U., *omit.*

Review Chap. X.

Review Chap. I–III.

General Review.

The list of experiments given above covers the ground of 42 of the Harvard elementary experiments, viz.: Nos. 1–4; 6–32; 34–42, and 44–45. It covers also the ground of 64 advanced experiments, viz.: Nos. 1–4; 7–18; 24–27; 29; 32; 35; 41–42; 45–46; 51–58; 60; 62–66; 68–71; 73–78; 81–82; 85–86; 89–90; 91–93; 95, and 98–99.

Two of the elementary experiments have practically been counted double, so that the real equivalent is 40 elementary experiments. To replace 11 of the advanced experiments anticipated by the elementary course, viz.: Nos. 17, 32, 41, 45, 46, 54, 56, 62, 63, 76, and 77, eleven extra experiments are suggested, namely, Nos. 9 A, 11 A, 23 A, 27 A, 33 A, 39 A, 59 A, 63 A, 68 A, 70 A, and 119 A. The exact correspondence of the regular experiments is shown in the schedule below. [The brackets indicate repetition.]

Elementary Course.

Harvard Elem. No.	First List No.	Harvard Elem. No.	First List No.	Harvard Elem. No.	First List No.
1	82 A	17	77 A	33	Omitted
2	81 A	18	74 A	34	49 A
3	78 A	19	72 A	35	53 A
4	79 A	20	73 A	36	50 A
5	Omitted	21	85 A	37	[52 A] & 54 A
6	28 A	22	37 A & 38 A	38	51 A
7	1 A & 2 A	23	31 A	39	[54 A]
8	6 A, 7 A	24	35 A	40	88 A
9	& 4 A	25	29 A	41	95 A
10	3 A [21 A & 32 A]	26	30 A	42	96 A
11	10 A	27	42 A	43	Omitted
12	76 A	28	44 A	44	104 A
13	84 A	29	45 A	45	105 A & 111 A
14	75 A	30	69 A	46	Omitted
15	80 A [79 A]	31	62 A		
16	[79 A]	32	60 A		

Advanced Course.

Harvard Adv. No.	First List No.	Harvard Adv. No.	First List No.	Harvard Adv. No.	First List No.
1	5 A	35	55 A & 56 A	71	97 A & 102 A
2	24 A	41	[53 A]	73	98 A
3	25 A	42	52 A	74	118 A
4	26 A	45	[51 A]	75	112 A
7	12 A	46	[51 A]	76	[104 A]
8	13 A	50	57 A	77	[105 A & 111 A]
9	8 A	52	58 A	78	103 A
10	14 A	53	32 A	81	107 A
11	16 A	54	[81 A]	82	108 A
12	15 A	55	83 A & 19 A	85	106 A
13	20 A	56	[31 A]	86	99 A
14	22 A	57	36 A	88	113 A
15	21 A	58	34 A	89	110 A
16	17 A & 18 A	60	40 A	90	109 A
17	[72 A]	62	[42 A]	91	114 A
18	71 A	63	[44 A]	92	117 A
24	64 A	64	43 A	93	115 A
25	66 A	65	87 A	95	116 A
26	61 A	66	92 A	98	100 A & 101 A
27	67 A	68	90 A & 91 A	99	48 A
29	65 A	69	93 A		
32	[49 A]	70	94 A		

APPENDIX VII.

SECOND LIST OF EXPERIMENTS IN PHYSICAL MEASUREMENT INTENDED TO COVER THE GROUND REQUIRED FOR ADMISSION IN ELEMENTARY PHYSICS TO HARVARD COLLEGE.

NOTE. The experiments in this list are designated by the letter B. The abbreviations are the same as in the first list (see Appendix VI., page 1035).

1 B. Find the length, breadth, and thickness in *cm.* of a block of wood by several measurements of each of its dimensions (¶ 3). Read §§ 1, 2 and 5. Calculate the volume in *cu. cm.* by multiplying the length, breadth, and thickness together.

Apparatus: — Block (wooden solid), and a Gauge (vernier).
H. U. Elem., 7, I.

2 B. Find the weight in grams of the block used in 1 B., as in ¶ 2. Read §§ 6 and 9. Calculate as in ¶ 1 the density of the block.

Apparatus: — Balance (*b*); Block (wooden solid); Weights (*g*). H. U. Elem., 7, II.

3 B. Find the density of water, or better that of a saline solution of unknown strength, by loading a block of wood until it floats or sinks, indifferently (foot-note, page 2), then finding as in 1 B and 2 B the volume, weight, and density of the block. The latter is equal to the density sought. Read § 64.

Apparatus: — Balance; Block (hollow); Gauge (vernier); Weights (*g*); Lead shot and (salt) water.
H. U. Elem., 10, II.

4 B. Find the specific gravity of a block of wood by flotation in water. Mark the water-line in pencil at each corner, and calculate, as in 4 A, the specific gravity of the block. Read §§ 3 and 69.

Apparatus: — Block (wooden, solid); a Metre Rod; a pencil and water. H. U. Elem., 9, III.

5 B.* Find the weight required to sink a Nicholson's hydrometer to a given mark in water, at, below, and above the temperature of the room (¶¶ 6 and 7). Plot a curve as in Fig. 7, page 12. Read § 59.

Apparatus: — Brush (camel's-hair); Nicholson's Hydrometer; Thermometer and Weights (*cg*); Hot and cold water.

H. U. Elem., *omit.*

6 B.*. Find the weight in air of some steel bicycle balls, also that of a small wooden block, by Nicholson's Hydrometer (¶ 8).

Apparatus: — Balls (steel); Block, (small wooden); Brush (camel's-hair); Nicholson's Hydrometer; Thermometer and Weights (*cg*).

H. U. Elem., 8, I., 9 I.

7 B.* Find the weight in water of objects used in 6 B (¶ 10), and calculate their apparent specific gravity (§ 66).

Apparatus same as in 6 B.

H. U. Elem., 8 II., 9 II.

See Note under 7 A.

8 B. Find the (apparent) specific gravity of kerosene as follows: Weigh a bottle when empty, when filled with water, and when filled with kerosene. Calculate (by subtracting the weight of the empty bottle) the weights of water and of kerosene required to fill the bottle. Divide the weight of

* Experiments 5 B, 6 B, and 7 B, may be performed with a Jolly (spring) balance instead of Nicholson's Hydrometer.

kerosene by the weight of water to find the specific gravity in question.

Apparatus: — Balance (*b*); Specific Gravity Flask, kerosene and water. (More exact methods are considered in Exps. XI. and XIV.) H. U. Elem. 10, I.

9 B. Find the (apparent) specific gravity of kerosene by the 1st method of balancing columns (¶ 42, page 63). Read the 1st and last paragraphs of ¶ 43, also §§ 62 and 63. Use formula, page 66.

Apparatus: — Metre Rod and *U*-tube, with glass tubes and rubber couplings. H. U. Elem., 10, III.

10 B. Find the (apparent) specific gravity of glycerine by the 2d method of balancing columns (¶ 42, page 64). Readings and calculation the same as in 9 B.

Apparatus: — Metre Rod, Stop-cock and *Y*-tube, with glass tubes and rubber couplings. H. U. Elem., 10, IV.

11 B. Find the readings of a densimeter in glycerine, water, and kerosene, and plot curve of corrections as in Exp. XV. (¶¶ 39, 40, and 41).

Apparatus: — A Densimeter with jars containing glycerine, water, and kerosene.

H. U. Elem., *omit.*

12 B. Find the density of three saline solutions by means of a densimeter, applying corrections found in 11 B. (Exp. XV., ¶¶ 39, 40, and 41.)

Apparatus: — A Densimeter with 3 jars, containing different saline solutions.

H. U. Elem., *omit.*

13 B. Find roughly the density of air (as in Exp. XVI.) (¶¶ 44 and 45). Calculate the degree of exhaustion.

Apparatus: — Balance (*b*); Pump (Richards); Rubber Stopper (1 hole); Specific Gravity Flask; Stopcock; Thermometer; Weights (*g*).

H. U. Elem., 11.

14 B. Find the density of some coal-gas, as in Exp. XVIII. (¶ 46). Read §§ 70 and 81. See Tables 18, *d* and *e*.

Apparatus: — Balance (*b*); Rubber Stopper; Specific Gravity Flask; Thermometer; Weights (*b*) and coal gas.

H. U. Elem., Extra.

15 B. Find the temperature of the air (¶ 15), and the dew-point (¶ 16). Read ¶ 17. Obtain the relative humidity (Table 14 A), and the pressure of aqueous vapor (Table 15).

Apparatus: — Cup (nickel-plated), and Thermometer, with ice and salt. H. U. Elem., 22, II.

16 B. Find the maximum pressure of aqueous vapor at about 40° (¶ 81).

Apparatus: — Balance (*b*); Rubber Stopper; Specific Gravity Flask; Thermometer; Weights (*g*), and hot water.

H. U. Elem., 22, I.

17 B. Find the maximum pressure of ether vapor at about 20° by the second method suggested in ¶ 80.

Apparatus: — Medicine Dropper, Rubber Stopper (2 holes); Specific Gravity Flask; Thermometer, glass tubes, ether, and mercury. H. U. Elem., *omit.*

18 B. Find the barometer pressure as in the first paragraph of ¶ 13, testing as in the first paragraph of ¶ 14, then find the pressure of ether vapor by the first method suggested in ¶ 80. Read ¶ 80.

Apparatus: — Barometer (aneroid); Barometer Tube; Medicine Dropper; Thermometer, glass tubes, and mercury.

H. U. Elem., *omit.*

19 B. Find readings of a manometer under two or more different pressures (¶ 78). Read ¶ 77, and §§ 77, 78, and 79.

Apparatus: — Air Thermometer and Manometric Apparatus, with mercury. H. U. Elem., 6.

20 B. Find the mercurial pressure required to keep air in manometer from expanding when heated from 0° to 100° (¶ 76, as far as line 17, page 130). Read §§ 74, 75 and 76; also ¶ 76. Calculate *e* by formula, page 131.

Apparatus: — Air Thermometer; Manometric Apparatus; Steam Boiler; Steam Jacket; Thermometer.

H. U. Elem., 25.

21 B. Find the fixed points of an air-thermometer (first paragraph, ¶ 73). Read § 80 and ¶ 74. Calculate *e* by formula X., page 126.

Apparatus: — Air Thermometer; Steam Boiler; Steam Jacket; Thermometer. H. U. Elem., 26.

22 B. Find the fixed points of a mercurial thermometer (¶ 69), estimating tenths of a degree (see Fig. 52, ¶ 68). Read §§ 4 and 26; also first paragraph of ¶ 70. Refer to Table 14. Calculate corrections for the thermometer at 0° and 100°.

Apparatus: — Barometer (aneroid); Steam Boiler; Thermometer; and ice. H. U. Elem., 23.

23 B. Find the coefficient of linear expansion of a brass rod from about 20° to 100° (¶ 57). Read §§ 82 and 83.

Apparatus: — Brass Rod; Micrometer Frame; Steam Boiler; Steam Jacket; Thermometer. H. U. Elem., 24.

24 B. Find the specific heat of lead shot (¶ 94, I.). Read §§ 84, 85, 86 and 90. Use Formula VII., page 194.

Apparatus: — Balance (*b*); Bottle (ice water); Calorimeter; Thermometer; Weights (*g*), shot, ice, and water.

H. U. Elem., 27.

25 B. Find the latent heat of liquefaction of water, as in 44 A (First List of Experiments). Read ¶ 102, also §§ 87 and 91.

Apparatus: — Balance (*b*); Shot-heater; Stirrer; Thermometer; Weights (*g*), ice, and warm water.

H. U. Elem., 28.

26 B. Find the latent heat of vaporization of water essentially as in ¶ 103, but find the temperature of the water by a single observation *before* pouring it into the calorimeter, and cut off the steam when the water reaches the temperature of the room (see note under 44 A). Read § 88. Calculate the result by the formula of ¶ 104, *neglecting* c. Read ¶ 104.

Apparatus: — Balance (b); Steam Boiler; Steam Trap; Stirrer; Thermometer; Weights (g). H. U. Elem., 29.

27 B. Find the candle-power of a kerosene lamp by Bunsen's photometer (¶ 114, I.). Read § 94, ¶¶ 109 and 113. Reduce the candle-power of the lamp to 8 grams per hour. Use formula and reasoning of ¶ 113.

Apparatus: — Candle; Kerosene Lamp; Optical Bench; and Photometer. H. U. Elem., 34.

28 B. Find the relative intensities of the red, green, and violet rays reflected by a colored and by a white surface (¶ 246). Read ¶ 115.

Apparatus: — Colored Glasses; Kerosene Lamp; Optical Bench; and Colored Paper. H. U. Elem., *omit.*

29 B. Find the principal focal length of a lens by two different methods (¶ 116, (1) (2)). Read § 103.

Apparatus: — Chimney (perforated); Kerosene Lamp; Lens (magnifying); Optical Bench. H. U. Elem., 36.

30 B. Find the equivalent focal length of a compound lens, as in 51 A (First List of Experiments). Calculate the focal length (F) of the compound lens from that (F') of the lens in 29 B, by the formula (see 51 A) —

$$F = F' \times \frac{d}{d'}.$$

Read first two paragraphs of § 104. See Harvard List of advanced Physical Experiments, No. 45.

Apparatus: — Candle; Kerosene Lamp; 2 Lenses (doublet and magnifying); Metre Rod; Optical Bench.

H. U. Elem., 38.

31 B. Find several conjugate focal lengths of a lens (¶ 117, (1) (2), and, (3)). Note the size of the images (see § 104). Calculate the principal focal lengths of the lens. Use formula page 238.

Apparatus: — Chimney (perforated); Kerosene Lamp; Lens (magnifying); Metre Rod; Optical Bench.

H. U. Elem., 37, I.

32 B. Find the virtual foci of several (nearly) plane mirrors (¶ 118). Tell which are convex and which concave, remembering that the virtual images of *convex* mirrors are *nearer* than the objects producing them. Read § 104 and ¶ 118.

Apparatus: — Mirror (small), and Optical Bench.

H. U., Elem., 35.

33 B. Find 3 virtual foci of a long-focus converging lens (¶ 119, I.). Calculate the principal focal length.

Apparatus: — Lens (long-focus), and Optical Bench.

H. U. Elem., 37, II.

34 B. Find 3 virtual foci of a diverging lens (¶ 119, II.). Calculate the virtual principal focal length by the formula of ¶ 119.

Apparatus: — Lens (diverging), and Optical Bench.

H. U. Elem., *omit.*

35 B. Find the wave-length of sound from a tuning-fork in a rubber tube (131, I.). Read § 100.

Apparatus: — Metre Rod; Rubber Tube; Tuning-fork; *Y*–tube.

H. U. Elem., 32.

36 B. Find the wave-length of sound from a tuning-fork in a resonance tube (¶ 132). Read §§ 98 and 99. Notice that the lengths of the tube responding to a given fork are nearly proportional to the *odd* integers 1, 3, 5, &c.

Apparatus: — Resonance Tube, and Tuning-fork (A = 220).

H. U. Elem., Extra.

37 B. Find the pitch of a tuning-fork by the graphical method (¶ 139). Read §§ 7 and 96.

Apparatus : — Bow (violin) ; Clock ; Smoked Glass Apparatus ; Tuning-fork ($c = 64$). H. U. Elem., 31.

38 B. Find the velocity of sound (¶ 135 (1), (2), (3); ¶ 136, first paragraph, ¶ 137, III.). Read ¶ 138 and ¶ 135 (4), also §§ 8, 10, 92, and 93. Use formula II., page 281.

Apparatus : — Clock ; Signalling Apparatus, and Tape Measure. H. U. Elem., 30.

39 B. Find the velocity of a bullet by a ballistic pendulum (¶ 147, (7)) as in 70 A. (First List of Experiments). Calculate the velocity V of the pendulum by the formula —

$$V = AB\sqrt{\frac{980}{AC}} \text{ (see § 109).}$$

and that (v) of the bullet by the formula —

$$v = \frac{(m + M)}{m} V \text{ (see ¶ 147 (7)).}$$

Read §§ 106 and 109.

Apparatus : — Clock, Metre Rod, and Pendulum (ballistic). H. U. Elem., *omit.*

40 B. Find the velocity acquired by a falling body (¶ 148). Read §§ 11, 107, and 108.

Apparatus : — Clock ; Falling Bodies' Apparatus ; Metre Rod. H. U. Elem., *omit.*

41 B. Find the length of a seconds, $\frac{1}{2}$ seconds, and $\frac{1}{4}$ seconds pendulum (¶ 149). Tabulate results as on page 319. Read §§ 110, 111.

Apparatus : — Clock ; Metre Rod ; Pendulum (simple). H. U. Elem., 19.

42 B. Find the relative masses of two billiard balls suspended by cords as suggested on pages 312–313. See 73 A. (First List of Experiments).

Apparatus : — Balls (billiard) ; Cords ; Metre Rod. H. U. Elem., 20.

43 B. Find the mass of a lead bullet by the method of oscillation (¶ 154). Read ¶ 155.

Apparatus: — Clock; Spiral Spring Apparatus; Weights (*cg*) and lead bullet.

H. U. Elem., 18.

44 B. Find corrections for a spring balance (¶ 158), and construct two tables (pages 339 and 340).

Apparatus: — Balances (spring 10 *k.*); Pulley; Weights. (safety-valve). H. U. Elem., *omit.*

45 B. Find the weight in kilograms of a 28 lb. weight (¶ 159, 1); a 56 lb. weight (¶ 159, 2, and ¶ 159, 3); and a 4 lb. weight (¶ 159, 4), using a lever and one or two spring balances of 10 kilos capacity.

Apparatus: — Balances (spring, 10 *k.*); Lever; Weights (safety-valve), with cords. H. U. Elem., 14.

46 B. Find with 1 or 2 spring balances of 10 kilograms capacity and a system of cords, the weight in kilograms of a 4 lb. weight (¶ 159, 5) and of a 56 lb. weight (¶ 159, 6). Read § 105.

Apparatus: — Balances (spring, 10 *k.*); Weights (safety-valve), with cords. H. U. Elem., 12.

47 B. Find the weight of a board as in ¶¶ 160 and 161. Read § 112.

Apparatus: — Board (loaded); Pendulum (simple); Triangular supports; Weights (safety-valve); a pencil.

H. U. Elem., 17.

48 B. Find the stiffness of 5 beams by bending them (¶ 162). Read §§ 114 and 115.

Apparatus: — Beam (steel); Micrometer; Triangular support; Weights (*kg*). H. U. Elem., 3.

49 B. Find the (torsional) stiffness of two or more rods by twisting them (¶ 164). Read §§ 113 and 116.

Apparatus: — Balances (spring, 10 *k.*), and Torsion Apparatus. H. U. Elem., 4.

50 B. Find the (longitudinal) stiffness of a wire by stretching it. Measure the force (f), the amount of stretching (e), the length of wire (l), and its weight (w). Take the density (d) from Table 9. Calculate q by formula II., page 361; then calculate Young's modulus as explained in ¶ 166.

Apparatus: — Balances (spring, 10 *k.*); Metre Rod; fine steel wire. H. U. Elem., 2.

51 B. Find the breaking strength of several wires (first paragraph, ¶ 168). Weigh a known length of the wire, and calculate the length which would break under its own weight. Read ¶ 168.

Apparatus: — Balance (spring, 10 *k.*); Bobbins and wires. H. U. Elem., 1.

52 B. Find by two methods the coefficient of friction of wood on wood (¶ 171, I. and II.). Review § 105.

Apparatus: — Balance (spring, 10 *k.*); Board and plank; Weights (*kg*).

H. U. Elem., 13.

53 B. Find the efficiency of a pulley (1) for raising heavy weights, and (2) for multiplying motion (¶ 173). Read § 117.

Apparatus: — Balance (spring. 10 *k.*); Metre Rod; Tackle; Weights (safety-valve). H. U. Elem., 21.

54 B. Find the poles of a magnet by means of iron filings, and confirm by a small compass needle (¶ 179) Read §§ 126 and 127.

Apparatus: — Compass (vibrating); Magnet (compound) Iron filings; Photographic paper and pencil.

H. U. Elem., 40.

55 B. Find the magnetic dip by a dipping needle (¶ 190). Read § 128.

Apparatus: — Dipping needle. H. U. Elem., Extra.

56 B. Find the relative strength of battery currents from a 1-fluid cell under given conditions (¶ 208, (1), to (8)). Read §§ 123, 124, 130, and ¶ 207. Reduce results as in ¶ 209.

Apparatus: — Battery (1 Daniell); Compass (vibrating); Galvanometer. (The porous cup is to be removed from the Daniell cell.) H. U. Elem., 41.

57 B. Find the deflection of a tangent compass at the centre of a coil of wire due to currents from a Daniell cell under the conditions of ¶ 208, (9) to (12). Weigh the zinc and the copper before and after the experiment. Read §§ 143, 144. Review ¶ 209.

Apparatus: — Balance (b); Battery (1 Daniell); Compass (surveying); 6 Galvanometers; Weights (g).

H. U. Elem. 42.

58 B. Find the length of copper wire about $\frac{1}{4}$ *mm.* in diameter (No. 31 B. W. G.), which can be substituted for a 1-ohm coil (C) in the circuit of a Daniell cell (B) and galvanometer (G), — see Fig. 243, page 476, — without changing the deflection. Repeat with a double wire, with a German silver wire of the same diameter, and with one of twice the diameter or 4 times the cross-section (about No. 25 B. W. G.). Read ¶ 218, also § 140.

Apparatus: — Battery (1 Daniell); Compass (surveying); Galvanometer; Resistance-Coil (1 ohm); and wires as stated.*

H. U. Elem., 44.

59 B. Find the (external) resistance of a circuit as follows: First, note the deflection of a galvanometer due to each one of two equal cells, then join the cells in series (Fig. 20, § 146), and include German silver wire enough in the circuit to give the same (average) deflection as before. Read § 138 and § 146. Calculate the resistance of this wire. This is equal to the value sought. For proof, see 105 A.

Apparatus: — Battery (2 Daniell); Compass (surveying); Galvanometer, with German silver wires.*

H. U. Elem., 45 I.

60 B. Find the resistance of a Daniell cell as follows: Note the deflection of each of two cells as in 59 B, and join them in multiple arc (Fig. 19, § 146). Include in the circuit enough German silver wire to give the same average deflection as before. Calculate the resistance of this wire, and multiply it by 2 to find the resistance sought. For Proof, see 111 A.

Apparatus: — Battery (2 Daniell); Compass (surveying); Galvanometer, with clamps and German silver wire.*

H. U. Elem., 45 II.

* It is not necessary to cut the wires in 58 B, 59 B, and 60 B. A greater or less length may be included between two clamps, as in ¶ 237. The wire should be kept straight, as in Fig. 249, page 486.

REVIEW.

Chapter I. (General Definitions), first 11 sections.

Chapter V. (Hydrostatics), omitting §§ 67, 68, 71, 72, and 73.

Chapter VI. (Heat), omitting § 89 on cooling.

Chapter VII. (Sound and Light), §§ 92, 93, 94, 96, 98, 99, 100, 103, and 104.

Chapter VIII. (Force and Work), as far as § 118.

Chapter IX. (Electricity and Magnetism), §§ 123, 124, 126, 127, 128, 130.

Chapter X. (Electromotive Force, and Resistance), §§ 138, 140, 143, 144, 146.

The Second List of Experiments is intended to cover the ground of 40 Exercises in Elementary Physics required for admission to Harvard College, *viz.* Nos. 1–4; 6–14; 17–32; 34–38; 40–42; and 44–45. In most cases the correspondence is exact; other cases are designated in the table below by an asterisk (*). The course of reading recommended covers the principles of at least three additional exercises, and three extra experiments are suggested. The ground covered for examination is therefore about equivalent to the 46 exercises of the Harvard elementary pamphlet. The laboratory work is divided into 50 experiments (assuming that 10 of the 60 are omitted as indicated). As these experiments all involve measurements, they are on the average fully as difficult as those recommended by the Harvard pamphlet. This course would be offered only by students who are ambitious to learn more about *physical measurement* than is thought desirable to require of all candidates for admission to Harvard College in elementary physics.

The exact correspondence of the second list of experiments with the "Descriptive List of Elementary Physical Experiments" published by Harvard University, October, 1889, is shown by the table below.

Harvard Elem., No.	Second List, No.	Harvard Elem., No	Second List, No.	Harvard Elem., No.	Second List, No.
1	51 B.	14	45 B.	32	35 B.
2	50 B.	15	[§ 113]	33	Omit
3	48 B.	16	[¶ 164]	Extra	36 B.
4	49 B	17	47 B.	34	27 B.
5	[§§ 62–63]	18	43 B.	35	32 B.
6	19 B.	19	41 B.	36	29 B.
7 I.	1 B.	20	42 B.	37 I.	31 B.
7 II.	2 B.	21*	53 B.*	37 II.	33 B.
8 I. & II. }	{ 6 B.	22 I. *	16 B.*	38	30 B.
9 I. & II. }	{ 7 B.	22 II.	15 B.	39	Omit
9 III.	4 B.	23	22 B.	40	54 B.
10 I.	8 B.	24	23 B.	Extra	55 B.
10 II.	3 B.	25	20 B.	41	56 B.
10 III.	9 B.	26	21 B.	42	57 B.
10 IV.	10 B.	27	24 B.	43	Omit
11	13 B.	28	25 B.	44	58 B.
Extra	14 B.	29	26 B.	45*	{ 59 B. &
12	46 B.	30	38 B.		{ 60 B.*
13	52 B.	31	37 B.	46	Omit

* Cases of only approximate correspondence (3 such cases in all).

APPENDIX VIII.

ADVANCED PHYSICS.

THIRD LIST OF EXPERIMENTS IN PHYSICAL MEASUREMENT INTENDED TO COVER THE GROUND REQUIRED FOR ADMISSION TO HARVARD COLLEGE IN ADVANCED PHYSICS.

NOTE. The experiments in this list are designated by the letter C. The abbreviations are the same as in the first list (Appendix VI., page 1035). Before beginning the experiments the student should review those sections in Part III., already mentioned (see Appendix VII., page 1077), and should read in addition Chapters II. and IV., omitting §§ 51, 52, and 61; also §§ 48 and 49 of Chapter III.

1 C. Find the sensitiveness of a balance with loads of 0, 20, 50, and 100 grams in each pan, (¶¶ 20, 21). Plot the results (Fig. 16). Read ¶ 22. Review §§ 26, 30, 59.

Apparatus: — Balance (*a*); Weights (*cg*).

H. U. Adv., 9.

2 C. Find the ratio of the arms of a balance (¶ 23). Repeat two or three times. Reduce as in ¶ 24. Read § 46. Estimate probable error (§ 50).

Apparatus: — Balance (*a*); Weights (*cg*).

H. U., Extra.

3 C. Find a correction for the reading of a barodeik (¶ 18), by means of a hygrodeik (¶ 15) and an aneroid barometer. Use Tables 19, 20. Read § 71.

Apparatus: — Balance (*a*); Barodeik; Barometer (aneroid); Hygrodeik; Thermometer; Weights (*cg*.).

H. U. Adv., 7.

4 C. Find the weight of a glass ball in air by a double weighing (¶ 28). Weigh also a piece of cork coated with varnish. Read §§ 35, 44, 67, and 72. Reduce the results to vacuo.

Apparatus: — Balance (*a*); Ball (glass); Rings (small); Weights (*cg*). H. U. Adv., 8.

5 C. Find the weight of a glass ball in water (¶ 29). Review §§ 64, 65, 66, and 67. Read § 68. Calculate the volume and density of the ball.

Apparatus: — Arch (hydrostatic); Balance (*a*); Ball (glass); Beaker; Brush (camel's-hair); Stirrer; Thermometer; Weights (*cg*). Supplies: Wire and water.

H. U. Adv., 10.

6 C. Find the weight of the cork (in No. 4 C) in water by attaching a sinker to it, and weighing the sinker in water with and without the cork (¶ 29). Calculate the density of the cork. Review § 34. Consider what assumptions you have made in this and in other experiments with the hydrostatic balance. Test the accuracy of one or more of these assumptions by reweighing the cork in air *after* weighing it in water.

Apparatus: — Arch (hydrostatic); Balance (*a*); Beaker; Brush (camel's-hair); Cork; Sinker; Weights (*cg*). Supplies: Wire and water. H. U. Adv., 12.

7 C. Find the weight of a glass ball (of No. 5 C) in alcohol at an observed temperature (¶ 30). Calculate the density of the alcohol (¶ 31).

Apparatus: — Arch (hydrostatic); Balance (*a*); Ball (glass); Beaker; Brush (camel's-hair); Stirrer; Thermometer; Weights (*cg*). Supplies: Wire and alcohol.

H. U. Adv., 11.

8 C. Find the capacity of a Specific Gravity Bottle (¶ 32). Read ¶ 33.

Apparatus. — Balance (*a*); Specific Gravity Bottle; Stirrer; Thermometer; Weights (*cg*); Water.

H. U. Adv., 13.

9 C. Find the density of alcohol by the Specific Gravity Bottle, and calculate the strength of the alcohol (¶ 38). Use Table 27.

Apparatus: — Balance (*a*); Specific Gravity Bottle; Stirrer; Thermometer; Weights (*cg*); Alcohol.

H. U. Adv., 15.

10 C. Find the volume of some steel balls by the Specific Gravity Bottle (¶ 34). Read ¶ 35; also § 38. Calculate the density of the balls.

Apparatus: Balance (*a*); Balls (steel); Specific Gravity Bottle; Stirrer; Thermometer; Weights (*cg*); Water.

H. U. Adv., 14.

11 C. Find the volume of some crystals of sulphate of copper by the use of alcohol (¶¶ 36, 37), and calculate their density.

Apparatus: — Balance (*a*); Specific Gravity Bottle; Stirrer; Thermometer; Weights (*cg*). Supplies: Alcohol and crystallized sulphate of copper.

H. U., *omit.*

12 C. Find the correction for one reading of a vernier gauge (¶ 50 I.). Read ¶ 47, but use Table 3 H. Read ¶¶ 48 and 49, also §§ 37, 43, and 73.

Apparatus: — Ball (glass); Gauge (vernier); Lens (magnifying).

H. U. Adv., 2.

13 C. Find the pitch of a screw (¶ 50, II.).

Apparatus: — Balls (steel); Micrometer gauge.

H. U. Adv., 3.

14 C. Find the constants of a spherometer (¶¶ 51 and 54).

Apparatus: — Ball (glass); Plate Glass; Spherometer.

H. U. Adv., 4.

15 C. Find the radii of curvature of 2 spherical surfaces (¶ 55). Read ¶ 56.

Apparatus: — Lens (magnifying); Spherometer.

H. U., Extra.

16 C. Find the capacity of a capillary tube by means of mercury. See ¶ 169, II., and ¶ 170. Read § 39.

Apparatus: — Balance (*a*); Capillary Tube; Weights (*cg*); Mercury. H. U. Adv., 55, II.

17 C. Find the fixed, middle, and quarter points of a mercurial thermometer (¶¶ 66, 67, 68, 69, and 70). Read § 36, (3).

Apparatus: — Beaker (for ice); Bunsen Burner; Steam Boiler; Thermometer. Supplies: Gas, ice and water (or steam). H. U. Adv., 56.

18 C. Find the coefficient of expansion of water between about 20° and 100° (¶ 59). Read ¶¶ 60 and 61. Review §§ 62 and 63.

Apparatus: — Expansion Apparatus with accessories, supply of water and steam. H. U. Adv., 53.

19 C. Find the coefficient of expansion of alcohol from about 20° to 40° or 50° by the Specific Gravity Bottle (¶¶ 62, 63). Review § 82.

Apparatus: — Balance (*a*); Specific Gravity Bottle; Stirrer; Thermometer; Weights (*cg*). Supplies: Alcohol and hot water. H. U., Extra.

20 C. Find the coefficient of expansion of glass by the weight thermometer (¶ 240). Review § 83.

Apparatus: — Balance (*a*); Bunsen Burner; Steam Boiler; Steam Jacket; Thermometer (weight); Weights (*cg*). Supplies: Gas, ice, mercury, and water (or steam).

H. U. Adv., 58.

21 C. Find the boiling point of one or more liquids, and the melting point of paraffine (¶¶ 83, 84).

Apparatus: — Stopper (1 hole); Test-tube; Thermometer. Supplies: Hot water, paraffine, alcohol, etc.

H. U. Adv., 57.

CALORIMETRY (Review §§ 84–91).

22 C. Find the rate of cooling of a calorimeter (¶¶ 85, 87). Read ¶ 86, also §§ 47, 89.

Apparatus: — Calorimeter; Clock; Stirrer; Thermometer. Supply of hot water. H. U., Extra.

23 C. Find the thermal capacity of a calorimeter with thermometer and stirrer (¶ 90 (1) I; ¶ 91, I). Read §§ 16. 45; Review § 85.

Apparatus: — Balance (*b*); Calorimeter; Clock; Stirrer; Thermometer; Weights (*g*). Supply of hot water.

H. U. Adv., 60.

24 C. Find the thermal capacity of a thermometer of a stirrer, and of a calorimeter, as in (¶ 90, 2). Use formula III., ¶ 91.

Apparatus: — A Balance (*b*); Calorimeter; Measuring glass; Stirrer; Thermometer and water.

H. U. Adv., 61.

25 C. Find the specific heat of turpentine by the method of mixture (¶ 96, I.). Read ¶ 95. Use formula VIII., ¶ 98. Review § 90.

Apparatus: — Balance (*b*); Calorimeter; Stirrer; Thermometer; Weights (*g*). Supplies: Turpentine cooled to 0°, and hot water.

NOTE. Students who have not already determined the specific heat of lead shot should substitute this determination (¶ 94, I.). H. U. Adv., 62.

26 C. Find the specific heat of alcohol by the use of lead shot (¶ 96, II.).

Apparatus: — Balance (*b*); Bunsen Burner; Calorimeter; Steam Shot-heater; Thermometer; Weights (*g*). Supplies: Gas, alcohol (at 0°), water (or steam) and lead shot.

H. U., *omit.*

27 C. Find the specific heat of alcohol or turpentine by an electrical method in 43 A. (First List of Experiments, see Appendix VI.)

Apparatus: — Balance (*b*); Battery (2 Bunsens); 2 Calorimeters; 2 Resistance-coils; 2 Stirrers; 2 Thermometers; Weights (*g*). Supplies: Alcohol and water, connecting wires. H. U. Adv., 64.

Review Exp. 25 B (H. U. Elem., 28 = H. U. Adv., 63).

28 C. Find the heat of combination of zinc and nitric acid (¶ 105, I., and ¶ 106).

Apparatus: — Balance (*a*); Calorimeter with glass lining; Clock; Stirrer; Thermometer; Weights (*cg*). Supplies: Zinc filings and dilute nitric acid. H. U., *omit.*

29 C. Find the heat of combination of zinc oxide and nitric acid (¶ 105, II., and ¶ 106).

Apparatus: — Balance (*a*); Calorimeter with glass lining; Clock; Stirrer; Thermometer; Weights (*cg*). Supplies: Zinc oxide and dilute nitric acid. H. U., *omit.*

RADIANT HEAT (Review §§ 93, 94; Read § 95).

30 C. Find the candle-heat-power of a kerosene lamp (¶ 111), and calculate that of a lamp burning 8 grams of kerosene per hour (¶ 113).

Apparatus: — Balances (*b*); Candle; Clock; Galvanometer (astatic); Kerosene Lamp; Optical Bench; Thermopile; Weights (*g*). H. U. Adv., 99.

Review Exp. 27 B (H. U. Elem., 34 = H. U. Adv., 32).

LIGHT.

Review Exps. 30 B, 31 B, 32 B, 33 B (H. U. Elem., 35–37 = H. U. Adv., 41, 42, 43, 45).

31 C. Find the zero-reading of a sextant (¶ 123). Read §§ 31, 32, 97.

Apparatus: — A Sextant. H. U. Adv., 35 I.

32 C. Find by a sextant the angular semidiameter of the sun (¶ 124, I.).

Apparatus: — A Sextant. H. U. Adv., 35, II.

33 C. Find by a sextant the distance of a terrestrial object of known magnitude (¶ 124, II., and ¶ 136).

Apparatus: — A Sextant. H. U., *omit.*

34 C. Find the latitude and longitude of a place (¶¶ 242, 243, Tables 44 A–44 G).

Apparatus: — An Artificial Horizon and a Sextant.
H. U., *omit.*

35 C. Find by a sextant the three angles of a prism (¶ 125, I.).

Apparatus: — A small Prism and a Sextant.
H. U. Adv., 51.

36 C. Find by a spectrometer the three angles of a prism (¶ 126).

Apparatus: — A small Prism; a (kerosene) Lamp; a Spectrometer. H. U. Adv., 50.

37 C. Find the angle of minimum deviation for a ray of sodium light passing through a prism angle of known magnitude (¶¶ 126, 127). Read ¶ 128, and § 102.

Apparatus: — Prism (used in No. 36 C); Sodium flame (with slit); Spectrometer (or sextant).
H. U. Adv., 52.

38 C. Find the distance between the lines of a diffraction grating (¶ 130). Review § 100. Read § 101 and ¶ 129.

Apparatus: — Diffraction Grating: Sodium flame (with slit); Spectrometer (or sextant). H. U., Extra.

SOUND (Read §§ 92 and 96).

Review Exps. 36 B and 37 B (H. U. Elem., 28 and 31 = H. U. Adv., 26 and 22).

39 C. Find the pitch of a tuning-fork by the toothed wheel (¶ 144). Read ¶ 145.

Apparatus: — A Toothed Wheel Apparatus, and a Tuning-fork (C = 64). H. U., Extra.

40 C. Find the musical interval between two tuning-forks by means of a monochord (¶ 133 III). Read ¶ 134.

Apparatus: — A Monochord and 2 Tuning-forks (A = 216 to 220, C = 256). See note under 64 A, First List of Experiments, Appendix VI. H. U. Adv, 24.

41 C. Find by Lissajous' curves (¶ 143) the musical interval between 2 C-forks 2 "octaves" apart; also find the musical interval between the higher of these forks and a G♯ fork, two "octaves" and a "third" below it. Read ¶¶ 134 and 142.

Apparatus: — Lens (small); 3 Tuning-forks (C = 256, C = 64, G♯ = 51.2). (Kerosene lamp for smoking, and sealing wax.)

See Note under 65 A, First List of Experiments, Appendix VI. H. U. Adv., 29.

42 C. Find the pitch of a set of forks, covering a known musical interval, by the method of beats (¶ 141). Read ¶ 140.

Apparatus: — A Clock and 5 Tuning-forks; G♯ = 51.2, A = 54, A♯ = 57, B = 60, C = 64. H. U. Adv., 25.

See Note under 66 A, First List of Experiments, Appendix VI.

43 C. Find the pitch of the note due to longitudinal vibration in a wire (¶ 248; I.), either by a pitch-pipe (Fig. 273), or (in the absence of a musical ear) by a resonance tube

(¶ 132, and ¶ 134, II.). Calculate the velocity of sound in the wire (¶ 248).

Apparatus: — A Pitch-pipe (or Resonance Tube); Tape measure; Wires; Cloth, resin, etc. H. U. Adv., 27.

44 C. Find the pitch of the note due to torsional vibrations in a wire (¶ 248, II.), either by a pitch-pipe or by a resonance tube. Calculate the velocity of these torsional vibrations in the wire.

Apparatus: — A Pitch-pipe (or Resonance Tube); Tape measure; Wires; Cloth, resin, etc. H. U., Extra.

DYNAMICS (Read §§ 28, 29, 111, ¶ 138).

45 C. Find the length and time of oscillation of an irrotational pendulum (¶ 151, II.). Read ¶ 150 and ¶ 152, §§ 40 and 61. Obtain g from table, ¶ 153.

Apparatus: — Clock; Gauge (vernier); Metre Rod; Pendulum (irrotational). H. U. Adv., 17.

46 C. Find the coefficient of torsion of a wire by a torsion balance (¶ 165). Read § 12; review §§ 13 and 116.

Apparatus: — Gauge (micrometer); Metre Rod; Torsion Balance; Torsion Head; Weights (*cg*). H. U., Extra.

47 C. Find Young's modulus of elasticity for a wire (¶ 167). Review § 114.

Apparatus: — Gauge (micrometer); Micrometer (electric) Tape measure; Weights (*kg*); Young's Modulus Apparatus. H. U. Adv., 54.

48 C. Find the surface tension of water by means of the capillary tube of No. 16 C (¶ 169, II.). Read ¶ 170.

Apparatus: — Beaker; Capillary Tube; Metre Rod; Thermometer. H. U. Adv., 55 I.

ENERGY (Read §§ 14, 15, 117–121).

49 C. Find the coefficient of hydraulic "resistance" for a rubber tube (¶ 172, page 378). Calculate the coefficient of friction for water.

Apparatus: — Balance (rough); Blocks; Clock; 2 Jars; Weights (*kg*). H. U., *omit.*

50 C. Find the efficiency of a water motor (¶ 174). Read ¶ 175. Read §§ 14, 15, 117, 118.

Apparatus: — Balance (rough); Clock; 2 Spring Balances; Jar; Tape measure; Water Motor (with pressure gauge); Weights (*kg*). H. U., *omit.*

51 C. Find (roughly) the mechanical equivalent of heat by means of lead shot (¶ 177, first paragraph). Read ¶ 176 and ¶ 178.

Apparatus: — Pasteboard Tube (with corks); a Thermometer and some Lead Shot. H. U. Adv., 65.

MAGNETISM.

52 C. Find the attraction and repulsion between two parallel magnets at a given distance (¶ 180). Estimate the strength of the poles (¶ 181). Read §§ 17, 129.

Apparatus: — Balance (*a*); Blocks (*cu. cm*); Gauge (vernier); 3 Magnets (compound); Weights (*cg*).

H. U., Extra.

NOTE. In this and in following experiments, the distance between the poles of the (short) compound magnets may be called equal to $\frac{7}{10}$ the length of the magnet. See ¶ 179.

53 C. Find the couple exerted by the earth's magnetism upon 3 magnets by means of torsion (¶ 182). Estimate "H."

Apparatus: — 3 Magnets (compound); Torsion Head and Wire tested in No. 46 C.; Wax and pins to serve as sights.

H. U. Adv., 67.

54 C. Find the deflection of a compass needle due to magnets of known strength (from No. 52 C.) at a given distance (¶ 183). Read ¶¶ 184, 185. Estimate "H." Calculate the true value of "H" from the estimates in Nos. 53 C. and 54 C.

Apparatus: — Compass (surveying); Magnets (compound); Metre Rod. H. U. Adv., 68.

55 C. Find the distribution of magnetism on a magnet by the method of vibrations (¶ 186). Plot a curve (Fig. 205). Estimate the distance between the poles.

Apparatus: — Clock; Magnet (vibrating needle); Magnet (long-bar); Metre Rod; Test Tube. H. U. Adv., 66.

56 C. Find the distribution of magnetism on a magnet by means of an induction coil (¶ 189). Plot the curve and estimate the distance between the poles as in No. 55 C. Read ¶¶ 187 and 188.

Apparatus: — Galvanometer (astatic); Helix (sliding); Magnet (long-bar); Metre Rod. H. U. Adv., 69.

57 C. Find the magnetic dip by the earth-inductor (¶ 192). Read ¶¶ 190, 191, and § 147.

Apparatus: — Earth-Inductor; Galvanometer (astatic, loaded so as to answer for a ballistic galvanometer), and a Level. H. U. Adv., 70.

ELECTRICAL CURRENT MEASURE (Read §§ 18, 19, 130–133).

58 C. Find the constant and reduction factor of a single ring tangent galvanometer (¶¶ 198 and 199, formulæ (5) and (6)).

Apparatus: — A Galvanometer (S. R.), a Gauge (long vernier), and a Tape Measure. H. U. Adv., 71, I.

59 C. Find the reduction factor of a double-ring galvanometer by the method of comparison (¶ 201). Read ¶ 200.

Apparatus: — Battery (2 Daniell); 2 Commutators; 2 Galvanometers (S. R. and D. R.), and connecting wire.

H. U. Adv., 73.

60 C. Find the reduction factor of an ammeter by the method of comparison (¶ 210).

Apparatus: — An Ammeter; Battery (2 or 3 Bunsen); 2 Tangent Galvanometers (S. R. and D. R.).

H. U. Adv., *omit.*

61 C. Find the reduction factor of an astatic galvanometer with shunt, by the method of comparison (¶ 201), as in 99 A (First List of Experiments, Appendix VI.). Note what plugs are removed from the rheostat, also the length, diameter, and material of the shunt. If the resistances R, G, and S of the rheostat, galvanometer, and shunt are known, calculate the reduction factor of the galvanometer without the shunt from that of the combination (I), by the formula

$$i = I \times \frac{S}{R + G + S}.$$

Apparatus: — A Battery (1 Daniell); 2 Galvanometers (astatic and D. R.); a Gauge (micrometer); a Metre Rod; a Resistance-box; 1 metre of German silver wire (about No. 25 B. W. G.). H. U. Adv., 86.

62 C. Find the reduction factor of a dynamometer by comparison with a single-ring galvanometer (¶ 204). Read ¶ 202. Let C be the current indicated by the galvanometer, and a the angle of torsion in the dynamometer; then the reduction factor (D) is

$$D = C \div \sqrt{a}.$$

Apparatus: — A Battery (3 Bunsen or 6 Daniell); 2 Commutators; a Dynamometer; a Galvanometer (S. R.), and connecting wires. H. U. Adv., 98, I.

63 C. Find by measurement the reduction factor of a dynamometer (¶ 203). Read §§ 134, 135. Review § 116. Use the formula

$$D = 10\sqrt{\frac{t}{KA}}.$$

Calculate the current C in No. 63 C. by the formula

$$C = D\sqrt{a}$$

then find I and H as in ¶ 204.

Apparatus: — A Dynamometer; a Gauge (vernier long); [a Torsion Balance, and Weights (cg)].

H. U. Adv., 98, II.

64 C. Find the reduction factor of a galvanometer by the electro-chemical method (¶ 205). Calculate "H" (¶ 206). Read § 142. Review §§ 143, 144.

Apparatus: — Balance (a); Battery (1 Daniell); Clock; Commutator; Galvanometer (S. R.); Weights (cg), and a spiral of copper wire. H. U. Adv., 71, II.

ELECTRICAL RESISTANCE (Read §§ 20, 136, 137).

65 C. Find the electrical resistance of a coil of wire by the method of heating (¶¶ 212, 213).

Apparatus: — Balance (b); Battery (2 Bunsen); Calorimeter; Resistance-coil; Stirrer; Thermometer; Weights (g).

H. U. Adv., 78.

Review Exp. 58 B (Elem. 44 = Adv. 76).

66 C. Find the electrical resistance of a conductor by means of a differential galvanometer (¶ 216).

Apparatus: — Battery (1 Daniell); a Galvanometer (astatic with differential connections); the Helix of No. 56 C.; a Key; and a Resistance-box. H. U. Adv., 85.

67 C. Find gross errors (if any) in a resistance-box by means of a Wheatstone's Bridge (¶ 217). Use as a (rough) standard of comparison the resistance-coil tested in No. 65 C. Read §§ 42 and 141. Review § 45.

Apparatus: — B. A. Bridge; Battery (1 Daniell); Galvanometer (astatic); Resistance-box and Resistance-coil.

H. U. Adv., 81.

68 C. Find by Wheatstone's Bridge the resistance of the shunt used in No. 61 C, and calculate the specific resistance of the material of which it is made (¶ 219). Read ¶ 217.

Apparatus: — B. A. Bridge; Battery (1 Daniell); Galvanometer (astatic), and Shunt. H. U. Adv., 82.

69 C. Find the resistance of the galvanometer used in 61 C by Thomson's method (¶ 220). Read ¶ 221.

Apparatus: — B. A. Bridge; Battery (1 Daniell, shunted); Galvanometer (astatic); Key; Magnet (small compound); Resistance-box. H. U. Adv., 90.

70 C. Find the resistance of a battery by Mance's method (¶ 222). Read ¶ 222 *a*.

Apparatus: — B. A. Bridge; a Battery (1 Daniell); a Galvanometer (astatic); a Key; a Magnet (compound, small); and a Resistance-box. H. U. Adv., 89.

71 C. Find the resistance of a tangent galvanometer by the use of a shunt (¶ 223, I.). Read ¶ 224, I.

Apparatus: — Battery (1 Daniell); 2 Galvanometers (S. R. and D. R.); Resistance-box (or shunt).

H. U., Extra.

Review Experiment 60 B (Elem. 45 = Adv. 77).

72 C. Find the resistance of a battery by Ohm's method (¶ 225). Review § 138.

Apparatus: — A Battery (1 Daniell); a Galvanometer (S. R.), and a Resistance-box. H. U. Adv., 75.

NOTE. The battery cell should be marked so that it can be identified later on.

73 C. Find the resistance of a battery by Thomson's method, as in 113 A (First List of Experiments, Appendix VI.).

Apparatus: — Battery (1 Daniell); Galvanometer (astatic); Resistance-box, and shunt. H. U. Adv., 88.

74 C. Find the resistance of a battery by Beetz' method (¶ 229). Read ¶¶ 226–228.

Apparatus: — 2 Batteries (2 Daniell, 1 Leclanché); Galvanometer (astatic); 2 Keys; Resistance-box.
H. U. Adv., 91.

ELECTROMOTIVE FORCE (Read §§ 21 and 139; Review §§ 137, 138, 145).

75 C. Find the electromotive force of a battery by the method of opposition (¶ 230 (7)). Use 5 or 6 Daniell cells and 3 Bunsen cells in series, with an astatic galvanometer and resistance box. Estimate the electromotive force of the Daniell cells from that of the single cell tested in No. 72 C. (see ¶ 230 (2)). From this find that of the Bunsen cells. Read § 41.

Apparatus: — Named above. H. U. Adv., 93.

NOTE. See note under 115 A (First List of Experiments, Appendix VI.).

76 C. Find the electromotive force of a Bunsen cell by Wiedemann's method (¶ 231).

Apparatus: — 2 Batteries (1 Bunsen, 2 Daniell); a Galvanometer (S. R. or D. R.). H. U. Adv., 95.

77 C. Find corrections for a volt-meter (¶ 234). Plot the results (Fig. 260).

Apparatus: — B. A. Bridge; Battery (2 Daniell); Galvanometer (astatic with extra slider); and a Resistance-box. H. U. Adv., 92.

78 C. Find the electromotive forces of a Bunsen and a Leclanché cell by a volt meter (¶ 235).

Apparatus: — Batteries (1 Bunsen, 1 Leclanché, &c.); a Galvanometer (astatic), and Resistance-box.

H. U. Adv., 94.

79 C. Find the electromotive force of a Daniell cell by Poggendorff's absolute method (¶ 237).

Apparatus: — 2 Batteries (1 Daniell, 1 or 2 Bunsen); 2 Galvanometers (astatic and S. R. or D. R.); Resistance-coil (in calorimeter). H. U. Adv., 96.

80 C. Find the efficiency of an electric motor (¶ 238).

Apparatus: — 2 Balances (spring); Battery (2 or 3 Bunsen); Clock; 2 Galvanometers (astatic and S. R. or D. R.); Motor (electric, small); Revolution Counter; Resistance-box. H. U., *omit.*

The "third list" of experiments given above contains 60 regular and 10 "extra" experiments. The latter are intended to take the place of 10 advanced experiments, the principles of which have probably been anticipated in an elementary course. The corresponding experiments in the elementary course are marked for review, with references to the Harvard elementary pamphlet, and to the "second list" (B, Appendix VII.), where they may be found. The student will do well in any case to prepare himself for examination upon the principles of these 10 elementary experiments, which together with the 60 regular experiments of the "third list" are thought to cover the ground of 66 of the 100 experiments in Physical Measurement published by Harvard University, June, 1890.

The 66 experiments have been selected as follows: —

13 in Mechanics and Hydrostatics; Nos. 2–4, 7–15, and 17.

6 in Sound; Nos. 22, 24–27, and 29.

9 in Light; Nos. 32, 35, 41–43, 45, and 50–52.

19 in Heat; Nos. 53–58; 60–71, and 73.

19 in Magnetism and Electricity; Nos. 75–78, 81–82, 85–86, 88–96, and 98–99.

The exact correspondence between these 66 experiments in "advanced physics" and those contained in the "third list" designated by the letter C is shown by the table below. The experiments marked B. are those taken from the "second list". (Appendix VII.). It is understood that these experiments are to be offered for admission to Harvard College either in elementary or in advanced physics. In the former case, they should be replaced by an equal number of experiments marked "extra" in the "third list," in order to meet the college requirements for admission in Advanced Physics.

Harvard Adv., No.	Third List, No.	Harvard Adv., No	Third List, No.	Harvard Adv., No.	Third List, No.
2	12 C	43	33 B	71 II.	64 C
3	13 C	45	30 B	73	59 C
4	14 C	50	36 C	75	72 C
7	3 C	51	35 C	76	58 B
8	4 C	52	37 C	77	60 B
9	1 C	53	18 C	78	65 C
10	5 C	54	47 C	81	67 C
11	7 C	55 I	48 C	82	68 C
12	6 C	55 II.	16 C	85	66 C
13	8 C	56	17 C	86	61 C*
14	10 C	57	21 C	88	73 C
15	9 C	58	20 C	89	70 C
17	45 C	60	23 C	90	69 C
22	37 B	61	24 C	91	74 C
24	40 C	62	25 C	92	77 C*
25	42 C	63	25 B	93	75 C
26	36 B	64	27 C	94	78 C*
27	43 C	65	51 C	95	76 C
29	41 C	66	55 C	96	79 C*
32	27 B	67	53 C*	98 I.	62 C
35 I.	31 C	68	54 C	98 II.	63 C
35 II.	32 C	69	56 C	99	30 C
41	32 B	70	57 C		
42	31 B	71 I	58 C		

* Cases of only approximate correspondence.

This and the preceding lists of experiments have been prepared by the author with the view of satisfying both the letter and the spirit of the latest Harvard requirements (1890–1891). In view, however, of the frequent and extensive changes which have taken place in these requirements, teachers will do well to consult members of the physical department before deciding what particular experiments they propose to have their pupils offer for admission to the University.

APPENDIX IX.

AVERAGES OF VARIABLE QUANTITIES.

The average value of a variable quantity is frequently required in physical measurement, as, for instance, in cases where the *average* atmospheric temperature or pressure affect the results. If a sufficient number of observations be taken there is no especial difficulty in computing their average; but the average of a variable quantity can be found in general only through formulæ established by the differential and integral calculus. It is doubtful whether experiments involving the use of such formulæ should be included in an elementary course; but if included, every effort should be made to explain the formulæ to the student.

The teacher may, in certain cases, find it advisable to anticipate some of the principles of the calculus rather than to defer an experiment until these principles would naturally be explained. This, however, is not generally necessary.

It will be seen from the following demonstrations that the averages of variable quantities may be obtained in a great many cases by simple arithmetic, algebraic, or geometric processes *without the aid of the calculus*, and when so obtained can be tabulated and employed in place of the integrals to which they correspond.

It is important to present new problems in their simplest possible form. The ideas involved in processes of averaging are not only more familiar, but also simpler than in integration; for the integral is a quantity which necessarily (unlike the average) *differs in kind* from the quantity operated upon. The use of averages will be found, accordingly, to have certain marked advantages over the use of integrals for the purposes of elementary demonstration.

(*a*) *Numerical Averages.* The average of a given number of terms is defined as the sum of the terms divided by that number. It may be assumed that students are already familiar with arithmetical processes by which averages are obtained. The same processes may be extended to cases in which it is desired to find the average of numerical functions, provided of course that all the values to be averaged are finite, and limited in number.

The average of all integral numbers between 0 and 10 inclusive is, for instance, 5; the average of the squares of these numbers is 35; the average of the cubes of these numbers is 275. A slightly different result would be obtained if intermediate values of these functions were also averaged. If, for instance, every integral number of tenths were considered, the average of the numbers would be 5 as before; but the average of the squares would be 33.5, and the average of the cubes 252.5.

If now we should consider every integral number of hundredths, the average would become 33.3 + and 250 + respectively. The same would be true if we considered thousandths or millionths of a unit. It would appear, accordingly, that the numbers which we have found represent with an increasing degree of accuracy the average value of the square and the cube of a quantity varying by small but equal steps between the values 0 and 10.

(*b*) *Limits of Error.* The truth of this statement is capable of demonstration. The average value of the square of a quantity between 0 and 1 cannot, for instance, be greater than 1 (the maximum value), nor less than 0 (the minimum value). In the same way the average value of the square of all numbers between 1 and 2 cannot be greater than 4 nor less than 1, &c. It follows that the mean square of a continuous variable between the values 0 and 10 cannot be greater than the average of the squares 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100, nor less than the average of the squares 0, 1, 4, 9, 16, 25, 36, 49, 64, and 81. That is, this mean square is necessarily greater than 28.5, and less than 38.5; hence equal to 33.5 *within* 5 *units.*

In the same way, by considering all possible numbers between 0 and 10 which can be expressed by an integral number of units and tenths, it can be proved that the mean square in question is equal to 33.335 *within* 5 *tenths of a unit;* and a still closer approximation is obtained by considering averages through intervals of one hundredth of a unit each.

The mean cube of a continuous variable between given limits can be found in the same way with any required degree of accuracy by purely arithmetical processes. The same methods are applicable to the case of any function whatsoever — always excluding the case of infinite or imaginary values. This fact may be made use of for the purpose of demonstrating to a class in elementary physics the value of certain mathematical constants which are usually determined only by the aid of higher mathematics. An example will be found in section *g* of Appendix X., relating to Probable Error, in which the "Coefficient of Probability" is determined roughly in this way.

(*c*) *Average of a variable x.* The use of purely arithmetical processes is confined to cases in which quantities are to be averaged between given limits. It has been shown, for instance, that the average of all the numbers between 0 and 10 inclusive is 5. It will be found that the average of all numbers between 0 and 100 inclusive is 50, &c. The question naturally arises, is the average of all numbers between 0 and a given number *always* equal to one half of the given number?

Let us call the number n; then there are n terms to be averaged. The first is 0; the last is n; these two give an average of $(n + 0) \div 2 = \frac{1}{2} n$. The second (1) and next to the last $(n - 1)$ give similarly an average $((n - 1) + 1) \div 2 = \frac{1}{2} n$. The numbers can evidently be thus combined in pairs, each averaging $\frac{1}{2} n$, until, if n is even, all the numbers are paired off; or if n is odd, a single number $(\frac{1}{2} n)$ remains. Obviously the average of all these averages is in any case $\frac{1}{2} n$; hence this must be the average of all the numbers.

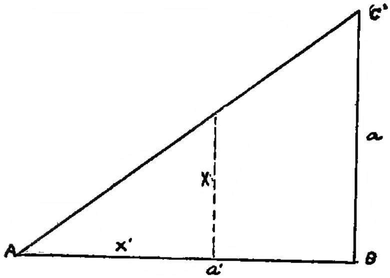
The same result would be obtained if we considered tenths or hundredths of a unit. We should have a greater number of pairs to be averaged, but as the value of each is $\frac{1}{2} n$, the average would be the same. We conclude, therefore, that the average value of a variable x between the value 0 and n is always equal to $\frac{1}{2} n$.

(*d*) *Notation of Averages.* The result obtained in the last section may be expressed as follows: —

$$\overset{o \;\rule[0.5ex]{2em}{0.4pt}\; n}{x} = \tfrac{1}{2} n. \qquad (1)$$

The line placed above the letter x denotes that an average is to be taken, and the limits of this average are indicated by the two values placed one at each end of the line. In such expressions variable quantities are customarily denoted by letters near the end of the alphabet (especially x, y, and z), while other letters, like numerals, denote constant quantities.

(*e*) *Geometrical Proof.* The area of a rectangle is found by multiplying together the base and altitude of the rectangle. The areas of other figures may similarly be found by multiplying together the base and *average* altitude of the figures. Thus the area (A) of an isosceles right-angle triangle ABC is equal to the product of the base ($AB = a' = a$) and its average altitude $x = x'$. That is:



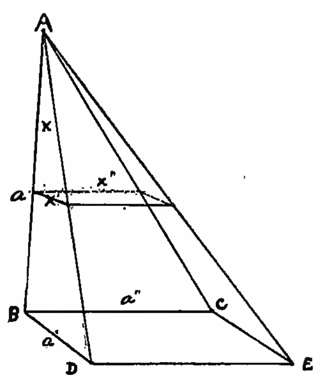
$$A = \overset{o\text{———}a}{x} \times a'$$

Now by geometry —

$$A = \tfrac{1}{2}\, a \times a';$$

hence, as before —

$$\overset{o\text{ —— }a}{x} = \tfrac{1}{2}\, a.$$



(*f*) *Average of* x^2. The volume of a rectangular block is found by multiplying together the cross-section and altitude of the block. In the same way the volume of other figures may be found by multiplying together the *average* cross-section and the altitude of these figures. Let the altitude of a pyramid $ABCDE$ be a, and its base $a' \times a'' = a^2$; then the cross-section of the pyramid at a distance x from the apex (A) is $x' \times x'' = x^2$. The volume V is therefore —

$$V = \overset{o \text{ —— } a}{x^2} \times a.$$

Now by geometry —

$$V = \tfrac{1}{3}\, a^2 \times a \text{; hence}$$

$$\overset{o \text{ —— } a}{x^2} = \tfrac{1}{3}\, a^2. \qquad (2)$$

We have already seen that the average of the squares of numbers from 0 to 10 is approximately equal to $33\frac{1}{3}$, and that when the squares are taken closer and closer together, there is a still closer approach to this value. The formula (2) shows that when all possible intermediate values are considered, the average is exactly $33\frac{1}{3}$. It also shows that in general the average value of the square of a quantity up to a given value is equal to one third of the square of this value.

(*g*) *Mechanical Proof.* To find the volume of a small prism, we multiply its length (l) by its (uniform) cross-section (q). To find the mass (m) of the prism, we multiply the result by the (uniform) density of the prism (d). To find the moment (M) of the prism about a distant point (o) in line with the axis of the prism, we multiply the result by the (nearly constant) arm (a) in question. That is —

$$M = l \times \bar{q} \times \bar{d} \times \bar{a} \text{ (nearly).}$$

Now if c is the distance from the centre of gravity to the point o, the moment of the prism about o is by definition —

$$M = m \times c;$$

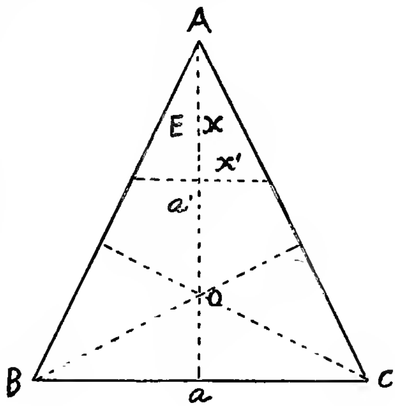
hence we have —

$$m \times c = l \times \bar{q} \times \bar{d} \times \bar{a}; \text{ or}$$

$$c = \bar{q} \times \bar{d} \times \bar{a} \times l \div m.$$

In a similar manner, the centre of gravity of any figure with respect to a given axis, is located by multiplying the quotient of the length of the figure by the mass into the *average* product of the cross-section, density, and arm corresponding to regularly increasing distances along the axis.

The distance from the apex A to the centre of gravity (O) of an isosceles triangle ABC, with base a and altitude a, and unit thickness and density throughout is found, accordingly, by averaging the products of the distances x from the apex by the cross-section, also equal to x, multiplying by the (vertical) length a, and dividing by the mass, $\frac{1}{2}a^2$. That is —



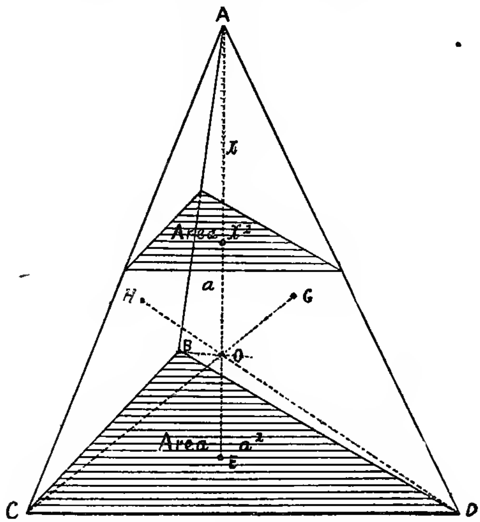
$$AO = \overset{o \text{ ——— } a}{x^2} \div (\tfrac{1}{2}a^2 \div a) = \overset{o \text{ ——— } a}{x^2} \div \tfrac{1}{2}a.$$

Now the centre of gravity of a triangle must lie at the common intersection of three lines, AO, BO, and CO, which bisect the sides BC, AC, and AB; for if the triangle be cut up into a series of bars parallel to either of the sides, each bar will balance about an axis which bisects it; hence the triangle as a whole must balance about such an axis, and this axis must contain the centre of gravity. Now the three lines in question can be shown by geometry to intersect at a point O, such that $AO = \frac{2}{3}a$. Hence we have, substituting, —

$$\overset{o \text{ —— } a}{x^2} \div \tfrac{1}{2}\, a = \tfrac{2}{3}\, a,$$

or as before $$\overset{o \text{ —— } a}{x^2} = \tfrac{1}{3}\, a^2.$$

(*h*) *Average of* x^3. In the same way the centre of gravity (O) of a pyramid $ABCD$, with altitude a, base a^2, and



density 1, is found by multiplying together the distances x and the cross-section x^2, and dividing the result by the ratio of the mass ($\frac{1}{3}\, a^3$) to the altitude (a). That is,

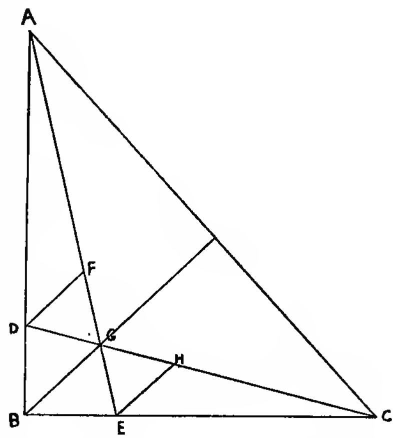
$$AO = \overset{o \text{ —— } a}{x^3} \div (\tfrac{1}{3}\, a^3 \div a) = \overset{o \text{ —— } a}{x^3} \div \tfrac{1}{3}\, a^2.$$

Now a pyramid must balance about any one of the four axes, AO, BO, CO, or DO, passing through the centre of gravity of the four sides, and hence through that of every section parallel to their sides; and since by geometry $AO = \frac{3}{4}\, a$, we have substituting —

$$\overset{o\,\text{———}\,a}{x^3} \div \tfrac{1}{3}\,a^2 = \tfrac{3}{4}\,a;\ \text{or}$$

$$\overset{o\,\text{———}\,a}{x^3} = \tfrac{1}{4}\,a^3. \qquad (3)$$

(*i*) *Average of* x^n. Let ABC be an isosceles right triangle, with density 0 along the line AC and density x^{n-1} at a distance x from AC measured *either* horizontally or vertically; then we have seen that the centre of gravity of a horizontal or vertical section of the triangle is (if $n = 1, 2$, or 3) at a point such that the distance between it and AB or BC is to the distance between it and AC as $1 : n$. The centres



of gravity of all sections lie, therefore, on the lines AE or CD, dividing the sides BC and AB in the same proportion. so that —

$$BE : EC :: BD : DA :: 1 : n.$$

The centre of gravity of the triangle is therefore at the intersection G of AE and CD. Drawing BG and, parallel to it, EH and DF, we have by similar triangles —

$$GH : HC :: GF : FA :: 1 : n.$$

Now by construction the triangles HGE and DGF are equal; hence $GE = GF = FA \div n$, and $DG = GH = HC \div n$.

It follows that the distance of the centre of gravity from the base of the triangle is to the distance of the apex from the base as $GE : AE$, or as $GE : AF + FG + GE$, or as

$$\frac{1}{n} : 1 + \frac{1}{n} + \frac{1}{n}, \text{ or as } 1 : n + 2.$$

Denoting the altitude of the triangle by a, we have, therefore, for the vertical distance d of the centre of gravity from the apex —

$$d = \frac{n+1}{n+2}\,a.$$

Now applying the ordinary methods we find the average density of a horizontal cross-section to be —

$$\overset{o\,\text{——}\,x}{x^{n-1}} = \frac{x^{n-1}}{n}$$

which multiplied by the cross-section (x) gives $\frac{x^n}{n}$ for the mass of the cross-section. The total mass is therefore (if $n = 1$, 2, or 3) —

$$a \times \overset{o\,\text{——}\,a}{\frac{x^n}{n}} = \frac{a^{n+1}}{n\,(n+1)}.$$

The ratio of the mass to the altitude is —

$$\frac{a^{n+1}}{n\,(n+1)} \div a = \frac{a^n}{n\,(n+1)}.$$

The moment of a given section about the apex is —

$$x \times \frac{x^n}{n} = \frac{x^{n+1}}{n}.$$

The distance of the centre of gravity is accordingly —

$$d = \overset{o \,\text{———}\, a}{\frac{x^{n+1}}{n}} \div \frac{a^n}{n\,(n+1)} = \frac{n+1}{n+2}\,a.$$

It follows that—

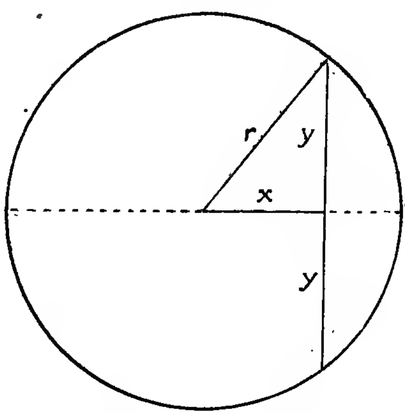
$$\overset{o \,\text{———}\, a}{x^{n+1}} = n \times \frac{a^n}{n\,(n+1)} \times \frac{n+1}{n+2}\,a = \frac{a^{n+1}}{n+2}.$$

It has been proved that if $n = 1$, 2, or 3,

$$\overset{o \,\text{———}\, a}{x^{n}} = \frac{a^n}{n+1};$$

it follows that the same expression holds good for x^{n+1}; hence it holds for x^4, hence for x^5, &c. In other words, we have in general (for positive integral powers of n),

$$\overset{o \,\text{———}\, a}{x^{n}} = \frac{a^n}{n+1}. \tag{4}$$



(*j*) *Average of Trigonometric Functions.* Passing now to the case of a circle of radius, r, and area, A, which we consider as the product of its diameter ($2r$) and *average* cross-section ($2\,y = 2\,\sqrt{r^2 - x^2}$), we find—

$$A = \pi\, r^2 = 2\, r \times \underset{2\,\sqrt{(r^2 - x^2)}}{-r \,\rule{8em}{0.4pt}\, + r}$$

whence, dividing through by $4\, r$,

$$\underset{(r^2 - x^2)^{\frac{1}{2}}}{-r \,\rule{6em}{0.4pt}\, + r} = \frac{\pi\, r}{4} \qquad (5)$$

Again, from the properties of the circle we have $\sin^2 x + \cos^2 x = 1$; hence

$$\overline{\sin^2 x + \cos^2} \;\; x = 1.$$

Now $\sin\, 0^\circ = \cos\, 90^\circ$; $\sin\, 1^\circ = \cos\, 89^\circ$, &c. Hence,

$$\underset{\sin^2 x}{0^\circ \,\rule{4em}{0.4pt}\, 90^\circ} = \underset{\cos^2 x}{0^\circ \,\rule{4em}{0.4pt}\, 90} \;,$$

and we have—

$$\underset{\sin^2 x}{0^\circ \,\rule{4em}{0.4pt}\, 90^\circ} = \underset{\cos^2 x}{0^\circ \,\rule{4em}{0.4pt}\, 90^\circ}$$

$$= \tfrac{1}{2} \underset{\sin^2 x + \cos^2 x}{0^\circ \,\rule{8em}{0.4pt}\, 90^\circ} = \tfrac{1}{2} \qquad (6)$$

The results already obtained are sufficient to illustrate certain methods by which functions may be averaged without the aid of the calculus when, through geometrical construction or otherwise, the averages of similar functions are known.

APPENDIX X.

PROBABILITY OF ERRORS.

(*a*) *Definitions.* The observed value (o) of a quantity differs, as has been pointed out in § 156, from the true value (q) by an amount (e) which is called the error of observation. Let the errors in a series of n observations be distinguished by subscript numerals, then the average error, $\bar{e}$, is defined by the equation

$$\bar{e} = [\Sigma](e_1 + e_2 + e_3 + \ . \ . \ . + e_n) \div n, \quad (1)$$

where the sign $[\Sigma]$ indicates that the numerical values of the errors are to be added together without regard to algebraic signs.

The mean square of the errors ($\overline{e^2}$) is defined by the equation

$$\overline{e^2} = \Sigma\ (e_1^2 + e_2^2 + e_3^2 + \ . \ . \ . + e_n^2) \div n. \quad (2)$$

The "error of the mean square" (ϵ) is defined simply as the square root of the mean square of the errors; that is

$$\epsilon = \sqrt{\overline{e^2}} = \sqrt{\Sigma(e_1^2 + e_2^2 + e_3^2 + \ . \ . \ . + e_n^2) \div n}. \quad (3)$$

In a long series of observations in which the errors are due to a great number of causes combining together in every possible way, the "probable error" is defined as one which is neither greater nor less than the majority. That is, if the errors are arranged in the order of their magnitude (without regard to signs), as follows, —

$$e_0,\ e_1,\ e_2,\ e_3\ \ . \ . \ .\ \ e_p,\ e_{p+1},\ e_{p+2},\ e_{p+3},\ \ . \ . \ .\ \ e_n,$$

the "probable error," e_p, is defined by the equation —

$$p = \tfrac{1}{2} n. \qquad (4)$$

The "coefficient of probability," π, is defined as the ratio of the probable to the mean error; that is,

$$e_p = \pi\epsilon. \qquad (5)$$

This definition is customarily extended to the case of short series of observations. In such cases the equation serves, however, to define the probable error, not the coefficient of probability. The value of this coefficient will be determined roughly in the course of the following elementary investigation (see (g)).

(b) *Distribution of errors.* The simplest case in the theory of errors is one in which practically a single source of error exists. Let us take as an example the indication of a spring balance which (from rust or other causes) is affected by friction much more than by any other cause. If the pointer of such an instrument be pulled down to a given reading, the reading is generally too small. If the pointer is first pulled down too far, then allowed to recover, the indication is generally too great. We will suppose for simplicity that the error is 1 unit in each case, — a kilogram for instance; then the average error, the mean square of the errors, and the error of the mean square, are all by definition equal to 1.

Now let it be required to find the weight of a body which rests, as in ¶ 159 (1) upon two such spring balances. Let us suppose that each balance is pulled down to a given reading in one half of the observations, but allowed to recover to the reading in the other half of the observations. Let us suppose, moreover, that the two balances are treated first in the same way, then in opposite ways. It follows that in one half of the observations the errors due to friction in the balances will offset each other; but that in the other half, they will combine together so as to give a resultant $\pm$ 2.

In point of fact, when a weight is thrown upon a spring balance, it usually makes several oscillations before it comes to rest. There is accordingly a (nearly) equal chance that the pointer may be arrested by friction either above or below its mean resting-point. We may assume, therefore, that in (about) half of the observations the readings will be increased, in the other half diminished by friction. Let us first consider those observations in which the readings of one spring balance are too great. In the absence of any necessary reason why the readings of the second spring balance should be affected by those of the first, we may assume that these readings are also too great in about half the cases under consideration, — that is, one fourth of the total number of cases, — and too small in the remaining fourth. In the same way, by considering that half of the observations in which the readings of the first spring balance are too small, we find a third fourth in which the readings of the second spring balance are also too small, and a fourth fourth in which they are too great. We find as before that the errors offset each other in one half (or two fourths) of the observations, and that they combine together in the other half. That is, the distribution of errors due to chance is (nearly) the same as that in which care is taken to bring about with equal frequency every possible combination.

The four ways in which two sources of error, each equal to $\pm$ 1, may combine together, may be written as follows:

$$A = +1 + 1 = +2; \qquad C = -1 + 1 = 0;$$
$$B = +1 - 1 = 0; \qquad D = -1 - 1 = -2.$$

Each of these combinations differs from the next simply in the fact that the sign of *one* error is reversed. It is always assumed in the treatment of accidental errors that positive and negative errors are equally probable.[1] It follows that each of the combinations named above is just as probable as the next; hence all are equally probable.

Now let us consider a third source of error, — a body, for instance, suspended in parts from three spring balances. Each of the four combinations named above gives two, in one of which the resultant is increased, in the other diminished by the new source of error. There are accordingly 8 combinations in all, namely,

$$\begin{array}{ll}
a = A + 1 = +3; & e = C + 1 = +1; \\
b + A - 1 = +1; & f = C - 1 = -1; \\
c = B + 1 = +1; & g = D + 1 = -1; \\
d + B - 1 = -1; & h = D - 1 = -3;
\end{array}$$

In one case the resultant is $+3$, in 3 cases $+1$, in 3 cases -1, in 1 case -3.

We come next to four sources of error. Each of the previous 8 combinations yields 2, so that there are 16 in all, namely,

$$\begin{array}{llll}
a + 1 = 4 & c + 1 = 2 & e + 1 = 2 & g + 1 = 0 \\
a - 1 = 2 & c - 1 = 0 & e - 1 = 0 & g - 1 = -2 \\
b + 1 = 2 & d + 1 = 0 & f + 1 = 0 & h + 1 = -2 \\
b - 1 = 0 & d - 1 = -2 & f - 1 = -2 & h - 1 = -4.
\end{array}$$

In 1 case we have a resultant $+4$, in 4 cases $+2$, in 6 cases 0, in 4 cases -2, in 1 case -4.

(*c*) *Table of Combinations.* In the same way, by purely arithmetical processes, the distribution of errors due to any number of sources may be obtained. The results already calculated for the case of 4 sources of error are compared in

[1] If this were not the case we should have a constant error as the result. It is supposed that all *constant* errors are eliminated.

the table below with similar results corresponding respectively to 16 and to 100 sources of error.

Magnitude of the Error.	No. of combinations giving a resultant of this magnitude when the No. of sources = 4.	No. of sources = 16.	No of sources = 100.	Magnitude of the Error.
0	6	12870	1009×10^{26}	0
+2	4	11440	989 "	−2
4	1	8008	932 "	−4
6	0	4368	844 "	−6
8	0	1820	734 "	−8
10	0	560	614 "	−10
12	0	120	494 "	−12
14	0	16	381 "	−14
16	0	1	282 "	−16
18	0	0	201 "	−18
20	0	0	137 "	−20
22	0	0	90 "	−22
24	0	0	57 "	−24
26	0	0	35 "	−26
28	0	0	20 "	−28
30	0	0	11 "	−30
32	0	0	6 "	−32
34	0	0	3 "	−34
36	0	0	1+ "	−36
38	0	0	1− "	−38
+40	0	0	0+ "	−40
Total	16	65,536	$12,677 \times 10^{26}$	Total

(*d*) *Probability Curve.* The results contained in the last table are represented graphically in the figure. In constructing this figure, the vertical distances were made proportional to the number of combinations in a given column, the first number in the column being taken equal to 1. The horizontal distances were made proportional to the magnitude of the resulting errors represented by a given curve; but in plotting the first curve, the results were divided by 2, in the second by 4, in the third by 10. The similarity of the curves when thus reduced to a common scale becomes apparent.

The three curves in the figure represent the relative probability of errors of a given magnitude resulting (1) from combinations of 4 sources, each equal to $\pm \frac{1}{2}$; (2) from combinations of 16 sources, each equal to $\pm \frac{1}{4}$; and (3) from combinations of 100 sources, each equal to $\pm \frac{1}{10}$, — because, in plotting the curves, we divided the resultants by 2, 4, and 10, respectively. The (approximate) agreement of the curves serves, therefore, to illustrate the truth of a general law, that the distribution of errors due to n sources, each equal to $\pm 1 \div \sqrt{n}$, is in all cases (approximately) the same.

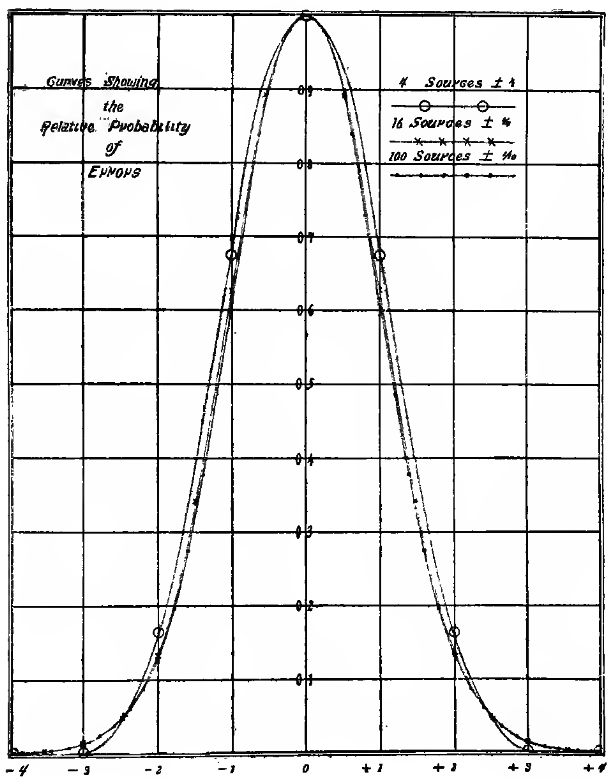
It follows from this law that the average error and the error of mean square must also, under the conditions named, be (approximately) the same. It also follows that the resultants due to *n unit* sources of error are, other things being equal, proportional (approximately) to the square root of n. Hence the average error and the error of the mean square are also (approximately) proportional to the square root of the number of sources from which they arise. This law, which holds only approximately for average errors, can be proved exactly in the case of the error of the mean square.

(*e*) *Calculation of Mean Squares.* The results which have been worked out in the distribution of errors enable us to calculate in certain cases the mean square of these errors. When a single unit source of error exists, the resultant is always ± 1, hence the mean square is $+1$, or $\epsilon_1^2 = 1$. With 2 unit sources, we have four combinations, in two of which the resultant is 0, in the other two ± 2. The mean square is accordingly

$$\epsilon_2^2 = (2 \times 0 + 2 \times 4) \div 4 = 2.$$

In the same way, in the case of three unit sources, we find

$$\epsilon_3^2 = (2 \times 9 + 6 \times 1) \div 8 = 3.$$


Curves Showing the Relative Probability of Errors
4 Sources ± 1
16 Sources ± ½
100 Sources ± 1/10
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
− 4
− 3
− 2
− 1
0
+ 1
+ 2
+ 3
+ 4

and in the case of four unit sources —

$$\epsilon_4^2 = (2 \times 16 + 8 \times 4 + 6 \times 0) \div 16 = 4.$$

We notice that the mean square of the resultant error is in each case equal to the number of unit sources of which it is composed.

It is easy to show that this law is perfectly general. Suppose we have found the distribution of errors due to n unit sources to be such as to give

x resultants of the magnitude e,
y resultants of the magnitude f, *&c.*,

and that a new unit source of error is now added, so as to increase half of the previous resultants by 1, and to diminish the other half by 1. We shall then have

$$\begin{cases} \frac{1}{2}\, x \text{ resultants of the magnitude } e + 1 \\ \frac{1}{2}\, x \text{ resultants of the magnitude } e - 1 \end{cases}$$
$$\begin{cases} \frac{1}{2}\, y \text{ resultants of the magnitude } f + 1 \\ \frac{1}{2}\, y \text{ resultants of the magnitude } f - 1, \text{ \&c.} \end{cases}$$

The squares of these errors will be, in

$$\begin{cases} \frac{1}{2}\, x \text{ cases, } e^2 + 2\,e + 1 \\ \frac{1}{2}\, x \text{ cases, } e^2 - 2\,e + 1, \text{ \&c.,} \end{cases}$$

the average in the x cases being $e^2 + 1$.

In the same way the average in y cases is $f^2 + 1$, &c. The mean square of all the errors is therefore

$$\begin{aligned} \epsilon^2_{n+1} &= [x\,(e^2 + 1) + y\,(f^2 + 1) + \text{\&c.}] \div [x + y + \text{\&c.}] \\ &= [x\,e^2 + y\,f^2 + \text{\&c.} + x + y + \text{\&c.}] \div [x + y + \text{\&c.}] \\ &= [x\,e^2 + y\,f^2 + \text{\&c.}] \div [x + y + \text{\&c.}] + 1. \end{aligned}$$

Now if it has been shown that in the case of n sources of error, the mean square of the resultant is equal to n, or if

$$\epsilon^2_n = [x\,e^2 + y\,f^2 + \&c.] \div [x + y + \&c\,] = n,$$

we have, substituting,

$$\epsilon_{n+1}^2 = \epsilon_n^2 + 1 = n + 1.$$

The law has been shown to hold in the case of four sources of error, hence it holds for 5, hence for 6, &c.; that is, it holds for any number of sources of error.

We have therefore, the following law: —

The mean square of the errors resulting from every possible combination of a given number of unit sources is equal to the number in question.

(*f*) *Law of Mean Squares.* If, instead of combining an error $\pm$ 1 with x errors of the magnitude e, we should combine an error of the magnitude $\pm\, e'$ with them, the mean square of the resultant would be $e^2 + (e')^2$.

If therefore a new source of error arises in which the resultants are in x' cases, $\pm\, e'$, in y' cases, $\pm\, f'$, &c., the mean square of the resultant will be

$$\begin{aligned}
&[x'\,(e^2 + (e')^2) + y'\,(e^2 + (f')^2) + \&c.] \div [x' + y' + \&c.] \\
&= [x'\,e^2 + y'\,e^2 + \&c. + x'\,(e')^2 + y'\,(f')^2 + \&c.] \div [x' + \\
&\qquad y' + \&c.] \\
&= e^2 + [x'\,(e')^2 + y'\,(f')^2 + \&c.] \div [x' + y' + \&c.] \\
&= e^2 + \epsilon^2,
\end{aligned}$$

where ϵ^2 represents the mean square of the resultant errors due to the new source. In the same way, the mean square of the resultant due to combining the new source of error with the resultants f, is to make the mean square of these resultants, $f^2 + \epsilon^2$; that is, the squares of all the previous resultants are increased on the average by the amount ϵ^2; hence also the mean square of these resultants.

The law of mean squares may now be stated in general as follows: —

The mean square of the errors resulting from combinations of accidental causes is equal to the sum of the mean squares due to each separate cause.

(*g*) *Coefficient of Probability.* The coefficient of probability, or ratio of the probable error to the error of the mean square depends upon the manner in which errors due to a great variety of sources are distributed in a long series of observations. A calculation of this coefficient may be based upon any of the curves plotted in the figure (see *d*). We will choose the curve due to 100 sources of error, not only because this curve is better defined than the others, but also because it has been found not to differ perceptibly from curves obtained with 1,000 or 1,000,000 sources of error. There is in fact a wide limit within which the results may be applicable. At the same time it must not be forgotten that the other curves yield results which in a great many cases would be more conformable to the facts.[1]

We find from the table (see *c*) that the number of combinations due to 100 unit sources which give resultants less than $\pm$ 6 is,

$$(932 + 989 + 1009 + 989 + 932) \times 10^{26} = 4851 \times 10^{26}.$$

In the same way we find the following numbers: —

Less than	$\pm$ 6,	4851×10^{26}	Less than	$\pm$ 8,	6539×10^{26}
Equal to	$\pm$ 6,	1688 "	Equal to	$\pm$ 8,	1468 "
Greater than	$\pm$ 6,	6138 "	Greater than	$\pm$ 8,	4670 "
Total		12677×10^{26}	Total		12677×10^{26}

[1] Most observations are affected by a few large sources of error, in comparison with which the smaller sources may be neglected. The distribution of errors may also be altered by the nature of the sources from which they arise.

Evidently the probable error is greater than $\pm$ 6, because there are more combinations giving a greater than a smaller resultant. The probable error is also evidently less than $\pm$ 8. It would be sufficiently accurate for most purposes to call the probable error in such a case 7 units. A more precise value may be found by interpolation.

In practice resultant errors are not confined to certain definite values such as $\pm$ 6 or $\pm$ 8, with gaps between them, but we find them, as indicated by the continuous curves of the figure, more or less uniformly distributed. We may assume therefore, as a first approximation, that the 1468×10^{26} combinations which in a hundred sources of error each equal to $\pm$ 1 would give resultants equal to $\pm$ 8, are in practice distributed evenly between $\pm$ 9 and $\pm$ 7; while the 1688×10^{26} resultants of $\pm$ 6 reach from $\pm$ 7 to $\pm$ 5. There would in this case be about 844×10^{26} combinations between $\pm$ 6 and $\pm$ 7, and $(844 + 4851) \times 10^{26}$, or 5695×10^{26} combinations below $\pm$ 6. The probable error (e_p or p) corresponds to half the total number of combinations, that is, to $\frac{1}{2}\, 12677 \times 10^{26} = 6338 \times 10^{26}$, nearly. Applying the ordinary rules for interpolation,[1] we find

$$p = 6 + (6338 - 5695) \div 844 = 6.7 +.$$

Now the error of the mean square due to 100 unit sources of error is, according to section (*e*), equal to $\sqrt{100}$ or 10 units; hence the coefficient of probability is 0.67 +.

(*h*) *Probable Errors of Sums and Differences.* When several quantities are added together, the sum is affected by

[1] A more exact value of this coefficient may be obtained by methods of interpolation involving 2d and 3d differences, or by considering a greater number of sources of error. Such a value of the coefficient is 0.67449. A similar investigation shows that the ratio of the probable to the average error is 0.8453.

the errors in each quantity. Since the probable error is always proportional to the error of mean square, we find, from the law of mean squares (see (*f*)), that the probable error (p) of the sum is equal to the square root of the sum of the squares of the probable errors (p_1 p_2, &c.) of all its components; that is,

$$p = \sqrt{p_1^2 + p_2^2 + \&c.}$$

The case is somewhat simpler when all the quantities added together are affected by the same sources of error. The number of sources of error in the result is then proportional to the number of the quantities, hence also the mean square of the errors. It follows that the probable error is proportional in such cases to the square root of the number of terms added together.

The same principles apply to the calculation of resultant errors in differences as in sums; for negative errors are supposed to be just as frequent as positive errors, hence it can make no difference in a long series of results, as far as the *errors* are concerned, whether one quantity is to be added or subtracted from another.

The probable error of the sum or difference of two quantities affected by the same sources of error is therefore greater than the probable error of the quantities themselves in the ratio of $\sqrt{2}$ to 1. Evidently the error bears a greater proportion to the difference of the two quantities than to their sum. Methods of difference are, therefore, relatively inaccurate (see § 38).

(*i*) *Multiplication and Division of Errors.* When an observed quantity is multiplied or divided by a constant, the error of observation is also evidently multiplied or divided by the same constant; hence the probable error is increased or diminished in the same proportion.

(*j*) *Probable Errors of Averages.* If the probable error of a single observed quantity is p, that of the sum of n similar quantities is, as we have seen (see (h)), $p\sqrt{n}$. In finding the average of n quantities, we divide their sum by n; hence, according to the last section, the probable error is also divided by n. It follows that the probable error of the average of n similar observations is $p\sqrt{n} \div n = p \div \sqrt{n}$. That is, *the probable error of the average of several observations affected by like sources of error varies inversely as the square root of the number of observations.*

(*k*) *Estimation of probable error from the differences between separate observations and their mean.*

Let $q + e_1$, $q + e_2$, $q + e_3$ . . . $q + e_n$ denote a series of n observations of a quantity q. The mean (m) of these observations is then

$$m = q + (e_1 + e_2 + e_3 + \ldots + e_n) \div n.$$

The difference (d_1) between the first observation and the mean is

$$d_1 = q + e_1 - m = e_1 - (e_1 + e_2 + e_3 + \ldots + e_n)\, n$$
$$= \frac{n-1}{n} e_1 + \frac{e_2}{n} + \frac{e_3}{n} + \ldots \frac{e_n}{n}.$$

That is, the difference between a given observation and the mean is found by multiplying one of the errors by $(n-1) \div n$, and the other $(n-1)$ errors by $1 \div n$. If ϵ^2 is the mean square of the errors, the mean square of one of the terms must be $\epsilon^2 (n-1)^2 \div n^2$; while the mean square of the other $(n-1)$ terms will be $\epsilon^2 \div n^2$; so that the sum of these mean squares, which is equal to the mean square of the differences

$$\overline{d^2} = \frac{\epsilon^2 (n-1)^2}{n^2} + (n-1)\frac{\epsilon^2}{n^2} = \frac{n-1}{n}\epsilon^2. \qquad (1)$$

Conversely we have

$$\epsilon^2 = \frac{n}{n-1}\,\overline{d^2}. \qquad (2)$$

From this and preceding formulæ, we find the following rule for the calculation of probable error: Add all the observations together, divide by their sum to find the mean, subtract the mean from each observation to find the "differences;" square all these differences, add the squares together, divide by the number of observations less one, to find the mean square of the error of observation; extract the square root of this result to find the error of the mean square, multiply the error of the mean square by 0.67449 (0.7, *nearly*) *to find the probable error of observation; divide by the square root of the number of observations to find the probable error of the mean of the observations.*

(*l*) *Example of the calculation of probable error* (see § 50).

A. No of observations.	B. Boiling point observed.	C. Differences from the average	D Squares of the differences.
1	78°.79	+ 0°.29	0.0841
2	78.33	− 0.17	289
3	78.02	− 0.48	2304
4	78.93	+ 0.43	1849
5	78.46	− 0.04	16
6	78.67	+ 0.17	289
7	78.00	− 0.50	2500
8	78.81	+ 0.31	961
9	78.43	− 0.07	49
10	78.56	+ 0.06	36
Sum	10) 785.00	10) [2 52]	9) 0.9134
Average	78.500	[0.252]	0.1015

Error of the mean square, $\sqrt{0.1015} =$ 0.318 +

Probable error of observation, (0.67 +) × (0.318 +) = 0.21 +

Probable error of the mean, $0.21 \div \sqrt{10} = 0.21 \div 3.16 =$ 0.07

Final result for the boiling-point of alcohol, 78°.50 ± 0°.07

(*m*) *Probable errors of products and quotients.* The probable errors of products and quotients are easily calculated by considering the proportion which the component and resultant errors bear to the quantities which they affect. We have, for instance, in the notation previously employed (§ 156): —

$$o_1\, o_2 = q_1 \times q_2 \times \left(1 + \frac{e_1}{q_1}\right)\left(1 + \frac{e_2}{q_2}\right) =$$

$$q_1 \times q_2 \times \left(1 + \frac{e_1}{q_1} + \frac{e_2}{q_2}\right) \text{ nearly,}$$

$$\frac{o_1}{o_2} = \frac{q_1}{q_2}\left(1 + \frac{e_1}{q_1}\right) \div \left(1 + \frac{e_2}{q_2}\right) = \frac{q_1}{q_2}\left(1 + \frac{e_1}{q_1} - \frac{e_2}{q_2}\right) \text{ nearly,}$$

neglecting in both cases terms which involve powers or products of the small ratios $(e : q)$; that is, neglecting terms of the second or higher degrees of smallness. It is evident from these formulæ that the proportional errors, $e_1 : q_1$, &c., are compounded in products and quotients just as ordinary errors are in sums and differences. The probable error (p) of a quantity q may, therefore, be calculated from the probable errors, p_1, p_2, p_3, &c., of its factors, q_1, q_2, q_3, &c., by the formula

$$\frac{p}{q} = \sqrt{\left(\frac{p_1}{q_1}\right)^2 + \left(\frac{p_2}{q_2}\right)^2 + \left(\frac{p_3}{q_3}\right)^2 + \text{\&c.}}$$

The same formula is to be employed whether the factors occur in the numerator or in the denominator of the fraction by which the quantity q is determined.

(*n*) *Probable Errors of Powers and Roots.* When an observed quantity, o, is raised to the power n, the result may be expressed: —

$$o^n = \left(q\left(1 + \frac{e}{q}\right)\right)^n = q^n\left(1 + \frac{e}{q}\right)^n =$$

$$q^n\left(1+\frac{ne}{q}+\frac{n\,(n-1)}{2}\,\frac{e_2}{q_2}+\&c.\right)=q_n\left(1+n\,\frac{e}{q}\right) \text{ nearly.}$$

The effect of raising an observed quantity to the power n is therefore to increase the proportional error, $e : q$, in the proportion $1 : n$. Since all such errors are increased in this proportion, the proportion which the probable error bears to the quantity which it affects must be increased in the same proportion.

The effect of extracting the n^{th} root of a quantity is the same as that of raising it to the power $1 \div n$; that is, the ratio of all errors, and hence that of the probable error to the quantities affected, is diminished in the ratio of $n : 1$.

(*o*) *General Method for finding the Probable Error of a Result.* The data from which results are calculated may generally be expressed in the form $q_1 = o_1 \pm e_1, q_2 = o_2 \pm e_2$, &c. The result r is then calculated from the values o_1, o_2, &c. Then the value $o_1 \pm e_1$ is substituted, and a new value of the result r_1 is calculated. Next the original value o_1 is employed, but $o_2 \pm e_2$ is substituted for o_2; and the corresponding value of the result r_2 is found, &c. The differences $d_1 = r_1 - r$; $d_2 = r_2 - r$, &c., represent the magnitude of the errors in the result due to the probable error in the several data. We have accordingly, from the law of mean squares, for the probable error (p) of the result

$$p = \sqrt{d_1^2 + d_2^2 + \&c.} \qquad (1)$$

(*p*) *Method for Determining the best possible Distribution of Time.* It is generally easy to see, from expressions for the probable error of a result, which of the data have the greatest influence upon the result. The number of observations upon which such data depend, should evidently be increased, other things being equal, in preference to observations for the less important data. Let us suppose, however, that such observations are exceedingly difficult, or that the number already made is so great that little comparative advantage can be gained by spending upon them an additional (limited) amount of time. The question then arises, would it not be better to spend the *same* amount of time upon some of the less important data?

The question is one to which it is easy in most cases to give at least an approximate answer. First decide how much time can be spent; estimate from the results of experience how many observations of each kind can be made in this length of time. Calculate the diminution of the probable error in the case of each of the data due to the additional number of observations, and find as in the last section, the corresponding reduction in the probable error of the result. That distribution of time is of course the best which gives the greatest reduction in this probable error. Certain practical rules concerning the distribution of time will be found in § 49.

(*q*) *Method of Least Squares.* We have seen that the mean square of the errors of observation is an indication either of the number or of the magnitude of the sources of error. We make use of this fact in estimating the relative accuracy or inaccuracy of different methods of observation. That method is, other things being equal, the best which makes the sum of the squares of the errors the least.

In calculating errors we have to assume more or less knowledge of the true value of the quantity observed. Any error in such an assumption introduces a new (apparent) source of error into the observations. It therefore tends, on the whole, to increase (apparently) the mean squares of the errors. Of two assumptions, we choose therefore, other things being equal, that which makes the sum of the squares of the errors of observation appear to be the least.

Let us take, for example, the case of a brass rod, the length of which was found to be 1000.0 *mm.* at 0°, 1001.7 *mm.* at 100°, and 1004.0 at 200°. The most probable value of the coefficient of expansion in such a case evidently lies between the maximum value observed (0.000023 from 100° to 200°) and the minimum value (0.000017 from 0° to 100°). The most probable value of the length of the rod at 0° lies moreover between 999.4 *mm.* (which would correspond to a length 1001.7 *mm.* at 100° and a coefficient of expansion 0.000023) and 1000.6 *mm.*; which would correspond to a length 1004.0 at 200° and a coefficient of expansion 0.000017. Let us assume that the length at 0° and the coefficient of expansion are half-way between these limits, that is 1000.0 *mm.* and 0.000020 respectively. The length at 0°, 100°, and 200°, should then be 1000.0, 1002.0, and 1004.0, respectively. This would make the errors of observation (expressed in tenths) 0, — 3, and 0, respectively; hence the sum of the squares would be 9. The sum of the squares in this and other cases is shown in the table below:

	.000017	.000018	.000019	.000020	.000021	.000022	.000023
999.4	216	161	116	81	56	41	36
999.5	171	122	83	54	35	26	27
999.6	132	89	56	33	20	17	24
999.7	99	62	35	18	11	14	27
999.8	72	41	20	9	8	17	36
999.9	51	26	11	6	11	26	51
1000.0	36	17	8	9	20	41	72
1000.1	27	14	11	18	35	62	99
1000.2	24	17	20	33	56	89	132
1000.3	27	26	35	54	83	122	171
1000.4	36	41	56	81	116	161	216
1000.5	51	62	83	114	155	206	267
1000.6	72	89	116	153	200	257	324

The smallest value in this table is 6, corresponding to the coefficient of expansion .000020, and a length at 0° equal to 999.9 *mm.* The most probable assumption which we can make, therefore [without considering values intermediate between those given in the table], is that the length of the bar really was 999.9 *mm.* at 0° and that its coefficient of expansion was .000020.

The validity of this conclusion evidently depends upon the truth of the assumption that the bar has a constant coefficient of expansion.

The three observations given above indicate that the coefficient of expansion is greater from 100° to 200° than from 0° to 100°; but in the absence of a greater number of observations, there is no way of testing the truth of this indication. Any theory in regard to the variation of the coefficient of expansion would in general be investigated by the method of least squares.

It is of course unnecessary in practice to tabulate more than a few values, in order to see where the least square lies. This method of approximation is not necessarily longer than the calculus methods ordinarily employed; and has the advantage (which the teacher will see by referring to examples in well known text-books) of giving in many cases a more accurate result.

(*r*) *Advantage of the Arithmetic Mean.* If m is the arithmetic mean of n observed quantities, $o_1, o_2, \ldots o_n$, affected by like sources of error, and if the differences of these quantities from the mean are $d_1, d_2, \ldots d_n$, we have $o_1 = m + d_1$; $o_2 = m + d_2$; . . $o_n = m + d_n$. Hence, adding and dividing by n, we have

$$m = (o_1 + o_2 + \ldots + o_n) \div n =$$
$$(m + d_1 + m + d_2 + \ldots + m + d_n) \div n$$
$$= m + (d_1 + d_2 + \ldots + d_n) \div n.$$

It follows that $d_1 + d_2 + \ldots + d_n = o$. The mean square of the differences from the mean m is

$$\epsilon^2 = (d_1^2 + d_2^2 + \ldots + d_n^2) \div n.$$

The mean square of the differences from any other value differing from m by the amount e is

$$\begin{aligned} E^2 &= [(d_1 + e)^2 + (d_2 + e)^2 + \ldots + (d_n + e)^2] \div n \\ &= [(d_1^2 + d_2^2 + \ldots + d_n^2) + (ne^2) + \\ &\quad (2d_1 e + 2d_2 e + \ldots + 2d_n e)] \div n \\ &= \epsilon^2 + e^2 + 2ne\,(d_1 + d_2 + \ldots + d_n) \div n. \end{aligned}$$

The last term in parenthesis is, as we have seen, equal to 0; hence we have simply

$$E^2 = \epsilon^2 + e^2.$$

That is, the mean square of the differences between the results of observation and their arithmetic mean is less than the mean square of the differences from any other value.

It follows from the principle of least squares that the arithmetic mean of a number of observations affected by like sources of error is the most probable value of the quantity observed which can be derived from these observations.

(*s*) *Weight of different results.* Let us suppose that on one day we have made 20 observations of the boiling-point of some alcohol, the result being 78°.58 ± .06; and that on another day we have made 80 observations in exactly the same manner, with the result 78°.48 ± .03. It follows from the last section that the most probable value of the boiling-point is that obtained by adding the total 100 results together, and dividing by their number (100). Let us suppose, however, that the original observations are lost. It is seen, nevertheless, that the sum of the first 20 must have been 20 × 78°.58, or 1571.60 and that the sum of the last 80 must have been 80×78.48, or 6278.40. The total of the 100 observations must, therefore, have been 7850.00, hence the average is 78.50.

The number of observations of a given sort represented in a result determines what is called the weight of the result. Evidently if the weights of several data r_1, r_2, &c., are w_1, w_2, &c., the most probable value of the result, R, is

$$R = \frac{w_1 r_1 + w_2 r_2 + \&c.}{w_1 + w_2 + \&c.}.$$

Now let us suppose that the number of observations upon which different data depend is unknown. We have, for instance,

$$r_1 = 78^\circ.58 \pm .06$$
$$r_2 = 78^\circ.48 \pm .03;$$

it follows from the rules of probable error that the last result, having one half the probable error of the first, represents four times the number of observations, hence if we call $w_1 = 1$, $w_2 = 4$. This gives

$$R = \frac{78^\circ.58 + 4 \times 78^\circ.48}{4 + 1} = 78^\circ.50$$

as before. In the absence of any better indication of the relative weights of different results we may accordingly estimate these weights w_1, w_2, &c., from their probable errors p_1, p_2, &c., by means of the formulæ,

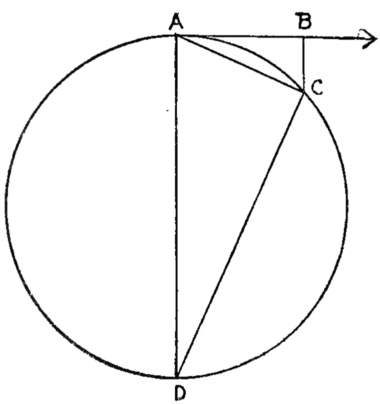
$$w_1 = 1 \div p_1^2; \; w_2 = 1 \div p_2^2, \; \&c. \qquad (2)$$

It is customary to extend this principle even to cases where results are known to be affected by unlike sources of error. The weight of a result is defined in general as the reciprocal of the square of its probable error.

APPENDIX XI.

PROOFS OF FORMULÆ.

(*a*) *Centrifugal Force.* When a body of mass m and velocity v in the direction AB is caused by a force f to move along the arc of a circle ACD, for a short time, t, it reaches a point C such that $AC = vt$. In the same time it is deflected in a direction at right angles to its original course through a distance BC, which by the law of falling bodies (§ 108) assuming the force to act constantly in the direction BC, is



$$BC = \tfrac{1}{2}\left(\frac{f}{m}\right)t^2.$$

the expression $(f \div m)$ taking the place of g. Drawing the diameter AD, and the chords AC and DC, we have by similar triangles

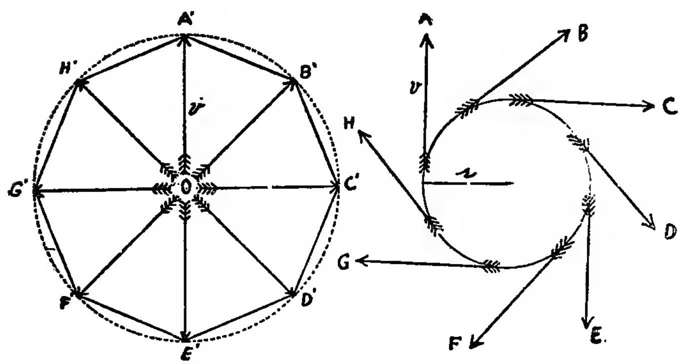
$$BC : AC :: AC : AD.$$

Substituting for BC its value from the formula, for AC its value (vt nearly), and for AD its value in terms of the radius ($2r$), we have

$$\tfrac{1}{2}\left(\frac{f}{m}\right)t^2 \times 2r = v^2t^2, \text{ or}$$

$$f = \frac{mv^2}{r}. \qquad (1)$$

It is evident that the direction of the force is not constant, and that, as the chord AC is less than the arc, the former cannot be exactly equal to vt. We may, however, consider arcs so small that the change in direction of the force and the difference in proportion between the arc and its chord become less than any assignable quantity. The formula above is therefore exact.



A second demonstration of this formula depends upon the method of representing changes in velocity (§ 105). Let A, B, C, &c., be the velocities of the moving body at different points in a circle of radius r, and let A', B', C', &c., be the same when arranged so as to start from a common point, O. The difference in velocity between A' and B' is represented by the line $A'B'$; between B' and C', by $B'C'$, &c. Hence the perimeter of the figure $A'B'C' \ldots A'$ represents the total change of velocity in one complete revolution.

This method of representation would be exact if the velocity changed *abruptly* from A' to B', from B' to C', &c. In point of fact, it goes through every possible intermediate value. The real change of velocity is evidently equal to the perimeter of the circle $A'B'C' \ldots A'$, rather than that of the polygon. Let $2t$ be the time of one complete revolution; then since the radius of the circle in question is v, the total change in the time $2t$ is $2\pi v$, and the acceleration (a) or change of velocity per unit of time is

$$a = \frac{2\pi v}{2t} = \frac{\pi v}{t}.$$

At the same time the velocity, v, of a body making the circuit of a circle with radius, r, in the time, $2t$, is

$$v = \frac{2\pi r}{2t} = \frac{\pi r}{t}; \text{ hence } \frac{\pi}{t} = \frac{v}{r}.$$

Substituting this value, we find

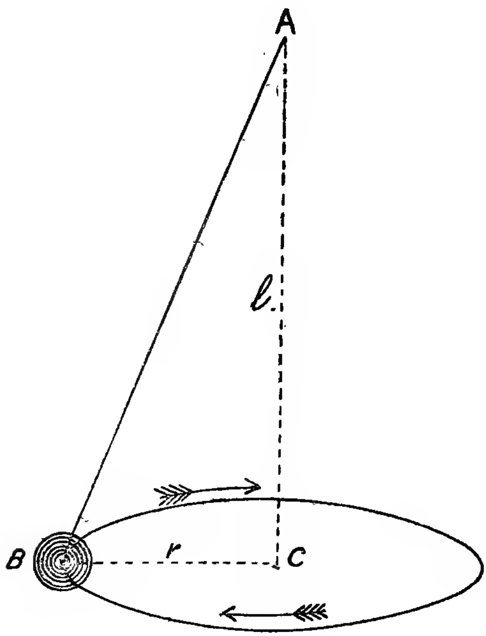
$$a = \left(\frac{\pi}{t}\right) v = \left(\frac{v}{r}\right) v = \frac{v^2}{r}.$$

The force (f) required to give this acceleration (a) to the mass (m) is according to the general definition (§ 12) equal to the product of the mass and acceleration; that is, as before

$$f = ma = \frac{mv^2}{r}.$$

The force f, which is exerted upon the body by some restraint, is evidently directed toward the centre of the circle in which the body is revolving, and is called accordingly a *centripetal force.* According to the general principle of action and reaction, an equal and opposite force is exerted by the body upon the restraint. This is called a *centrifugal force.*

(*b*) *Rotary Pendulum.* When a body of mass m, suspended by a cord AB (as in the figure), revolves about the centre, C, of a circle of radius, r, under a force of gravity equal to g dynes per gram, the centripetal force exerted by gravity is, by the principles of the composition and resolution



stated in § 105, equal to $mg \times \overline{BC} \div \overline{AC}$, or, calling $AC = l$, and $BC = r$,

$$f = \frac{mgr}{l}. \tag{1}$$

Now from the last section, if the time of a complete revolution is $2t$, the velocity v is

$$v = \frac{2\pi r}{2t} = \frac{\pi r}{t};$$

and the centripetal force is

$$f = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{\pi r}{t}\right)^2 = \frac{\pi^2 mr}{t^2}.$$

Equating the two values of f, we have

$$\frac{mgr}{l} = \frac{\pi^2 mr}{t^2},$$

or, cancelling m and r, and multiplying by l,

$$g = \frac{\pi^2 l}{t^2}, \qquad (2)$$

from which we find

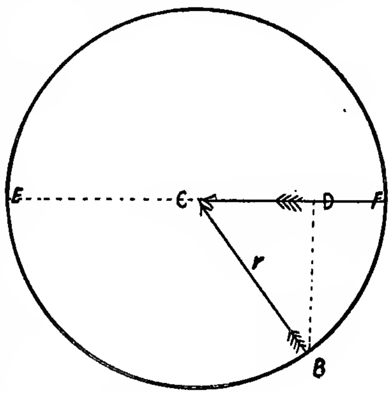
$$t = \pi\sqrt{\frac{l}{g}} \qquad (3) \qquad\qquad \text{and} \quad l = \frac{gt^2}{\pi^2} \qquad (4)$$

We note that the distance l is not the length of the pendulum, but its vertical component. When the displacement (r) is small, l may, however, be considered (practically) equal to the length of the pendulum.

(*c*) *Simple Pendulum.* The force f acting upon a rotary pendulum at any point B of its orbit is from the last section equal to $mg \times \overline{BC} \div \overline{AC}$. The component of this force in the direction of a given diameter $\overline{FE}$ is represented by the line $\overline{DC}$ in the figure. The distance passed over between the points F and B is equal to the arc $\overline{FB}$; the component of this distance in the direction of the diameter is $\overline{FD}$.

In the case of a simple pendulum, vibrating in the direction of the diameter $\overline{FE}$, the distance passed over is (neglecting the difference between a small arc and a straight line) equal to $\overline{FD}$ (nearly). The horizontal component of the force urging it in the direction $\overline{DC}$ (again neglecting small differences owing to curvature in the path $\overline{FE}$) is equal to $mg \times \overline{DC} \div \overline{AC}$ (nearly).

Now let the simple and rotary pendula start together at F; since both are urged toward E with the same force, the components of velocity acquired in a given length of time will be the same. The projection D of the conical pendulum upon the diameter $\overline{FE}$ will therefore coincide with that of the simple pendulum. Starting at any point with the same component velocity, and urged forward by the same component forces, and having the same components of distance to traverse, the two pendula will evidently arrive at or opposite C at the same time, or allowing for curvature of the path $\overline{FC}$, at *nearly* the same point of time; and in the same way, both will arrive at E at (nearly) the same instant. In other words, the time of a simple pendulum is nearly the same as that of a rotary pendulum.



When the arc of oscillation is very small, we may consider the oscillations both of the simple and of the rotary pendulum to be confined to a given plane, hence we have as before

$$g = \frac{\pi^2 l}{t^2} \quad (1) \qquad t = \pi\sqrt{\frac{l}{g}} \quad (2) \quad \text{and} \quad l = \frac{gt^2}{\pi^2}. \quad (3)$$

(*d*) *Compound Pendulum.* Multiplying both g and l in the last two equations by $m\,l$, we have

$$m'g = \frac{\pi^2 ml^2}{t^2}, \text{ and } t = \pi\sqrt{\frac{ml^2}{mlg}}. \qquad (1)$$

The quantity mlg, or $mg \times l$, is called the *directive force* (D) exerted by gravity upon the pendulum; the quantity ml^2 is called the moment of inertia (K) of the pendulum. Substituting these values, we have

$$D = \frac{\pi^2 K}{t^2} \quad (1) \qquad \text{or } t = \pi\sqrt{\frac{K}{D}}. \qquad (2)$$

When a pendulum of length l and mass m is deflected through a small distance r, the force brought to bear upon it is, as we have seen, equal to $mgr \div l$. This corresponds to a couple $(mgr \div l) \times l$, or $mlg \times \left(\frac{r}{l}\right)$. Substituting for the angle $(r \div l)$ its value in circular measure, α, we have for the couple c,

$$c = mlg \times \alpha = D \times \alpha.$$

The "directive force" (D) determines accordingly the couple which is brought to bear by gravity (g) when a mass (m) at the end of a lever-arm (l) is deflected through a given angle. The "moment of inertia" (ml^2) represents the couple necessary to give to a mass (m) at the end of a lever-arm (l) a unit angular velocity. This couple is evidently equal to the product of the mass (m) and the square of the lever-arm (l), because the force which must be exerted is the product of the mass (m) and the velocity (l) acquired; hence the couple, or product of the force (ml) and lever-arm (l), is equal to ml^2.

It is evident that if two bodies when deflected through a given angle are subjected to the same couple, and if the same couple produces the same angular acceleration, the time of oscillation must be the same. The formula above must apply, therefore, to compound as well as to simple pendula.

(*e*) *Reversible Pendula.* Let a pendulum of mass M, and moment of inertia $K = Mk^2$ about its centre of gravity, be suspended from a point at the distance x above the centre of gravity; then the directive force is

$$D = Mxg,$$

and the moment of inertia about the point of suspension is, as will be proved under (j).

$$K = Mk^2 + Mx^2,$$

where k represents the radius of gyration about the centre of gravity. The time of oscillation, t, is then

$$t = \pi \sqrt{\frac{K}{D}} = \pi \sqrt{\frac{Mk^2 + Mx^2}{Mxg}}.$$

Substituting $l = \dfrac{k^2 + x^2}{x}$, we have

$$t = \pi \sqrt{\frac{l}{g}},$$

where from the resemblance of the formula to that of a simple pendulum, l is called the length of an equivalent simple pendulum.[1]

Let us now suspend the pendulum at a distance y from the centre of gravity such that

[1] The length l is equal (by definition) to the distance between the centres of suspension and oscillation.

$$x + y = l, \text{ or } y = l - x = \frac{k^2 + x^2}{x} - x = \frac{k^2}{x}.$$

Then we have

$$t = \pi \sqrt{\frac{Mk^2 + My^2}{Myg}} = \pi \sqrt{\frac{Mk^2 + M\left(\frac{k^2}{x}\right)^2}{M\frac{k^2}{x}g}}$$

$$= \pi \sqrt{\frac{Mk^2x^2 + Mk^4}{Mk^2xg}} = \pi \sqrt{\frac{x^2 + k^2}{xg}} = \pi \sqrt{\frac{l}{g}}.$$

There are, accordingly, two distances, x and y, from the centre of gravity which give the same time of oscillation. We notice that $xy = k^2$, also that $x + y = l$. If, therefore, two points of suspension at the *unequal* distances x and y from the centre of gravity and on *opposite sides* of it are found to give the same time of oscillation, the length l of the equivalent simple pendulum may be found by measuring the distance between the points in question.

(*f*) *Errors of Adjustment in the Reversible Pendulum.* Let the pendulum of the last section be suspended from a point at a distance $x' = x + e$ from the centre of gravity. The corresponding value of y is

$$y' = \frac{k^2}{x'} = \frac{k^2}{x + e} = \frac{k^2}{x}\left(1 - \frac{e}{x}\right) \text{ nearly,}$$

if e is a small quantity. Hence the length (l') of the equivalent simple pendulum is

$$l' = x' + y' = x + e + y\left(1 - \frac{e}{x}\right) = x + y + \frac{ex - ey}{x}.$$

In the same way, if the pendulum be suspended from a point at a distance $y'' = y - e$ from the centre of gravity, the length of the equivalent simple pendulum is

$$l'' = x'' + y'' = x + y + \frac{ex - ey}{y}.$$

Calling the distance between the two points of suspension l, we have

$$l = x' + y'' = (x + e) + (y - e) = x + y;$$

hence we find

$$l' - l = x + y + \frac{ex - ey}{x} - (x + y) = \frac{ex - ey}{x}.$$

$$l'' - l = x + y + \frac{ex - ey}{y} - (x + y) = \frac{ex - ey}{y}, \text{ and}$$

$$\frac{l' - l}{l'' - l} = \frac{\dfrac{ex - ey}{x}}{\dfrac{ex - ey}{y}} = \frac{y}{x} = \frac{t' - t}{t'' - t} \text{ (nearly)},$$

assuming that the effects of small errors upon the time of oscillation are proportional to the effects upon the length of an equivalent simple pendulum.

We have, accordingly,

$$yt'' - yt = xt' - xt,$$

$$xt - yt = xt' - yt'' = (x - y)\ t' + yt' - yt''.$$

Hence finally,

$$t = t' + \frac{y}{x - y}\ (t' - t'').$$

We notice that

$$\text{either } t'' > t' > t, \text{ or } t'' < t' < t;$$

in no case is t between t' and t''.

It may be observed that if x and y are equal, the expressions for l, l', and l'' become identical. In other words, the pendulum does not respond under these conditions to a slight change in the points of suspension. To obtain accurate results, x should be several times greater (or less) than y.

The formulæ for the moment of inertia of a compound pendulum are established only for *parallel axes*; hence it is important that the axes passing through the two centres of suspension should be parallel. It is also important that the centre of gravity should lie in the plane of these two axes; if it does not, the dislocation should be allowed for in calculating the sum of the distances x and y.

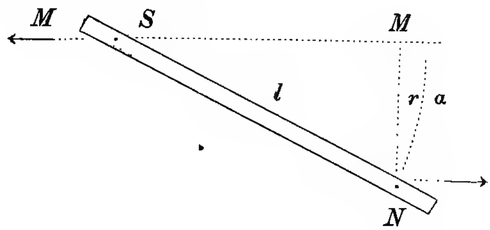
(*g*) *Torsion Pendulum.* The moment of inertia of a (thin) ring about its axis is evidently equal to the mass M of the ring multiplied by the square of its mean radius R, for the whole mass of the ring is situated practically at the distance R from the axis (see (*k*) (1)). The directive force, D, of a wire which gives to a ring of mass M and mean radius R a time of oscillation, t, is therefore, according to section (*d*) formula (1):—

$$D = \frac{\pi^2 MR^2}{t^2} \qquad (1)$$

The directive force, D, corresponds, as we have seen, to the couple required to produce a unit deflection in circular measure. The "coefficient of torsion" for 1° is evidently found by dividing the directive force D by the number of degrees in 1 circular unit of angle — that is 57°.29578, or 57°.3, nearly. We have, accordingly, in general

$$T = \frac{D}{57.3} = \frac{\pi^3 K}{180\, t^2} \qquad (2)$$

(*h*) *Magnetic Pendulum.* When a magnet with poles of the strength $\pm s$, separated by a distance l, is suspended so that it is free to move only in a horizontal plane, we need only to consider the forces exerted upon it by the horizontal component of the earth's magnetic field, which we will say is equal to H dynes per unit of magnetism. The two forces are accordingly $\pm s \times H$.



If the poles are relatively deflected east and west through a distance r, the couple tending to restore them to the magnetic meridian, MM, is $s \times H \times r$, or $(s \times l \times H) \times (r \div l)$. The ratio $r : l$ here determines (approximately) the small angle a in circular measure through which the magnet is deflected. The product $s \times l \times H$ corresponds accordingly to the product, $D = mlg$, in the case of a compound pendulum. Since the product of the strength (s) and distance (l) between the poles of a magnet determines its moment M, we have, substituting (see (d), (1)),

$$D = MH = \pi^2 \frac{K}{t^2},$$

where K is the moment of inertia (see k) and t the time of oscillation of the magnet.

(i) *Magnetometer.* Let a magnet with poles of the strength $\pm s$, separated by a distance l, be brought near a compass needle as in Fig. 200, ¶ 183. Let d be the mean distance of the poles from the needle. The nearer pole, being at a distance $d - \frac{1}{2}l$, will create a field of force f', such that

$$f' = \frac{\pm s}{(d - \frac{1}{2}l)^2}.$$

The further pole will create a field f'', such that

$$f'' = \frac{\mp s}{(d + \frac{1}{2}l)^2}.$$

The resultant field F is

$$F = f' + f'' = \pm s \left(\frac{1}{(d - \frac{1}{2}l)^2} - \frac{1}{(d + \frac{1}{2}l)^2}\right) =$$

$$\pm s \left(\frac{(d + \frac{1}{2}l)^2 - (d - \frac{1}{2}l)^2}{(d - \frac{1}{2}l)^2 \times (d + \frac{1}{2}l)^2}\right)$$

$$= \pm s \frac{d^2 + dl + \frac{1}{4}l^2 - (d^2 - dl + \frac{1}{4}l^2)}{d^4 - d^2 l^2 + \frac{1}{4}l^4} = \frac{\pm 2sl}{d^3} \text{ nearly,}$$

neglecting l^2 in comparison with d^2.

Now if H is the horizontal component of the earth's field, and a the deflection, we have

$$F = H \tan a.$$

Equating the two values of F and substituting M for sl we have

$$H \tan a = \frac{2M}{d^3},$$

$$\text{or } \frac{M}{H} = \frac{1}{2}d^3 \tan a. \qquad (1)$$

We have already found in the last section,

$$MH = \frac{\pi^2 K}{t^2};$$

multiplying the values of $\frac{M}{H}$ and MH together, we find

$$\frac{M}{H} \times MH = M^2 = \frac{\pi^2}{2} \frac{Kd^3 \tan a}{t^2}$$

whence $$M = \frac{\pi}{t} \sqrt{\tfrac{1}{2}Kd^3 \tan a}.$$

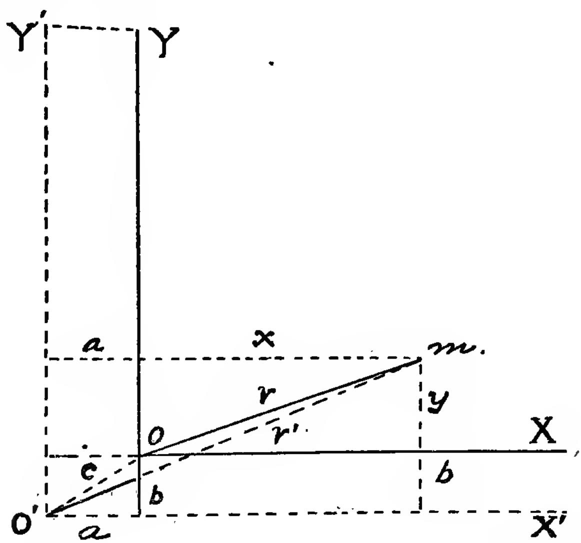
Dividing the value MH by that of $\frac{M}{H}$, we find

$$MH \div \frac{M}{H} = H^2 = \frac{2\,\pi^2 K}{t^2 d^3 \tan \alpha},$$

whence
$$H = \frac{\pi}{t}\sqrt{\frac{2\,K}{d^3 \tan \alpha}}. \qquad (2)$$

Instead of d^3 in these formulæ, we should strictly substitute $d^3 - dl^2 + \frac{1}{4}l^4 \div d$. The last term may almost always be neglected.

(*j*) *Moments of Inertia about Parallel Axes.* The moment of inertia (K) of a body about a given axis may be defined as the sum of the moments of inertia of the separate



masses of which it is composed. That is, denoting a given mass by m, and its distance from the axis by r,

$$K = \Sigma\, mr^2; \qquad (1)$$

or, if we consider the total mass, M, of the body as divided into small *equal* masses, m,

$$K = M\overline{r^2} = Mk^2, \tag{2}$$

where k is the "radius of gyration" about the axis O (or the radius of the mean square). Denoting by x and y the distance of m from two rectangular planes passing through O, we have, substituting

$$K = M\overline{r^2} = M\overline{(x^2+y^2)} = M(\overline{x^2} + \overline{y^2}).$$

In the same way, the moment of inertia about a parallel axis O', at a distance $OO' = c$ from O, is determined by the distances $x + a$ and $y + b$ from two planes $O'X'$ and $O'Y'$ at distance b and a from OX and OY, so that $a^2 + b^2 = c^2$. We have accordingly

$$K' = M\overline{(r')^2} = M\overline{(x')^2 + (y')^2} = M\overline{(x+a)^2 + (y+b)^2}$$

$$= M\overline{x^2 + 2ax + a^2 + y^2 + 2by + b^2}$$

$$= M(\overline{x^2} + \overline{y^2} + \overline{2ax} + \overline{2by} + \overline{a^2} + \overline{b^2}).$$

Now if O contains the centre of gravity, we have by definition $\overline{x} = 0$ and $\overline{y} = 0$, hence $\overline{2ax} = 0$ and $\overline{2by} = 0$; this gives

$$K' = M(\overline{x^2} + \overline{y^2} + a^2 + b^2) = M\overline{r^2} + Mc^2 =$$

$$Mk^2 + Mc^2 = K + Mc^2 \tag{3}$$

It follows that the moment of inertia (K) of a body of mass M about an axis passing through the centre of gravity is less than that (K') about any other parallel axis at the distance c by the amount Mc^2, which is equal to the moment of inertia of the whole mass M concentrated at a single point at the distance c from the axis.

The quantity Mc^2 represents the difficulty of causing the centre of gravity of a body to begin to rotate in an arc of radius c. The quantity $M\overline{r^2}$ or Mk^2 represents the difficulty of making a body begin to rotate about its own centre of gravity. When a body begins to rotate about a given axis and also about its centre of gravity at the same time, both sources of difficulty are met.

(*k*) *Calculation of Moments of Inertia.* The moment of inertia of a small mass m, at a distance l from its axis of revolution, is by definition ml^2. The moment of inertia of a thin ring of mass M and mean radius R about its axis is accordingly MR^2, for the ring may be thought of as composed of n small masses of the magnitude m, and such that $nm = M$, each of the masses being situated at a distance R from the axis of revolution. Since the moment of inertia for each mass is mR^2, the total is $n \times mR^2$ or $(nm)\ R^2$ or MR^2. Hence we have

$$K = MR^2. \qquad (1)$$

The moment of inertia of a long thin bar of length l and mass M is evidently less than $M \times (\frac{1}{2}L)^2$, because the whole of the weight is not situated at the end of the bar. The moment of inertia increases in fact according to the square of the distance x measured from the centre of the bar outward. Hence, applying the method of averages, we find for either end of the bar

$$\frac{K}{2} = \frac{M}{2}\ 0\overline{\quad x^2 \quad}\tfrac{1}{2} = \tfrac{1}{3}\,\frac{M}{2}\left(\frac{L}{2}\right)^2,$$

or

$$K = \tfrac{1}{12}\,ML^2 \qquad (2)$$

The moment of inertia of such a bar about an axis at a distance a from its middle point is, according to the last section,

$$K = \tfrac{1}{12}ML^2 + Ma^2. \tag{3}$$

A bar of length L and breadth B may be considered as composed of a series of thin parallel bars of length l, each having a moment of inertia $\frac{1}{12}ML^2 + My^2$, depending upon its distance y from the axis. Imagining such a bar to be divided longitudinally into halves by a plane passing through the axis, we find, averaging for either half,

$$\frac{K}{2} = \overline{\tfrac{1}{12}ML^2 + My^2}^{\,0 \text{---} \frac{B}{2}} = \overline{\tfrac{1}{12}ML^2}^{\,0 \text{---} \frac{B}{2}} + \overline{My^2}^{\,0 \text{---} \frac{B}{2}}$$

$$= \tfrac{1}{12}ML^2 + \tfrac{1}{3}M\left(\frac{B}{2}\right)^2,$$

whence
$$K = \tfrac{1}{12}M(L^2 + B^2). \tag{4}$$

The moment of inertia of a thin ring of radius r about one of its diameters as an axis is found by averaging the square of the distance $(r \sin x)^2$ of points subtending all possible angles x from the centre of the ring, and multiplying by the mass M of the ring. Since $\overline{\sin^2 x} = \frac{1}{2}$, (see IX. (j)), we find simply

$$K = \tfrac{1}{2}MR^2. \tag{5}$$

A thin disc of mass M and radius R can be regarded as a series of rings with increasing radius and mass. The mass of a ring of radius x bears to one of radius R (of the same breadth, thickness and density) the ratio $x : R$; and since the moments of inertia are proportional to the masses and to the squares of the radii, they are to each other in the ratio $x^3 : R^3$; hence on the average the moment of inertia of a series of rings occupying a total breadth from 0 to R is to that of a series of the same breadth, all having the radius R, in the proportion (see IX. (h)),

$$\overset{0\,\text{————}\,r}{x^3 \div R^3} = \tfrac{1}{4}. \tag{6}$$

The area covered by a series of rings of the breadth R and radius R would, however, be $2\pi R \times R = 2\pi R^2$; while the area actually covered by the rings is πR^2; hence (assuming that masses and areas are proportional) the mass with which we have compared the disc is $2M$. We conclude that the moment of inertia of the disc about its axis is

$$K = \tfrac{1}{4}2MR^2 = \tfrac{1}{2}MR^2; \tag{7}$$

and for the moment of inertia of a disc about a diameter,

$$K = \tfrac{1}{4}2\frac{M}{2}R^2 = \tfrac{1}{4}MR^2. \tag{8}$$

The moment of inertia of a disc of mass M about a diameter can also be found by averaging moments of inertia corresponding to a given distance x from the diameter. The moment of inertia of the disc about an axis parallel to the diameter and passing through the axis of the disc at a distance L from it is, according to the last section,

$$K = \tfrac{1}{4}MR^2 + ML^2. \tag{9}$$

A cylinder with a transverse axis passing through its middle point may be regarded as a series of discs situated at regularly increasing distances x from the axis. The moment of inertia of any such disc is

$$K = \tfrac{1}{4}MR^2 + Mx^2;$$

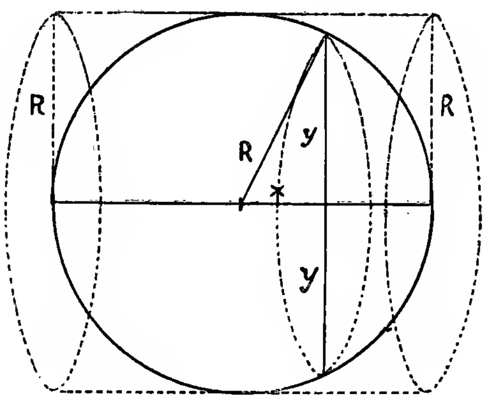
hence, averaging (see IX. (f)), we find for either half of the cylinder,

$$\frac{K}{2} = \overset{0\,\text{——————————}\,\frac{L}{2}}{\tfrac{1}{4}\frac{M}{2}R^2 + \frac{M}{2}x^2} = \overset{0\,\text{———}\,\frac{L}{2}}{\tfrac{1}{4}\frac{M}{2}R^2} + \overset{0\,\text{——}\,\frac{L}{2}}{\frac{M}{2}x^2}.$$

$$= \tfrac{1}{4}\frac{M}{2}R^2 + \tfrac{1}{3}\frac{M}{2}\left(\frac{L}{2}\right)^2;$$

whence $$K = \tfrac{1}{12} ML^2 + \tfrac{1}{4} MR^2. \tag{10}$$

A sphere of radius R may be regarded as a series of discs of varying weight and radius. The weight of a disc at the distance x from the centre of the sphere bears to that of one



of the same thickness at the centre the ratio $y^2 : R^2$: the moments of inertia are proportional to the weight multiplied by the square of the radii, hence are to each other as $y^4 : R^4$. The moment of inertia of a sphere compared with that of a cylinder of the same length and diameter is therefore (see IX. (*i*)),

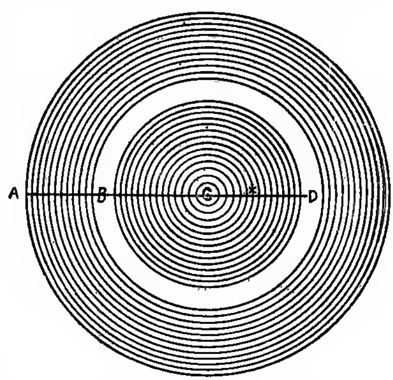
$$\int_{-R}^{+R} \frac{y^4}{R^4} = \int_{-R}^{+R} \frac{(R^2 - x^2)^2}{R^4} = \int_{-R}^{+R} 1 - 2\frac{x^2}{R^2} + \frac{x^4}{R^4}$$

$$= 1 - \tfrac{2}{3} + \tfrac{1}{5} = \tfrac{8}{15}. \tag{11}$$

The volume of the sphere is $\frac{4\pi}{3}R^3$; that of the cylinder is $2R \times \pi R^2 = 2\pi R^3$; that is, $1\frac{1}{2}$ times as great as the sphere. Hence if the mass of the sphere is M, that of the cylinder is $1\frac{1}{2}M$, and its moment of inertia about its axis is $\frac{1}{2} \times 1\frac{1}{2}M \times R^2 = \frac{3}{4}MR^2$. The moment of inertia of the sphere, being $\frac{8}{15}$ that of the cylinder, is accordingly,

$$K = \tfrac{8}{15} \times \tfrac{3}{4}MR^2 = \tfrac{2}{5}MR^2 \qquad (12)$$

(*l*) *Coefficient of Viscosity.* Let a tube of radius r and length l be filled with a liquid of which all is frozen except a tubular section of (mean) radius x, and unit thickness. Let p be the difference of pressure at the two ends of the tube, then the force on the core is $p \times \pi x^2$ (nearly). This is resisted by a force equal to the velocity v of the core multiplied by the (mean) area of the opposite surfaces of the tubular section, $2\pi xl$, and multiplied by the coefficient of viscosity, η. That is, —



$$2\pi x\eta vl = p\pi x^2, \text{ whence}$$

$$v = \frac{px}{2\eta l}.$$

The quantity (or volume) q of (principally frozen) liquid which flows through the tube in the time t is

$$q = \pi x^2 vt; \text{ hence, substituting,}$$

$$q = \pi x^2 vt = \frac{\pi x^2 pxt}{2\eta l} = \frac{\pi pt}{2\eta l}\, x^3.$$

If now a new tubular section be melted, either inside or outside of the former section, the relative velocity of all points within the new section will be increased as much as if the former section were solid, hence the *increase* in the flow will be represented by the same formula as before. We may suppose all the sections to be melted one by one, each contributing a certain amount to the flow. If Q is the total flow, $Q \div r$ must be the *average* flow for each section; hence we have (see IX. (h)),

$$\overset{o\,\text{———}\,r}{\frac{Q}{r} = \frac{\pi pt}{2\eta l}\cdot\, x^3} = \frac{\pi pt}{2\eta l}\cdot\ \tfrac{1}{4} x^3 = \frac{\pi ptr^3}{2\eta l}.$$

Now if the pressure p is due to a hydrostatic column of height h and density d, and if the acceleration of gravity is g, we have

$$p = ghd.$$

The weight of liquid delivered is, moreover, $Q \times d$, so that

$$Q = \frac{w}{d}.$$

Making these substitutions, we have, solving for η,

$$\eta = \frac{\pi g d^2 h r^4 t}{8wl}.$$

(m) *Coefficients of Elasticity.*[1] When a unit cube is subjected on all sides to a pressure P, its volume is reduced by an amount v, given by the equation

$$v = \frac{P}{M},$$

[1] The formulæ here derived apply only to "isotropic" substances.

where M is by definition the "coefficient of resilience of volume." The increase (l, b, and t,) of the length, breadth, and thickness, are of course each equal to $\frac{1}{3}$ v. That is,

$$l = b = t = \tfrac{1}{3}\,\frac{P}{M}.$$

When a unit cube is subjected to a pressure P on two opposite faces so as to diminish its length, and an equal tension on two other faces so as to increase its breadth, its volume remains the same (nearly); but the length and breadth are altered so that

$$\frac{1+b}{1-l} - 1 = \frac{P}{S},$$

where S is by definition the "coefficient of simple rigidity."

Since b and l are small and nearly equal, we have

$$\frac{1+b}{1-l} - 1 = 1 + b + l - 1 \text{ (nearly)} = 2b = 2l = \frac{P}{S}.$$

whence $$b = l = \tfrac{1}{2}\,\frac{P}{S}.$$

When a unit cube is subjected simply to a pressure P in the direction of its length, the latter becomes shortened by an amount l, such that

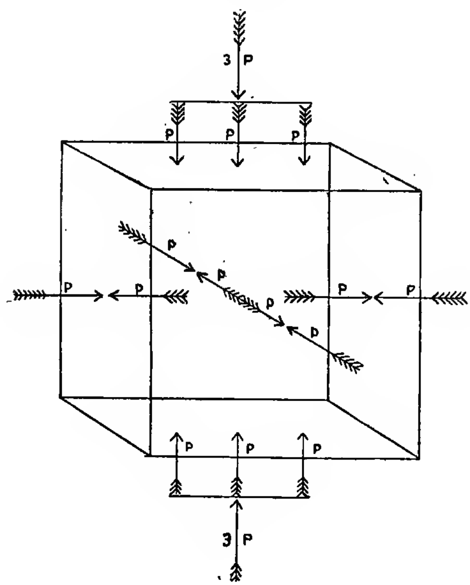
$$l = \frac{P}{Y},$$

where Y is by definition "Young's modulus of elasticity." At the same time the breadth and thickness increase by equal amounts, b and t,

$$b = t = \mu l = \mu\,\frac{P}{Y},$$

where μ is by definition "Poisson's ratio."

Now let a unit cube be subjected to the pressure $3P$ in the direction of its length; and let equal and opposite pressures $\pm P$ be applied to each of its sides. We shall then have a uniform pressure, P, on each surface, tending to diminish the volume, combined with two pairs of equal and opposite pressures, P, one pair tending to increase the breadth, the other pair tending to increase the thickness, and both pairs tending



to diminish the length. Hence we have, adding the three effects upon the length together,

$$l' = \tfrac{1}{3}\frac{P}{M} + \tfrac{1}{2}\frac{P}{S} + \tfrac{1}{2}\frac{P}{S} = \tfrac{1}{3}\frac{P}{M} + \frac{P}{S}.$$

We have also, remembering that the total longitudinal pressure is $3P$,

$$l' = 3 \times \frac{P}{Y};$$

hence, equating the two values of l', and dividing through by $3P$,

$$\frac{1}{Y} = \frac{1}{9M} + \frac{1}{3S}. \qquad (1)$$

By means of this equation either one of the coefficients Y, M, or S can be found if the other two are given. If S and M are proportional to the numbers 6 and 10, for instance, Y is represented by the number 15.

The increase of breadth and thickness can be found in the same way as the length, remembering that only one pair of equal and opposite forces tends to increase each, and that the effects of compression tend to diminish the result. We have

$$b' = t' = \tfrac{1}{2}\frac{P}{S} - \tfrac{1}{3}\frac{P}{M}.$$

Dividing b' (or t') by l', we find

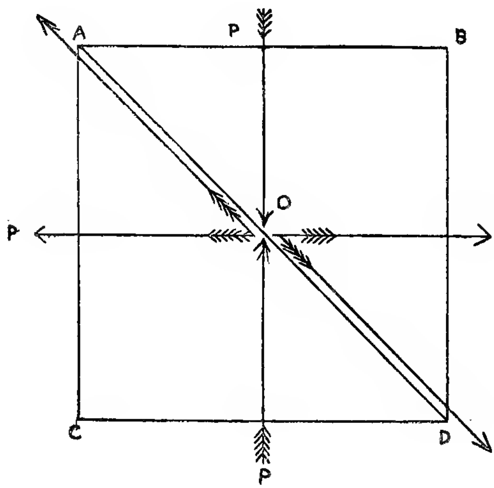
$$\mu = \frac{b'}{l'} = \frac{t'}{l'} = \frac{\tfrac{1}{2}\frac{P}{S} - \tfrac{1}{3}\frac{P}{M}}{\tfrac{1}{3}\frac{P}{M} + \frac{P}{S}} = \frac{3M - 2S}{6M + 2S}. \qquad (2)$$

It is seen that if S and M are proportional to the numbers 6 and 10, $\mu = \frac{1}{4}$.

(*n*) *Shearing Stresses and Strains.* Let a unit cube be acted upon by two equal and opposite pressures, P, tending to reduce its length, and by two equal and opposite tensions, also equal to P, tending to increase its breadth. The resultants may evidently be represented by two forces, OD . . . and OA, . . . each equal to $\sqrt{2} \times P$. These tend to make the two halves of the cube slide relatively in the directions AD

and DA. This tendency is resisted by the plane AD, the area of which is $\sqrt{2}$. Hence the intensity of the tangential or "shearing" stress is $\sqrt{2} \times P \div \sqrt{2} = P$.

We have seen that if S is the simple rigidity of a body subjected to a pair of stresses at right angles equal to $\pm P$,



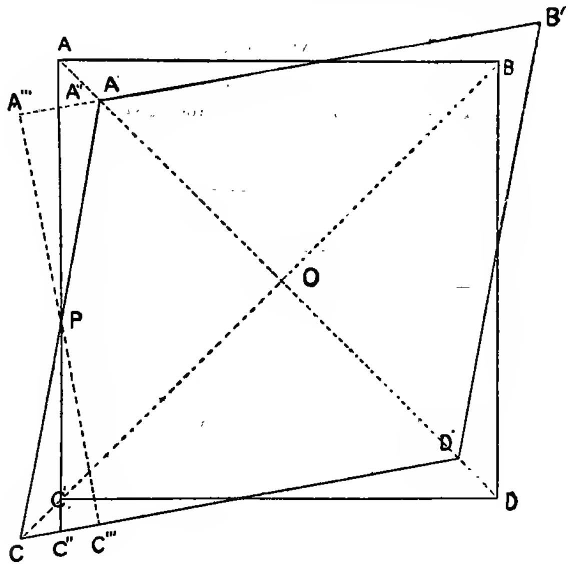
the dimensions of a unit of length become altered by the amount

$$l = \pm \tfrac{1}{2} \frac{P}{S}.$$

If, therefore, the stresses are exerted along the *diagonals* of the cube (AOD and BOC) one of these diagonals, AOD for instance, will be shortened, the other lengthened by the amount $\sqrt{2} \times l$ (since the change of length is proportional to the distance affected). It follows that AA', DD', &c., being equal to one half the change of length in each case, are all equal to $\frac{1}{2} \sqrt{2} \times l$, or

$$AA' = \tfrac{1}{2}\sqrt{\tfrac{1}{2}}\,\frac{P}{S}.$$

From P, where AC and $A'C'$ intersect, draw a perpendicular PA''' to $B'A'$ and PC''' to $D'C'$; and let AC cut $B'A'$



and $D'C'$ at A'' and C''. Then, by construction, in the (nearly) isosceles right-angled triangle $AA''A'$,

$$A'A'' = \sqrt{\tfrac{1}{2}} \times AA' = \sqrt{\tfrac{1}{2}} \times \tfrac{1}{2}\sqrt{\tfrac{1}{2}}\,\frac{P}{S} = \tfrac{1}{4}\frac{P}{S}\,\text{(nearly)}.$$

We have also, by construction,

$$A'A'' = A''A''' = C'C'' = C''C'''\ \text{(nearly)},\ = \tfrac{1}{4}d,$$

where d is the total dislocation of the side $A'B'$ with respect to $C'D'$, which, since the distance between the sides is 1, is equal to the tangental or "shearing" strain. Hence we have

$$d = 4AA' = \frac{P}{S} \text{ (nearly).}$$

or
$$S = \frac{P}{d}.$$

We have seen that P represents the shearing stress, d the shearing strain; S is the "simple rigidity." It might also be called the "modulus of shearing."

The constant S in formulæ involving transverse stresses and strains evidently takes the place of Young's modulus in formulæ where these stresses and strains are longitudinal.

(*o*) *Coëfficients of Torsion.* In a thin tube of length l, thickness t, and mean radius r, the cross-section is $2\pi rt$, and if the angle of torsion is α in circular measure, the twist per unit of length is $\alpha \div l$, so that two points at the unit distance (measured longitudinally) are dislocated through the distance $r \times \alpha \div l$. The force necessary to produce such a dislocation between surfaces of the area $2\pi rt$ is

$$f = 2\pi rt \times r \times \alpha \div l \times S,$$

where S is the "coefficient of simple rigidity." The couple required is accordingly

$$c = f \times r = 2\pi Sr^3 t\alpha \div l.$$

The "directive force" (d), or ratio of the couple to the angle of torsion in circular measure is

$$d = \frac{c}{\alpha} = \frac{2\pi Sr^3 t}{l}.$$

A cylindrical rod (or wire) may be considered as a series of tubes with radii varying from 0 to r. The directive force for each tube is less than that of a tube with the radius r in the proportion $x^3 : r^3$; hence the directive force of a rod is less than that of a tube with a radius and thickness equal to the radius of the rod in the proportion (see IX. (h)),

$$\underset{x^2 \div r^3}{0 \text{———} r} = \tfrac{1}{4}.$$

Now the directive force of tube of radius r and thickness r would be

$$d = \frac{2\pi S r^4}{l};$$

hence that of the rod is (see section (d) formula (1));

$$D = \tfrac{1}{4} \cdot \frac{2\pi S r^4}{l} = \frac{\pi}{2} \quad \frac{S r^4}{l}. \tag{1}$$

Dividing the directive force by $\dfrac{360}{2\pi}$ (the number of degrees in 1 unit of angle), we find the coefficient of torsion per degree,

$$T = \frac{2\pi D}{360} = \frac{\pi^2}{360} \cdot \frac{S r^4}{l}. \tag{2}$$

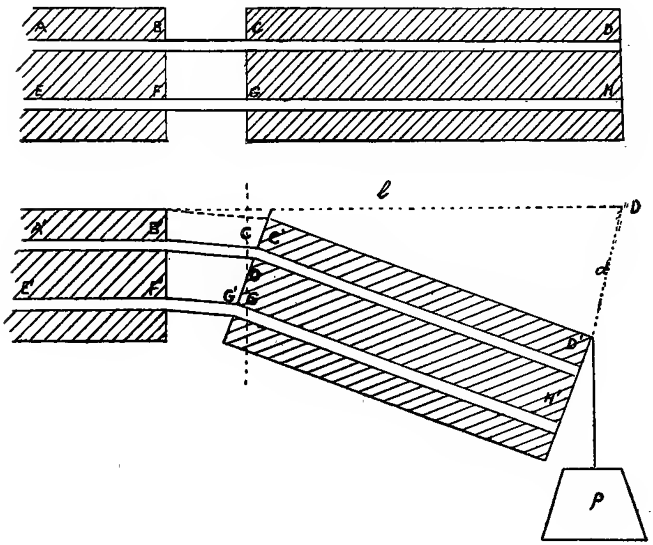
Given D or T, the coefficient of simple rigidity, S, may evidently be found by the formula

$$S = \frac{2lD}{\pi r^4} = \frac{360 l T}{\pi^2 r^4}. \tag{3}$$

If the directive force D is determined by the time of oscillation t of a body of moment of inertia K, we have, substituting the value of D, namely $\pi^2 K \div t^2$ (see section (d) formula (1)),

$$S = \frac{2\pi l K}{r^4 t^2}. \tag{4}$$

(p) *Transverse Elasticity.* Let a beam consisting of two thin rods, AD and EH, of length L, breadth B, and unit thickness, be bound in some light inelastic material except-ing the unit lengths BC and FG, which are at a distance l from the ends of the rod, and at a mean distance t from each other; and let a transverse force p applied at the end of the rod produce a deflection $\overline{DD'} = d$.



Drawing COG parallel to $B'F'$ and bisecting $C'O'G'$, so that $OC' = OG' = \frac{1}{2}t$, we have from (nearly) similar triangles,

$$CC' : OC' :: GG' : OG' :: d : l,$$

or
$$CC' = GG' = \tfrac{1}{2}td \div l.$$

The forces brought into play by stretching the rods BC and FG of unit length and with the cross-section B are, if Young's modulus is Y,

$$f = \pm \overline{CC'} \times BY = \pm \tfrac{1}{2}tBYd \div l,$$

and the couple produced is

$$C = f \times t = (\tfrac{1}{2} tBYd \div l) \times t = \tfrac{1}{2} t^2 BYd \div l.$$

This couple must be equal to that due to the force p on the arm l, hence

$$p \times l = \tfrac{1}{2} t^2 BYd \div l, \text{ whence}$$

$$Y = \frac{2pl^2}{t^2 Bd}.$$

It would be possible to find Young's modulus by this formula (remembering that the result is to be multiplied by the length BC and divided by the thickness of the rods, if these are not unity); in practice we employ, however, a solid rod, of thickness T, which we may consider as composed of a series of pairs of rods of the unit thickness, equal in number to $\frac{1}{2}T$. If the total couple produced by these rods is C, the average couple is evidently $C \div \frac{1}{2}T$, or $2C \div T$. Hence we have (see IX. (f)),

$$\frac{2C}{T} = \int_0^T \frac{1}{2} \frac{t^2 BYd}{l} = \frac{1}{2} \cdot \frac{1}{3} \frac{T^2 BYd}{l} = \frac{1}{6} \frac{T^2 BYd}{l},$$

from which,

$$d = \frac{12Cl}{BT^3 Y} = \frac{12(F \times l)\,l}{BT^3 Y} = \frac{12Fl^2}{BT^3 Y},$$

where F is the force producing the couple C. Now suppose that the rod is released from its restraint to a distance L from the free end, each portion contributing an amount d to the total deflection D, due to the bending of all the portions. The average deflection due to each unit of length of the rod being $D \div L$, we have

$$\frac{D}{L} = \overset{0\,\text{———}\,L}{\frac{12Fl^2}{BT^3Y}} = \frac{12 \times \frac{1}{3}L^2}{BT^3Y} = \frac{4FL^2}{BT^3Y},$$

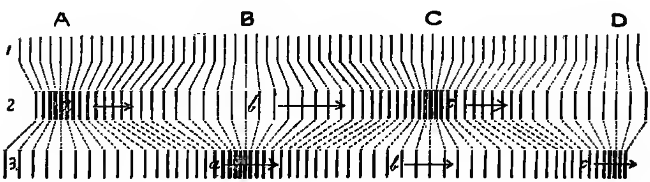
from which we find

$$Y = \frac{4FL^3}{BT^3D}.$$

When a rod of length l, breadth b, and thickness t, supported at both ends and loaded in the middle by a force f, is deflected through a distance d, each support reacts with a force $F = \frac{1}{2}f$, and since the middle of the rod remains horizontal, the length bent is $L = \frac{1}{2}l$. Substituting these values we find

$$Y = \frac{4 \cdot \frac{1}{2}f(\frac{1}{2}l)^3}{bt^3d} = \frac{fl^3}{4bt^3d}$$

(*q*) *Longitudinal Wave Motion.* The strata of a medium which in a state of rest would be equally spaced, as in the series (1), in the figure, are when transmitting a wave of



sound, crowded together is some places, as A 2, C 2, B 3, and D 3, and more or less separated in others. It is seen that a comparatively small distance traversed by the strata between (2) and (3) accounts for the apparent movement of the condensation from A to B. We will suppose this apparent movement to continue indefinitely with the velocity v, and that several imaginary points, a, b, c, &c., move with the same velocity and in the same direction, so that one of them, a for instance, is always in the denser portion of the "wave," while another point, b, is in a comparatively rarefied portion. The number of strata traversed by a in a given length of time must be approximately the same as that traversed by b, for if a left many more strata behind it than b, the strata would soon become exhausted from between them, and if b left more behind it than a, there would be an indefinite condensation of strata. Both of these suppositions are contrary to the conditions which we have assumed. Now if n' is the number of strata per unit of distance at a, n'' the number at b, v the velocity of the points a and b, v' that of the strata at a, and v'' that of the strata at b, the relative velocities are $v - v'$ and $v - v''$ respectively; the number passed by a in the time t is $(v - v')\, n't$; and that passed by b is $(v - v'')\, n''t$; hence

$$(v - v')\, n't = (v - v'')\, n''t.$$

Now the densities of the medium, d' at a, and d'' at b, are evidently proportional to the number of strata per unit of distance, hence

$$\frac{v - v'}{v - v''} = \frac{n''}{n'} = \frac{d''}{d'},$$

from which we find

$$\frac{v - v'}{v - v''} - 1 = \frac{d''}{d'} - 1 = \frac{v'' - v'}{v - v'} = \frac{d'' - d'}{d'},$$

or

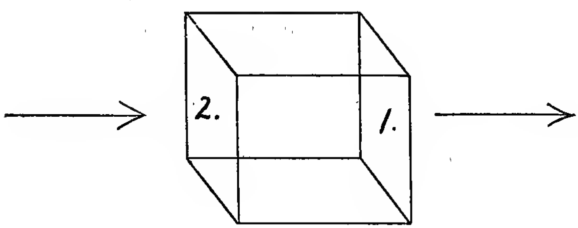
$$\frac{v'' - v'}{d'' - d'} = \frac{v - v'}{d'}.$$

Now if d' represents the mean density, d, of the medium and v' the corresponding velocity, which, in the absence of any motion of translation [e. g. wind] we will assume to be 0, we have, substituting,

$$\frac{v'' - 0}{d'' - d} = \frac{v - 0}{d}, \text{ or } \frac{v''}{v} = \frac{d'' - d}{d}. \qquad (1)$$

That is, the velocity of a given particle is to the velocity of the wave as the difference in density from the mean density is to the mean density of the medium. We notice that in the denser portions of a wave the particles are moving with it; but in the rarer portions they are moving against it. Under these conditions only is longitudinal wave-motion possible.

(*r*) *Velocity of Sound in Air.* Let 1 and 2 in the figure be two points on opposite faces of a centimetre cube of air,



where the pressures are p_1 and p_2, the densities d_1 and d_2, the velocities of the particles v_1 and v_2, respectively. Then if a wave of sound moves from 2 to 1 with the velocity v, it will occupy a time t in traversing the (unit) distance in question such that

$$t = \frac{1}{v}.$$

The forces acting upon the two faces of the cube p_2 and p_1, being opposite in direction, have a resultant f in the direction of the wave motion,

$$f = p_2 - p_1.$$

The mass acted upon is numerically equal to the density of the air,

$$m = d \text{ (nearly)}.$$

The velocity acquired is equal to the difference between the original velocity, v_1 at the point 1 and the final velocity, v_2, which with the other properties of the point 2 are carried to the point 1 by the progression of the wave. We have, therefore,

$$\triangle\, v = v_2 - v_1.$$

Substituting the values of t, f, m, and $\triangle\, v$, in the formula expressing the general law of motion,

$$f \times t = m \times \triangle\, v, \text{ we have}$$

$$(p - p_1) \times \frac{1}{v} = d \times (v_2 - v_1).$$

Substituting for v_2 and v_1 their values from the last section, namely

$$v_2 = \frac{v}{d}\,(d_2 - d) \text{ and } v_1 = \frac{v}{d}\,(d_1 - d) \text{ we have}$$

$$(p_2 - p_1) \times \frac{1}{v} = d \times \left(\frac{v}{d}\,(d_2 - d) - \frac{v}{d}\,(d_1 - d)\right)$$

$$= v\,(d_2 - d_1).$$

Hence we find

$$v^2 = \frac{p_2 - p_1}{d_2 - d_1}. \qquad (1)$$

If air *suddenly compressed* obeyed the law of Boyle and Mariotte (as Newton wrongly supposed), we should have

$$P : D :: p_2 : d_2 :: p_1 : d_1 :: p_2 - p_1 :: d_2 - d_1, \text{ \&c.}$$

In fact, however, so much heat is developed by sudden compression that the increase of pressure is, in the case of air, about 1.408 times, and in general κ times as great as it would be according to the law of Boyle and Mariotte. We have accordingly,

$$v^2 = \frac{p_2 - p}{d_2 - d} = \kappa \frac{P}{D}. \tag{2}$$

Substituting for κP the symbol E, representing the "coefficient of volume resilience" we have finally,

$$v = \sqrt{\frac{E}{D}}. \tag{3}$$

This formula applies to the velocity of sound in any medium, provided that E represents that modulus of elasticity which resists the dislocation of strata accompanying the propagation of the sound.

In the case of a thin wire, we substitute for E "Young's modulus of elasticity," if the vibrations are longitudinal, or the "simple rigidity" if the vibrations are torsional.

(*s*) *Index of Refraction of a Prism.* When a ray of light, *FGHI*, passes through an equilateral prism, *AJL*, in a direction *GH*, parallel to the base, *JL*, the angles *KGH* and *KHG* between the ray and the normals *BK* and *CK*, are evidently each equal to the angle of refraction r. The sum of these angles ($2r$) is the supplement of *BKC*; and the prism angle A is also the supplement of *BKC*; hence

$$r = \tfrac{1}{2} A. \tag{1}$$

From the equality of the angles *KGH* and *KHG* within the prism, follows that of the angles *BGF* and *CHI* outside of the prism; these are accordingly each equal to the angle of incidence, i. Now the ray of light is deviated at the point G through an angle *DGF*, and at H through an equal angle; hence the total angle of deviation,

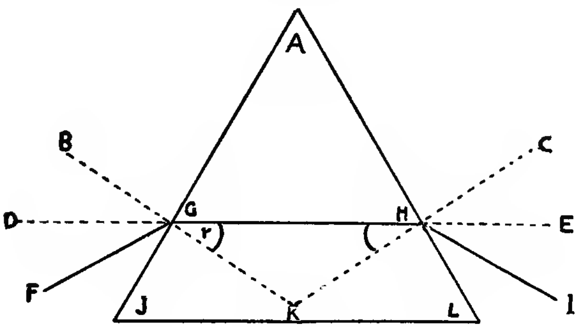
$$d = 2\,DGF = 2(BGF - BGD) = 2(BGF - KGH)$$
$$= 2\,(i - r).$$

Hence

$$i - r = \tfrac{1}{2}d, \text{ or } i = r + \tfrac{1}{2}d = \tfrac{1}{2}A + \tfrac{1}{2}d. \qquad (2)$$

Substituting these values of i and r in the formula § 102, we have

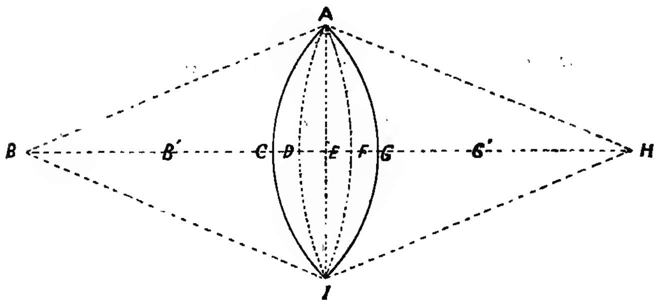
$$\mu = \frac{\sin\,(\frac{1}{2}A + \frac{1}{2}d)}{\sin\,(\frac{1}{2}A)}. \qquad (3)$$



(*t*) *Index of Refraction of a Lens.* When waves of light from a point B are brought to a focus at H, it is evident that in a given length of time different distances are traversed by different portions of the wave. Drawing the arcs AFI and ADI with B and H as centres, also the straight line AEI, we see that the path BAH is longer than BEH by the amount DF. In the same time that light traverses a distance CG through the lens it passes accordingly through a distance $CG + DF$ in air. The index of refraction is, accordingly,

$$\mu = \frac{CG + DF}{CG}.$$

The object of the present investigation is simply to express DF and CG in terms of the radii of curvature $B'G$ and $G'C$) and focal lengths (BE and HE) of the lens. We have by geometry



$$(EF) = (AE)^2 \div (BE) \text{ and } (DE) = (AE)^2 \div (HE).$$

Hence

$$DF = DE + EF = (AE)^2 \times (1 \div f_1 + 1 \div f_2),$$

where f_1 and f_2 represent the conjugate focal lengths.

We have similarly,

$$(CG) = (AE)^2 \times (1 \div B'E + 1 \div G'E) =$$

$$(AE)^2 \times (1 \div R_1 + 1 \div R_2) \text{ nearly,}$$

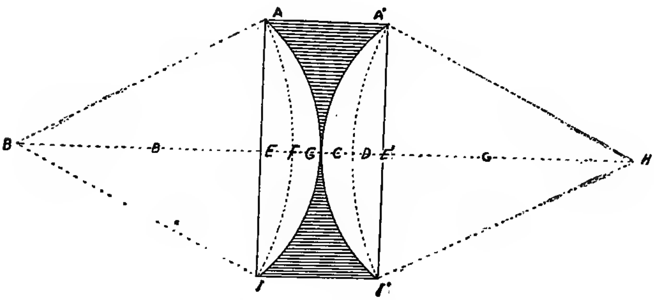
neglecting the relatively small distances CE and EG in comparison with the radii R_1 and R_2. Substituting these values and cancelling $(AE)^2$ we find

$$\mu = \frac{CG + DF}{CG} = \frac{\left(\frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{R_1} + \frac{1}{R_2}\right)}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}.$$

Substituting $\frac{1}{F}$ for $\frac{1}{f_1} + \frac{1}{f_2}$ (see § 103), we have, if $R_1 = R_2 = R$,

$$\mu = \frac{\frac{1}{F} + \frac{2}{R}}{\frac{2}{R}} = \frac{R + 2F}{2F} = 1 + \tfrac{1}{2}\,\frac{R}{F}.$$

(*u*) *Compound Lenses.* Let the lens studied in the last section be cut in two in the plane AI, and the two halves made tangent at G-C, also let the space between the two



halves be filled with a liquid having an index of refraction μ' less than μ. Then if v is the velocity of light in air $v \div \mu$ is its velocity in the lens and $v \div \mu'$ is its velocity in the liquid. The time occupied in passing through the distance $EG + CE'$ is accordingly

$$(EG + CE') \div (\mu \div v) = \mu\,(EG + CE') \div v.$$

The time occupied in passing through an equal distance (from A to A') through the liquid is similarly $\mu'\,(EG + CE') \div v$. The difference between these two times is compensated by the difference in the time required to pass through the distances $BA + A'H$ and $BE + E'H$ in air; that is, the time required to pass through the distance $EF + DE'$ in air. That is,

$$\mu\,(EG + CE') \div v - \mu'\,(EG + CE') \div v$$
$$= (EF + DE') \div v,$$

whence $$\mu - \mu' = \frac{EF + DE'}{EG + CE'}.$$

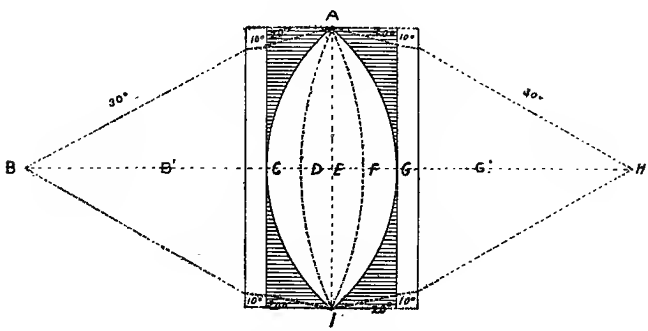
Substituting as in the last section, and cancelling $(AE)^2$ we have

$$EF + DE' = (AE)^2\,(1 \div f_1 + 1 \div f_2) \text{ and}$$
$$EG + CF' = (AE)^2\,(1 \div R_1 + 1 \div R_2)$$

$$\mu - \mu' = \frac{\dfrac{1}{f_1} + \dfrac{1}{f_2}}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \frac{\dfrac{1}{F}}{\dfrac{2}{R}} = \tfrac{1}{2}\,\frac{R}{F}.$$

It follows that

$$\mu' = \mu - \tfrac{1}{2}\,\frac{R}{F}.$$



The same formula holds approximately for a lens mounted between two plates, the spaces being filled with liquid, for the distances traversed through the lens, the liquid and the air are (nearly) the same.

(*v*) *Electrostatic Potential.* Two bodies, each charged with a unit of positive electricity, repel each other at the distance d with a force (f) such that

$$f = \frac{1}{d^2}.$$

The work (w) required to change the distance between the bodies from d_1 to d_2 is

$$w = f \times (d_1 - d_2) = \frac{d_1 - d_2}{\delta^2},$$

where δ represents some mean between the distances d_1 and d_2.

If we assume that the work W necessary to bring the two bodies together from an infinite distance to the distance d is in general

$$W = \frac{1}{d},$$

we have $W_1 = 1 \div d_1$, $W_2 = 1 \div d_2$, &c., whence by difference

$$w = W_2 - W_1 = \frac{1}{d_2} - \frac{1}{d_1} = \frac{d_1 - d_2}{d_1 d_2} = \frac{d_1 - d_2}{d^2},$$

where d is the geometric mean between d_1 and d_2. There can evidently be no great error in using the geometric or any other mean when the distances are very small; and by dividing a given motion into a sufficient number of steps the proportional error in the estimation of w can be indefinitely diminished. Now the proportional error in sums cannot be greater than in the separate terms; hence the general formula, $W = 1 \div d$, must be exact. The work W required to bring a unit of positive electricity to a given point is called the electrostatic potential of that point. We have seen that the electrostatic potential due to one unit of positive electricity at the distance d is $1 \div d$; that due to q units is accordingly

$$e = \frac{q}{d}.$$

When q units are distributed uniformly over the surface of a sphere of radius r, they act upon points outside of the sphere as if they were at the centre of the sphere. The potential of the sphere is determined by the work necessary to bring a unit of positive electricity up to the surface of the sphere, that is, to within a distance r of the charge; hence we have

$$e = \frac{q}{r}, \text{ and } q = er.$$

Let two spheres, suspended as in ¶ 258, be charged to the potential e; then we have

$$q = q' = er = er' = \tfrac{1}{2}ed = \tfrac{1}{2}ed'.$$

The force of repulsion is

$$f = \frac{qq'}{s^2} = \frac{(\frac{1}{2}ed)^2}{s^2} = \frac{1}{4}\,\frac{e^2d^2}{s^2},$$

where s is the distance between the spheres. This is balanced by a force $w \times g \times \frac{1}{2}s \div l$; hence we have

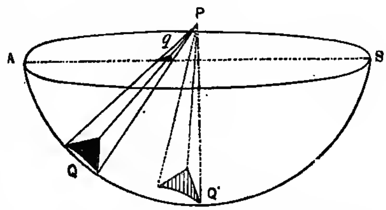
$$\frac{1}{4}\,\frac{e^2d^2}{s^2} = w \times g \times \tfrac{1}{2}s \div l, \text{ or}$$

$$e^2 = \frac{2wgs^3}{ld^2},$$

whence,

$$e = \sqrt{\frac{2wgs^8}{ld^2}}.$$

(*w*) *Absolute Electrometer.* Let P be a point charged with a unit of positive electricity, and AB an electrified plane near P charged with s units of electricity per unit of surface. Draw the hemisphere $A\ Q\ Q'\ B$, with unit radius and with P as a centre; let



q and Q be sections of the plane and hemisphere included in a small solid angle Q, and let Q' be a similar section such that PQ' is normal to AB. We have, by geometry,

$$\frac{q \cos QPQ'}{(Pq)^2} = \frac{Q'}{(PQ')^2}.$$

Assuming that the hemisphere is also charged with s units of electricity per unit of area, the attractions of q and Q' resolved in the direction PQ' become

$$s \times \frac{q \cos QPQ'}{(Pq)^2}, \text{ and } s \times \frac{Q'}{(PQ')^2}$$

respectively. We have seen that these two quantities into which s is multiplied are equal. Since any two portions of the plate and hemisphere occupying the same solid angle exert the same attraction on the point P, supposing the section of the hemisphere to be transferred to Q', the whole plate must exert the same attraction as the whole hemisphere, neglecting a small portion near the edges, and supposing the whole transferred to Q'. Since the surface of a hemisphere of unit radius is 2π, and the quantity of electricity is s units per unit of surface, the attraction in question is

$$2\pi s \div (PQ)^2 = 2\pi s.$$

In the absolute electrometer in ¶ 270, we consider the point P to be between two plates with equal and opposite charges of $\pm\ s$ units per unit of surface. Hence the force (f) on P is $f = 4\pi s$. If the distance between the plates is d, the work required to take P from one to the other is

$$f \times d = 4\pi sd = e,$$

where e is the difference in electrostatic potential. The charge on the upper plate, of area a, is $s \times a$, hence the force F is

$$F = wg = s \times a \times 2\pi s = 2\pi s^2 a.$$

whence
$$s = \sqrt{\frac{wg}{2\pi a}}.$$

Substituting this value of s we find

$$e = 4\pi sd = 4\ \pi \sqrt{\frac{wg}{2\pi a}} = \sqrt{\frac{8\pi wg}{a}}.$$

(x) *Capacity of Condensers.* The electrical capacity c of a body is defined as the ratio of the charge (q) to the electromotive force (e); that is the difference of potential which it produces, or by which it is produced. Since, in the case of a sphere of radius r, $q = er$ (see (v)), we have

$$c = \frac{q}{e} = \frac{er}{e} = r. \tag{1}$$

We have seen in the last section that the difference of potential (e) between two plates charged with $\pm\ s$ units of electricity per unit of surface and at a distance d is $4\pi sd$. If the area of either plate is A, the charge is

$$q = \pm\ As,$$

This (q) is the available charge of the condenser formed by the two plates; for a flow of q units from one plate to the other would reduce the charge of each to 0. It follows that the capacity C, or ratio of the charge to the difference of potential or electromotive force is

$$C = \frac{q}{e} = \frac{As}{4\pi sd} = \frac{A}{4\pi d}. \tag{2}$$

The plates are here supposed to be separated by air, or by other material the specific inductive capacity of which may be taken as unity.

APPENDIX XII.

USEFUL FORMULÆ.

(a) Interpolation.

Let $+x =$ response of instrument to value A; $-y =$ response to value $A + a$; $s =$ sensitiveness $= x + y$,

$$q = A + \frac{xa}{s}. \qquad (\S\ 41)$$

(b) Geometrical Formulæ.

Circumference c of circle of radius r,

$$c = 2\pi r. \qquad \text{(Table 3 } F\text{)}$$

Cross-section (q) of cylinder of radius r, weight w, density d, and length l,

$$q = \frac{w}{ld} = \pi r^2; \qquad \text{(Table 3 } G\text{)}$$

whence
$$r = \sqrt{\frac{q}{\pi}} = \sqrt{\frac{w}{\pi l d}}.$$

Volume v of sphere of radius r, and diameter d,

$$v = \tfrac{4}{3}\pi r^3 = \tfrac{1}{6}\pi d^3; \qquad \text{(Table 3 } H\text{)}$$

whence
$$d = 1.2407\ \sqrt[3]{v}.$$

(c) Hydrostatics.

Mass M, Density D, Volume V, and Specific Volume S,

$$S = \frac{1}{D}\,; \quad S = \frac{V}{M}\,; \; V = MS\,; \; M = \frac{S}{V}\,; \qquad (\S\ 155)$$

$$D = \frac{M}{V}\,; M = VD\,; \; V = \frac{M}{D}.$$

Pressure p, due to vertical height h, of column of liquid of density d,

$$p = hgd. \qquad (\S\ 63)$$

(d) Expansion.

Reduction of volume V, and density D, of a gas at temperature t, and pressure p, to volume V_0 and density D_0 at 0° and 76 *cm.*,

$$D_0 = D \times \frac{76}{p} \times \frac{273 + t}{273}. \quad \text{(Tables 18, } d\text{–}e\text{; § 81))}$$

$$v_0 = v \times \frac{p}{76} \times \frac{273}{273 + t}. \quad \text{(Tables 18, } f\text{–}g\text{)}$$

Laws of gases, $T,\ T_0,\ T_1,\ T_2 =$ absolute temperatures,
$v,\ v_0,\ v_1,\ v_2 =$ corresponding volumes,
$p,\ p_0,\ p_1\ p_2 =$ corresponding pressures,
(0) at 0°, (1) at 100°, &c.

$$vp = v_0 p_0 = v_1 p_1 = v_2 p_2 \text{ \&c.}$$

(Law of Boyle and Mariotte, § 79)

$$T : T_0 : T_1 : T_2 :: v : v_0 : v_1 : v_2 \text{ \&c.}$$

(Law of Charles, ¶ 76, § 74)

If $z =$ absolute zero,

$$z = -\ 100° \frac{p_0}{p_1 - p_0} = -100° \frac{v_0}{v_1 - v_0} = -\ 273°.$$

(¶¶ 74–76)

Coefficient of expansion e (mean from temperature t_1 to t_2),

$$e = \frac{v_2 - v_1}{v_0 (t_2 - t_1)}. \qquad (\P\P\ 63\text{–}74)$$

Linear coefficient ϵ (mean relative from t_1 to t_2),

$$\epsilon = \tfrac{1}{3}\, e = \tfrac{1}{3}\, \frac{v_2 - v_1}{v_1 (t_2 - t_1)}. \qquad (\P\ 240)$$

(e) Calorimetry.

s_1 s_2, &c., = specific heats,
w_1 w_2, &c., = corresponding weights,
t_1 t_2, &c., = corresponding temperatures before mixture,
l_1 l_2, &c., = corresponding latent heats,
c = capacity of calorimeter, t_3 its temperature before mixture, t the temperature of the mixture, and q the no. of units of heat lost,

$$c = w_1 s_1 \times \frac{t_1 - t}{t - t_3}.$$

$$c = w_1 s_1 + w_2 s_2 + w_3 s_3 + \&c. \qquad (\P\ 91)$$

$$w_1 s_1 (t - t_1) + w_2 s_2 (t - t_2) + c (t - t_3) + q + l_1 w_1 = 0. \qquad (\P\ 100)$$

(f) Light.

Law of inverse squares,

$$x : y :: \left(\frac{1}{a}\right)^2 : \left(\frac{1}{b}\right)^2.$$

Photometric law, $x : y :: a^2 : b^2$. (¶ 109)

Principal focal length = F, conjugate focal lengths = f_1 and f_2,

$$F = \frac{1}{\frac{1}{f_1} + \frac{1}{f_2}} = \frac{f_1 \times f_2}{f_1 + f_2}. \quad \text{(For real foci, ¶ 117)}$$

$$F = \frac{1}{\frac{1}{f_1} - \frac{1}{f_2}} = \frac{f_1 \times f_2}{f_2 - f_1}. \quad \text{(For virtual foci, ¶ 119)}$$

A = angle of prism, D = angle of minimum deviation. *Index of refraction,*

$$\mu = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2} A} \qquad \text{(Appendix XI. (}s\text{))}$$

R = mean radius of curvature of double convex lens,
F = principal focal length,

$$\mu = 1 + \frac{1}{2} \frac{R}{F}. \qquad \text{(Appendix XI. (}t\text{))}$$

Double convex lens (of index μ) between plates filled with liquid (of index μ'),

$$\mu' = \mu - \frac{1}{2} \frac{R}{F}. \qquad \text{(Appendix XI. (}u\text{))}$$

Rotation of plane of polarization (α) of sodium light in degrees due to depth d of sugar solution containing s grams *per cu. cm.*

$$\alpha = 6.65\, d \,.\, s \text{ or } s = 0.150\, \alpha \div d. \qquad \text{(¶ 245)}$$

Wave-length l, angle of diffraction α, and distance between lines of grating (d) in position of minimum deviation,

$$l = 2\, d \sin \tfrac{1}{2} \alpha. \qquad \text{(¶ 129)}$$

Correction of observed altitude, A.

s = semidiameter (sun = 16′, nearly),
h = dip of horizon (from point m metres high, $1\frac{3}{4}' \sqrt{m}$ nearly),

r = refraction ($1' \times$ cotan A nearly),
p = parallax ($0'$ for the sun),

$$a = A + s - h - r + p. \qquad (\P\ 242)$$

Latitude, l, from altitude a and declination d,

$$l = 90° - a \pm d. \qquad (\P\ 242)$$

Longitude T, in hours, minutes, and seconds, from standard times t' and t'' of equal altitudes, and equation of time e,

$$T = \tfrac{1}{2}(t' + t'') \pm e. \qquad (\P\ 243)$$

(g) Sound.

Lissajous' curves ; P, p = pitch ; n = No. of lobes, c = No. of cycles per second,

$$P = np \pm c. \qquad (\P\ 143)$$

Velocity v, pitch p, wave-length l, distances traversed d, d_1, d_2, corresponding times, t, t_1, t_2,

$$v = pl\ (\P\ 133) = \frac{d}{t} = \frac{d_1 - d_2}{t_1 - t_2}. \qquad (\P\ 136)$$

$$v = \sqrt{\frac{E}{D}} = \sqrt{\frac{\kappa P}{D}} = \sqrt{\frac{1.41\, P}{D}} \text{ for air.}$$

(Appendix XI. (r))

Velocity (v_1) *of longitudinal vibrations* (Young's modulus = Y),

$$v_1 = \sqrt{\frac{Y}{D}}. \qquad (\P\ 248;\ \text{Appendix XI.}\ (r))$$

Velocity (v_2) *of torsional vibrations* (Simple Rigidity = S)

$$v_2 = \sqrt{\frac{S}{D}}. \qquad (\P\ 248;\ \text{Appendix XI.}\ (r))$$

Pitch of a string, of length l, and mass $m \times l$, stretched by force f,

$$p = \tfrac{1}{2} l \sqrt{\frac{f}{m}}. \quad \text{(Appendix VI., 64 } A\text{)}$$

(h) Moments of Inertia.

Moment of inertia of mass M about axis through centre of gravity $= K$; about parallel axis at distance $c = K'$,

$$K' = K + Mc^2. \quad \text{(Appendix XI. } (j)(3)\text{)}$$

Small mass M at distance l from axis,

$$K = ml^2. \quad \text{(Appendix XI. } (d)\text{)}$$

Thin ring (or tube) of mass M and mean radius R about its axis (*e.g.* rim of wheel),

$$K = MR^2. \quad \text{(Appendix XI. } (k)\ (1)\text{)}$$

Thin ring about a diameter (as in spinning),

$$K = \tfrac{1}{2} MR^2. \quad \text{(Appendix XI. } (k)\ (5)\text{)}$$

Square bar of length L, breadth B, and mass M, about a central transverse axis (*e.g.* suspended magnet),

$$K = \tfrac{1}{12} M(L^2 + B^2). \quad \text{(Appendix XI. } (k)\ (4)\text{)}$$

Round bar of length L, and radius R, about central transverse axis (*e.g.* suspended magnet),

$$K = \tfrac{1}{12} ML^2 + \tfrac{1}{4} MR^2. \quad \text{(Appendix XI. } (k)\ (10)\text{)}$$

Disc or cylinder of mass M and radius R about axis (*e.g.* a wheel),

$$K = \tfrac{1}{2} MR^2. \quad \text{(Appendix XI. } (k)\ (7)\text{)}$$

Thin disc about diameter (*e. g.* a coin spinning),

$$K = \tfrac{1}{4} MR^2. \quad \text{(Appendix XI. (}k\text{) (8))}$$

Sphere of mass M and radius R,

$$K = \tfrac{2}{5} MR^2. \quad \text{(Appendix XI. (}k\text{) (12))}$$

(i) Dynamics.

Force f, acting for time t, gives mass m velocity v,

$$ft = mv. \qquad (\S\ 106)$$

$$f = \frac{mv}{t}\ ;\ t = \frac{mv}{f};\ m = \frac{ft}{v}\ ;\ v = \frac{ft}{m}.$$

Gravity (g), time t, velocity v, distance d,

$$v = gt. \qquad (\S\ 108)$$

$$d = \tfrac{1}{2} gt^2. \qquad (\S\ 108)$$

Ballistic pendulum,

$$v = AB\sqrt{\frac{g}{AC}}. \qquad (\S\ 109)$$

Pendulum of length l, time t (latitude λ),

$$t = \pi\sqrt{\frac{l}{g}}. \quad \text{(Appendix XI. (}c\text{) (2))}$$

$$l = \frac{gt^2}{\pi^2}. \quad \text{(Appendix XI. (}c\text{) (3))}$$

$$l = 99.3562 - 0.2536 \cos 2\lambda. \quad \text{(Table 48)}$$

$$g = \frac{\pi^2 l}{t^2}. \quad \text{(Appendix XI. (}c\text{) (1))}$$

$$g = 980.6056 - 2.5028 \cos 2\lambda. \quad \text{(Table 47)}$$

Compound pendulum with directive force D, and moment of inertia K,

$$t = \pi \sqrt{\frac{K}{D}}. \quad \text{(Appendix XI. } (d) \text{ (2))}$$

Directive force D of magnet of moment M due to horizontal component H of earth's magnetic field,

$$D = MH = \pi^2 \frac{K}{t^2}. \quad \text{(Appendix XI. } (d) \text{ (1))}$$

(j) Elasticity.

Coefficient of torsion T,

$$T = \frac{D}{57^\circ.3} = \frac{\pi^3 K}{180\, t^2}. \quad \text{(Appendix XI. } (g) \text{ (2))}$$

Simple rigidity S, of a wire of length l, radius r, and coefficient of torsion T,

$$S = \frac{360\, Tl}{\pi^2 r^4} = \frac{2\pi l K}{r^4 t^2}.$$

(Appendix XI. (o) (3) (4))

Young's modulus Y, for a beam of length l, breadth b, thickness t, suffering deflection d, from force f at middle of beam.

$$Y = \frac{f l^3}{4 b d t^3} = \tfrac{1}{4} F. \quad \text{(¶ 163; Appendix XI. } (p))$$

Resilience of volume, with coefficient or modulus M,

$$M = \frac{SY}{9S - 3Y}.$$

(¶ 240; Appendix XI. (m) (1))

Poisson's ratio (μ) of lateral contraction to longitudinal extension,

$$\mu = \frac{3M - 2S}{6M + 2S}. \quad \text{(Appendix XI. } (m) \text{ (2))}$$

(k) Friction.

Coefficient of friction in fluids f, creating force F on area a through velocity v,

$$f = \frac{F}{av^2}. \qquad (¶\ 172)$$

Viscosity coefficient η, in capillary tube of length l and radius r, transmitting in time t, a weight w, of liquid of density d, under a pressure $p = hgd$,

$$\eta = \frac{\pi g d^2 h r^4 t}{8\,wl}. \quad (¶\ 251\text{; Appendix XI. } (l))$$

Efficiency e of water motor with wheel of circumference c making n revolutions per unit of time against tangential force f, while consuming a volume v of water under the pressure p,

$$e = \frac{cnf}{vp}. \qquad (¶\ 175)$$

Mechanical equivalent of heat J, in terms of number of times (n) that a material of specific heat s must fall through a distance d, under gravity (g) to warm itself $t°$,

$$J = \frac{ndg}{st}. \qquad (¶\ 178).$$

(l) Magnetism.

Mean strength s, of the poles of two parallel magnets, the attraction of which at the distance d is greater than the repulsion by amount Δ,

$$s = \tfrac{1}{2}\, d \sqrt{\Delta} \text{ (nearly)}. \qquad (\S\ 129)$$

Moment of magnet with poles of strength $\pm s$ and distance l between poles,

$$M = s \times l. \qquad (\P\ 185)$$

Magnetic couple (c) deflecting a wire of coefficient of torsion T, $a°$, or giving body with moment of inertia K, a time of vibration t, in earth's horizontal field H,

$$MH = Ta\,(\P\ 182) = \frac{\pi^2 K}{t^2}. \qquad \text{(Appendix XI. }(h))$$

Maximum deflection a, due to magnet of moment M at mean distance d, in earth's horizontal field H,

$$\frac{M}{H} = \tfrac{1}{2}\, d^3 \tan a. \qquad \text{(Appendix XI. }(i)\,(1)$$

Horizontal intensity H of earth's magnetism,

$$H = \frac{\pi}{t}\sqrt{\frac{2\,K}{d^3 \tan a}}. \qquad \text{(Appendix XI. }(i)\ (2))$$

Dip (d), estimated by throws of ballistic galvanometer; a' due to vertical, a'' due to horizontal components,

$$\tan d = \frac{\text{chord } a'}{\text{chord } a''}. \qquad (\P\ 101)$$

(m) Magnetic Current Measure.

Constant K of a coil with n turns of radius r,

$$K = \frac{2\pi n}{r}. \qquad (\P\ 199,\ \S\ 133)$$

Reduction factor of galvanometer with constant K in magnetic field H, deflected $a°$ by current C,

$$i = \frac{H}{K} = \frac{c}{tan\ u} \text{ for absolute units,}$$

$$I = 10\frac{H}{K} = \frac{C}{tan\ u} \text{ for ampères.} \qquad (¶\ 190)$$

Comparison of tangent galvanometers with reduction factors I and I', giving deflections a and a',

$$\frac{I}{I'} = \frac{tan\ a}{tan\ a'}. \qquad (¶\ 201)$$

Shunt of resistance S increases reduction factor of galvanometer of resistance $R + G$ in the ratio

$$\frac{i}{I} = \frac{S}{R + G + S}. \text{ (Appendix VIII., 61 } C\text{).}$$

Dynamometer with large coil of constant K, and small coil of magnetic area A, gives deflection a, under current in ampères C, against torsion of wire having coefficient t, such that

$$C = 10\sqrt{\frac{ta}{KA}}. \qquad (¶\ 204)$$

Electro-chemical current measure in terms of weight w, of substance having electro-chemical equivalent q, acted upon in time t,

$$c = \frac{w}{qt}.$$

For copper,

$$C = \frac{3050\,w}{t} \text{ ampères.} \qquad (¶\ 206\ (2))$$

Current C, of heat or of electricity in terms of quantity Q, in time t,

$$C = \frac{Q}{t} \text{ (by definition).}$$

Specific conductivity ς, in terms of current C, length of conductor L, area of its cross-section A, and difference of potential or temperature, E or T,

$$\varsigma = \frac{CL}{AT} = \frac{CL}{AE}. \qquad (\P\ 241)$$

(n) Electrical Resistance.

Resistance R, of conductor, in which a current C in ampères, generates heat enough in the time T, to raise a weight w of water, and a calorimeter of thermal capacity c, from $t_1°$ to $t_2°$,

$$R = \frac{4.17\,(w+c)\ (t_2 - t_1)}{C^2\,T} \qquad (\P\ 213)$$

Specific resistance S, of conductor of length L, cross-section A, and resistance R,

$$S = \frac{RA}{L}. \qquad (\P\ 219)$$

Wheatstone's Bridge (see Fig. 18, page 732),

$$AB : BC :: AD : DC. \qquad (\S\ 141)$$

Resistance (R) *in multiple arc* of conductors having resistances R_1 R_2, &c.,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \text{\&c.} \qquad (\S\ 140)$$

Thomson's Method. Battery resistance B, galvanometer resistance G; if external resistance R gives same deflection as r gives with battery shunt of resistance S,

$$B = S\,\frac{R - r}{r + G}. \qquad \text{(Appendix VI., 113 } A\text{)}$$

Ohm's Method. Battery resistance B, electromotive force E, deflection a; with added resistance R_2, deflection a_2,

$$B = \frac{R_2 \tan a_2}{\tan a_1 - \tan a_2}. \qquad (\P\ 225)$$

$$E = \frac{R_2 \tan a_1 \tan a_2}{\tan a_1 - \tan a_2}. \qquad (\P\ 230)$$

Beetz' Method. Resistance of stronger battery B, electromotive force E', external resistance r_1' and r_2', corresponding resistances of shunt r_1 and r_2,

$$B = \frac{r_1 r_2' - r_1' r_2}{r_2 - r_1}. \qquad (\P\ 228)$$

$$\frac{E'}{E''} = \frac{B + r_1 + r_1'}{r_1} = \frac{B + r_2 + r_2'}{r_2}. \qquad (\P\ 228)$$

(o) Electromotive Force.

Electro-chemical equivalent q, heat of combustion h, electromotive force E, and mechanical equivalent J,

$$E = Jqh. \qquad (\S\ 145)$$

Electrical power (P) in terms of electromotive force E, and current C,

$$P = CE = C^2R; \quad (\P\ 230;\ \S\S\ 136,\ 137)$$

whence $C = P \div E$, and $E = P \div C$.

Electromotive force in terms of the current C and resistance R,

$$E = CR; \qquad (\S\ 139)$$

Whence *Ohm's Law*,

$$C = \frac{E}{R}. \qquad (\S\ 138)$$

Wiedemann's Method. Electromotive forces E and e in conjunction and opposition, corresponding deflections A and a,

$$\frac{E}{e} = \frac{\tan A + \tan a}{\tan A - \tan a}. \qquad (¶\ 231)$$

Electromotive forces E and e produce equal currents with given external resistances; also with the resistances R and r added to respective circuits; then

$$e : E :: r : R. \qquad (¶\ 233)$$

Differences of Potential e_1 and e_2, corresponding to distances d_1 and d_2 on uniform straight wire carrying a current,

$$e_1 : e_2 :: d_1 : d_2. \qquad (¶¶\ 235, 236)$$

(p) Electrostatics.

Capacity c of sphere of radius r;

$$c = r. \qquad \text{(Appendix XI. } (x)\ (1))$$

Capacity of condenser with insulating layers of area A, thickness t, and specific inductive capacity s,

$$c = \frac{As}{4\pi t}. \qquad \text{(Appendix XI. } (x)\ (2))$$

Charge q, in condenser of capacity c, due to electromotive force e (by definition),

$$q = ce.$$

Electromotive force e in electrostatic measure causing two pith-balls of diameter d, weight wg, suspended by cords of length l, to diverge through distance s,

$$e = \sqrt{\frac{2wgs^8}{ld^2}}. \qquad \text{(Appendix XI. } (v))$$

Electromotive force e in electrostatic measure causing a plate of area a to be attracted or repelled by a large plate at a distance d, with a force wg,

$$e = d\sqrt{\frac{8\pi g w}{a}}. \qquad \text{(Appendix XI. } (w))$$

(q) Average.

$$\frac{o \text{———} a}{x^n} = \frac{a^n}{n+1}. \qquad \text{(Appendix IX. } (i))$$

(r) Probable Error.

$P =$ probable error of single observations,
$p =$ probable error of mean of n observations,
$d^2 =$ mean square of the differences,

(Appendix X. (k))

$$P = 0.67449\, d\sqrt{1 \div (n-1)} = p\sqrt{n}.$$

Probable error (p) of a result, in terms of variations d_1 d_2, &c., introduced by changing the separate data by an amount equal to probable error of each,

$$p = \sqrt{d_1^2 + d_2^2 + \text{\&c.}} \qquad \text{(Appendix X. } (o))$$

(s) Weight of Results.

Weights $= w_1\ w_2\ w_3$, &c. (Appendix X. (s))
probable errors $= p_1, p_2, p_3$, &c.

$$\text{then } w_1 : w_2 : w_3, \text{\&c.}, :: \left(\frac{1}{p_1}\right)^2 : \left(\frac{1}{p_2}\right)^2 : \left(\frac{1}{p_3}\right)^2 \text{\&c.}$$

Most probable result R, in terms of several results, $r_1\ r_2\ r_3$, &c., with weights $w_1\ w_2\ w_3$, &c.,

$$R = \frac{w_1 r_1 + w_2 r_2 + w_3 r_3 + \text{\&c.}}{w_1 + w_2 + w_3 + \text{\&c.}}.$$

(t) Dimensions.

NOTE. The dimensions of a quantity may be defined as a mathematical expression for the number of times that multiples of the three fundamental units of length (L) mass (M) and time (T) must be employed as factors to express the quantity in question. The dimensions are usually represented by ordinary "exponents."[1]

Dimensions are useful in reducing results from one system to another. Let L be the value in centimetres of the unit of length in any system, M the value in grams of the unit of mass, T the value in seconds of the unit of time; then the dimensions of a given quantity, let us say $L^x M^y T^z$, give at once the factor for reducing that quantity from the given system to $C.\ G.\ S.$ units.

Dimensions obey the following laws: —

(1) Only quantities of a given kind can be added or subtracted, and the sum has the same dimensions as the separate quantities.

(2) The dimensions of the product or quotient of two quantities are equal to the product or quotient of their separate dimensions treated as algebraic quantities. It is through this law that dimensions are calculated.

(3) The two sides of an equation must always have the same dimensions; for quantities differing no matter how slightly in dimensions are, like surfaces and volumes, essentially different in kind, and hence cannot be numerically or quantitatively compared. This equality of dimensions, being a condition which every rational formula must satisfy, furnishes a useful test of the accuracy of mathematical work.

Angles, strains, specific gravity, temperature, and all *rela-*

[1] For proofs and illustrations, see Kohlrausch, Physical Measurement, Appendix A.

tive magnitudes, having no dependence upon the fundamental units, are of dimensions 0.

The dimensions of other quantities are expressed as follows:

Length	L
Surface	L^2
Volume	L^3
Time	T
Velocity	$L \div T$ or LT^{-1}
Acceleration	$(L \div T) \div T$ or LT^{-2}
Mass	M
Density	$L^{-3}M$
Force	LMT^{-2}
Work (or kinetic energy)	L^2MT^{-2}
Couple	L^2MT^{-2}
Directive Force	L^2MT^{-2}
Power	L^2MT^{-3}
Moment of inertia	L^2M
Stress, or pressure	$L^{-1}MT^{-2}$
Modulus of elasticity	$L^{-1}MT^{-2}$
Electrostatic or magnetic unit	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}$
Electrostatic potential	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$
Electrostatic capacity	L
Magnetic moment	$L^{\frac{5}{2}}M^{\frac{1}{2}}T^{-1}$
Magnetic field	$L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$
Electrical current (magnetic measure)	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$
Electro-magnetic unit of quantity	$L^{\frac{1}{2}}M^{\frac{1}{2}}$
Electromotive force	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}$
Electro-magnetic capacity	$L^{-1}T^2$
Resistance	LT^{-1}

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ABBREVIATIONS.

app., apparatus.
B. W. G., Birmingham wire gauge.
C. G. S., centimetre-gram-second system.
cm., centimetre [s].
coef., coefficient [of].
const., constant.
cu., cubic.
det., determination [of].
e. m. f., electromotive force.
eq, equivalent.
ex., exercise.
exp., experiment.
g., gram [s]; also acceleration of gravity.
gr., gravity; also grain.
h., hour [s].
H. U., Harvard University.
kilo., kilogram [s].
kilom., kilometre [s].
m., metre [s]; also minute [s].
meas., measurement [of].
min., minute [s].
mm., millimetre [s].
obd., observed.
obs., observation [s].
s. or *sec.*, second [s].
sp., specific.
sp. gr., specific gravity.
sp. ht., specific heat.
sq., square.
temp., temperature.
vol., volume.
wt., weight.

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ERRATA.

Pages 21–22. The figures quoted from Tables 18 a, 18 b, and 18 c, should be corrected to correspond with later and more accurate values introduced into these tables.

Page 78. References to "§ 40" and "§ 41" should read "§ 37."

Page 170. For "final temperature" read "average temperature."

Page 372. To "this weight" add "neglecting the buoyancy of the atmosphere."

Page 551. For "Table 31 D," read "Table 31 E."

Pages 643 and 646. For "§ 38" and "§ 39" read "§ 37."

Page 653. For "Metnod" read "Method."

Page 785. For "Kupfer" read "Kupffer."

Page 861. For "ammoniac" read "ammoniacal gas."

Page 890. Under the "positive pole" of "Clark" cell read "mercury," not "carbon."

Page 975. For "$A*$" read "A."

Page 1102. For "n^2" read "a^2."

Page 1105. For "$\frac{}{3}$" read "$\frac{1}{3}$."

Page 1108. For

"$0° \frac{}{\sin^2 x + \cos^2 x} 90°$" read "$\frac{1}{2}\, 0° \frac{}{\sin^2 x + \cos^2 x} 90°$"

Page 1110. For "coeficient" read "coefficient."

Page 1148. For "x^4" read "$\frac{x^4}{K^4}$"

Page 1172. For "force (f) or P" read "force (f) on P."

Page 1209. For "Leclauché" read "Leclanché."